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Comprobar que $\int_0^1 \sqrt{1-x^2} \xrightarrow{\text{con}}$ fdp óptima $(f^{o}_{\hat{x}}(x))$

↳ el error del método de muestreo es **0**.

Para encontrar el estimador óptimo debemos minimizar:

$$\sigma^2[\tilde{g}(x)] = \int \frac{g(x)^2}{f_{\hat{x}}}\, dx - I^2 \equiv$$

↓ minimización → multiplicadores de Lagrange $(I \neq I(f_{\hat{x}}))$

$$\mathcal{L}[f_{\hat{x}}] = \int \frac{g(x)^2}{f_{\hat{x}}(x)}\, dx + \lambda \int f_{\hat{x}}(x)\, dx$$

↓ minimizamos $\left.\frac{\delta L}{\delta f_x}\right|_{f_{\hat{x}} = f^{o}_{\hat{x}}} = 0$

$$f^{o}_{x}(x) = \lambda^{-1/2} |g(x)|$$

↓ condición normalización $\int f^{o}_{\hat{x}}(x)\, dx = 1 \rightarrow \int \lambda^{-1/2} |g(x)|\, dx = 1$

$$\boxed{\lambda^{1/2} = \int |g(x)|\, dx}$$

$$\boxed{f^{o}_{x}(x) = \frac{|g(x)|}{\int |g(x)|\, dx}} \rightarrow \tilde{g}^{o}(x) = \frac{g(x)}{f^{o}_{\hat{x}}(x)}$$

↳ Aplicado a nuestro caso: $g(x) = |g(x)|$; $g(x) \geq 0 \quad \forall x\ [0,1]$

- $f^{o}_{x}(x) = \frac{\sqrt{1-x^2}}{\pi/4} = \frac{4}{\pi}\sqrt{1-x^2}$ • $\tilde{g}^{o}(x) = \frac{\pi}{4}$

↳ varianza: $\sigma^2[\tilde{g}^{o}(x)] = \left(\int |g(x)|\, dx\right)^2 - I^2 = 0$

$$\frac{\left(\int |g(x)|\, dx\right)^2}{(\pi/4)^2} - \left(\frac{\pi}{4}\right)^2$$

Aplicado al método de muestreo de importancia la varianza asociada al estimador $\hat{Q}_o$ es:

$$\hat{\sigma}_N[\hat{Q}_o] = \frac{N}{N-1}\left[\frac{1}{N}\sum_{i=1}^{N}\tilde{g}^o(x_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}\tilde{g}^o(x_i)\right)^2\right] =$$

nuestro caso:

$$\hat{\sigma}_N[\hat{Q}_o] = \frac{N}{N-1}\left[\frac{1}{N}\cdot\Big(\underbrace{\frac{\pi^2}{4^2}+\frac{\pi^2}{4^2}+\cdots\frac{\pi^2}{4^2}}_{\text{"N veces"}}\Big) - \Big(\frac{1}{N}\Big(\underbrace{\frac{\pi}{4}+\frac{\pi}{4}+\frac{\pi}{4}\cdots\frac{\pi}{4}}_{\text{"N veces"}}\Big)\Big)^2\right]$$

$$= \frac{N}{N-1}\left[\frac{1}{N}\cdot\left[N\left(\frac{\pi^2}{4^2}\right)\right] - \left(\frac{1}{N}\left(N\cdot\frac{\pi}{4}\right)\right)^2\right] =$$

$$= \frac{N}{N-1}\left[\frac{\pi^2}{4^2}-\frac{\pi^2}{4^2}\right] = 0$$