Interface depinning in the absence of an external driving force

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We study the pinning-depinning phase transition of interfaces in the quenched Kardar-Parisi-Zhang model as the external driving force F goes towards zero. For a fixed value of the driving force, we induce depinning by increasing the nonlinear term coefficient λ, which is related to lateral growth, up to a critical threshold. We focus on the case in which there is no external force applied ($F=0$) and find that, contrary to a simple scaling prediction, there is a finite value of λ that makes the interface to become depinned. The critical exponents at the transition are consistent with directed percolation depinning.

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I. INTRODUCTION

The dynamics of random interfaces in the presence of noise is an interesting example of critical phenomena and generic scale-free behavior in systems far from equilibrium. In the case of surface growth dominated by thermal fluctuations, the Kardar-Parisi-Zhang (KPZ) equation [1] has been very much studied for it represents a whole universality class of growth, which includes many well-known discrete computer models [2]. In many experimental situations, however, interface motion is affected by the existence of random pinning forces (see [2] and references therein). In this case, the simplest way to model interface roughening is to replace the noise term $\eta(\mathbf{x},t)$ in KPZ by a quenched disorder $\eta(\mathbf{x},h)$,

$$\frac{\partial h}{\partial t}=\nu\boldsymbol{\nabla}^2 h+\lambda(\nabla h)^2+F+\eta(\mathbf{x},h), \tag{1}$$

which is often referred to as the quenched Kardar-Parisi-Zhang (QKPZ) equation. The first term on the right-hand side describes the smoothening effect of surface tension, F is the driving force that pushes the interface through the disorder, and the term $\lambda(\nabla h)^2$ comes from lateral growth and represents the nonlinear most relevant correction. The quenched disorder has short-range correlations $\langle \eta(\mathbf{x},h)\eta(\mathbf{x}',h')\rangle=\delta(\mathbf{x}-\mathbf{x}')\Delta(h-h')$, where the correlator $\Delta(u)$ is a very rapidly decreasing function of $|u|$ and is the term actually responsible for the pinning of the interface. This equation is expected to describe interface roughening in many disordered systems, including the nonequilibrium dynamics of magnetic domain walls in disordered materials [3–6], an elastic chain in a quenched disorder [7], fracture cracks propagation [8], etc. Its applicability to describing luid-fluid displacement in porous media might be less justified though [9].

The QKPZ model described by Eq. (1) exhibits a continuous phase transition at a certain critical value F_c of the external driving force F. For F larger than F_c, the interface moves with a finite velocity. However, the interface remains pinned by the disorder for $F<F_c$. The critical point $F=F_c$ is known as depinning transition. The interface velocity scales as $v\sim(F-F_c)^{\theta}$ near and above the transition and plays the role of an order parameter.

The value of the critical force depends on the parameters of the model, in particular, it depends on the value of the coefficient λ of the nonlinear term. Therefore, by keeping constant the rest of the equation parameters, one may find a critical line $F_c=f(\lambda)$ separating the pinned from the depinned phase. Alternatively, we can see this critical line the other way around and let $\lambda_c=f^{-1}(F)$ be the critical value of the KPZ nonlinearity above which the interface gets depinned. The driving force F favors the advance of the interface and thus, the lower the driving force is, the larger the critical value λ_c of the nonlinearity that is needed in order to get the interface depinned. Indeed, one would expect that as $F\to 0$ depinning becomes more and more difficult until eventually, at $F=0$, the threshold $\lambda_c\to\infty$ and depinning becomes impossible. This intuitive picture can be justified by means of a simple scaling argument as follows. Consider a typical region of size l pinned by the disorder. Equation (1) applied to that region reads

$$\nu h l^{-2}+\lambda h^2 l^{-2}+F-\Delta(0)^{1/2}l^{-d/2}=0. \tag{2}$$

If one supposes that the nonlinear term dominates over the diffusion, the interface remains pinned whenever $\lambda a^2 l^{-2}\ll\Delta(0)^{1/2}l^{-d/2}$, where a is the lattice spacing in the growth direction. This defines a characteristic length, $l_c=[\lambda^2 a^4/\Delta(0)]^{1/(4-d)}$, such that for $l\ll l_c$ the interface gets pinned. Now to estimate the critical force that is necessary to depin a region of typical size l_c, one equates the force term with the disorder in Eq. (2) to get to an expression for the critical line, $F_c\sim\Delta(0)^{2/(4-d)}(\lambda a^2)^{-d/(4-d)}$. Inverting the latter, one finds

$$\lambda_c\sim\frac{\Delta(0)^{2/d}}{a^2}F^{-(4-d)/d} \tag{3}$$

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for the critical line of the depinning transition [10]. In $1+1$ dimensions for instance, Eq. (3) predicts a diverging $\lambda_c \sim F^{-3}$ as $F \to 0$ [11].

In this paper, we show that, contrary to this scaling picture, there is always a finite critical value λ_c of the KPZ nonlinearity such that the interface gets depinned even for $F=0$. Our conclusions are based upon numerical integration of Eq. (1) in $d=1$. We numerically calculate the critical line and find that $\lambda_c(F=0)=3.60\pm0.01$ (in natural units) for the QKPZ equation. Our results support the somehow counterintuitive conclusion that an interface may get depinned in the absence of the external driving force by the sole effect of nonlinearities.

II. NUMERICAL RESULTS

In order to numerically integrate Eq. (1), the equation parameters can easily be rescaled to have only two independent tuning parameters—namely, the nonlinear KPZ coefficient λ and the driving force F. We have used a standard finite-difference scheme for integrating the QKPZ equation given (in natural units) by

$$\begin{aligned} h(i,t+\Delta t) = {} & h(i,t) + \Delta t F + \Delta t\, \eta[i,\tilde{h}(i,t)] + \Delta t\,[h(i+1,t) \\ & + h(i-1,t) - 2h(i,t)] \\ & + \Delta t \lambda \left[\frac{h(i+1,t)-h(i-1,t)}{2}\right]^2, \end{aligned} \tag{4}$$

where the lattice spacing has been set to unity. We start our simulation from a flat initial condition $h(x,0)=0$ and periodic boundary conditions, i.e., $h(0,t)=h(L,t)$ and $h(L+1,t)=h(1,t)$, are imposed on the interface. $\tilde{h}(i,t)$ stands for the integer part of $h(i,t)$, and the quenched disorder is Gaussian distributed and has correlations $\langle \eta(i,\tilde{h})\eta(j,\tilde{h}')\rangle = \delta_{i,j}\delta_{\tilde{h},\tilde{h}'}$. Simulations with different time steps were carried out, and the scheme proved to be stable and well behaved for a time step $\Delta=0.01$ (or smaller) for the range of tuning parameters simulated. Following Newman and Bray [12], who found some numerical instabilities when numerically integrating KPZ, we took special care in checking that no numerical instabilities appear (i.e., surface cusps are effectively smoothened by the Laplacian term) even for the large values of λ used here.

We carried out simulations in systems of size $L=128,256,\ldots,8192$. For each value of the of the nonlinear coefficient λ, we computed the critical value of force needed to get the interface depinned. Our results are summarized in Fig. 1. As expected, we find that as the driving force is smaller, the critical value λ_c of the nonlinear coefficient required in order to depin the interface becomes larger. However, as anticipated above, the critical point λ_c always remains finite, even for $F=0$. At a purely phenomenological level, we find that the critical line can be fitted very nicely by

$$\left(\frac{\lambda}{b_1}\right)^{2/3} + \left(\frac{F}{b_2}\right)^{2/3} = 1, \tag{5}$$

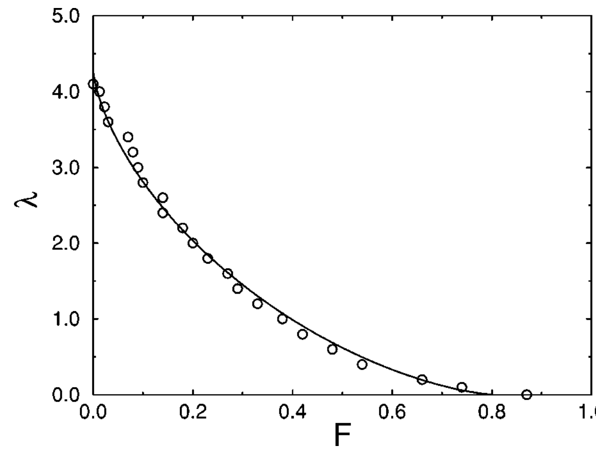

FIG. 1. Critical line $\lambda_c = f(F)$ for the QKPZ equation. Symbols are points obtained from numerical simulations in a system of size $L=1024$. The line is a fit according to Eq. (5). Note that λ_c remains finite, even at $F=0$.

where the constants $b_1=4.31\pm0.04$ and $b_2=0.81\pm0.03$ (see Fig. 1). To our knowledge, this is the first formula for the critical line and demands theoretical explanation.

In the following, we focus on the case in which no external driving $F=0$ pushes the interface and depinning is due solely to nonlinear lateral growth. We have studied the critical behavior in the vicinity of $\lambda_c(F=0)=3.60\pm0.01$ in order to address the problem of the nature of the critical point. First, we have computed the scaling behavior of the stationary interface velocity at $F=0$ as the transition is approached. In Fig. 2 (inset) we plot v vs λ for $F=0$ and a system of size $L=8192$ showing that the transition is continuous. The critical behavior of the order parameter v is shown in Fig. 2. We find that close to the depinning threshold, the interface velocity scales as $v \sim (\lambda-\lambda_c)^{\theta}$ with a critical exponent $\theta=0.635\pm0.007$.

The depinning mechanism for $F=0$ is the following. Starting from a flat initial condition $h(x,t=0)=0$, all the

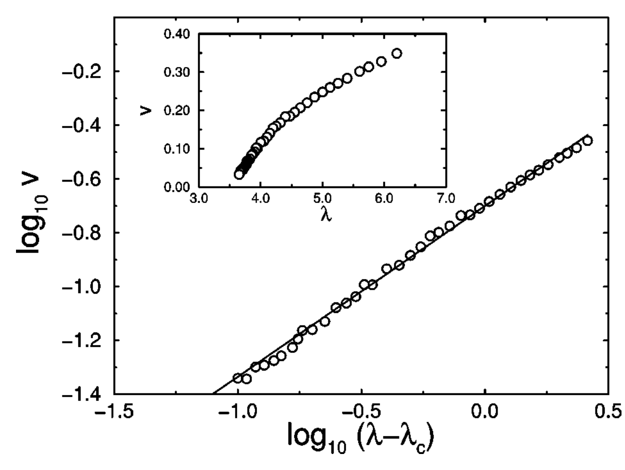

FIG. 2. Interface velocity vs coefficient λ for the QKPZ equation at $F=0$ (inset) close to the threshold $\lambda_c(F=0)$. The critical behavior of the velocity $v \sim (\lambda-\lambda_c)^{\nu}$ is shown in the main panel. A straight line is found for $\lambda_c=3.60\pm0.01$ and the slope corresponds to the velocity critical exponent $\nu=0.635\pm0.007$.

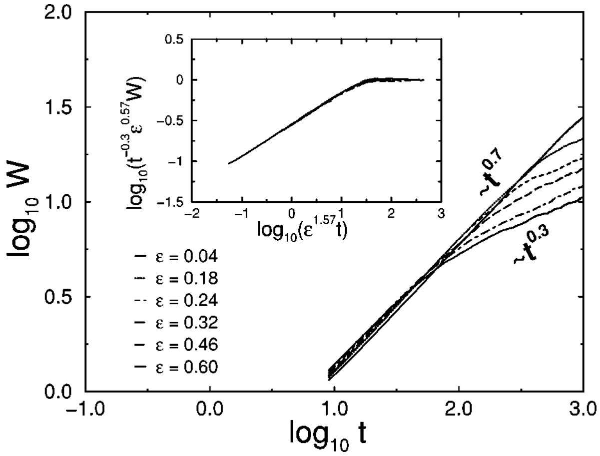

FIG. 3. In the main panel, we plot the global width for different distances (as shown) $\epsilon=(\lambda-\lambda_c)/\lambda_c$ to the threshold for $F=0$ in a system of size $L=8192$. The crossover from $t^{0.7}$ to $t^{0.3}$ occurs at times that scale as $t_c\sim\epsilon^{-\gamma}$ with the distance to the threshold. Inset shows a data collapse according to Eq. (8) of the sets shown in the main panel. A good collapse is found for the exponents $\beta_{kpz}=0.3$, $\kappa=0.57$, and $\gamma=1.57$.

terms in Eq. (1) are zero except for the disorder. At time $t=0$, the quenched random term $\eta(x,h)$ generates inhomogeneities in the front, which in turn produce a finite value of $(\nabla h)^2$. For small values of λ, these inhomogeneities smear out and the interface gets pinned by the disorder at one of the infinite pinning paths. However, for $\lambda>\lambda_c$ these initial inhomogeneities are effectively amplified by the nonlinearity and the interface gets moving with a finite velocity.

As occurs in the standard case of depinning driven by an external force, we find that the depinned phase $\lambda\gg\lambda_c$ is rough and belongs to the universality class of KPZ. This can be seen by studying the scaling behavior of the the global width $W(L,t)=[\langle h(x,t)^2\rangle-\langle h(x,t)\rangle^2]^{1/2}$, where the average is over all x and different realizations of disorder [13]. We obtain that the global width scales as [14]

$$W(L,t)\sim\begin{cases} t^{\beta} & \text{if} \quad t\ll t_\times \\ L^{\alpha} & \text{if} \quad t\gg t_\times, \end{cases} \tag{6}$$

with a time exponent $\beta=0.33\pm0.01$ and a roughness exponent $\alpha=0.50\pm0.01$ in agreement with the KPZ class of growth.

However, when approaching the depinning transition from above, $\epsilon=(\lambda-\lambda_c)/\lambda_c\to0^+$, the scaling of the global width is affected by the existence of a diverging correlation length $\xi\sim\epsilon^{-\nu}$. This is the typical size of the fluctuations of the majority phase, i.e., the characteristic size of connected regions formed by pinned sites. As we show in Fig. 3, the global width (and similarly, the local width) displays a crossover from $\sim t^{0.7}$ to KPZ-like behavior $\sim t^{0.33}$. More precisely, one can see in Fig. 3 that the width approximately behaves as

$$W(t,\epsilon)\sim\begin{cases} t^{\beta_c}\epsilon^{\kappa_c} & \text{if} \quad t\ll t_c \\ t^{\beta_{kpz}}\epsilon^{-\kappa} & \text{if} \quad t\gg t_c, \end{cases} \tag{7}$$

where κ_c, in view of the dependence of the curves on ϵ, must be very small. These two regimes are separated by a crossover time t_c that depends on ϵ. Indeed, following Kertesz and Wolf [15], near a roughening phase transition, one expects the crossover time to scale with the distance to the threshold as $t_c\sim\xi^z\sim\epsilon^{-\gamma}$, where $\gamma=z\nu$. Direct examination of Fig. 3 immediately suggests the scaling ansatz

$$W(t,\epsilon)\sim t^{\beta_{kpz}}\epsilon^{-\kappa}g(t/t_c), \tag{8}$$

which is characteristic of systems close to a roughening transition [15–17]. The scaling function is given by

$$g(u)\sim\begin{cases} u^{\beta_c-\beta_{kpz}} & \text{if} \quad u\ll1 \\ \text{const.} & \text{if} \quad u\gg1, \end{cases} \tag{9}$$

and the scaling relation

$$\kappa_c+\kappa=(\beta_c-\beta_{kpz})\gamma \tag{10}$$

among critical exponents must be fulfilled so that both regimes match.

In Fig. 3 (inset) we show a data collapse of $t^{-\beta_{kpz}}\epsilon^{\kappa}W(t,\epsilon)$ vs $\epsilon^{\gamma}t$. A good data collapse is obtained for the exponents $\beta_{kpz}=0.3$, $\kappa=0.57$, and $\gamma=1.57$, the error in estimating these exponents being of about 10%. From the scaling relation (10), one also gets $\beta_c=0.73$ in good agreement with our previous estimate.

The value of the critical exponents is consistent with those of the DPD model [18,19] just above the transition [16,2]. We thus conclude that the lateral growth-driven depinning point at $F=0$ and $\lambda=\lambda_c$ also belongs to the universality class of DPD.

III. DISCUSSION

Our results indicate that in the absence of any external driving field, an interface may get depinned by increasing the nonlinear term λ in Eq. (1) up to its critical value. From the experimental point of view, this implies that, assuming the parameter λ is tunable in the laboratory, an interface could become depinned even when no external driving force is applied. In the following, we discuss the role of anisotropy of the background random medium in generating the KPZ term $\lambda(\nabla h)^2$, and how this mechanism may be used to raise the value of λ in experiments by increasing the degree of disorder anisotropy.

The QKPZ equation for $\lambda=0$ is known as the quenched Edwards-Wilkinson (QEW) equation and has been much studied in recent years. The critical exponents at the depinning transition have been well determined by several authors [20–23,7]. In (1+1)-dimensions one finds $\alpha\sim1.25$ and $\beta\sim0.85$ at the threshold $F=F_c$ and $\alpha=1/2$ and $\beta=1/4$ in the moving phase for $F\gg F_c$, where the disorder $\eta(x,h)$ may be replaced by $\eta(x,vt)$ and the exponents of the EW universality class [2] are recovered. The QEW equation arises naturally as the Langevin equation for the Hamiltonian $H=\int d\mathbf{x}[\sqrt{1+(\nabla h)^2}+V(\mathbf{x},h)]$ describing the elastic energy of an interface in a disordered potential $V(\mathbf{x},y)$ [3–5]. The term $\lambda(\nabla h)^2$ cannot be deduced as a variation of any Hamiltonian and is added as the most relevant nonlinear correction [2]. Geometrically, it accounts for growth in a direction locally normal to the interface and is referred to as nonlinear lateral growth term.

In the past, the physical origin of the KPZ nonlinearity in interface depinning has been found to be related to two distinct mechanisms for different models [24]. On the one hand, in the spirit of the original work of KPZ [1], the λ term may have a purely kinematic origin, so that $\lambda \propto v$ [20,24]. In this case, the term $\lambda(\nabla h)^2$ goes to zero at the depinning transition, $F=F_c$, and the system thus belongs to the QEW universality class. On the other hand, there are models [24] for which λ remains finite at the transition [25]. These models have exponents that correspond to the DPD universality class [20,26]. Tang, Kardar, and Dhar [27] have shown that this finite λ term may arise in some models because of an underlying anisotropy in the random medium, i.e., models that have a growth direction determined by the random medium. A further numerical step on this direction has recently been achieved by Park, Kim, and Kim [28] by studying a model with an anisotropic disorder correlator. The effect of anisotropy on real experiments has also been successfully tested by Albert *et. al.* [29]. Experiments on fluid flow in a random medium formed by packed glass beads [30] are now known to belong to the isotropic QEW universality class [29]. However, the scaling exponents obtained for paper wetting [19,31,32] are close to the prediction of the anisotropic DPD universality class. A definite identification of paper wetting with DPD is still an open question though [9,33]. In paper-wetting experiments, a sheet of paper is vertically suspended over a reservoir of liquid (usually black ink). The fluid then wets the paper and the interface between wet and dry phases rises until it eventually stops. The interface grows upwards because of capillary forces in the paper pores. Notice that there is no external driving force. The anisotropic paper fiber distribution determines the quenched disorder term. Disorder in these systems is thus highly anisotropic. Pressure difference between the reservoir and the paper pores leads to a coarse-grained effective nonlinear term, which depends on viscosity of the invading fluid and microstructure of the medium. Whenever the effectively generated λ term is large enough to be above λ_c, depinning of the wetting front occurs.

In summary, we have studied the QKPZ equation focusing on the case in which there is no external driving force ($F=0$). We have shown that there exists a depinning transition for a finite value of the KPZ coefficient $\lambda=\lambda_c(F=0)$ and that transition belongs to the DPD universality class. Moreover, we find that the interface velocity scales as $v\sim(\lambda-\lambda_c)^\theta$ with a critical exponent $\theta=0.635\pm0.007$, which is identical to the scaling in the case of depining driven by an external force. This seems to indicate that the λ term upon renormalization gives rise to a constant term in a linear fashion that makes the role of a finite driving force. A finite value of the nonlinear coefficient λ appears in systems with anisotropic disorder, such as for instance in paper wetting experiments. In this system, there is no external driving force and depinning occurs due to local capillary forces, which drive the interface through the anisotropic lateral growth term $\lambda(\nabla h)^2$. We conclude that by varying the anisotropy degree of the corresponding random medium in other experimental systems, depinning is possible even with no external driving.

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