Comment on "Macroscopic Equation for the Roughness of Growing Interfaces in Quenched Disordered Media"

In a recent Letter [1] Braunstein and Buceta introduced a "macroscopic" equation for the time evolution of the width of interfaces belonging to the directed percolation depinning (DPD) universality class [2]. From numerical simulations of the DPD model, they inferred an ansatz [Eq. (1) in Ref. [1])] for the time derivative of the interface width (called DSIW in Ref. [1]) at the depinning transition. Braunstein and Buceta found that their formula fitted the numerical data at the depinning transition, for $q_c = 0.539$ and $\beta = 0.63$, with the appropriate election of some arbitrary constants.

Here we argue that, contrary to what is claimed in Ref. [1], Braunstein and Buceta's formula does not describe the macroscopic behavior of the interface. The formula proposed in Ref. [1] for the DSIW is an approximation to the very short times regime (when less than one layer has been completed), which is not significant for the description of the surface dynamics at large scales. We obtain analytically the short time behavior of the DPD model, which is valid for any q and explains the appearance of an exponential term in the formula of Ref. [1] for the DSIW.

Let us consider the DPD model in a system of size L and a density q of blocked cells ($p = 1 - q$ density of free cells). We are interested in the very short times regime when the first monolayer still has not been completed, i.e., the number of growth attempts N is $N \ll L$ (this corresponds to times $t = N/L \ll 1$). In this regime, the probability of having a column i with height $h_i > \min(h_{i-1}, h_{i+1}) + 2$ is negligible and the columns are growing almost independently. The growth at this early stage can be seen as a random deposition (RD) process [3] in which every column grows in one unit with probability p/L. The short time regime of the DPD model is then like RD, which is solvable exactly, but with the additional ingredient of a density q of blocked sites.

One can see that, within this approximation, the probability of having a column with height h after N growth attempts is given by

$$P(N,h) = \frac{(Nsp)^h}{h!}\, e^{-Ns} + qp^h \sum_{r=h+1}^{N} \frac{(Ns)^r}{r!}\, e^{-Ns}, \quad (1)$$

where $s = 1/L$ is the probability of attempting to grow a column and the usual approximation $s^r(1-s)^{N-r}N!/[(N-r)!r!] \approx (Ns)^r \exp(-Ns)/r!$ has been made.

From the probability (1), one can calculate the interface width $W^2 = \langle h^2 \rangle - \langle h \rangle^2$ and then the time derivative, whose leading terms are

$$\frac{dW^2}{dt} = pe^{-qt} + 2p^2 e^{-qt}\left(\frac{e^{-qt}-1}{q} + t\right), \quad (2)$$

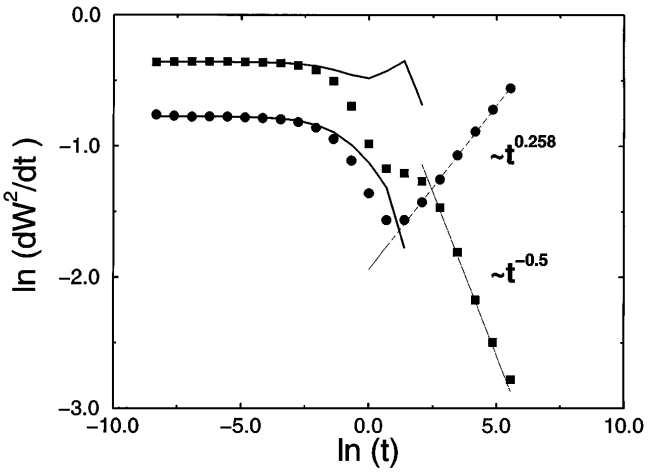

FIG. 1. Numerical results for the DPD model in a system of size $L = 2^{13}$ for $q_c = 0.539$ (circles) and $q = 0.3$ (squares). Continuous lines correspond to Eq. (2) and fit the data for $t \ll 1$. For larger times our approximation is not valid any longer and the power law $t^{2\beta-1}$ takes over with $\beta = 0.623$ and $\beta = 0.3$ for $q_c = 0.539$ and $q = 0.3$, respectively (dashed lines).

where $t = Ns = N/L$ is the time in the units used in Ref. [1]. This formula gives the exact time evolution of $\frac{dW^2}{dt}$ for any q (not only at $q_c = 0.539$) and is valid for times $t \ll 1$. For times $t > 1$ differences between neighboring columns are likely to be larger than 2 resulting in horizontal correlations and the breakdown of (2). A comparison of Eq. (2) with numerical simulations of the DPD model is presented in Fig. 1.

Our calculation suggests that the exponential term in the ansatz of Ref. [1] is actually produced by the usual random depositionlike dynamics, which occurs in any growth model [3] for short times.

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