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Research paper

Effects of update rules on networked *N*-player trust game dynamics

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ABSTRACT

We investigate the effects of update rules on the dynamics of an evolutionary game-theoretic model – the *N*-player evolutionary trust game – consisting of three types of players: investors (trusters), trustworthy trustees, and untrustworthy trustees. Interactions among players are constrained by local neighborhoods predefined by spatial or social network topologies. We compare different evolutionary update rules with behaviors that rely on the level of payoffs obtained by neighbors. In particular, we study the dynamics resulting from players using a deterministic rule (i.e., unconditional imitation with and without using a noise process induced by a voter model), a stochastic pairwise payoff-based strategy (i.e., proportional imitation), and stochastic local Wright-Fisher processes. We study these dynamics with different social network structures and varying levels of game difficulty. We see that there are significant differences on the level of promoted trust and global net wealth depending on the update rule. Under 'harder' game settings, rules based on unconditional imitation achieve the highest global net wealth in the population. We observe that there are key spatio-temporal correlations in the system for all rules. The update rules lead to the formation of fractal structures on a lattice, and low frequencies in the output signal of the system (i.e., long-term memory) when the rules are stochastic.

1. Introduction

Evolutionary game theory is a mathematical framework for investigating the dynamics of strategies in populations ranging from two players to structured societies [1]. Trust between players plays a major role in the evolution of games in a social context, and has deep implications for the collective action of social and human systems [2,3]. Players rely on trust to handle complex problems and maintain relationships. This, in turn, gives rise to opportunities and allows trust to spread [4]. A recent study found that players are more trustworthy when making uncalculating cooperation [5].

In terms of models, the most well-known version of the trust game considers interactions between two types of players: *investors* (or *trusters*) and *trustees* [6–9]. Here, the investor must first decide whether to trust the trustee. If the decision is

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positive, the trustee must then decide whether to be trustworthy or not. Although two-player games are the most common configuration for trust games in the literature, they have their limitations. For instance, we cannot generalize some of the insights found in pairwise interactions to games with multiple players having more than two strategies [10].

Abbass et al. [4] therefore proposed an evolutionary *N*-player trust game to generalize the concept of trust to a well-mixed population of individuals who play a trust game concurrently. Chica et al. [11] subsequently showed that promoting trust in this *N*-player trust game depends on the social network structure. In their model, a structured population of players have three possible strategies to choose from: to be an investor (*I*), a trustworthy trustee (*T*), or an untrustworthy trustee (*U*). Players interact with their direct neighbors on the social network, and obtain their payoff values through a fixed number of time steps. They evolve their strategies according to the obtained payoffs in a non-deterministic way.

In this paper, we extend the work of Chica et al. [11] by studying the implications in terms of spatio-temporal correlations of different update rules in the same networked *N*-player trust game. In addition to a proportional imitation update rule, we also consider unconditional imitation, a Wright-Fisher process, and a hybrid approach using unconditional imitation and a voter model in a stochastic function. In three-strategy games, systems can exhibit repetitive succession of oscillatory and stationary states [12]. This is because when having three strategies, the outcome can be considerably more complicated and the game can end in ever-increasing oscillations or with the elimination of some of the existing strategies [1]. It is thus important for us to examine the effects of different update rules and how the system dynamics would change in this context.

In our simulations, we consider the influence of different social network topologies (i.e., a square regular lattice [13] and scale-free (SF) networks [14,15]) on the evolutionary outcome. We also explore the effects of the update rules for different values of the temptation to defect ratio, r_{UT}, in the game. This r_{UT} ratio defines the temptation to 'defect' investors' trust by the trustees and measures the level of difficulty of the game. We evaluate the final steady state (surviving strategies) and global net wealth (total payoffs) of the population to determine which update rules and under which conditions players can better promote trust and obtain the highest net wealth.

The rest of this paper is organized as follows. In Section 2, details about the model and update rules are described. The effects of the different update rules are then examined in Section 3 to better understand how trustworthy and untrustworthy behaviors emerge and what is their corresponding global net wealth. After that, we study the model dynamics in Section 4, first from a spatial perspective. It is known that spatial games of evolution can generate dynamic fractals, gliders and kaleidoscopes [1,13], and so we analyze the spatial formations as well as those spatial correlations between the players' behaviors and strategies. Next, we show the temporal dynamics of the model by analyzing the signal of the system and investigate if the majority of the rules generate those correlations. Finally, we also explore if more elaborated correlations with both spatial and temporal types occur in the model's output.

2. Model

2.1. Model description

The multi-player trust game consists of a finite set of agents (players) occupying the nodes of a social network, with edges denoting interactions between them (both for accumulating payoffs and strategy updating [16]). Each player *i* chooses a strategy *s* from three possibilities at every time step ($s(i) = \{I, T, U\}$) [11]: being an investor (strategy *I*), being a trustworthy trustee (strategy *T*), or being an untrustworthy trustee (strategy *U*). The agents' strategies are initially assigned at random.

During the game, all players interact with their directly connected neighbors at the same time in a group setting (i.e., group interaction [17]). As shown in Fig. 1, each agent would be involved in not just a single but multiple groups, one as the focal agent while others as a group member in its neighbors' groups (as done in [18]). Let us look at focal agent *i* in Fig. 1. The game is played between agent *i* and all its direct neighbors. It is clear that agent *i* participates not just in one but two groups, as highlighted in red and green in the figure.

We denote the numbers of investors, trustworthy trustees and untrustworthy trustees in a local neighborhood (including the focal agent itself) with k_I, k_T, and k_U, respectively. In addition, we have k_{TU} as the total number of both types of trustees: $k_{TU} = k_T + k_U$. The equality $k_I + k_T + k_U = \langle k \rangle_i + 1$ must therefore always be satisfied for consistency's sake, where $\langle k \rangle_i$ is the degree (number of connections) of the focal agent.

Every investor releases, in each of its interaction groups, a unit of payoff per time step. Since all the agents in the neighborhood are playing the game as a group, this means there are k_I units. Each of the k_{TU} trustees of the group receives an equal division of this total quantity: k_I/k_{TU}. A trustworthy trustee (strategy *T*) then returns the received fund multiplied by R_T to the investors of the group: $R_T\, k_I/k_{TU}$ (i.e., R_T/k_{TU} to each of the k_I investors in the interaction group). Each of the trustworthy trustees also receives the same amount of fund as the investors (i.e., $R_T\, k_I/k_{TU}$ for each *T* player in the interaction group). Untrustworthy trustees (strategy *U*) return nothing but keep for themselves the received fund multiplied by R_U: $R_U\, k_I/k_{TU}$.

The parameter $r_{UT} \in (0,\ 1)$ acts as the temptation to defect ratio (to be untrustworthy), defined by $r_{UT} = (R_U - R_T)/R_T$, and it is used to control the difficulty level of the trust game. Also note that $R_U = (1 + r_{UT})R_T$. R_T and R_U values must fulfill the restriction $1 < R_T < R_U < 2\,R_T$. Taking into account the number of players in the group and r_{UT}, we can determine the net wealth of individual agents, based on their payoffs, and according to the strategy adopted by themselves and their direct

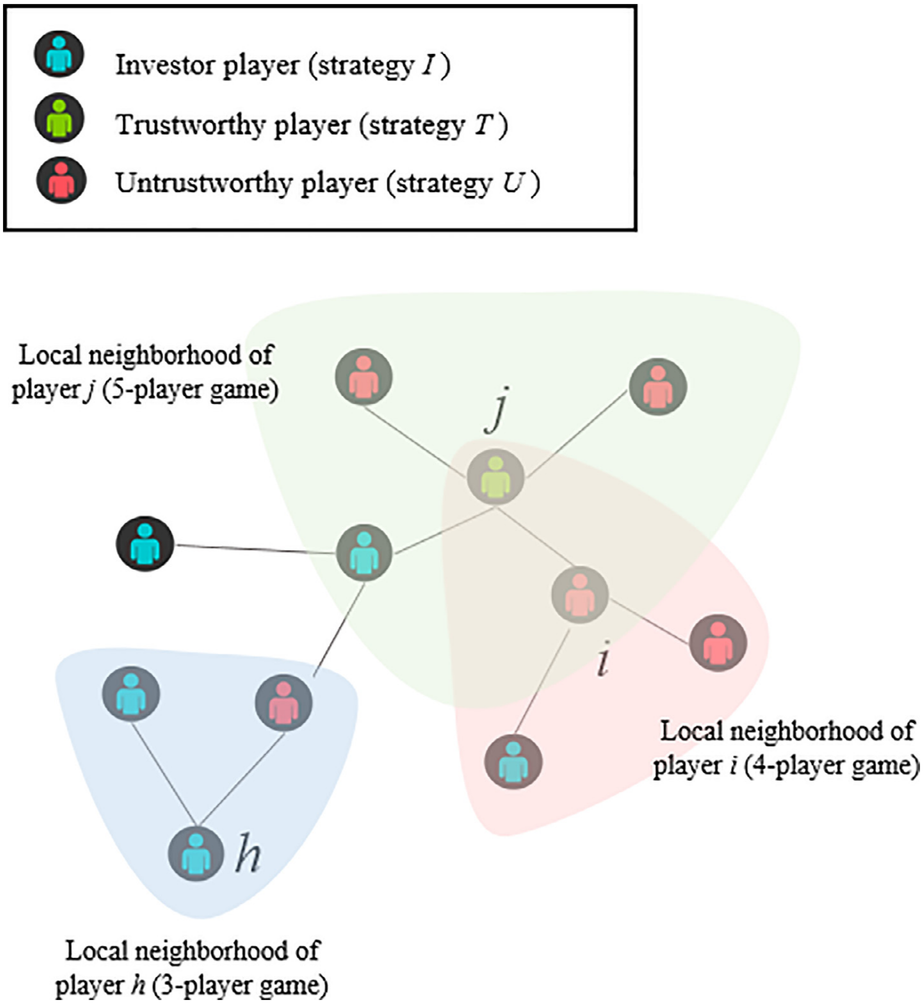

Fig. 1. An illustration of players' group diversity in the game on a scale-free social network. We have highlighted three different groups of 4, 5, and 3 players, for focal players i, j, and h, respectively.

neighbors. The net wealth w_i of focal agent i is obtained as follows:

$$w_i = \begin{cases} \dfrac{R_T \cdot k_T}{k_{TU}} - 1, & \text{if } s(i) = I \text{ (investor)}, \\ \dfrac{R_T \cdot k_I}{k_{TU}}, & if s(i) = T \text{ (trustworthy)}, \\ \dfrac{(1 + r_{UT}) \cdot R_T \cdot k_I}{k_{TU}}, & if\, s(i) = U \text{ (untrustworthy)}. \end{cases} \tag{1}$$

Focal agent i's net wealth is 0 when there is no trustee in the neighborhood (i.e., $w_i = 0$ when $k_{TU} = 0$). As players participate in multiple groups, they can obtain net wealth from all the groups they play. In order to set the net wealth of a player in a specific time step, we sum it over all their groups' obtained net wealth. Finally, the global net wealth W of the population is calculated as $W = \sum_{i=1}^{pop} w_i$.

2.2. Update rules

After playing a game and calculating their payoffs, the agents have an opportunity to update their strategies according to the payoffs received. Agents then decide which strategy to choose based on the population strategies in the previous time step (i.e., $t - 1$) and their own payoff [19]. The strategy update follows an evolutionary procedure based on neighbor

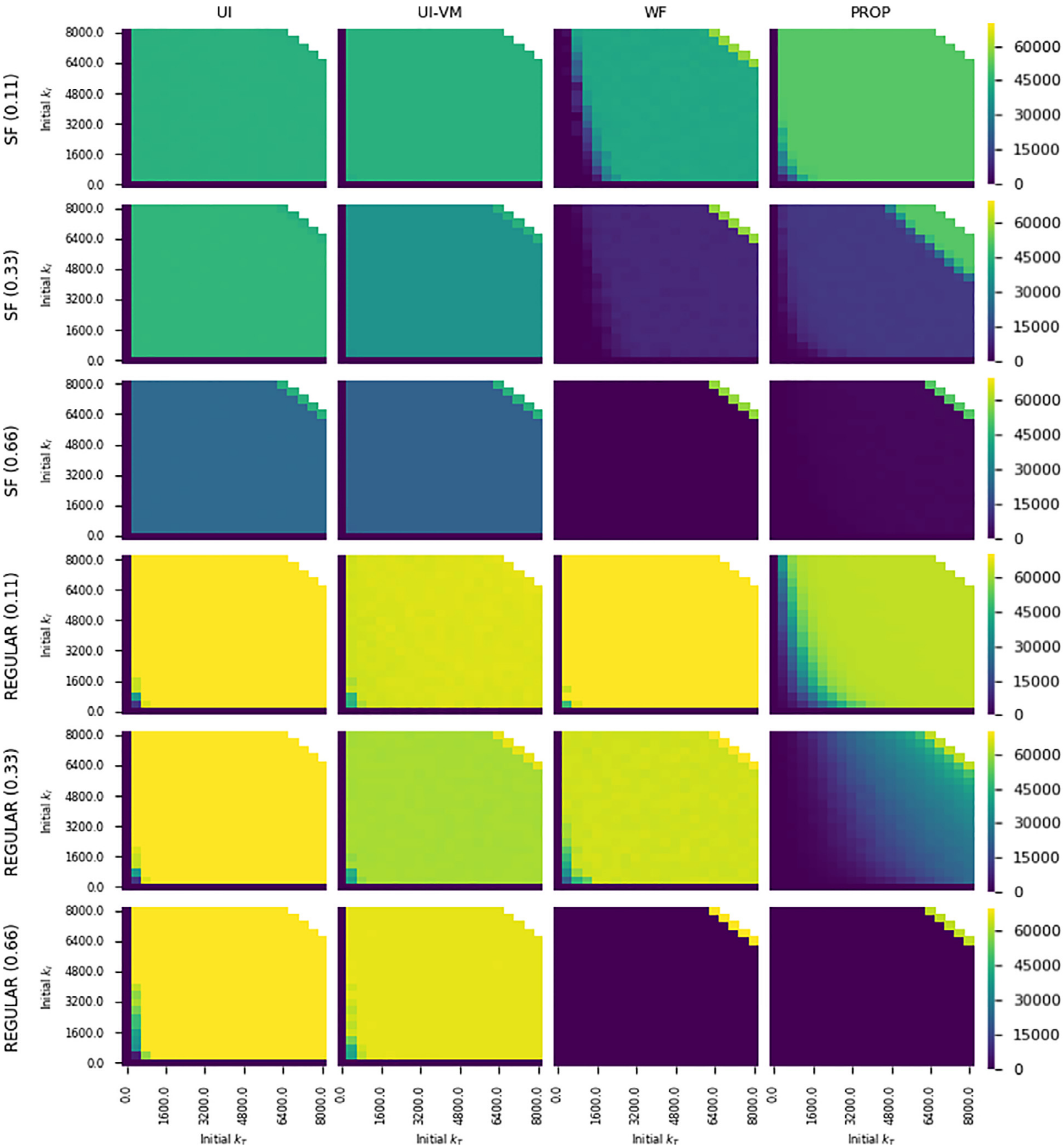

Fig. 2. Sensitivity analysis on different configurations of the initial population for the update rules, network topologies, and three difficulty levels of the game (r_{UT} values). Simulations on regular lattices better promote trust. Also, UI and UI-VM can better promote trust, while PROP is the one achieving the lowest net wealth values. Note that blank spaces on the top right corner mean combinations with more players than allowed in the 14,400-player game.

imitation that can be interpreted as information exchange within a social learning process [16]. To be more precise, at time step t, focal agent i (independent from its strategy) evaluates its previous payoff results in $t-1$ and decides whether to imitate a direct neighbor j's strategy or not by using an evolutionary update rule. These rules of imitative nature are widely employed in the literature and represent a situation where bounded rationality or lack of information forces players to copy (imitate) others' strategies [20].

In this work, we have considered four different update rules, all applied synchronously. Some of them are of deterministic nature while others stochastic; and some of them evaluate all the neighbors and can make mistakes while others cannot. These four update rules are:

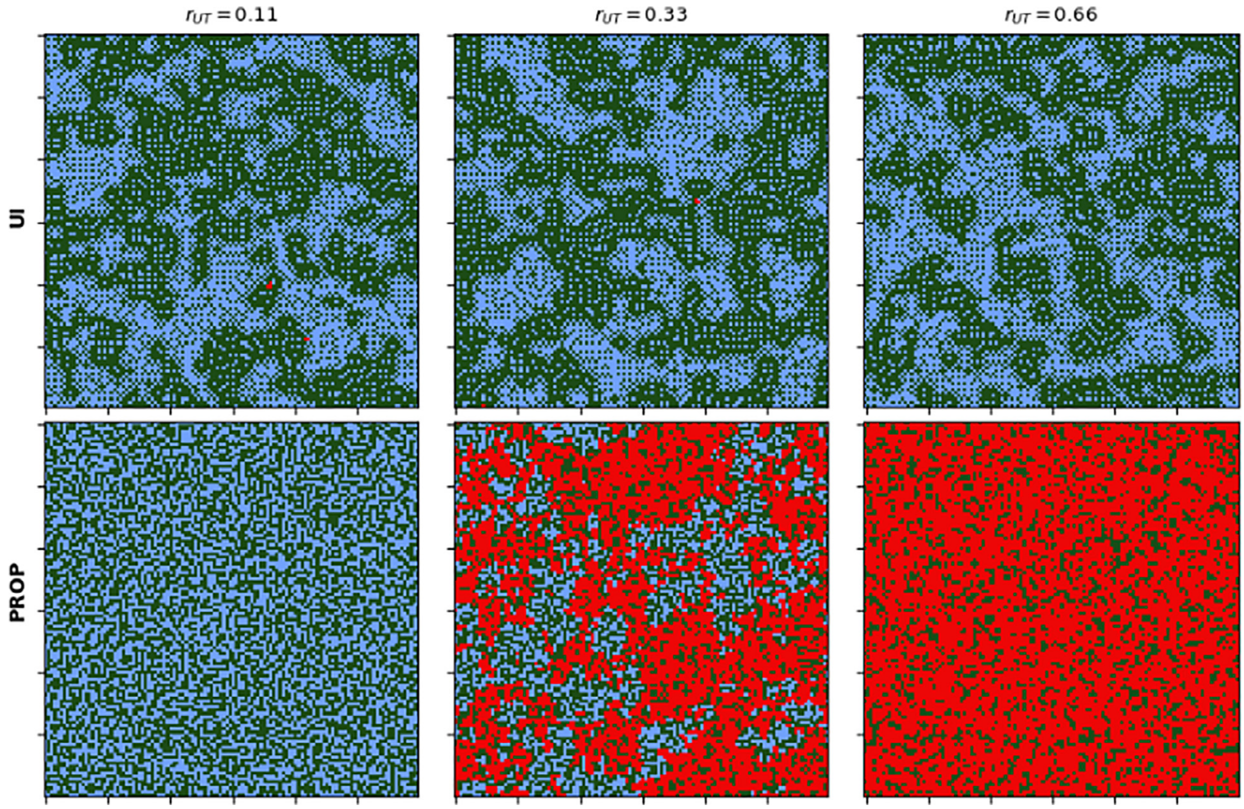

Fig. 3. Spatial pattern formations on a 120×120 lattice for the UI and PROP update rules. Blue cells are players with strategy *I*, green cells are players with strategy *T*, and red cells are players with strategy *U*. Trustworthiness is spread all over the lattice with the UI rule. It is not possible to promote trust when using the PROP microscopic rule under difficult game conditions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

- Unconditional imitation (UI) [13]. Agent *i* directly copies the strategy of the best performing neighbor *j* only if the payoff w_j^{t-1} is better than w_i^{t-1}. UI is a deterministic and strategic rule based only on payoff maximization when imitating others.
- Hybridization of UI and a voter model (UI-VM). By using this mechanism, players imitate others by hybridizing two rules: UI, a totally deterministic and payoff-based rule, and the voter model [21], a random decision-making mechanism based on just picking the strategy of one of the neighbors, *j*, at random. As done in [22], our model uses a probability parameter *q* for players to select between the two update rules: the voter model is chosen as the update rule by agent *i* with probability *q* while UI is chosen by agent *i* with probability $1 - q$. We set $q = 0.1$ for our experiments after some preliminary analysis across a range of values.
- The Wright-Fisher (WF) process [23,24]. With this update rule, focal agent *i* evaluates all its neighbors' payoffs and assigns probabilities to them proportionally according to their payoff values obtained in the last time step. This update rule is also stochastic, but based on imitation dependent on the payoffs. At each time step, there are possibilities to imitate each of the neighbors. Agent *i* thus might make mistakes by imitating a worse-performing neighbor in terms of payoffs.
- Proportional imitation (PROP) [25]. It is a pairwise and stochastic update rule where players cannot make mistakes during the imitation process (i.e., never imitate players with a lower payoff value). Agent *i* first evaluates if the individual net wealth of neighbor *j* in the previous time step $t-1$ (w_j^{t-1}) is higher than its own (w_i^{t-1}). If the net wealth of *j* is higher, *i* will adopt the strategy of *j* with a probability depending on the difference between their payoffs:

$$prob_i^t j = \frac{\max\{0, w_j^{t-1} - w_i^{t-1}\}}{\phi}, \tag{2}$$

where $\phi = max_w - min_w$ is the maximum payoff distance between two players to have $prob_i^t j$ properly normalized. The minimum possible net wealth for the game, min_w, is -1 when the focal agent is an investor and its neighborhood is formed entirely of untrustworthy trustees. The maximum possible net wealth for the game, max_w, occurs when the focal agent is untrustworthy and all its neighbors are investors. In this case, max_w is equal to $(1 + r_{UT})R_T k_I$.

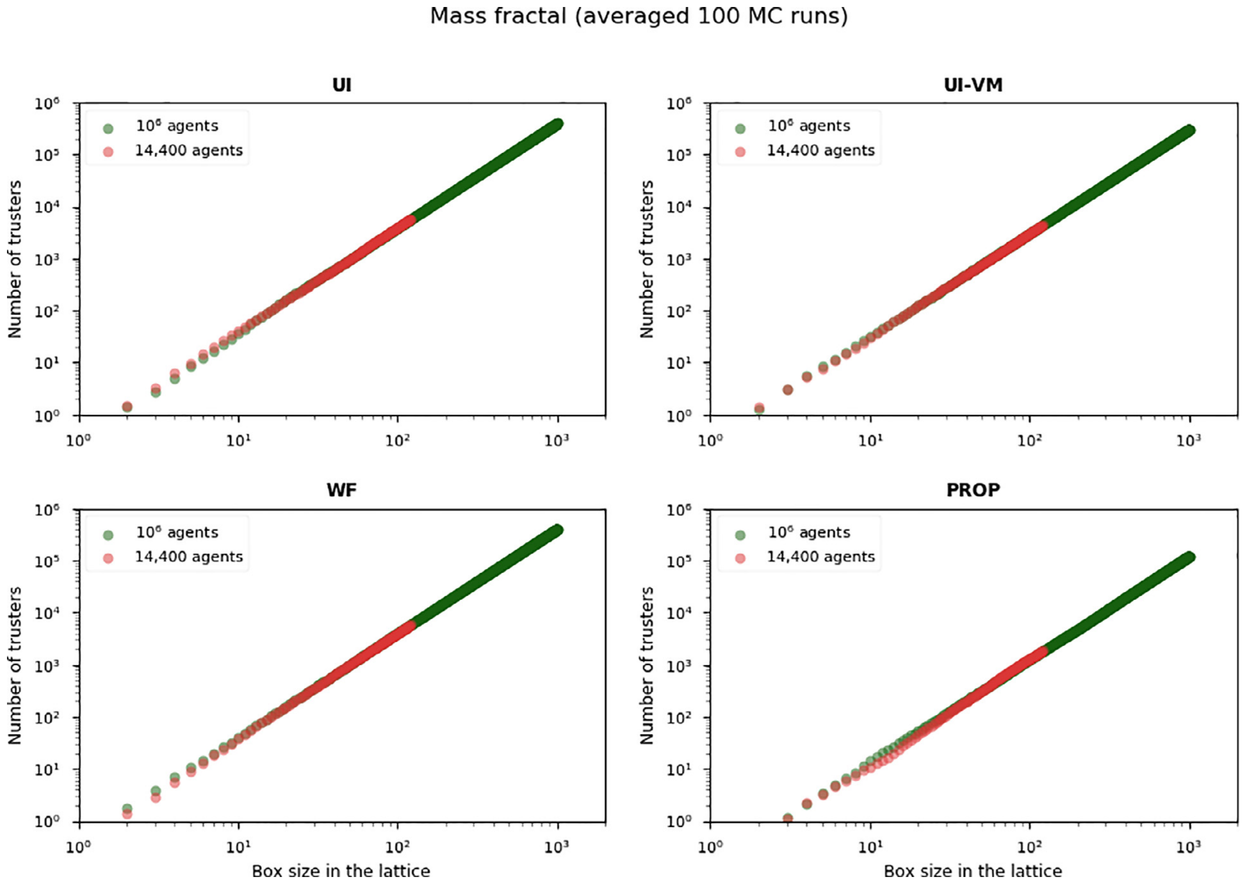

Fig. 4. A mass study showing fractal structures of the update rules in 120×120 (14,400 agents) and $1,000 \times 1,000$ (10^6 agents) lattices, with $r_{UT} = 0.33$ and the INIT-0 configuration. For this configuration, exponents' values a for a $f(x) \approx x^a$ function are 1.72 for UI, 1.38 for UI-VM, 1.735 for WF, and 1.63 for PROP.

3. Results on trust promotion and global net wealth

First, we will analyze the results on global net wealth of the population (i.e., the conditions and update rules to promote the highest sum of payoffs of all players in the population). The simulations were run for 5,000 time steps until the model reached a stationary state, with a population of 14,400 players. We set R_T to 6, as in [4,11]. Three different values of the temptation to defect ratio, r_{UT}, were used: 0.11, 0.33, and 0.66, which cover three different trust scenarios from 'easier' to 'harder' evolutionary trust games. All experiments were repeated for 100 independent Monte Carlo (MC) realizations.

We performed sensitivity analysis on the initial population conditions where the global net wealth was averaged over the last 25% of the 5000 time steps. Fig. 2 shows heatmaps of the results for the four update rules (individual columns of the panel) and three r_{UT} values with two social network topologies (see the six rows of the panel). Specifically, we present results for a square regular lattice (REGULAR) and a SF network with average degree $\langle k \rangle = 4$, generated using the Barabasi-Albert algorithm. The first three rows report on the net wealth using SF, with the game level rising from easy to difficult ($r_{UT} = \{0.11, 0.33, 0.66\}$). The last three rows report on the same results when considering a regular lattice for the same game conditions.

We can see how, even for difficult game conditions ($r_{UT} = 0.66$), both UI and UI-VM can promote high 'cooperation' levels under a wide range of initial population conditions when using a regular lattice (see the bottom left plots of the panel). Both UI-based update rules (first two columns of the panel) perform equally well in terms of promoting trust, and the influence of r_{UT} is almost insignificant. We attribute this to the oscillatory behavior UI-based rules exhibit, in which the system is trapped in a steady-state stage of the simulation (as we will see in Section 4.2). In terms of network topologies, the net wealth is higher when using a regular lattice, and bigger differences between the two topologies can be found with high r_{UT} values. Comparing UI and UI-VM, we see that the latter can generate a slightly wider region of high global net wealth than the former.

Trust cooperation results obtained by UI-based rules, however, are not as good as WF and PROP update rules in those scenarios when SF is used under easy game conditions (the first row of Fig. 2). The WF rule is able to obtain higher global

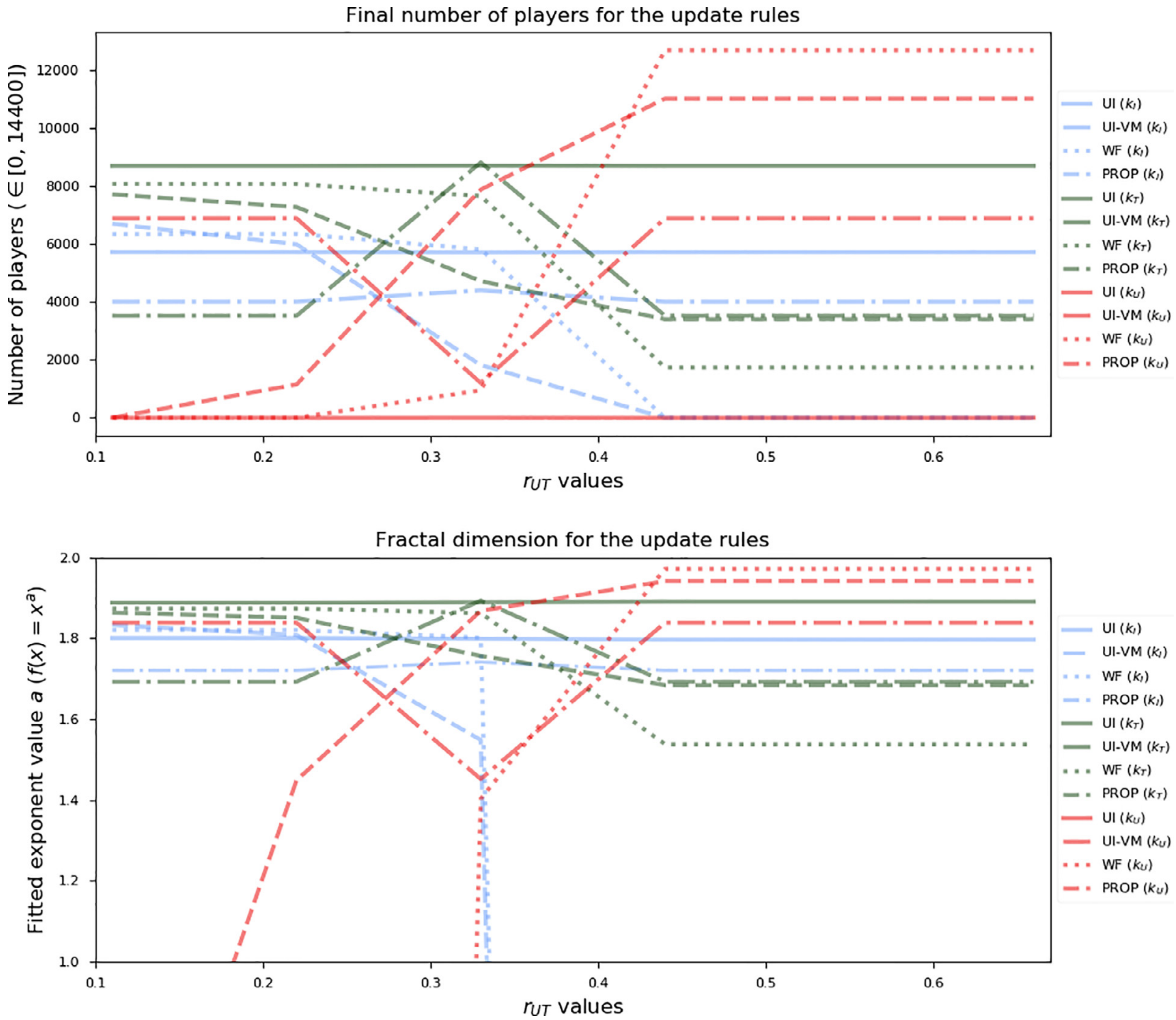

Fig. 5. The top plot shows the number of players of each strategy (k_I, k_T, and k_U) at the end of the simulations. The bottom plot shows fitted fractal exponents of the three variables (i.e., the three playing strategies k_I, k_T, and k_U) for the four considered update rules. This experimentation was conducted using a 120×120 lattice and INIT-0 as the initial population of players.

net wealth values but under narrower initial conditions (see the heatmap plot at position (1,3) of Fig. 2). When the game becomes harder, both the WF and PROP update rules cannot obtain high net wealth values when using a SF topology and trust cannot be promoted (same in the case of UI-based rules). See for example those heatmaps in the third row of the panel, which correspond to the case of $r_{UT} = 0.66$. Both WF and PROP are not able to promote non-zero global net wealth at the end of the simulations.

4. Model dynamics

In this section, we focus on spatial and temporal dynamics of the game using different update rules. For all the analyses, we ran 100 independent MC realizations and considered square regular lattices of sizes 120×120 (i.e., 14,400 players) and 1000×1000 (i.e., 10^6 players) to better understand the spatial correlations between them. We also tested four different initial population settings:

- INIT-0 (baseline): 30% of players with strategy *I*, 25% with strategy *T*, and 45% with strategy *U*.
- INIT-1: 25% of players with strategy *I*, 10% with strategy *T*, and 65% with strategy *U*.
- INIT-2: 10% of players with strategy *I*, 50% with strategy *T*, and 40% with strategy *U*.
- INIT-3: 50% of players with strategy *I*, 5% with strategy *T*, and 45% with strategy *U*.

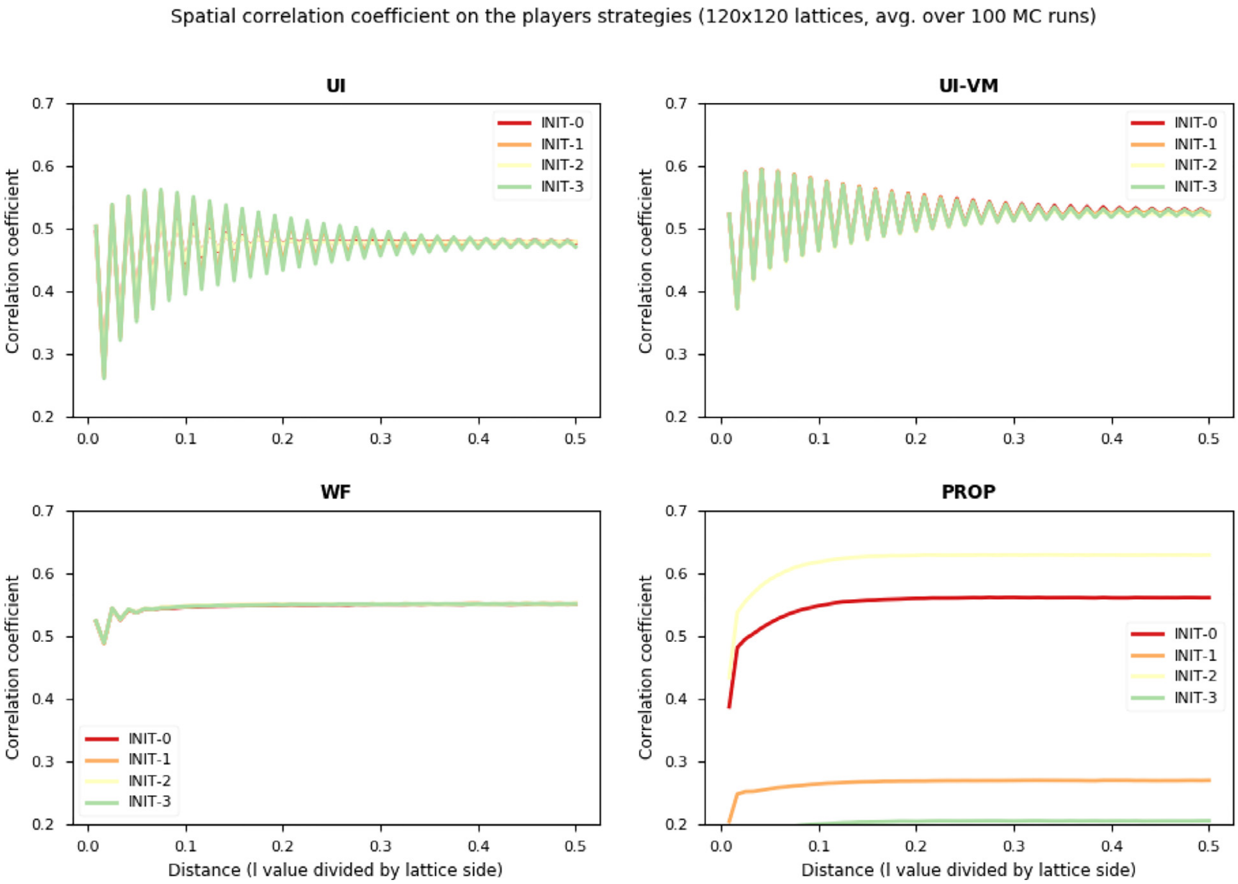

Fig. 6. Spatial correlation coefficients between players separated at several distances l (on a 120×120 lattice, with $r_{UT} = 0.33$, and four initial population configurations). Spatial correlations are observed for all the configurations. Correlation differences in the bottom right plot are found because initial configurations with the PROP update rule are more sensitive to trust promotion, as observed in the sensitivity analysis of Fig. 2. Similar results are obtained for triangular lattices.

4.1. Spatial effects

We first present spatial pattern formations of the lattices after reaching an equilibrium state. Fig. 3 shows the spatial layout of 120×120 lattices with the three possible strategies, *I, T,* and *U*, for two update rules and three r_{UT} values. Nodes in blue are those with strategy *I*, nodes in green are those with strategy *T*, and nodes in red are those with strategy *U*. For all the update rules and difficulty levels we can see similar cluster and spatial formations. Also, the existing playing strategies are in line with the global net wealth observed in the previous section and Fig. 2. Even with tough cooperation conditions, the UI and UI-VM rules are able to have all *U* players eliminated (only blue and green cells remained). With $r_{UT} = 0.66$, however, untrustworthy players k_U partially dominate the population when the PROP rule is in place.

The spatial layout also suggested a fractal structure. This is especially clear when considering the UI rule, which provokes the emergence of ‘organic’ clusters in the lattice. In order to further investigate the fractal properties of the game, we studied, in a steady-state stage of the simulation, the relation of k_I values (i.e., number of investors) when increasing a box size within the lattice. Fig. 4 shows this relationship for lattice sizes of 120×120 and 1000×1000. Again, in this analysis we see that the number of players (i.e., the size of the lattice) does not affect the fractal properties of the game.

We have fitted the exponents of these curves for the fractal variables (i.e., the number of players playing each of the three strategies), and found their exponents’ values a for a $f(x) \approx x^a$ function. The a exponents’ values for the fractal structures of Fig. 4 are specified in its caption, being all between 1.38 and 1.735. Additionally, the bottom plot of Fig. 5 shows the a values for different settings of r_{UT}, ranging from 0.11 to 0.66 (see the *x*-axis). The top plot of the same figure shows the number of existing playing strategies to complement the fractal analysis. As we can see in these plots, for all the update rules, the output of the spatial model presents a fractal structure with a values between 1.6 and 1.9 for the majority of the r_{UT} values. This fractal structure appears when the difficulty level of the game allows more than one strategy and for the three variables of the game (i.e., the numbers of investors and both types of trustees). However, the fractal structure is clearer for the number of investors (k_I).

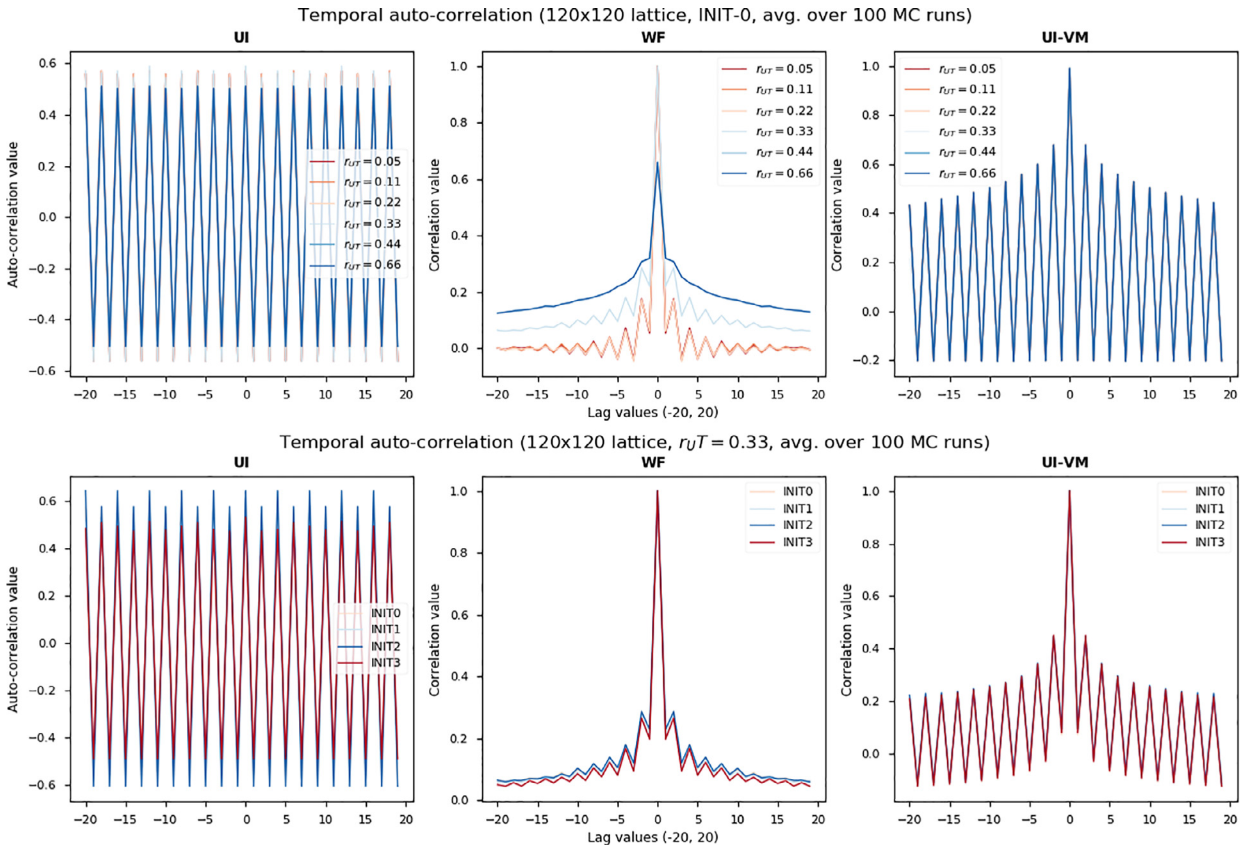

Fig. 7. Temporal auto-correlation of a player in a steady-state stage of the simulation under different r_{UT} configurations and INIT-0 (the top row) as well as under different initial conditions while setting r_{UT} to 0.33 (the bottom row). Temporal correlations are clear for all the settings, having only slight differences with the WF update rule and the highest r_{UT} values, where it is harder to promote trust.

Finally, we calculated the spatial correlation coefficients for all players in the lattice, which are located at a distance l. We followed a similar approach in [26] by computing a spatial correlation function $G(l)$ for all the agents in the lattice as follows:

$$G(l) = \frac{1}{n} \sum_{\forall i,j/d(i,j)=l} \delta_{str(i),str(j)}, \tag{3}$$

where n is the number of pairs of agents at distance l in the lattice, $d(i, j)$ is the distance in the lattice between agents i and j, δ is Kronecker's delta, and $str(i) \in \{1, 2, 3\}$ represents the three possible strategies of agent i.

Fig. 6 shows all the $G(l)$ values for the 120×120 lattice and for the four update rules under four different initial population conditions. We observe similar trends for all initial configurations except with the PROP rule. This is because some initial conditions (mainly INIT-1 and INIT-3) in the PROP case do not facilitate a trustworthy stationary state at the end of the simulation with $r_{UT} = 0.33$. For the two UI-based rules and the WF rule, there is cyclic oscillatory behavior. Additional experiments applying the same analysis to a triangular lattice of 14,400 players provided us with similar results and insights like those shown in Fig. 6.

4.2. Temporal correlations

Next, we explore temporal correlations of the model. Fig. 7 shows temporal auto-correlations for three of the update rules with oscillatory behavior under different levels of game difficulty and initial population conditions. We see that temporal oscillations are regularly observed for the UI rule between two different states. Oscillations also appear for the WF and UI-VM rules, although there is a decay in the correlations for WF with high lag time values.

Fig. 8 shows the power spectrum of signal in a 120×120 square lattice for non-deterministic rules WF, PROP and UI-VM under four different initial configurations. The signal here refers to the total number of players playing strategy I (i.e., k_I) at each step of the simulation, obtained when the simulation reaches a steady state. We do not show the power spectrum of the UI rule, as it is a strictly periodic function. Specifically, the signal obtained from the UI rule has a high frequency of

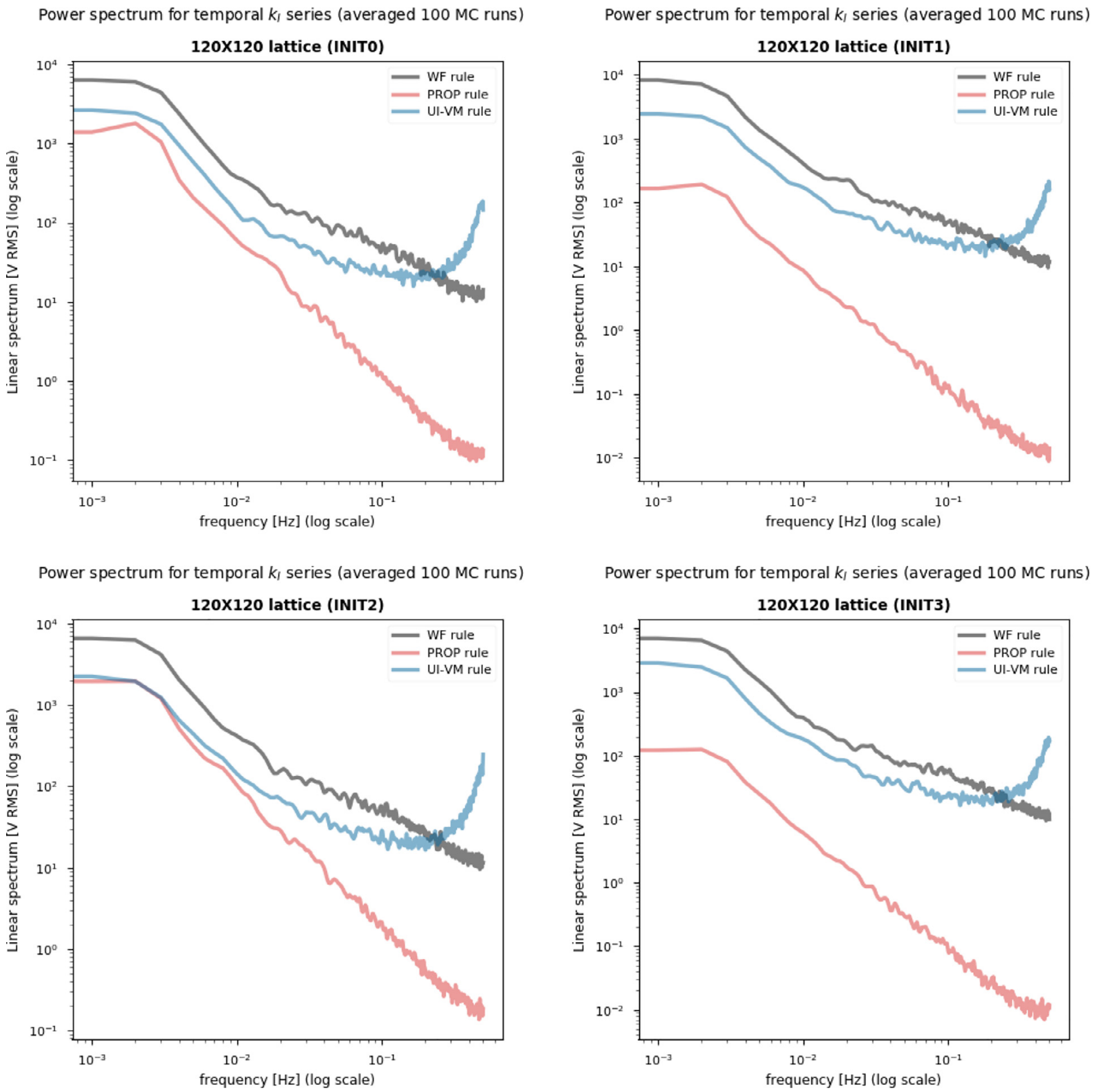

Fig. 8. Power spectrum of evolution in k_I for the UI-VM, WF, and PROP rules in a 120×120 lattice for the four different initial population configurations and $r_{UT} = 0.33$. We can clearly see the same power-law distribution for the four considered configurations.

0.5 Hz, which means that there is a complete temporal correlation with a period of 2 steps. This is because of the clear oscillatory behavior of UI within the model, which can also be seen in the spatial analysis in the next section.

The plots in Fig. 8 show the power spectrum of WF, PROP and UI-VM in a log-log scale for the four initial configurations. The three spectra of the rules in all the initial conditions have a clear power-law distribution, showing a linear drop for the intermediate and tail areas. The power-law distributions of the spectra of the rules indicate that the model's output has many different low frequencies. Therefore, when using non-deterministic update rules, the model has a long-term memory (i.e., long temporal correlations). This behavior is not common in this kind of models, as many of them exhibit stochastic dynamics, where there is only a short memory and the decision does not depend on previous history [27].

4.3. Spatio-temporal correlations

As the final stage of our experiments, we calculated the spatio-temporal correlations among pairs of players in the lattice. The calculation process is as follows. We first set a focal agent at random and four additional players situated at a distance

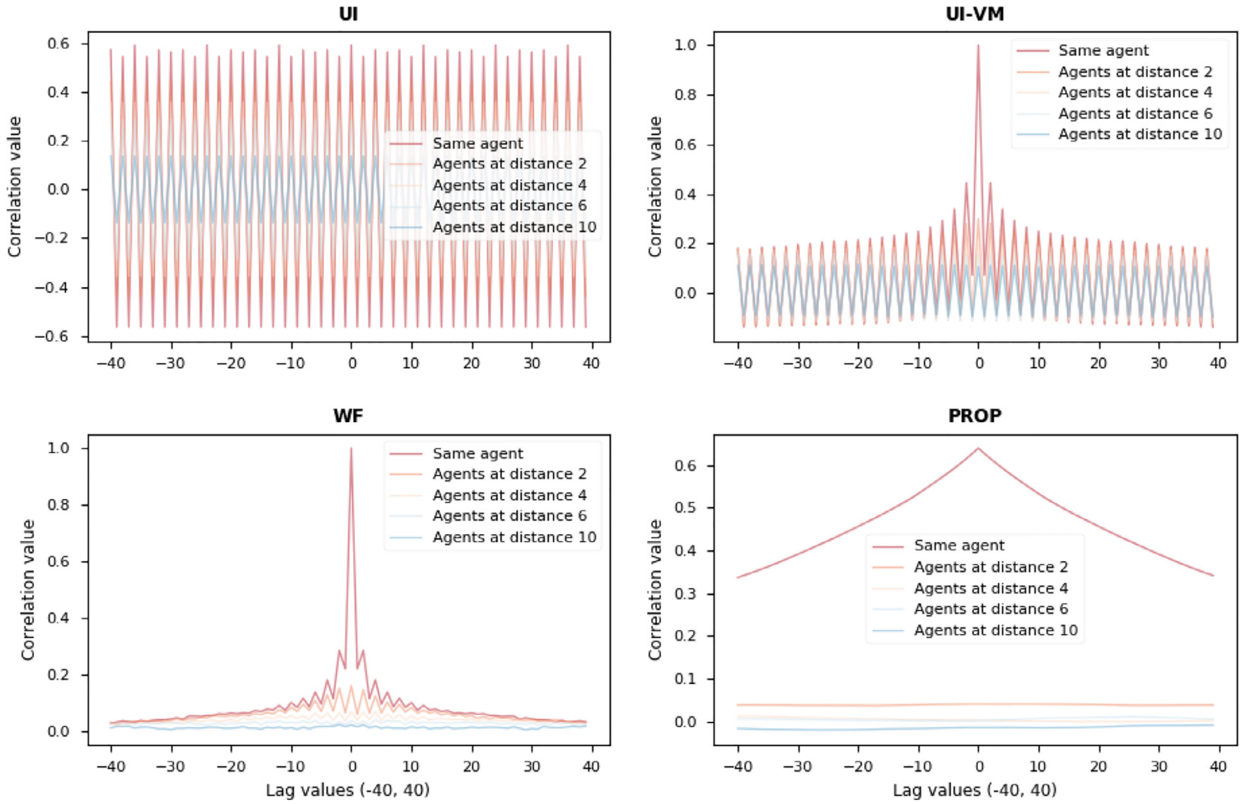

Fig. 9. Spatio-temporal correlations between the same focal player and players at distances 2, 4, 6, and 10 in a 120 × 120 lattice, with $r_{UT} = 0.33$ and INIT-0 as the initial population configuration. UI-based rules have strong oscillatory correlations. The WF rule also generates spatio-temporal correlations but decaying exponentially. Finally, the PROP update rule shows only correlations with the closest players. Similar results are observed with INIT-1, INIT-2 and INIT-3 configurations, hence details are not shown here.

of 2, 4, 6, and 10 in the lattice with reference to the focal one. We obtain the strategy evolution of the focal agent and the other four players in the last 50% of the steps. Later, the focal agent's time series is compared to the other four players' time series to find spatio-temporal correlations.

Fig. 9 shows plots with time-lag Pearson correlation values of four players' time series with respect to a focal agent for the INIT-O initial configuration. Plots for the other three initial configurations are not shown, as their results are identical to that shown in Fig. 9. Here, we see similar behavior from the results compared to those observed when analyzing the spatial and temporal correlations independently. There are high correlation values for the UI deterministic rules (both UI and UI-VM), and these correlations are cyclic and for all the considered distances. This is related to the oscillatory behavior seen for the UI rule. In the case of PROP (the bottom right plot of Fig. 9), the rule generates the highest correlation with the closest players (i.e., at distance 2). When using the WF update rule, we see that there are oscillatory dynamics in time and space but they are absorbed with high lag values (i.e., decaying exponentially).

5. Final discussion

In this work, we have shown the dynamics of different update rules for a networked multi-player trust game of three strategies. The game also represents the mutualism that appears when promoting trust, because investors and trustworthy trustees are obligate mutualists [28]. We first investigated which rules and social network topologies can obtain the highest trust level among players in the population. Based on the simulation results, we are able to divide the update rules into two groups: (1) the UI and its hybrid update rule with a voter model; and (2) the PROP and WF rules. PROP and especially the WF update rule have the best results in terms of global trust promotion when the game is easy ($r_{UT} = 0.11$), and the network topology in place is SF. When the game is more challenging, however, both UI-based rules are able to promote higher levels of trust and global net wealth for different initial population settings with the use of a square regular lattice. In contrast, WF and PROP are not able to promote trust under difficult game conditions. These findings are in line with previous work showing that different microscopic update rules can lead to different evolutionary outcomes [29,30].

Apart from evaluating trust promotion by the rules, we also studied the spatial and temporal model dynamics in lattices. We found that the UI-based rules form spatial clusters of sponge-like areas. These patterns are created to enable net wealth maximization of each playing group by just including trustworthy trustees (strategy *T*) with at least one investor (strategy *I*). Under this condition, being an untrustworthy trustee is not worthwhile, and they cannot invade the network. More interestingly, we found that not only the fully deterministic UI rule but all the update rules considered can generate a fractal space on the lattice when reaching a steady-state stage. Generally speaking, spatial correlations are important and occur mainly between closer players within the lattice. By studying the power spectrum of the model's output, we also observed that when introducing stochasticity in the rules, a long-term memory appears in the system. This means that for the UI-VM, WF and PROP rules, there are signals at low frequencies and their spectra are of power-law nature.

Another interesting behavior is that the system is not 'frozen' when reaching a steady state for deterministic rules such as the UI, and there are spatial pattern formations in the network [30]. Specifically, a phenomenon related to 'switching' or cyclic oscillatory behavior emerges (e.g., from investor strategy *I* to trustworthy trustee *T* or the other way round). This behavior has been studied in other 3-player games such as the prisoner's dilemma with voluntary participation [31,32]. From our temporal and spatio-temporal analyses we found these oscillations and maximum correlations with a 2-step period (a high frequency of 0.5*Hz*). It is also clear that all of these switching nodes must have more than one connection to create a chain of switching nodes. This switching behavior was not observed in the PROP update rule. The reason may be that PROP is the only rule among those considered that evaluates just one neighbor within a group to make a decision. When considering rules such as UI, UI-VM, and WF, if one neighbor *j* changes the strategy, the decision of focal player *i* changes totally, as all the neighbors are evaluated to make a decision to imitate.

These important findings about the hidden dynamics and spatio-temporal correlations in the multi-player trust game show the complexity of managing trust in social networks in the long run. Our findings also open doors for future research on evolutionary game theory in general and for other evolutionary trust games in particular, such as considering the quantity to invest in others (e.g., the investor can choose whether and how much to invest, as done in [9]) and using genetic algorithms to enrich the evolutionary study (as done in [33]).

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