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Control of chaos in unidimensional maps

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For the case of one-dimensional iterated maps we present a new method for controlling deterministic chaos by the stabilization of one of the underlying unstable periodic orbits. The method works by applying a series of regular proportional feedbacks in the variable and does not require that the particular dynamical law is known. The method is illustrated with an application to the logistic and exponential maps.

The last 30 years have seen a growing interest in the study of low-dimensional non-linear dynamical systems. The study of systems whose interaction laws are no longer linear offers a new integrated framework for the study and interpretation of many natural phenomena. One of the best known effects is deterministic chaos, characterized by the sensitive dependence of the dynamics on the initial conditions and which has a pictorial expression in terms of the so-called butterfly effect [1]. In many practical situations one is interested in enhancing the appearance of chaos in order to favour processes such as mixing of fluids – through the onset of turbulence – or heat transfer (e.g. in the chemical industry [2]).

However, in many other places chaos may be undesirable, as chaotic vibrations may produce irregular operation and fatigue failure in mechanical systems, temperature oscillations outside safe margins in thermal systems, etc. Plapp and Hübler [3] have deviced a chaos control method that allows the control of chaotic systems by applying *large* perturbations to some system parameters. More interesting, perhaps, is the method put forward by Ott, Grebogi and Yorke in their seminal contribution to this problem [4]. Their proposal relies on a chaotic attractor having typically embedded within it an infinite number of unstable periodic orbits, the latter being dense in the attractor [5]. This method needs the knowledge of a Poincaré return map of the system and works by perturbing the system state in such a way that it is led to the fixed point, and has been applied to control some experimental systems [6–8].

An interesting extension of the method has been developed by Showalter et al. [9] for such dissipative systems as can be represented well by a one-dimensional return map. This simplified procedure allows one to concentrate on the evolution of a single parameter, and allows control by the application of a feedback proportional to the difference with the reference value. There exist also some experimental implementations of this method [10–13].

Another interesting technique, that presents the advantage of having a much easier implementation, consists of the application of resonant periodic parametric perturbations [14,15] (e.g. sinusoidal) that effectively stabilize some unstable periodic orbit of the system. It is interesting to point out that the issue of chaos control has been applied to other situations such as the synchronization of chaotic systems [16], the direction of trajectories to specified targets [17]

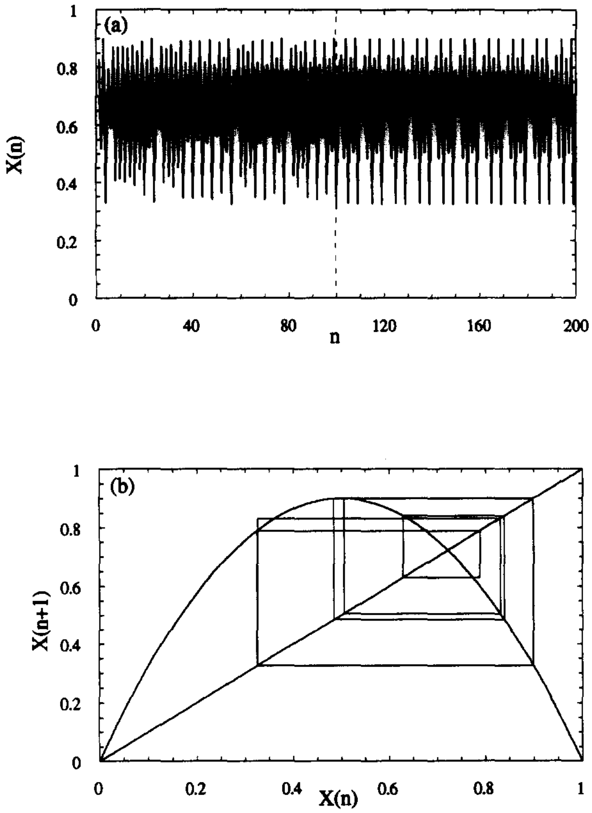

Fig. 1. Logistic map (1) with $\lambda=0.9$, $\gamma=0.05$ and $\Delta n=5$.

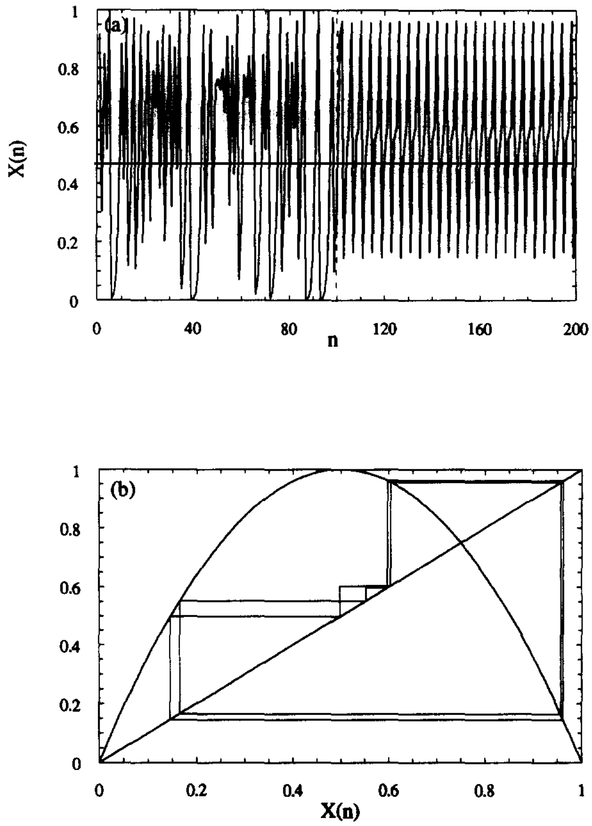

Fig. 2. Logistic map (1) with $\lambda=1.0$, $\gamma=-0.396$ and $\Delta n=4$.

and the transmission of information [18].

In the present contribution we present a different strategy to gain control over chaotic systems, and apply it to the case of iterated maps. It has in common with other approaches that it also acts by stabilizing a given unstable periodic orbit. If differs from them in the fact that it does not change system parameters, but rather performs a feedback in the population, i.e. the variable of the map, every Δn iterations. Iterated maps are interesting because they are good models of the growing of populations of individuals, being also useful as simple models of more complex dynamical systems. In particular, in this work we have considered the logistic map,

$$x_{n+1}=4\lambda x_n(1-x_n)\ , \qquad (1)$$

together with the exponential map,

$$x_{n+1}=x_n \exp[\lambda(1-x_n)]\ . \qquad (2)$$

Our control algorithm consists of the application every Δn iterations of a feedback to the variable x having the form

$$x'_n=x_n(1+\gamma)\ , \qquad (3)$$

where γ represents the strength of the feedback. In other words, the map is controlled by changing x_n in such a way that a proportional feedback is applied in the form of pulses. This means that, depending on the sign of γ, some part of x is injected or withdrawn depending on the value of x_n at that moment.

For both models (1) and (2) we have considered two different conditions, that can be found in figs. 1 and 2 (for the logistic map) and in figs. 3 and 4 (for the exponential map). In every figure, part (a) shows a plot of x_n versus the number of iterations, while in part (b) we have plotted x_{n+1} versus x_n. In the former part of the plots the dotted vertical line separates a region (at the left), where the control al-

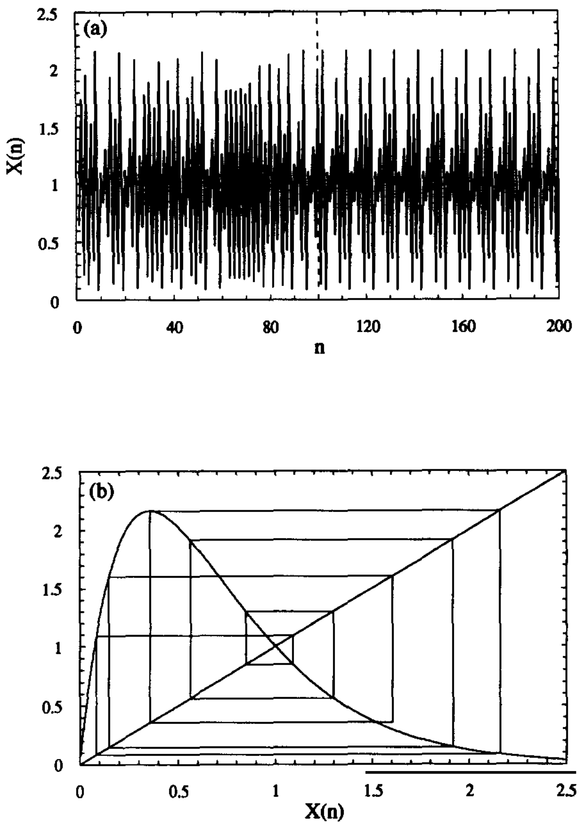

Fig. 3. Exponential map (2) with $\lambda=2.8$, $\gamma=0.2$ and $\Delta n=10$.

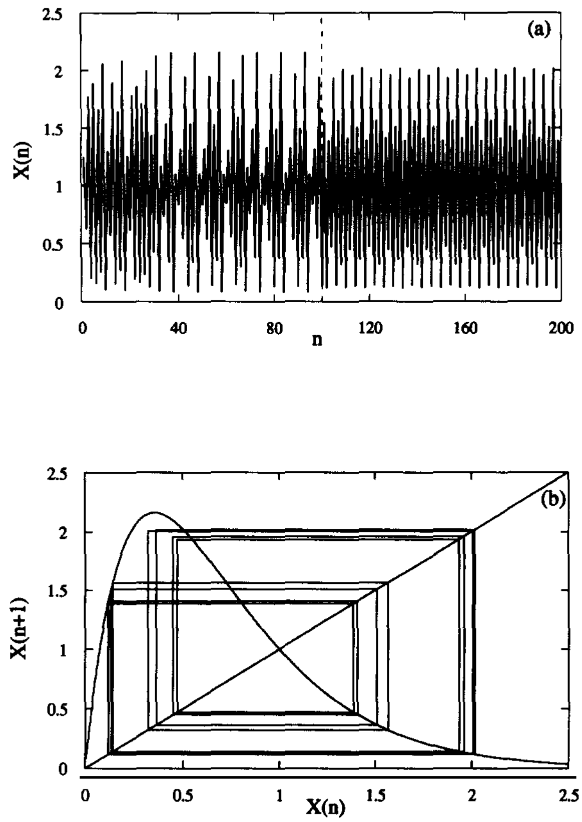

Fig. 4. Exponential map (2) with $\lambda=2.8$, $\gamma=-0.066$ and $\Delta n=4$.

gorithm is not acting, and another one (the right part), which shows the periodic behavior of the system when control is applied. It must be pointed out that periodicity does not set in immediately, and so a transient of 30000 iterations is left in the calculations to assure convergence, although a smaller number would also suffice in all cases.

The x_{n+1} versus x_n plots can be easily interpreted by using a straightforward geometric interpretation. First of all, one has the bisecting line $y=x$ (or $x_{n+1}=x_n$) and the curve that represents the function studied. Operating with some x_n value amounts to starting on the axis of abscissae and going vertically until intersection with the function (one gets then x_{n+1}). In this process, next one goes horizontally until the $y=x$ line is found, allowing one to use this value for the next iteration, which will consist again of moving vertically until intersection with the function. Repeating this process for a number of iterations, one can easily discover discrete jumps in the variable at the moment the injecting control algorithm is acting, and this interpretation is easier than the one found for continuous systems [19]. Thus, it is very clear that the effect of the control algorithm is to stabilize a particular unstable periodic orbit by changing its value every Δn iterations, allowing the orbit to match and yield a cycle.

In our procedure, the ability of controlling chaos depends critically on the values of γ and Δn. The latter is closely related to the periodicity of the observed cycle, this being a multiple of Δn. On the other hand, if one does not use the appropriate value of γ periodic dynamics is not obtained. It appears that the effect of our method is very similar to the use of resonance parametric perturbations [14,15], with the difference that one only changes the values of the variables.

Regarding the possible applications of these ideas, a first case are dissipative systems that can be represented well by a one-dimensional return map [9], as this map can be controlled with the techniques of this work. One could think that this kind of techniques could be applied to controlling the economy, which is a complex system [20] driven by highly non-linear effects – particularly if one considers positive feedbacks – and so one can observe deterministic chaos. However, this complexity offers the possibility of stabilizing an infinite number of unstable periodic orbits. A possible mechanism would be a control process performed by the suitable authority consisting of pulsewise injection – or withdrawal – of the appropriate macroeconomical quantity.

In conclusion, we have shown that it is possible to control one-dimensional non-linear maps in a range of parameters that makes the system exhibit deterministic chaos. The suggested method is based on a series of proportional feedbacks on the system variable, performed every Δn iterations. The algorithm is applied to the logistic and exponential maps for various values of the parameters.

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