A New Conceptual Model for Forest Fires Based on Percolation Theory

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INTRODUCTION

The question of what relatively simple model might describe forest fire propagation and occurrence as well as frequency–area statistics has occupied a number of authors [1-12] (not an exhaustive list). Except for the first citation, these models fall into two classes: self-organized criticality (SOC) (second through the sixth) [13] and percolation (seventh through twelfth) [14] models. A review [15] is rather pessimistic regarding the interpretation of forest fire dynamics in the context of percolation theory. The present conceptual development is based on percolation theory but has some characteristics of SOC, and is not directly related to existing percolation models.

There is certainly considerable evidence to support the SOC model that suggests that the following equation for the (noncumulative) frequency f of forest fires of burn area A is universal,

$$f(A) \propto A^{-\alpha}, \qquad (1)$$

with the value of α being 1.3 (values of 1 to 1.3 are apparently considered acceptable) [6]. Note that Eq. (1) is valid for binning discretely on a logarithmic scale, such as would be obtained by integrating a continuous distribution from A' to $2A'$; thus, the exponent for a continuous distribution would be $\alpha + 1$. Grassberger [4] obtains for the Drossel–Schwabl [3] model $\alpha + 1 = 2.15$, in the middle of the range cited in [6]. A (frequently) published graph [6] of 15,308 forest fires from Ontario, Canada (0.002 $\text{km}^2 < A < 1330\ \text{km}^2$) clearly yields $\alpha = 1.38$. These authors then state that "a number of other authors (including several articles of Minnich and coauthors) have found good correlation of the frequency-area distributions of forest fires and wildfires with the power-law relation [Eq. (1)], although others [16, 17] disagree." (An important point of reference [16] is the introduction of a maximum size due to large-scale heterogeneity in species distributions.) In Figure 1, I reproduce the statistics of fire occurrence in Baja California from [18] (from Minnich and coauthors, but not specifically cited in [6]), which also shows a power-law distribution of forest fire areas as in Eq. (1), but with an exponent of $\alpha = 0.69$. In this case, fire perimeters were determined by analysis of series of aerial photographs. Analysis disclosed 865 fires of size greater than 5 hectares. The authors [18] divided their data into four different

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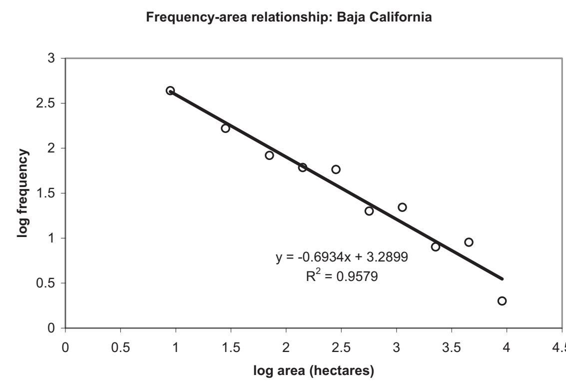
time periods, 1925–1941, 1942–1955, 1956–1971, and 1972–1990, with exponents of 0.65, 0.56, 0.82, and 0.72, respectively, and an arithmetic average of these four values being 0.69. (The aforementioned values were determined by Excel fits of a slope on a log–log plot.) A difference of this magnitude suggests a possible relevance of other physics.

Consider the following quote from the abstract of [19]. "Each site [] is either vacant or occupied by a tree. Vacant sites become occupied at rate 1. Further, each site is hit by lightning at rate λ. This lightning instantaneously destroys (makes vacant) the occupied cluster of the site. This model is closely related to the Drossel-Schwabl forest fire model, which has received much attention in the physics literature. The most interesting behavior seems to occur when the lightning rate goes to zero. In the physics literature it is believed that then the system has so-called self-organized critical behavior." Later, in the introduction, these authors complete the reasoning, "This is a continuous-time version of the Drossel-Schwabl model [. . .]. The most interesting questions are related to the asymptotic behavior when the lightning rate tends to 0. . . . it is believed that, asymptotically, the cluster size distribution has a power-law behavior." In contrast, however, consider the conclusions from [20] regarding lightning strikes. In the Sierra San Jacinto and the Sierra San Bernardino, in southern California, only 3.1% and 3.8% of all lightning strikes generated fires of measured size. "The efficiency of lightning in initiating fires required to account for recent fire history is only 5% of discharges in the San Jacinto and 2% of discharges in the San Pedro Martir [Baja California]." Given that the occurrence of lightning (in a region where lightning strike frequency is two orders of magnitude lower than many areas east of the Rocky Mountains) is nevertheless almost two orders of magnitude higher than that required to set the observed fires, it seems likely that the "most interesting" limit is *not* the zero limit of lightning occurrence, at least not if it is intended to be relevant to observed fire frequency relationships. In fact, the number of lightning-caused fires is argued to be so high that the authors [20] suggest that most lightning-caused fires burn only very small areas, called "spot fires," (frequently as small as a single tree) and that the landscape-scale burns, which account for "most of the disturbance and consumption of fuels," are infrequent. Thus, most of the lightning strikes occur at a time when the fuel content of neighboring trees is too small to burn and no spreading occurs.

FIGURE 1



Logarithmic plot of the frequency of burns as a function of burn size for data from Baja California collected for fires of size 5 hectares or larger [18].

Note that an important element of power-law statistics is the lack of a characteristic size. Nevertheless, as long ago as 1983 [21] it was clear that typical southern California burns and northern Baja California burns (with essentially the same topography and climate) were approximately an order of magnitude different in size. This difference is even evident visually on fire mosaic maps, where individual US fires were sometimes large enough to span the montane ecotone bounded by the coastal plain to the west and the inland desert to the east. The difference was traced to the effects of fire suppression on the US side of the border [21]. By virtue of the fundamental similarity of the landforms and the climate, which do not respect political boundaries, this distinct difference is not due solely to finite-size effects (which do introduce a characteristic size), though on the US side of the border such effects do appear to be important.

Thus, an important issue regarding fire occurrence is suppression. By systematically suppressing small fires when weather conditions are moderate (relatively high humidity and low winds in summer), the effects of fires under extreme conditions (Santa Ana winds in fall), when fire is impossible to suppress, are accentuated [21]. The number of destructive lightning strikes (or matches dropped) is reduced. At the same time, such management policies tend to homogenize the landscape, making areas of stands of (particularly) chaparral of the same age much larger and less complex in shape. But, as the same author points out, the most important factor on whether chaparral will burn when exposed to flame is the age of the individual; young plants do not ignite under summer conditions. In both its fundamental aspects, such a suppression policy could well drive a system toward SOC.

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FUNDAMENTAL HYPOTHESIS

The question of whether percolation theory is quantitatively suited to predicting the observed forest fire frequency–area distributions is rather complex and will not be answered in general here, although some tests, general and specific, are proposed. Percolation theory is based on the concept that if bonds between grid sites are established with probability $p > p_c$, where p_c is a critical value, an infinite cluster of connected grid sites will be established. Critical path analysis [22, 23] is a means to calculate the dominant contribution to flow or conduction by analyzing a fraction of an entire system just at the percolation threshold, which is composed of the most conductive elements. In the present case, an analogue to critical path analysis is proposed; one considers a time interval such that the mosaic of all burns just percolates. Thus, instead of a landscape evolution perpetually at critical conditions, one proposes looking at a time window for which the system is precisely at percolation. First, this concept suggests definition of a landscape scale recurrence time (discussed later) rather than a typical burn recurrence time for an individual, or a stand. But, whether system dynamics are actually constrained by percolation at any given time is a question that is, in this picture, difficult to evaluate. If the hypothesis is quantitatively accurate, a necessary result is that forest fire area statistics, at least over the appropriate time interval, should be consistent with percolation cluster statistics in two dimensions. To test the predictions of this model precisely, however, one must either know or be able to determine from observation what the value of p_c is (making possible the choice of the appropriate window in time), necessitating some discussion of finite-size and anisotropy effects on extracted values of p_c. This question will be addressed later, but first consider the implications of this chief hypothesis.

In any case, there should be three consequences of the hypothesis: (1) forest fire burn statistics taken over the proper window in time should follow percolation cluster statistics in two dimensions, (2) finite-size effects on the percolation probability could, in principle, be relevant to apparent observed values, and (3) in the case of either wind (or topography)-controlled anisotropy or elongated ecotones, the apparent percolation probability would be different for percolation in the orthogonal directions.

In truth, one of the main factors governing the size of a given burn is its proximity to prior burns. Here is a quote from [18]: "the occurrence of fires tends to be self-limiting, spatially nonrandom, and with recurrence intervals of 70 years related to the gradual, cumulative development of fire hazard during successions. Mosaics of fire-created patches assume a nonrandom and self-organizing spatial process where the occurrence of fire is affected by preexisting burns" [note by the present author this implies the effect of spatial correlations and *memory*, not SOC]. Note that in many individual cases where suppression is not employed fire boundaries are defined by the boundaries of drainage basins, partly because it is difficult for fires burning under nonideal conditions (in the absence of suppression, the typical condition) to burn across divides.

A frequently used statistic in fire ecology is a burn recurrence time. These time scales clearly vary from a few years in grass fires through a few decades in chaparral fires to centuries in forest fires in the Pacific Northwest. Burn recurrence times that one might *put into* a model would relate to characteristics of individuals, such as at what age chaparral has accumulated sufficient fuel to burn easily. But burn recurrence times derived from large scale observation may be more closely related to the burn recurrence interval proposed earlier and discussed later. In any case, burn recurrence times are related in a complex way to climate variability on seasonal scales (grass fires) to decades or centuries (in rainforests), and thus also to drought statistics. For typical forest and chaparral fires, there may be an interaction of processes occurring on these time scales, "In Californian mixed-conifer forest, the Mediterranean climate of winter storms and dry summers results in unfavourable temperature and moisture conditions for decomposition, [24] leading to fuel build-up and fire hazard. [25, 26]" Burn recurrence times may also be influenced by climate change. If climate changes significantly within a recurrence time, it may be impossible to define such a time consistently.

To provide a clear prediction of what time scale to expect for burn recurrence, a consistent definition of this time scale is required that is not influenced by, e.g., extreme value statistics. What I propose here is relatively straightforward, though its most obviously useful application is to the California peninsular and transverse ranges (southern California and northern Baja California mountain ranges). In this region, either previously burned areas or the boundary of the region susceptible to fire (in California, and particularly in Baja California, these are basically the coastal plain and the desert) limit spatial extent of fires. The basic hypothesis is that when, in a given isotropic and quadratic climate and topographic region, the total burned area extends from north to south and east to west without breaks (i.e., the burned region percolates), a recurrence time has been defined. However, in California, the mixed-conifer forest and chaparral regions occur mostly in the mountain regions mentioned, meaning that the spatial distribution of fires is not uniform and the regions of interest are not quadratic. Furthermore, fires occur under predominant climatic conditions subject to prevailing surface winds, most frequently southwesterly winds, leading to the tendency for burns to be elongated across the mountain chains rather than along them, accentuating the anisotropy. A recent treatment of anisotropy in critical path analysis [27] provides a possible analytic means of treating such anisotropy in fire mosaics (discussed in more detail in the next section).

When the burn recurrence time frame has been identified, one can predict that the statistics of the size of forest fires that occur within that time

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frame in an infinite system should follow the cluster statistics of percolation theory (in two dimensions), which predicts a universal power law decay out to infinite sizes. (At other time scales, even in infinite systems, the cluster statistics of percolation theory would predict an exponential cut-off of burn sizes at a size inversely proportional to the "distance" from the percolation threshold, and it is useful to eliminate this complication.) In two dimensions, the critical exponent for cluster areas is 2.05, which is rather poor compared with the observed value of $\alpha + 1 = 1.69$. However, SOC model values of $\alpha + 1$ [4, 6] are little bit further off at 2.15 or 2.3, respectively. While it was not possible to check the statistics at a time interval verified to be appropriate for the definition of critical percolation as proposed here, a procedure designed to investigate this hypothesis was designed. In particular, the entire time interval investigated (65 years) was divided into subintervals (of about 20 years each) and the power was analyzed for both the subintervals and the entire time period. There was no obvious change in the power, i.e., the power of 0.69 obtained (see Figure 1) was approximately the average value of the powers in the subintervals.

APPLYING PERCOLATION THEORY TO LANDSCAPES: VALUE OF P_C

The present application of percolation theory to forest fires, hoped to generate both a burn recurrence time and fire area relationships, relates thus to both the critical percolation probability, which is system dependent, and to cluster statistics of percolation theory, believed to be universal. It is supposed that the percolation threshold is to be obtained visually from fire mosaics constructed for various time intervals. If a landscape is isotropic, winds are random, and ecotones are equidimensional, percolation will occur in an infinite system from north to south and from east to west simultaneously. None of these conditions is met in the California transverse and peninsular ranges, however. So, some discussion of how a critical percolation probability should be independently obtained is warranted.

Critical percolation probabilities for a number of grid characteristics (or lattice types) have been tabulated, but such values are not so relevant to natural landscape applications, for which continuum percolation is better adapted conceptually. So, even though site percolation on a square lattice has often been used as a basis to interpret ecological problems (e.g., [28]) it is proposed here to use continuum percolation theory formulation to represent real landscapes. Continuum percolation theory applied to two-dimensional (2D) problems employs fractional surface area (or coverage) as the variable p. The difficulty is that in continuum percolation there is little theoretical guidance for predicting a percolation threshold, p_c. This particular parameter must be left as an unknown, to be revealed by analysis.

The next fundamental question that arises is, "to what scale percolation theory should be applied, whether to the individual tree (or bush) scale, or to a larger scale." If to the individual tree scale, then the fundamental length scale, χ_0, which appears in the quantities from percolation theory, is simply a typical distance between trees. For such an application, finite-size effects may not be visible even in relatively small systems. However, this does not appear to be the appropriate scale of application in real systems, particularly in the context of the definition of (landscape) burn recurrence times proposed here. Consider that, e.g., many lightning-caused fires burn only a single tree, and other analyses [20] have suggested that the statistics of single tree burns do not have any relationship with the statistics of larger burns, occurring far more frequently than otherwise expected. In a percolation sense, this appears to indicate that the majority of lightning-caused fires originate under conditions that are not conducive to the spread of fires (when $p = 0$). Thus, I suggest using χ_0 also as a parameter to be extracted from analysis. It would be very convenient to have a simple procedure to extract both χ_0 and p_c simultaneously, but such a procedure is not given here.

Given χ_0 as a parameter, it is known [29] that the correlation length scale from percolation theory diverges according to the following power law:

$$\chi = \chi_0 |p - p_c|^{-\nu}, \qquad (2)$$

where p is the fractional area of coverage, p_c is the critical area fraction for percolation, and ν is a universal exponent from percolation theory, which takes the value of 1.33 in two dimensions[30]. If p is effectively a linearly increasing function of time, then it would be possible, in principle, to extract χ_0 independently from spatial images representing the total burnt area as a function of time. However, aerial photos are only available at widely spaced intervals, leaving little intermediate data to perform the analysis. Physically, the correlation length represents the size of the largest cluster of interconnected area of a particular type, e.g., either burned or unburned (permitted by using the absolute value of $p - p_c$), depending on the value of p_c applied. The correlation length is thus particularly useful for defining finite area effects on percolation because of the confinement of a particular type of vegetation to the elongated shapes of climate zones in mountainous topography, such as in the Basin and Range province, or in the peninsular and transverse ranges of Southern California or Baja California. In the case of anisotropy (because of the effects of prevailing winds), χ_0 cannot be used as a single parameter, valid for either direction. Nevertheless, one can in this case use the shape anisotropy of individual burns to define a typical ratio R of burn lengths in the long and short directions. If, as in Southern and Baja California, the long burn direction is across the mountain range, then the product $R\chi_0$ must be compared with the width of the ecotone.

When p is close enough to p_c (but less than p_c) and χ is greater than the width of the peninsular ranges, then percolation across the mountain range will occur somewhere in a long enough

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range. Such a value of p would be an observed p_c for transverse percolation, and can be called p_c'.

$$W = R\chi_0(p_c - p_c')^{-\nu}. \qquad (3)$$

Here W is the width of the mountain range (more accurately, the ecotone).

p must be larger than p_c for percolation to occur along the length of such mountain ranges. Above p_c, the correlation length represents the largest unburned break in the percolation cluster. If such a hole can be larger than the width of an ecotone, then percolation along its length is unlikely. Thus, for percolation to occur along the length of the ecotone, the correlation length must be smaller than or equal to the width of the ecotone,

$$W = R\chi_0(p_c'' - p_c)^{-\nu}, \qquad (4)$$

where p_c'' is the fractional area for which longitudinal percolation occurs.

Identification of a time scale, for which the fractional burn area would be p_c (in a quadratic region under isotropic climatic conditions), then yields the proposed landscape burn recurrence time, t_c. Clearly, such an analysis requires that the climate zones be stationary. If one is in possession of a sufficiently long record of burn areas, violation of the conditions of this analysis might be used to identify nonstationary statistics and thus perhaps the influence of climate change in that the relationship of p_c to t_c is not constant.

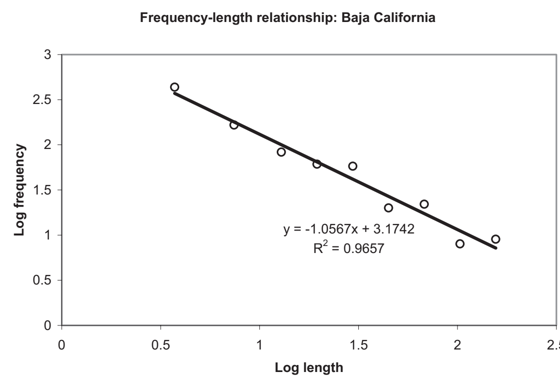
If finite-size effects are important, then one also has complications because percolation does not occur for the same value of p_c (thus p' or p'') in every realization, but this complication is neglected.

p' and p'' as determined from spatial analysis may then be related through Eqs. (3) and (4) to p_c,

$$p' = p_c - \left(\frac{R\chi_0}{w}\right)^{\frac{1}{\nu}}$$

$$p'' = p_c + \left(\frac{R\chi_0}{w}\right)^{\frac{1}{\nu}}. \qquad (5)$$

FIGURE 2



Logarithmic plot of the frequency of burns (as in Figure 1), but with respect to a length scale appropriate for a river drainage (proportional to $A^{0.6}$).

RESULTS AND DISCUSSION

The potential problem with the aforementioned analysis is that if the forest fire burn areas just percolate, but the burn areas do not obey the cluster statistics of percolation theory (with an exponent of $\alpha + 1 = 2.05$), we appear to have a contradiction. In fact, the burn area statistics in Baja California (see Figure 1) at least appear to follow a power law, but with exponent approximately $\alpha + 1 = 1.7$ rather than 2.05. The value of this exponent does not appear to have any dependence on the length of the time interval chosen (results not shown), though there was a limited potential to investigate this variable. Any cause for this discrepancy, other than the failure of the percolation model, has not been identified here, and it is possible that a completely different model, such as random fragmentation, would be relevant. Note, however, that one of the conditions for the derivation of a critical exponent greater than 2 is the requirement in an infinite system that the fractional area covered by burns (as given by an integral over $Af(A)$ out to infinite A) does not diverge[29]. Such an argument applies equally to SOC. Moreover, the discrepancy is not smaller there; rather, it is slightly larger.

Alternate Hypotheses

As noted, an alternate possibility is that a landscape fragmentation model would be more appropriate. A second alternate hypothesis could be that what is actually being observed relates to the statistics of drainage basins rather than to fire characteristics, since the divides between basins restrict the spread of fire. One potential means to investigate this is to check what the statistics of fire occurrence in terms of the length of a drainage basin turns out to be. The length of a drainage basin is not necessarily a well-defined object, but the length of a stream can be defined. The most commonly quoted experimental relationship between stream length, S, and drainage basin area, A, is $S \propto A^{0.6}$, a relationship known as Hack's law[31]. Using S as the independent variable (see Figure 2) generates a power of $\alpha + 1 = 2.06$ for the frequency of forest fire lengths. Note that the cluster statistics of percolation theory in 1D imply an exponent of 2, however, rather than 2.05. This could mean that percolation theory is relevant to the arrangement of at least the smaller drainage basins along 1D transects across the range and that the behavior of fires is largely controlled by the interaction of this topography with the prevailing winds.

CONCLUSIONS

A new conceptual framework for analyzing the effects of anisotropy in landscape as well as climatic conditions on fire statistics has been proposed. This framework is analogous to critical path analysis for the electrical or hydraulic

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conductivity, and is thus based on percolation theory. In a test to determine whether it is applicable to burn area statistics in Baja California, however, it was impossible to confirm that the cluster statistics of percolation theory are relevant to the fire frequency–area statistics observed.

On the other hand, it also appears obvious that there are serious conceptual problems with the application of concepts from SOC to forest fire burn areas, at least in Baja California, thus implying that a search for an appropriate conceptual understanding is still relevant. Further, discrepancy between the predicted power law from SOC and the observed statistics appears to be somewhat greater than for percolation theory. The principal conceptual objections here are (1) that the observed fires do appear to have a relevant length scale and (2) that the "interesting" limit of a zero lightning rate frequency is never approached in nature.

A third hypothesis, that the chief controlling factors were topography and wind direction, and that the fire area statistics might actually relate to percolation in basin lengths, appeared to perform a little better than the first two, but this alternative should probably be regarded at this time as an unsubstantiated conjecture.

Related Web sites

- http://www1.coe.neu.edu/~emelas/forestFire.htm (site with simple applet tool).
- http://polymer.bu.edu/java/java/blaze/blaze.html (laboratory protocol, evidently from Gene Stanley's group).
- http://www.cof.orst.edu/org/usiale/lasvegas2004/swapmeet.htm.

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