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Globalization, polarization and cultural drift

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Abstract

We study a one-dimensional version of Axelrod's model of cultural transmission. We classify the equilibrium configurations and analyze their stability. Below a critical threshold, an initially diverse population will converge to a monocultural equilibrium, or ordered state. Above this threshold, the dynamics settle to a multicultural or polarized state. These multicultural attractors are not stable, so that small local perturbations can drive the system towards a monocultural state. Cultural drift is modeled by perturbations (noise) acting at a finite rate. If the noise rate is small, the system reaches a monocultural state. However, if the noise rate is above a size-dependent critical value, noise sustains a polarized dynamical state.

JEL classification: Z10; C63; C1

Keywords: Agent-based models; Culture dynamics; Noise

1. Introduction

In this paper, we discuss the Axelrod model for culture transmission (Axelrod, 1997a,b). In this model, culture is defined as a set of attributes subject to social influence. An individual is characterized by F cultural *features*, each of which can take q values that represent the possible *traits* of that feature. Each individual occupies one site of a regular lattice and interacts with its immediate neighbors. The model incorporates two interesting extensions of familiar models of interacting agents. First,

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instead of an individual being characterized by a single attribute with binary values, culture is characterized by an array of features, each with q possible values. Second, the dynamics of the model explicitly rely on the interaction between these different cultural features. The basic premise of the model is that the more similar an actor is to one of its neighbors, the more likely the actor will be to adopt one of the neighbor's cultural traits. This *similarity criterion* for social influence is an example of social comparison theory in which individuals are most influenced by others who are similar.

Axelrod's model illustrates how local convergence can generate global polarization. In a typical dynamical evolution, the system freezes in a multicultural state with co-existing spatial domains of different cultures. The number of these domains is taken as a measure of cultural diversity. It is interesting to notice that it is precisely the dynamics of local imitation that lead to polarization and stops the evolution towards global monoculture. Indeed, if similarity is not used to weight the probability of social interaction, the system always reaches a uniform (monocultural) state (Kennedy, 1998). Axelrod explored how the number of different cultural domains in equilibrium (the frozen states) changes with different values of F and q, with the size of the interaction neighborhoods and with the size of the system. The robustness of the predictions of Axelrod's model has been checked by *alignment* (Axtell et al., 1996) with the Sugarscape model developed by Epstein and Axtell (1996). In addition, the model has been extended in a number of ways, including its use as an algorithm for optimizing cognition (Kennedy, 1998). Further, a study that increased the range of agents' interactions with each other (Greig, 2002) suggests that an increase in communication promotes the emergence of a global culture that is not simply composed of the initially dominant traits, but is a hybrid of the initial population of cultures. In another extension, the effect of mass media in the cultural evolution has also been incorporated into the model (Shibanai et al., 2001). A systematic analysis of the dependence on q of the original model was carried out by Castellano et al. (2000) through extensive numerical simulations. Analyzing the relative size of the largest cultural domain, these authors unveil an order–disorder transition: There exists a threshold value q_c, such that for $q < q_c$ the system orders in a monocultural uniform state, while for $q > q_c$ the system freezes in a polarized or multicultural state. This result partially modifies the original conclusions of Axelrod, in the sense that globalization or polarization is determined by the parameter q which measures the degree of initial disorder in the system.

The questions that we address in this paper are, first, how robust is the above result that local imitation dynamics trap the system in a multicultural state for $q > q_c$, and, second, what is the stability of such multicultural states. We do this in a one-dimensional version of the model considered by Axelrod. In the one-dimensional setup, individuals are distributed at regular intervals along a line. The advantage of considering the one-dimensional case is that it can be proved that the total number of shared cultural features in neighboring sites is non-increasing during the dynamical interactions. This fact allows us to classify systematically the different equilibrium configurations of the model. We show that the uniform monocultural states are stable equilibria, while the other equilibrium configurations, corresponding to multicultural states, are not stable equilibria: when the system is trapped in one of these multicultural equilibria, any small perturbation will take the system away from the multicultural attractor. Such a perturbation can be seen as the effect of cultural drift. Thus, this result answers the question posed by Axelrod: "*Perhaps the most interesting extension and, at the same time, the most difficult to analyze is cultural drift*". In this sense, cultural drift, against the naive expectation of promoting differentiation, and within the limitations of the present model, turns out to be an efficient mechanism to take the system to the uniform monocultural state. These results imply that the order–disorder transition described by Castellano et al. (2000) is not robust: exogenous perturbations will drive the system to a monocultural state. However, if perturbations act at a sufficiently high rate, the system cannot settle to a monocultural state and noise dominated dynamics persist. These two competing effects of noise, namely, helping to find the path to a stable equilibrium or producing a noisy disordered dynamics, were previously recognized in studies of social impact theory (Latane et al., 1994).

The paper is organized as follows. Section 2 reviews the formal definitions and some general properties of the model. In Section 3, an order–disorder transition with respect to the parameter q is discussed in terms of global collective properties. In Section 4, we characterize the equilibrium states and their stability. Section 5 shows that by iterated perturbation and subsequent relaxation the system is driven towards the monocultural equilibrium state. Section 5 also contains a discussion of the effect of cultural drift. Concluding remarks are given in Section 6.

2. Axelrod model

Axelrod's cultural diffusion model (Axelrod, 1997a, b) structures a population of N individuals or agents as the sites of a lattice. Each agent i has a cultural state vector $(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iF})$ with F components. Each component (*cultural feature*) σ_{if} can take any of the values $1, \ldots, q$ (*cultural traits*). These values are initially assigned to each agent independently and with an equal probability of $1/q$. The discrete time dynamics of the model are governed by the principle that the probability of trait transmission from one agent to another increases with the number of features that they already have in common. Thus, agents that are similar will tend to become more similar. The dynamics are defined by the following iterative steps:

1. Select at random a site i and any of its neighbors j.
2. Calculate the *overlap* (number of common features) $l_{ij} = \sum_{f=1}^{F} \delta_{\sigma_{if}, \sigma_{jf}}$. The bond (i,j) is said to be *active* if there is at least one common feature, $l_{ij} > 0$.
3. In the case of an active bond, if there exist any features that agent i and agent j do not share in common, agent j, with probability l_{ij}/F, will change the value of one of these features (chosen randomly) to that of i.

In the remainder of the paper, the topology is that of a one-dimensional lattice with bonds only between nearest neighbors. Unless stated otherwise, boundaries are open, i.e. the boundary sites $i = 1$ and $i = N$ have only one neighbor, $j = 2$ and $j = N - 1$, respectively.

A useful description of a *state* or *configuration* $\{\sigma\}$ of the system is in terms of *cultural domains*. A cultural domain is a contiguous set $D \subseteq \{1, \ldots, N\}$ of sites all

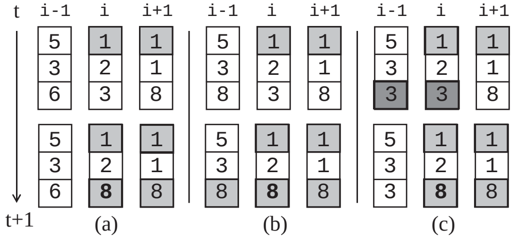

Fig. 1. Three possible outcomes of an interaction between agents i and $i+1$ for a system with $F=3$ features and $q=10$. Shared features are indicated by gray background. The trait of feature $\sigma_{i3}=3$ is switched to $\sigma_{i3}=\sigma_{(i+1)3}=8$. The new acquired trait by agent i increases the overlap with its $i+1$ neighbor and (a) has no effect on $i-1[L(t+1)=L(t)-1]$; (b) increases the overlap with $i-1[L(t+1)=L(t)-2]$; (c) decreases the overlap with $i-1[L(t+1)=L(t)]$.

sharing the same culture, i.e., $\sigma_{if}=\sigma_{jf}$, for all $i,j\in D$ and for all f. A configuration is assigned a degree of order (social homogeneity) defined as the relative size of the largest cultural domain $S_{\max}=\max\{|D|, D \text{ is cultural domain}\}$ (Castellano et al., 2000). There are q^F monocultural configurations with maximal order $S_{\max}/N=1$, where one culture extends over the whole system. If none of the cultural domains reaches a size that is appreciable on the scale of the system size, $S_{\max}\ll N$, the configuration is extremely polarized and agents have shared cultural attributes with only a small neighborhood.

For the one-dimensional topology considered here, the total overlap $L=\sum_{i=1}^{N-1} l_{i,i+1}$ never decreases when the agents change their features.[1] This is seen easily by considering an interaction where agent i adopts a trait from one of its neighbors $i\pm 1$, such that the overlap in features associated with this bond increases by one. At the same time, the overlap between agent i and its other neighbor $i\mp 1$ cannot change by more than one unit. Since overlaps of all other bonds remain the same, the total overlap L cannot decrease (see Fig. 1). Note that this argument is valid only in the given topology where agents do not have more than two neighbors. See the concluding section for a discussion of other topologies.

The maximum value of L is reached for a single cultural domain for which $L=(N-1)F\equiv L_0$. The comparison between results for different parameter values of F and N is facilitated by using the normalized negative overlap $\rho=(L_0-L)/L_0$. This is a non-increasing function that adopts its minimum value, $\rho=0$, in a monocultural configuration. For the random initial configurations, $\rho(t=0)=1-1/q$ is the expectation value.

Most of the results in the following sections are based on the study of an ensemble of identical systems. In order to reduce fluctuations, we plot data as mean values over the ensemble and indicate the averaging by angle brackets $\langle . \rangle$.

3. Globalization–polarization transition

In this section, we present simulation results under variations in the parameter q. All dynamical runs reach an *equilibrium* after a finite time. We can characterize the

[1] From the point of view of dynamical systems, the total overlap L is said to be a *Lyapunov function*.

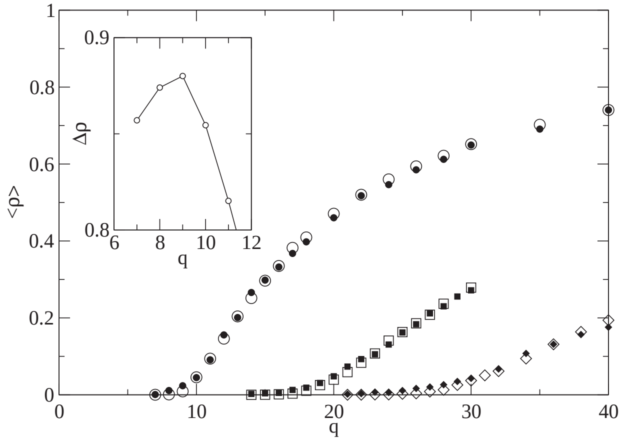

Fig. 2. Averaged normalized negative overlap $\langle\rho\rangle$ in the equilibrium as a function of q for $F = 10$ (circles), $F = 20$ (squares) and $F = 30$ (diamonds). Number of agents is $N = 100$ (filled symbols) and $N = 1000$ (open symbols). The inset shows the difference $\Delta\rho$ between the initial value and the value reached in the equilibrium for $N = 1000$ and $F = 10$.

equilibria in terms of the number of overlapping features, as shown in Fig. 2. We observe that, as a function of q, the overlap decreases continuously from its maximum value. It is apparent in this figure that a change in the system's behavior, in which there is a transition from equilibria with maximum overlap to equilibria with smaller overlap, occurs at $q \simeq F$. This change in system dynamics is also manifest in the difference $\Delta\rho = \langle\rho\rangle - \langle\rho(t=0)\rangle$ between the values in the initial random configuration and the final equilibrium. The maximum of $\Delta\rho$ is also observed at $F \simeq q$ (see inset of Fig. 2), suggesting that F and q are not two independent relevant parameters, but rather than the combination q/F is the proper parameter. This is corroborated in Fig. 3 where we observe that ρ is indeed a function only of q/F. These results indicate the existence of a transition at $q = q_c \simeq F$. If the initial number of traits is below the critical value q_c, the system evolves towards an equilibrium where one culture spans a system-wide domain. However, if the initial diversity is above the critical value, the system will evolve towards an equilibrium with the coexistence of small cultural domains. Therefore, $q \approx q_c$ identifies a globalization–polarization transition. It is also interesting to note that the convergence to the equilibrium is faster as the initial diversity is farther from the critical value.

This transition is also captured by $S_{\max}/N$. Fig. 4 shows, for $F = 10$, the values of the average $\langle S_{\max}\rangle/N$ in the equilibrium as a function of the number of available traits q. For $q < 9$ we always find a monocultural equilibrium. Increasing q beyond 9, $\langle S_{\max}\rangle/N$ drops towards zero, more rapidly with increasing system size, indicating the existence of a transition for $q \simeq 9$. This change of behavior between monocultural and polarized equilibria is highlighted by looking at the outcomes of the simulations (without averaging) in Fig. 5. We observe that this transition is not accompanied by a regime of bistability close to q_c. That is, there is not a finite range of q-values

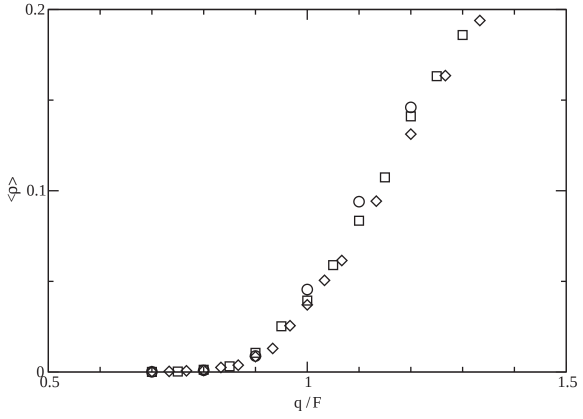

Fig. 3. The data from Fig. 2 with $N = 1000$ and $F = 10, 20, 30$ collapse when plotted as a function of the rescaled parameter q/F.

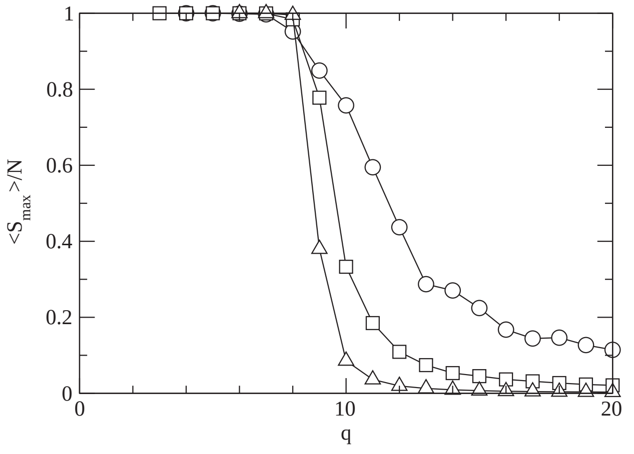

Fig. 4. The average $\langle S_{\max}\rangle/N$ in one-dimensional lattices as a function of q for $N = 100$ (circles), 1000 (squares), 10 000 (triangles) agents. Each plotted value is an average over 100 runs with independent initial conditions. Number of features $F = 10$.

for which a similar number of simulations finish either in a monocultural or in a multicultural equilibrium. The absence of bistability suggests that the transition can be classified as continuous. A similar type of transition observed in two-dimensional lattices is accompanied by a bistable regime, indicating that in the two dimensional case the transition is discontinuous or first order (Castellano et al., 2000, Klemm et al., 2003b, 2003c).

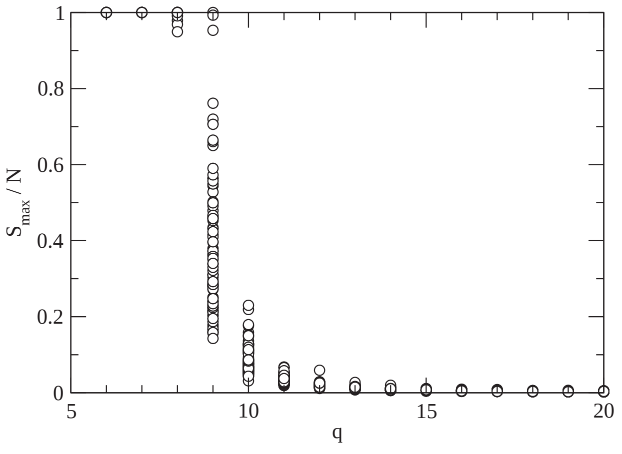

Fig. 5. Scatter plot of the $S_{\max}/N$ in one-dimensional lattices as a function of q for $N = 1000$ agents and $F = 10$ features. For each value of q the outcome of 100 independent runs is plotted.

4. Characterization of the equilibria

Thus far, the asymptotic presence of order or disorder in the system has been understood in terms of equilibrium configurations. In order to obtain some analytical understanding of our numerical derivations of a phase transition and its robustness, we now examine the connection between the number of overlapping features and the equilibrium configurations of the system. For a general (not necessarily equilibrium) configuration, the overlap can be derived from the number n_k of bonds with overlap k by

$$L = \sum_{k=0}^{k=F} k\, n_k, \tag{1}$$

with $\sum_{k=0}^{F} n_k = N - 1$. The equilibrium configurations correspond to the case $n_k = 0$, for $0 < k < F$. For these configurations we obtain

$$L_{\mathrm{eq}} = n_F F = (N - 1 - n_0)F, \tag{2}$$

where n_0 is the number of barriers (bonds with zero overlap). Therefore, the equilibria can be ordered according to the number of barriers.

1. In the monocultural equilibria all the bonds have overlap F and thus $n_k = 0$, $\forall k \neq F$ and $n_F = N - 1$. They correspond to the global maxima of the overlap with $L_0 = (N - 1)F$. There is a multiplicity of these maxima corresponding to the $\eta_0 = q^F$ possible different cultures. Which one of these monocultural configurations is selected depends solely on the initial conditions and the stochastic realization of the dynamics.
2. Multicultural states consisting of two or more cultural domains separated by barriers are equilibria as well. The first level corresponds to the $\eta_1 = [q(q-1)]^F N$

configurations in which two different cultural domains coexist separated by one barrier. In this case $n_0 = 1$, $n_F = N - 2$ and all other $n_k = 0$. The overlap is $L_1 = (N-2)F$. The next level corresponds to the $\eta_2 = [q(q-1)^2]^F N(N-1)/2$ configurations with three cultural domains (and two bonds of zero overlap), and with an overlap $L_2 = (N-3)F (n_0 = 2, n_F = N-3$ and all other $n_k = 0)$. In general there will be

$$\eta_K = [q(q-1)^K]^F \binom{N}{K} \tag{3}$$

equilibrium configurations with $K+1$ cultural domains and K barriers, and with an overlap

$$L_K = (N - 1 - K)F. \tag{4}$$

Let us analyze the stability of the equilibria against perturbations. We concentrate first on *single feature perturbations*, defined as randomly choosing an agent i and one of its features f, and replacing trait σ_{if} by a new value randomly chosen from $\{1,\ldots,q\}$. When performed in a monocultural configuration, such a perturbation introduces an island of one agent with a single deviant feature. The subsequent relaxation process is treated in the following section. The important fact with respect to stability is that this island may grow, and there is a finite probability that the deviating trait will take over the whole system. From any monocultural configuration a small perturbation may cause the system to settle into a different monocultural configuration. A perturbation cannot take the system from a monocultural configuration to a multicultural one. This would require at least one barrier in the final equilibrium. Barriers cannot be introduced by a single feature perturbation because all features but one remain unaltered, and thus deviant agents can still interact with all of their neighbors. Consequently, the set of monocultural configurations is *stable*, but a particular monocultural configuration is *metastable*.

Now we turn to the multicultural equilibria. These configurations are not local maxima of the overlap function and they consist of two or more domains separated by barriers. Let us first consider the equilibria where each domain contains at least two agents. After a perturbation, the introduced deviant trait may spread in the given domain and eventually reach one of the barriers. If this deviant trait is part of the feature set of the culture across the barrier, the members of adjacent cultures suddenly become able to share active bonds. This way a barrier may be dissolved or moved. Note that the outcome of the relaxation process is probabilistic. In general, the spreading of the perturbation is reversible. So, for any perturbation there is a non-zero probability to return to the original state. We call *marginally stable* these multicultural equilibria without domains of size one, because there are always neighboring configurations with the same value of the overlap. Neighboring configurations are those that differ by a single feature perturbation.

There is a special class of multicultural configurations, namely those with 'double' barriers. They contain at least one domain of size one – a single agent with a unique culture. Perturbing such an agent is an irreversible step because the overwritten trait cannot be recovered. Multicultural configurations with double barriers are *unstable* because there are perturbations that, with certainty, will drive the system towards a different configuration. This is so because they have at least one neighboring configuration with a larger value of the overlap.

We expect that by repetition of cycles of perturbation and relaxation the number of domains is decreased, and the overlap is increased, until a monocultural configuration with the global maximum value of overlap L is reached. If this is correct, the phase transition described in Section 3 would not be robust in presence of exogenous perturbations. This is the topic we address in the next section.

5. Effect of random perturbations

5.1. Exogenous perturbations

In order to describe the consequences of the stability properties of the multicultural states, we have simulated subjecting equilibrium states to the single feature perturbations defined above. The simulations are designed as follows:

(A) Draw a random initial configuration.
(B) Run the dynamics by iterating steps 1–3 of Section 2, until an equilibrium is reached.
(C) Perform a single feature perturbation of the equilibrium and resume at (B).

Whenever an equilibrium configuration has been reached, we measure L and S_{max}, perform a perturbation and restart the dynamics from the perturbed configuration. This process models the effect of a random influence on the system which acts on a much slower timescale than the dynamics of cultural imitation. We find that under these conditions the system is driven to complete order, i.e., L gradually increases to the maximum value $(N-1)F$ and S_{max} gradually increases to the maximum value N. In summary, the exogenous perturbations allow the system to exit the multicultural configuration towards a monocultural stable state. For a typical simulation run, Fig. 6 displays the evolution towards the monocultural state. One observes that L decays exponentially, which implies that the number of barriers also decreases exponentially. This reflects the fact that barriers dissolve independently, i.e., the probability for a given barrier to vanish does not depend on the number of barriers present in the system.

5.2. Cultural drift

We stress that in the above discussion of the effect of exogenous perturbations, the system is always allowed to relax to an equilibrium configuration before a new perturbation is performed. However, if perturbations can occur at any time during the dynamics, their effects may accumulate. At a sufficiently large perturbation rate, this may result in a disordered system with many cultures. For a very low rate of perturbation, the system will be close to the process of alternating perturbation and relaxation that we have discussed, resulting in a monocultural state. As a way to approximate cultural drift, we consider the effect of random perturbations acting at a

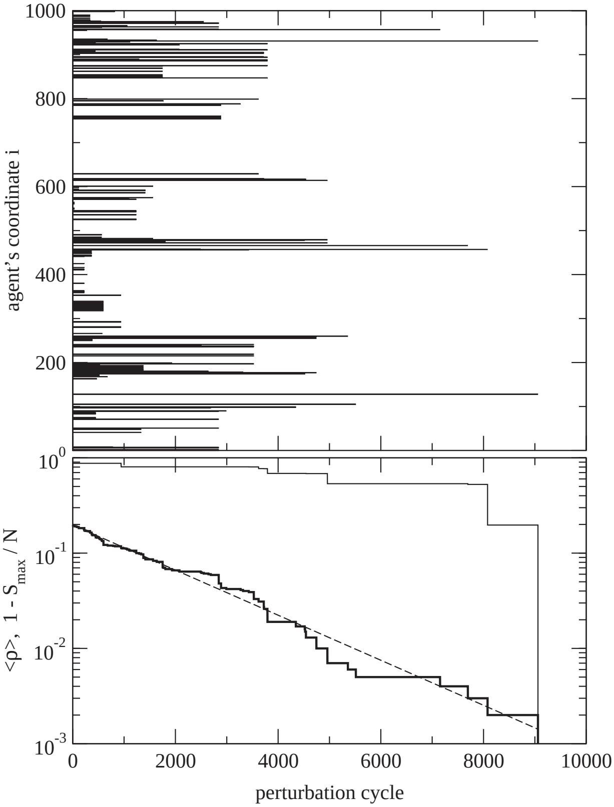

Fig. 6. Ordering of the system by iterated cycles of perturbation and relaxation. The upper panel shows the cultural barriers (bonds with zero overlap) after a given perturbation cycle. For the same dynamical run the lower panel shows the values of the size of the largest cultural domain $S_{\max}$ (thin curve) and the normalized negative overlap $\langle \rho \rangle$ (thick curve). Parameter choices are $F = 10$, $q = 13$ and $N = 1000$.

constant rate r. To be more specific, we implement cultural drift by adding a fourth step in the iterated loop of the model defined in Section 2.

4. With probability r, perform a single feature perturbation.

This is intended to be a more realistic approximation of uncertainty in the agent's behavior than the perturbation and relaxation procedure. Notice that we are using asynchronous updating in which a single agent is perturbed in each time step. The important difference between the continuous noise case and the perturbation and relaxation case explored above is that in this case the system is not necessarily in an equilibrium configuration when a perturbation occurs. Therefore, it is not straightforward to generalize to this case from the previous results. We will see, however, that an analytical result in the study of random walks is able to give us some quantitative predictions.

We first show the results of the numerical simulations of the model with cultural drift. Fig. 7 shows the variation of the size of the largest cultural domain $\langle S_{\max}\rangle/N$

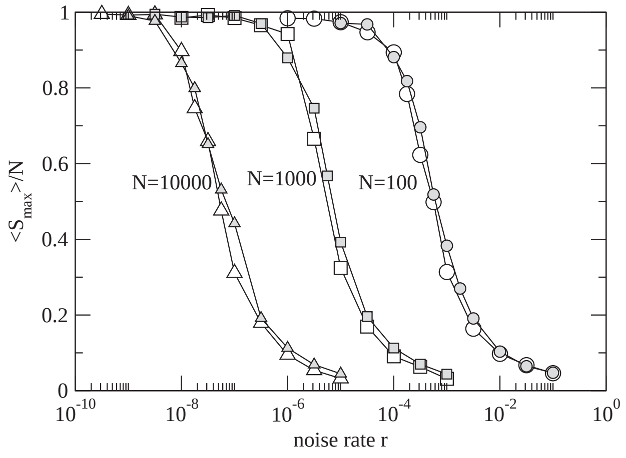

Fig. 7. Dependence of the relative size of the largest cultural domain with noise rate r in one-dimensional lattices of size $N = 100$ (circles), $N = 1000$ (squares), $N = 10\,000$ (diamonds), for $q = 5$ (filled symbols) and $q = 50$ (open symbols). Agents have $F = 10$ features.

with the noise rate r. As expected, disorder appears for sufficiently large noise rate r where noise should dominate the dynamics. But for small values of r the system settles into a monocultural state. Therefore we find a transition, controlled by the parameter r, from an ordered state to a disordered state. The exact location of the transition point strongly depends on the number of agents N, but it is only weakly dependent on the number of traits q. In fact, the variation of q from $q = 5$ to 50, which in the absence of noise or perturbations leads to qualitatively different outcomes, causes an almost negligible shift of the transition towards slightly lower values of r.

To understand these changes to the system's behavior, we offer a dynamical explanation for the transition from ordered to disordered states in the presence of cultural drift. Let us define T as the average time the system takes to reach equilibrium after a single feature perturbation. If the noise rate is such that the typical time $1/r$ between perturbations is shorter than T, the effect of the perturbations adds up in the system and disorder appears. The system is in a polarized, 'noisy' dynamical state. On the contrary, if the noise rate is small, noise becomes an efficient way to take the system to explore nearby configurations, and the system eventually escapes from multicultural equilibrium states to find a monocultural state. This simple picture tells us that disorder will set in when the average relaxation time T of perturbations of a monocultural state satisfies $r \approx 1/T$.

It is possible to obtain analytical estimation of T. Imagine a completely ordered state as the initial condition at $t = 0$. A single feature perturbation of this state induces 'damage' of size $x(t = 0) = 1$ in one of the features. In the subsequent time steps, the damage may spread until an ordered state is reached again by $x(t) = 0$ or $x(t) = N$. Therefore, we can envisage the system as a damaged cluster and an undamaged background separated by two active bonds (interfaces). These interfaces execute a

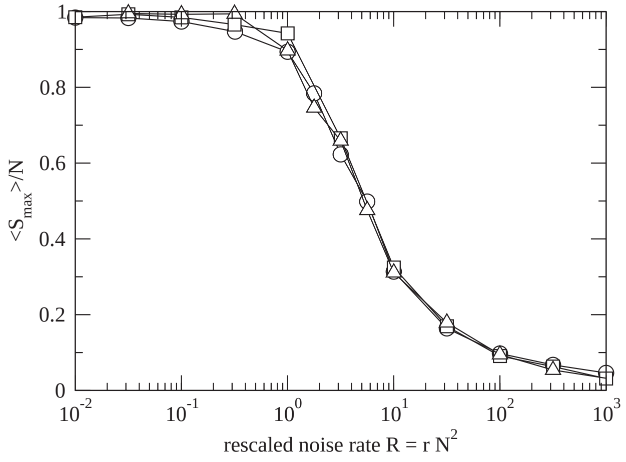

Fig. 8. Scaling of the relative size of the largest cultural domain in one-dimensional lattices. Symbols as in Fig. 4, $q = 50$.

random walk type of diffusion and the average time needed for them to merge into an ordered region that spans the whole system is well known (Grimmett and Stirzaker, 1982) to scale as

$$T \sim N^2, \tag{5}$$

so that the average relaxation time of perturbations diverges quadratically with increasing number of agents. This result implies that the number of agents is a relevant parameter (as already seen in Fig. 7). The N^2 dependence is confirmed, see Fig. 8, by showing that the data of Fig. 7 collapse into a single curve when the noise rate is measured in units of T^{-1}. That is, when it is plotted as a function of a rescaled noise rate rN^2, which incorporates the noise rate r and the number of agents N.

Cultural drift, as modeled by continuous random perturbations, has a crucial role in the behavior of Axelrod's model: it induces a transition from a state of global culture to a polarized state. For any finite size population, there is a critical value of the noise rate below which cultural drift induces the dominance of a single culture. Conversely, if the noise rate is large enough, polarization prevails. In the limit of large number of agents ($N \to \infty$) polarization always prevails, recovering Axelrod's original idea of polarization in spite of a local mechanism of convergence at work. However, these polarized states no longer correspond to a frozen configuration. Rather they present a noise-sustained dynamical system.

6. Conclusions and outlook

We have shown that Axelrod's model of cultural diffusion in a one-dimensional world can be understood as a system in which global monoculture corresponds to a global maximum of overlap among features. When the initial cultural diversity is large enough, the system finds an attractor corresponding to a culturally polarized state. The system can always escape from these attractors by any small perturbation, since there are always nearby configurations with the same or higher number of overlapping features. Cultural drift, as modeled by continuous perturbations, can provide a mechanism for promoting cultural globalization. However, if cultural drift acts at high enough rate, it can have the opposite effect of promoting a dynamical state of global polarization.

As we have previously mentioned, our results about the maximization of overlap among features can be rigorously proven only for the one-dimensional topology. Most of our qualitative findings for the one-dimensional world are the same as those obtained from simulations in a two-dimensional regular network (Klemm et al., 2003a). In the two-dimensional world, there is a transition between the uniform and multicultural states for a threshold value q_c, but the attractors are not easily classified in terms of the number of barriers. The dynamical stability of the other attractors is also unknown. However, simulations indicate that, as in the one-dimensional case, perturbations acting on the multicultural states take the system to a monocultural state. Further, in the presence of cultural drift there is also a transition from uniform states to a polarized multicultural state controled by the noise rate.

Our discussion has been restricted to regular networks with interactions between nearest neighbors. However, social networks are known in many cases to be different from regular or random networks. In fact, the idea that network topologies might reflect social cleavages was already posed by Axelrod (Axelrod, 1997a, b). Two types of networks that have received a lot of attention recently are the small world networks (Watts and Strogatz, 1998), representing an intermediate situation between regular and random networks, and the scale free networks (Barabási and Albert, 1999), characterized by a power law tail in the probability distribution for the number of links to each site in the network. Such a power law indicates the presence of few sites with a very large number of links. Simulations of the Axelrod model in these networks (Klemm et al., 2003b) indicate that small world networks favor cultural globalization, in the sense that the value of q_c for the transition to a polarized multicultural state is larger than in the regular network. For small world networks, the maximum value of q_c is obtained in the limit of a random network; however, scale free connectivity is more efficient than random connectivity in promoting global culture, and thus scale free networks have an even larger value for q_c. In fact, for scale free networks the value of q_c depends on the number of agents N, and in the limit of very large N, the system reaches the uniform multicultural state for any value of q. An interesting unsolved question is whether the dynamics of cultural evolution can introduce dynamics into the social networks of the agents that will in turn effect the original dynamics of the model. Models that better approximate social dynamics will not take networks as given a priori or as fixed for the duration of the model (Lazer, 2001). As a first step, the co-evolution of individual culture and social network could be modeled similarly to studies of cooperation in which the social network emerges from the results of the dynamics of cooperation (Zimmermann et al., 2001).

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