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System capacity model and algorithm for urban multimodal transport network with transfer

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ABSTRACT

This paper proposes a method for calculating transport networks capacity in dealing with multimodal transfers. Multimodal networks are represented by a modified 'supernetwork', while the passenger's travels are defined as 'superpaths'. Within this framework, the relation between the travel demand from O-D matrices and the resulting link flows in the supernetwork is modelled as a relationship matrix to describe urban mobility by using a logit-based stochastic user equilibrium. Based on this relationship matrix, an approximate iteration algorithm (AIA) is developed. Our numerical results show that the AIA performs better than the sensitivity analysis-based algorithm (SAB) and genetic algorithm (GA) regarding the execution-time, and that the capacity of multimodal transport networks can be underestimated if the combined travels are neglected.

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1. Introduction

In large urban areas, commuting is a common feature of everyday residential life. Such trips often require multimodal combinations to cover all the distance from origin to the final destination. In contrast to single-mode travel, multimodal travels show an enhanced performance by taking the complementary advantages of various transportation modes (Huang et al. 2018). The capacity of a multimodal transport system, indicating its supply performance under certain travel behaviours, has become a concern of transportation planning authorities (Liu et al. 2021). With the same multimodal nature, however, the capacity analysis is more challenging than that of a single-mode transport system or a multimodal system without considering combined travel (Romero et al. 2015). This paper attempts to model the capacity of a multimodal transport network while accounting for combined travel behaviour, and to devise a dedicated algorithm tailored to this model.

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In traffic engineering, capacity generally refers to the maximum hourly traffic volume that can traverse a specific road section. Initially introduced by Smeed, area-level capacity defines the maximum number of vehicles that can pass through a road network per unit of time without congestion (Smeed 1966). Subsequently, Yang, Bell, and Meng (2000) defined the capacity of transport networks as the maximum attainable throughput of the network, which is calculated by aggregating the Origin-Destination (O-D) demands while considering the capacity constraints of individual road sections. However, since the constraint of the model is about link flows, it is necessary to transform O-D demand into flow links for comparison. Typically, this transformation is achieved based on equilibrium theory with the assumption that all O-D trips make self-interested choices, thus leading to a state of equilibrium in flows along all road sections in the network (Sheffi 1985).

The equilibrium theory-based method has been applied to capacity evaluation of transport networks in many studies. For example, Asakura (1992) first proposed a general method, in which O-D demands increase until the volume of any link's volume exceeds its capacity and the sum of the maximum tolerable O-D is the network capacity. Consecutive studies (Wong and Yang 1997) followed Asakura's idea and took the demand ratio between different O-D pairs as constants. However, the evaluated capacity is highly dependent on the given O-D demand ratio. An unreasonable ratio can lead to significant deviations from the actual capacity. To solve such problems, several works (Chen and Kasikitwiwat 2011; Yang, Bell, and Meng 2000) evaluated the capacity with a variable O-D structure. These studies focus on single-mode transport systems, especially road networks.

Modern urban traffic systems must be seen as composite networks with multiple subnetworks, one representing car traffic, one for buses, one for subway, and so on. In multimodal transport systems a representation with heterogeneous network structures is required (Liang, Huang, and Zhao 2022) and more complex travel behaviours are involved (Liao et al. 2020), where modelling needs to consider: (1) high-dimensional choices, implying that travellers' decisions are made by combining path choice, mode choice and transfer choice; (2) multi-index travel cost, which means travellers will consider not only link-specific costs such as travel time but also mode-specific costs like pricing, waiting time, transfer time, in-vehicle crowding and other factors. As a result, equilibrium in a multimodal transport network is considerably more intricate compared to that in a single-mode transport network. Although some studies (Liu et al. 2018; Wu et al. 2012) have combined traveller's mode choice and path choice, the multimodal equilibrium issue associated with complex travel behaviour has received limited attention.

A few works have investigated equilibrium-based capacity measures for multimodal transport networks. For example, Cheng et al. (2014) estimated the capacity of a network comprised of automobiles and transit by extending the road network capacity model (Yang, Bell, and Meng 2000). In this model, travel time equates to travel cost, and a numerical case study shows that the network capacity is increased due to rapid transit lines. Xu et al. (2018) formulated the (spare) capacity of a multimodal network including roads and metro lines. The cost of metro travel is assumed to remain constant, and the cost of road links is measured using the standard BPR function. Liu et al. (2021) dealt with travel costs in the same way as Xu et al. (2018) and explored network capacity with second-best constraints. Recently, Du, Jiang, and Chen (2022a) formulated travel time for various link types using BPR and established a capacity model to identify critical links in multimodal transport networks. While these studies integrated combined mode split and traffic assignment models to describe travellers' choice of mode and path, they disregarded the intricacies of combined travel. Additionally, only travel time is considered as the travel cost but mode-specific costs are neglected. Zheng, Zhang, and Liang (2020) addressed the car-to-metro mode but overlooked the general case of combined travel, such as travel with multiple transfers. Du et al. (2022b) modelled the capacity of networks with emerging travel modes but did not account for in-vehicle crowding. Overall, existing studies on multimodal network capacity have not fully explained the complexities of travel behaviour in multimodal transport networks.

Algorithm complexity is another important factor to consider in the equilibrium-based capacity approach. This problem is mostly formulated as a bi-level programming model, known for its NP-hard nature (Bard 1991). Heuristic algorithms have been used as solutions to these problems (Meng 2000). The solution methods can be categorized into two primary groups: (1) sensitivity analysis-based methods (SAB), and (2) other heuristic methods. In most existing studies, SAB simplifies the bi-level model by Taylor expansion (Du, Jiang, and Chen 2022a; Du et al. 2022b; Liu et al. 2021; Xu et al. 2018). However, the applicability of the Taylor approximation hinges on the proximity of solutions between successive iterations, potentially resulting in diminished algorithmic convergence speed. Moreover, sensitivity analysis for equilibrium problems is essential in SAB, yet with a large computational burden. Other heuristic methods, such as genetic algorithms and simulated annealing algorithms, have been used to solve bi-level programming problems (Fan and Machemehl 2006; Long et al. 2010). These methods use universal stochastic optimization mechanisms to search among a huge number of potential solutions. With each solution solving an equilibrium problem, equilibrium problems require many runs and thus are intrinsically time-consuming, particularly when dealing with large-scale networks. Therefore, the effectiveness of heuristics is diminished in addressing large-scale problems.

To highlight the differences between our work and previous studies, Table 1 lists the characteristics of relevant works. In summary, combined travel is missing in the model of equilibrium-based capacity for multimodal transport networks, and the solving algorithms remain to be systematically explored and improved. The goal of this study is to develop a novel framework for evaluating the capacity of multimodal transport networks considering combined travel. The combined travel and multi-index travel costs are considered based on a supernetwork, and the capacity of such supernetwork is evaluated. To

Table 1. Characteristics of relevant works in comparison with our work.

	Modelling				
Reference	Variable O-D structure	Multimodal	Combined travel	Multi-index travel costs	Algorithm
Wong and Yang (1997)	×	×	×	×	SAB
Yang, Bell, and Meng (2000)	√	×	×	×	SAB
Chen and Kasikitwiwat (2011)	√	×	×	×	SAB + GA
Xu et al. (2018);Du, Jiang, and Chen (2022a)	×	√	×	×	SAB
Du et al. (2022b)	×	√	√	×	SAB
Cheng et al. (2014)Liu et al. (2021)	√	√	×	×	SAB
This paper	√	√	√	√	AIA

solve the capacity evaluation model, we approximate the relationship between O-D demands and link flows based on the optimality condition of equilibrium problems, and propose an approximate iteration algorithm (AIA). The main contributions of this paper are as follows:

(1) Combined travels are described based on superpaths and subpaths. Transfer behaviour in the multimodal network is considered by introducing transfer links into the supernetwork. Superpaths and subpaths are defined to describe trips in the multimodal network. A generalized travel cost function for superpaths is constructed, taking into account in-vehicle crowding.
(2) AIA is designed to solve the proposed model. The relationship between the lower-level variables and the upper-level variables is approximated using the equilibrium problem's solution. An iterative mechanism with mutual feedback between the upper-level and lower-level is employed to converge a satisfactory solution.

The rest of the paper is organized as follows: Section 2 presents the improved supernetwork representation for multimodal transport systems. Section 3 establishes the relationship between the O-D demands and link flow based on the equilibrium model of the supernetwork. Section 4 models the capacity of multimodal transport networks with combined travel and presents AIA solution algorithms for the proposed model. Two illustrative numerical examples are described in Section 5. Section 6 further discusses the proposed model and algorithm. Section 7 concludes the paper.

2. Supernetwork for multimodal transport system

2.1. List of symbols

We provide a brief explanation of the symbol notation (as shown in Table 2) to be used throughout the paper.

2.2. Supernetwork and superpath

Generally, $G = (N, A)$ is used to denote a single-mode transport network, where N is a node set, representing a traffic zone, intersection, parking lot, etc.; A is a link set, representing the road section connecting two adjacent nodes. Due to the mode attribute, the network model for single-mode transport systems cannot be used for multimodal transport systems. The supernetwork model was first proposed by Sheffi (1985) to describe the simplified multimodal transport system, which is a modified network by augmenting multiple traffic mode subnets with dummy links. However, it cannot describe the combined travel problems, because various subnets are not connected and transfer is not allowed. In this paper, an improved supernetwork is proposed to express the multimodal transport system involving combined travel, in which the heterogeneous transport subnets are interconnected by transfer links.

$SG = (M, N, A)$ is defined as a supernetwork to represent the multimodal transport system, where M, N, and A denote sets of transport modes, nodes, and links, respectively.

Table 2. Symbol explanation table.

Num	Symbols	Explanation		
1	SG	supernetwork for the multimodal transport system		
2	M	sets of transportation modes		
3	N	N_1	sets of nodes	set of origins and destinations
4		N_2		set of nodes other than the origins and destinations
5	A	A_1	sets of links	set of boarding links
6		A_2		set of links that serve vehicle running
7		A_3		set of transfer links
8		A_4		set of alighting links
9	r		the node representing origin, $r \in R$	
10	s		the node representing destination, $s \in S$	
11	m		transportation mode, $m \in M$	
12	i		physical location of a node, $i \in I$	
13	K^{rs}		set of superpaths between O-D pairs rs, $k \in K^{rs}$	
14	S_k^{rs}		set of subpaths belonging to the superpath k between O-D pairs rs	
15	C_k^{rs}		generalized costs on superpath k between O-D pair rs	
16	c_a		generalized costs on the out-of-vehicle link a, $a \in A_1 \cup A_3 \cup A_4$	
17	l^m		a subpath of mode m	
18	$w_{l^m}^{rs}$		generalized costs on subpath l^m belongs to superpath $k$between O-D pair rs	
19	$\delta_{a,k}^{rs}$		correlation coefficients between link a and superpath k of O-D pair rs	
20	l_a		length of link a, $a \in A_1 \cup A_3 \cup A_4$	
21	v		walking speed	
22	η		amplification parameter	
23	$c_{l^m}^{rs}$		mode-specific costs on subpath l^m belongs to superpath k between O-D pair rs	
24	$t_{m,a}$		travel time on vehicle running link a in the subnet of mode m, $a \in A_2$, $\boldsymbol{t} = (t_{m,a})_{m \in M, a \in A_2}$	
25	δ_{a,l^m}^{rs}		correlation coefficients between vehicle running link a and subpath l^m of the superpath k between O-D pair rs	
26	t_m		travel time excluding vehicle running time of mode m, a constant	
27	τ_m		conversion coefficient from price to time for mode m	
28	ρ_m		the fare per mileage of mode m	
29	$d_{l^m}^{rs}$		distance of subpath l^m	
30	$t_{m,a}^0$		free-flow travel time of mode m on link a, $a \in A_2$	
31	$x_{m,a}$		traveller volume of mode m on link a, $a \in A_2$, $\boldsymbol{x} = [\cdots, x_{m,a}, \cdots]^T$	
32	K_m		vehicle capacity of mode m (person)	
33	$C_{m,a}$		traffic capacity of link a in subnet m (vehicle)	
34	α_m, β_m		parameters indicating congestion between vehicles of mode m	
35	ϕ_m, φ_m		parameters indicating crowding between passengers in vehicle of mode m	

N consists of two subsets, $\{N_1, N_2\}$. N_1 represents the set of nodes comprising both origins and destinations, while N_2 is the set of other nodes. N_1 consists of two subsets, R and S, representing the set of origins and destinations, respectively.

$$N = N_1 \cup N_2 \tag{1}$$

$$N_1 = R \cup S \tag{2}$$

The elements in N_1 can be represented by a single variable $r(r \in R)$ or $s(s \in S)$. The elements in N_2 can be represented by bivariate $(m, i)(m \in M, i \in I)$, where m denotes the traffic mode, i denotes the physical location of a node, and I is the set of physical locations.

According to the properties of connected nodes, the links in a supernetwork can be divided into four types: boarding link A_1, vehicle running link A_2, transfer link A_3 and alighting link A_4.

$$A = A_1 \cup A_2 \cup A_3 \cup A_4 \tag{3}$$

The link can be represented by its connected nodes. For example, the four types of links can be respectively expressed as $\{r, (m, i)\}$, $\{(m, i), (m, j)\}$, $\{(m, j), (n, j)\}$ and $\{(n, j), s\}$. Note that $m, n \in M$ and $i, j \in I$.

An example of a multimodal transportation supernetwork is shown in Figure 1.

In the supernetwork above, we describe a traveller's travel process between O-D pairs using a superpath, which is defined as follows.

Definition 1: the sequence of nodes connecting an O-D pair in the supernetwork is referred to as the superpath, and the set of superpaths between O-D pairs rs is denoted as K^{rs}.

For example, the two superpaths between r and s in the supernetwork shown in Figure 1 can be expressed as:

$$r \rightarrow (1, 1) \rightarrow (1, 2) \rightarrow (1, 4) \rightarrow (1, 6) \rightarrow s \tag{4}$$

$$r \rightarrow (1, 1) \rightarrow (1, 3) \rightarrow (1, 5) \rightarrow (2, 5) \rightarrow (2, 10) \rightarrow s \tag{5}$$

It can be seen that a superpath contains much more information than a path in a single-mode transport network. A superpath not only depicts the physical path taken by a vehicle within the subnets but also encompasses transfer-related information. For example, the superpath in Eq (4) represents a single-mode travel path, while the superpath described by Equation (5) is a combined travel path.

The travel behaviour on a superpath usually involves four processes: boarding subnet, vehicle running, transfer and alighting subnet. Such travel behaviour can also be categorized as either in-vehicle behaviour or out-of-vehicle behaviour. In a superpath, the traveller's in-vehicle behaviour occurs on the vehicle running links, while out-of-vehicle

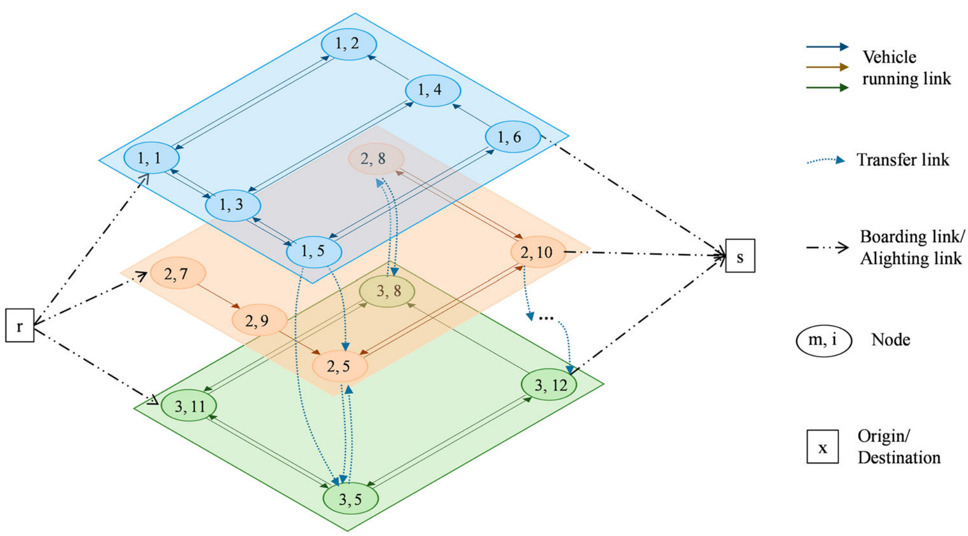


Figure 1. Supernetwork of multimodal transport system.

behaviour occurs on the other links. Apparently, a traveller completes the in-vehicle behaviour through the physical path in the subnet, after taking a vehicle to board a subnet and before transferring or alighting. To better describe these complex traveller behaviours, we define the subpath as follows:

Definition 2: In a superpath, the set of consecutive vehicle running links between a boarding link (or a transfer link) and an alighting link is referred to as the subpath of a superpath, and the set of subpaths of the superpath k between O-D pairs rs is denoted as S_k^{rs}.

Obviously, a superpath may contain several subpaths, especially in long-distance travel. For the superpath in Equation (5), one of its subpaths can be expressed as:

$$(1, 1) \rightarrow (1, 3) \rightarrow (1, 5) \tag{6}$$

2.3. Generalized travel costs

Travellers make choices based on generalized travel costs. The generalized costs of combined travel include link-specific costs and mode-specific costs. Figure 2 illustrates the superpath in Equation (5) and the composition of its generalized costs. We formulate the generalized costs of a superpath incorporating that of subpaths.

As shown in Figure 2, the generalized costs of a superpath equal the sum of the costs of the associated links and subpaths, which can be written as:

$$C_k^{rs} = \sum_{a \in A_1 \cup A_3 \cup A_4} c_a \delta_{a,k}^{rs} + \sum_{l^m \in S_k^{rs}} w_{l^m}^{rs} \quad r \in R, s \in S, m \in M, k \in K^{rs} \tag{7}$$

where C_k^{rs} is the generalized costs on superpath k between O-D pair rs; c_a is the generalized costs on the out-of-vehicle link a; $w_{l^m}^{rs}$ is the generalized costs on subpath l^m of the superpath k between O-D pair rs; $\delta_{a,k}^{rs}$ is the correlation coefficients between link a and superpath k of O-D pair rs, if the link a is on superpath k, $\delta_{a,k}^{rs} = 1$; otherwise, $\delta_{a,k}^{rs} = 0$.

As stated above, boarding, transfer and alighting links capture a traveller's out-of-vehicle behaviour, and the costs on these links are link-specific. Since the main cost on such links is walking time, to which travellers tend to attach higher importance (Arentze and Molin 2013), the walking time is amplified as the cost of these links.

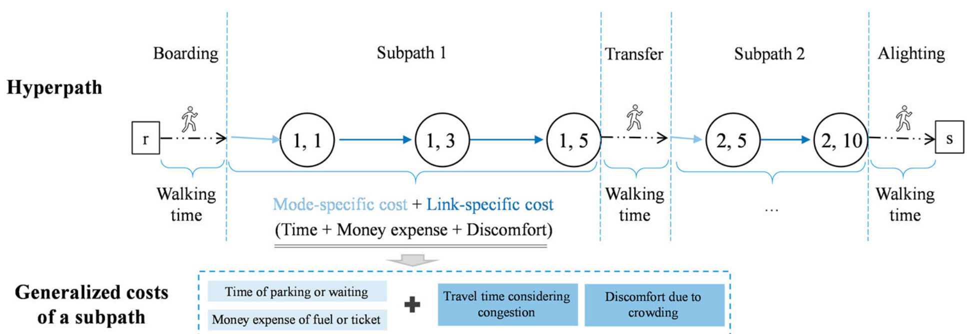


Figure 2. Illustration of a superpath and its generalized costs.

Assuming there is no walking congestion, travel times on these links are fixed and independent of link flows. Therefore, the generalized costs function applicable to boarding, transfer and alighting links can be expressed as follows:

$$c_a = \eta \cdot \frac{l_a}{v} \quad a \in A_1 \cup A_3 \cup A_4 \tag{8}$$

where l_a is the length of link a; v is the walking speed; η is an amplification parameter.

For a subpath, generalized costs consist of two parts: mode-specific costs and link-specific costs. The former mainly includes non-vehicle running time and pricing factors that influence a traveller's mode choice, such as parking time, waiting time, transit fares, and more factors. The latter refers to the vehicle running time along the subpath after the traveller boards a vehicle. This can be quantified by considering the travel time on the vehicle running links constituting the subpath. Consequently, the generalized costs on the subpath l^m of the superpath k between O-D pair rs can be written as:

$$w_{l^m}^{rs} = c_{l^m}^{rs} + \sum_{a \in A_2} t_{m,a} \cdot \delta_{a,l^m}^{rs} \quad r \in R, s \in S, m \in M, l^m \in S_k^{rs}, k \in K^{rs} \tag{9}$$

where $c_{l^m}^{rs}$ is the mode-specific cost on subpath l^m of superpath k between O-D pair rs; $t_{m,a}$ is the travel time on vehicle running link a in the subnet of mode m, $a \in A_2$; δ_{a,l^m}^{rs} is the correlation coefficient between vehicle running link a and subpath l^m of superpath k of O-D pair rs. If link a is on subpath l^m, $\delta_{a,l^m}^{rs} = 1$; otherwise, $\delta_{a,l^m}^{rs} = 0$.

The mode-specific costs consist of fixed time and length-based fares, such as fuel consumption and public transport fares (Li et al. 2018), which can be formulated as:

$$c_{l^m}^{rs} = t_m + \tau_m \cdot \rho_m \cdot d_{l^m}^{rs} \quad r \in R, s \in S, m \in M, l^m \in S_k^{rs}, k \in K^{rs} \tag{10}$$

where t_m is the time excluding vehicle running time of mode m, which is expressed as a constant; τ_m is a conversion coefficient from price to time; ρ_m is the fare per mileage of mode m; $d_{l^m}^{rs}$ is the length of subpath l^m.

Generally, travellers may perceive a longer in-vehicle time than the actual time, as they experience discomfort due to crowding (Lo, Yip, and Wan 2003). As a result, the travel time on an in-vehicle link comprises two components: the actual travel time and the additional time penalty due to in-vehicle discomfort. The actual travel time is primarily influenced by traffic flow on the link (Bureau of Public Roads 1964), while in-vehicle crowding arises when the number of travellers onboard reaches the vehicle's capacity (Si et al. 2020). In this paper, we propose a travel time function for the vehicle running link that considers in-vehicle crowding.

$$t_{m,a}(x_{m,a}) = t_{m,a}^0 \left[1 + \alpha_m \left(\frac{x_{m,a}}{K_m \cdot C_{m,a}}\right)^{\beta_m}\right]\left[1 + \phi_m \left(\frac{x_{m,a}}{K_m}\right)^{\varphi_m}\right] \quad m \in M, a \in A_2 \tag{11}$$

where $t_{m,a}^0$ and $x_{m,a}$ are respectively the free-flow travel time and the traveller volume of mode m on link a, $a \in A_2$; K_m is the vehicle capacity of mode m; $C_{m,a}$ is the traffic capacity of link a in subnet m; α_m, β_m, ϕ_m and φ_m are parameters related to mode m.

Note that in a multimodal supernetwork, travel times for various link types are set differently: boarding, alighting, and transfer links are assigned constant travel times, while travel times for vehicle running links are variable under the influence of link flow.

3. Relationship between O-D demands and link flows

As is well-known, the traffic distributed on the transport network is a macroscopic phenomenon resulting from the interaction between the travellers' choice and the service level of the transport system. Assignment models typically focus on travellers' behaviours such as mode choice and path choice. These models explain how link flows are determined under specific conditions, including O-D demands and network structure. Obviously, such a model can be used to establish the relationship between O-D demands and link flows.

Let $P^{rs}_{m,a}$ express the probability that demand q^{rs} between O-D pair rs travels through vehicle running link a in the subnet of mode m. Therefore, the relationship between O-D demands and link flows can be written as:

$$x_{m,a} = \sum_{s \in S} \sum_{r \in R} q^{rs} P^{rs}_{m,a} \quad m \in M,\ a \in A_2 \tag{12}$$

$\boldsymbol{x}$ and $\boldsymbol{q}$ denote the vector of link flows and O-D demands respectively, that is $\boldsymbol{x} = [\cdots, x_{m,a}, \cdots]^{\mathrm{T}}$, $\boldsymbol{q} = [\cdots, q^{rs}, \cdots]^{\mathrm{T}}$. Equation (12) can be rewritten as follows (Cascetta and Nguyen 1988; Yang et al. 1992):

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{q} \tag{13}$$

where $\boldsymbol{A}$ is a relationship matrix between O-D demands $\boldsymbol{q}$ and link flows $\boldsymbol{x}$, that is:

$$\boldsymbol{A} = \begin{bmatrix} P^{1}_{1,1} & \cdots & P^{rs}_{1,1} \\ \vdots & \vdots & \vdots \\ P^{1}_{m,a} & \cdots & P^{rs}_{m,a} \end{bmatrix} \tag{14}$$

Theorem: If the traffic assignment problem is expressed as the following logit-based stochastic user equilibrium (SUE) model (Daganzo 1982):

$$\min Z(\boldsymbol{t}) = \sum_{m} \sum_{a} \int_{t^0_{m,a}}^{t_{m,a}} x_{m,a}(w)dw - \sum_{r \in R} \sum_{s \in S} q^{rs} \cdot U^{rs}[\boldsymbol{C}^{rs}(\boldsymbol{t})] \tag{15}$$

where $x_{m,a}(w)$ is the inverse function of (11) and $U^{rs}[\boldsymbol{C}^{rs}(\boldsymbol{t})]$ is the satisfaction function of O-D pair rs. We assume that the traveller's choice probability follows the logit model, so the satisfaction function can be expressed as:

$$U^{rs}[\boldsymbol{C}^{rs}(\boldsymbol{t})] = -\frac{1}{\theta} \ln \sum_{k \in K^{rs}} \exp\left(-\theta C^{rs}_{k}\right) \tag{16}$$

where θ is the parameter of the Logit model, and $\theta \geq 0$.

Then, the probability that demand q^{rs} between O-D pair rs travels through vehicle running link a in the subnet of mode m, $P^{rs}_{m,a}$ can be written as:

$$P^{rs}_{m,a} = \frac{\sum_{k \in K^{rs}, l^m \in S^{rs}_k} \exp\left(-\theta C^{rs}_{k}\right) \delta^{rs}_{a,l^m}}{\sum_{p \in K^{rs}} \exp\left(-\theta C^{rs}_{p}\right)} \quad r \in R,\ s \in S,\ m \in M,\ a \in A_2 \tag{17}$$

Proof: For mathematical optimization model (15), the minimizing condition for the model is $\nabla Z(\boldsymbol{t}) = 0$. Take $\boldsymbol{t} = (t_{m,a})_{m\in M, a\in A_2}$ as its minimizing point, there is:

$$\frac{\partial Z}{\partial t_{m,a}} = x_{m,a}(t_{m,a}) - \sum_{r\in R}\sum_{s\in S} q^{rs} \sum_{k\in K^{rs}} \frac{\partial U^{rs}}{\partial C_k^{rs}} \frac{\partial C_k^{rs}}{\partial t_{m,a}} = 0 \tag{18}$$

From Equation (16), the following equation can be derived:

$$\frac{\partial U^{rs}}{\partial C_k^{rs}} = \frac{\exp(-\theta C_k^{rs})}{\sum_{p\in K^{rs}} \exp(-\theta C_p^{rs})} \quad r\in R, s\in S, k\in K^{rs} \tag{19}$$

According to Equations (7) and (9), there is:

$$\frac{\partial C_k^{rs}}{\partial t_{m,a}} = \sum_{l^m\in S_k^{rs}} \delta_{a,l^m}^{rs} \quad r\in R, s\in S, k\in K^{rs}, m\in M, a\in A_2 \tag{20}$$

Then the minimizing condition of model (15) can be shown as:

$$x_{m,a}(t_{m,a}) = \sum_{r\in R}\sum_{s\in S} q^{rs} \frac{\sum_{k\in K^{rs}, l^m\in S_k^{rs}} \exp(-\theta C_k^{rs})\delta_{a,l^m}^{rs}}{\sum_{p\in K^{rs}} \exp(-\theta C_p^{rs})} \quad m\in M, a\in A_2 \tag{21}$$

which expresses the link flows at the logit-based SUE state.

Because the flow on a link is the sum of the demands assigned to it from each O-D, the link flows can also be expressed as:

$$x_{m,a} = \sum_{r\in R}\sum_{s\in S} x_{m,a}^{rs} \quad m\in M, a\in A_2 \tag{22}$$

Equations (21) and (22) imply the relationship between O-D demands and link flows, that is:

$$P_{m,a}^{rs} = \frac{x_{m,a}^{rs}}{q^{rs}} = \frac{\sum_{k\in K^{rs}, l^m\in S_k^{rs}} \exp(-\theta C_k^{rs})\delta_{a,l^m}^{rs}}{\sum_{p\in K^{rs}} \exp(-\theta C_p^{rs})} \quad r\in R, s\in S, m\in M, a\in A_2 \tag{23}$$

where $P_{m,a}^{rs}$ is the flows contributed by O-D pair rs to link a in the subnet of mode m.

Note that we model this SUE problem based on the supernetwork described in subsection 2, so that the high-dimensional choice and multi-index travel costs are all characterized.

4. Model and solution algorithm for multimodal transport network capacity

Due to the availability of transportation capacity within each mode, there exists an upper limit on the total travel demand that the multimodal transport network infrastructure can accommodate. This upper limit on travel demand can be used to quantify network capacity (Yang, Bell, and Meng 2000). Since travel demand can be broken down into multiple O-D demands (those with the same origin and destination), the capacity of a multimodal transport network can be expressed as the maximum summation of O-D demands. That is

$$\max \sum_{s \in S} \sum_{r \in R} q^{rs} \tag{24}$$

When a network reaches its capacity, it cannot serve additional travel demand. In this state, the path between any O-D pair becomes unavailable, because at least one constituent vehicle running link can no longer accommodate additional traffic flow. Accordingly, the traffic flow on a vehicle running link does not exceed its capacity when the network is at capacity, that is:

$$x_{m,a} \leq K_m \cdot C_{m,a} \quad m \in M, a \in A_2 \tag{25}$$

Based on Equations (12) and (25) can be rewritten as:

$$\sum_{s \in S} \sum_{r \in R} q^{rs} P_{m,a}^{rs} \leq K_m \cdot C_{m,a} \quad m \in M, a \in A_2 \tag{26}$$

The capacity of a multimodal transport network can be calculated by the following programming model:

$$\max \sum_{s \in S} \sum_{r \in R} q^{rs} \tag{27}$$

$$\text{s.t.} \quad \sum_{s \in S} \sum_{r \in R} q^{rs} P_{m,a}^{rs} \leq K_m \cdot C_{m,a} \quad m \in M, a \in A_2 \tag{28}$$

where $P_{m,a}^{rs}$ is obtained by solving Equation (15).

The proposed model composed of (27) and (28) constitutes a bi-level programming model, as its constraints (Equation (28)) encompass an optimization problem (Equation (15)). The decision variables for the upper-level problem are O-D demands, while for the lower-level problem, they are link flows. The multimodal transport network is expected to accommodate as many O-D demands as possible. These O-D demands are distributed throughout the network, forming traffic flows on links. The link flows are checked to ensure that they meet capacity constraints.

Solving bi-level programming problems is challenging because of the complex relationship between upper and lower-level variables. The relationship can be expressed as the response function of lower-level variables to upper-level variables. To simplify the model, we approximate the reaction functions by solving the logit-based SUE problem. On this base, AIA is proposed to solve the proposed capacity model. The main idea of the AIA method is as follows: (1) for the given O-D demands, matrix $\boldsymbol{A}$ can be obtained by solving the lower problem; (2) according to $\boldsymbol{A}$, the new $\boldsymbol{q}$ can be obtained by solving a monolayer linear programming problem; (3) based on the new O-D demand, the lower SUE problem can be solved and the new matrix $\boldsymbol{A}$ can be obtained again. The above process is repeated until a stable objective value is reached.

The detailed procedures of the AIA method are as follows:

Approximate iteration algorithm	
Step 1.	Let $j = 1$, and set the initial O-D demands $\boldsymbol{q}^{(j)}$.
Step 2.	Solve the logit-based equilibrium problem (15) by MSWA.
Step 3.	Calculate the contribution of O-D demands to links $P^{rs(j)}_{m,a}(r \in R, s \in S, m \in M, a \in A_2)$ by (23).
Step 4.	Solve model (27) $\sim$ (28), and obtain a new $\boldsymbol{q}^{(j+1)}$.
Step 5.	Check whether $\max_{r\in R, s\in S} \lvert q^{rs}_t - q^{rs}_{t+1} \rvert \le \varepsilon_{AIA}$; if yes, stop; otherwise, set $j = j + 1$, and go to step 2.

The following Dial-based MSWA (Liu et al. 2009) can be adopted to solve the logit-based equilibrium problem expressed in Equation (15). Since the Dial algorithm (Dial 1971) directly assigns O-D demands to links, the links' flow from each O-D can be easily recorded. In addition, the Dial algorithm avoids path enumeration, which contributes to computational time savings, particularly in the context of large-scale networks. The detailed procedure of Dial-based MSWA is given as follows.

The Dial-based MSWA.	
Step 1.	Initialization. Let $n = 0$, $x^{(n)}_{m,a} = 0$.
Step 2.	Update links costs. Obtain $t^{(n)}_{m,a}$ at flow $x^{(n)}_{m,a}$ according to Equation (11).
Step 3.	For an O-D pair rs, $(r \in R, s \in S)$, perform the Dial algorithm to get $y^{rs(n)}_{m,a}$.
Step 4.	Accumulate the flows from all O-D pairs to a link and then obtain its flow $y^{(n)}_{m,a}$.
Step 5.	Update the link flows according to $\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n)} + \dfrac{n}{(1+2+3+\cdots+n)}(\boldsymbol{x}^{(n)} - \boldsymbol{y}^{(n)})$.
Step 6.	Check whether $\max_{m\in M, a\in A_2} \lvert x^{(n+1)}_{m,a} - x^{(n)}_{m,a} \rvert \le \varepsilon_{MSWA}$ is met; if yes, stop; otherwise, let $n = n + 1$, and go to Step 2.

Note that the shortest path search in supernetworks differs from that in traditional networks. The shortest superpath is searched according to link costs, and we account for mode-specific travel costs in multimodal networks. However, mode-specific costs are independent of any specific link, which brings challenges to path searches based on link costs. The proposed supernetwork shows two important characteristics: (1) mode-specific costs are involved only when a traveller starts a subpath, and (2) a subpath can be started only through a boarding link or a transfer link. These characteristics enable us to associate mode-specific costs with boarding or transfer links for the same mode. Then the Dijkstra can be applied in supernetworks.

5. Numerical experiments

This section presents computational experiments based on two numerical cases to illustrate the proposed methodology. Detailed network attributes and parameter values are given in Appendix A. All experiments are done using the Python 3.7 programming tool on the Microsoft Windows 10 operating system with a 2.10 GHz CPU and 16 GB RAM. Three aspects are investigated:

(1) Algorithm efficiency (simple network case and Sioux Falls (SF) network case). This group of experiments compares the performance of the AIA, sensitivity analysis-based algorithm (SAB) (Ying and Miyagi 2001) (see Appendix B) and genetic algorithm (GA), to illustrate the merits of the proposed AIA.

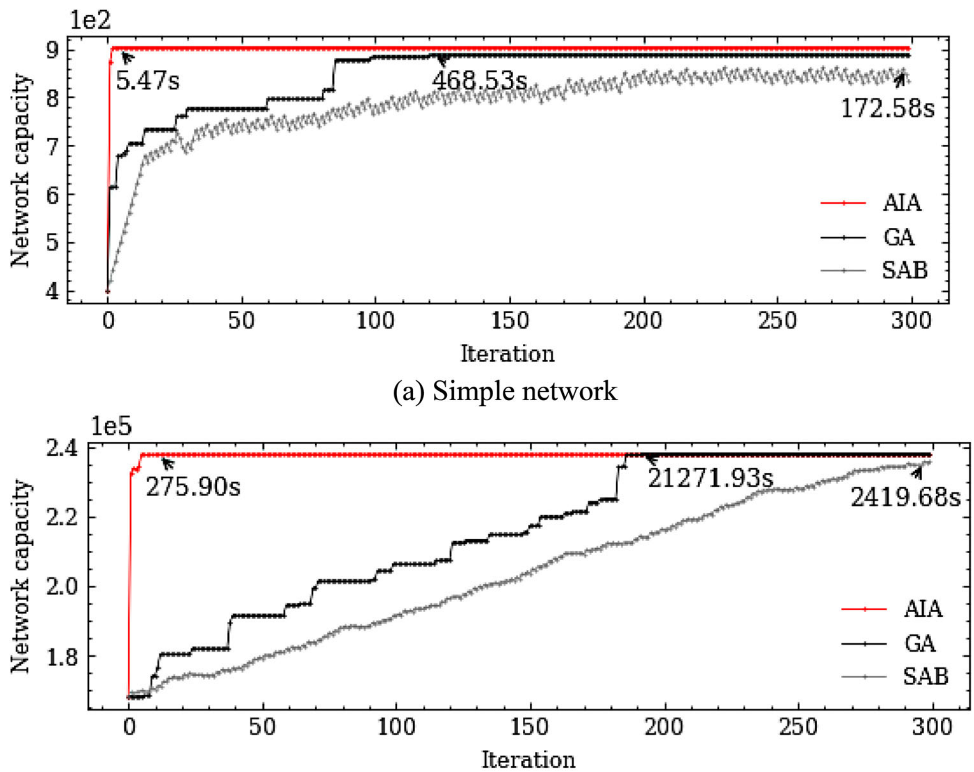
(2) Combined travel versus no combined travel (SF network case). We compare the capacity of the large-scale network with and without considering combined travel. Further analysis of travel costs is carried out to clarify the impact of combined travel on travellers.
(3) Capacity under changes in parameters (SF network case). Network capacity under changes in transfer cost and public transport frequency are calculated. The travel cost and share of public transport modes are analyzed.

5.1. Algorithm efficiency

We evaluated the performance of AIA, GA, and SAB using both the simple network and the SF network. The results are presented in Figure 3 and Table 2, with detailed information available in Appendix C.

Figure 3 illustrates the iterative process of the three algorithms, with their convergence points and corresponding time marked with black arrows. In the case of the simple network, as shown in (a), AIA converges in just 5.47 s, outperforming both GA and SAB. For the SF network, as depicted in (b), AIA reaches the convergence point in 169.62 s, yielding a superior value compared to SAB, but lower than GA.

As shown in Table 3, in the case of the simple network, AIA achieves a capacity value that is 4.87% higher than SAB and 1.69% higher than GA. However, this advantage



(a) Simple network

(b) SF network

Figure 3. Convergence trend under different solution methods.

Table 3. Comparison of solution methods performance.

	Network			
	Simple network		SF network	
Algorithm	CPU times (s)	Capacity	CPU times (s)	Capacity
AIA	5.47	903	160.62	237867
GA	468.53	889	21271.93	238091
SAB	172.58	862	2419.68	235714

diminishes in the SF network, where AIA capacity value is only 0.91% higher than SAB but falls 0.09% lower than GA. Notably, AIA shows significant advantages in terms of computation time for both network sizes presented in the case study.

The shorter computation time of AIA is attributed to its approximation mechanism by approximating the relationship between variables using equilibrium conditions. Compared to GA, this approximation mechanism solves an approximate linear programming problem at each iteration, yielding a certain optimization direction, as opposed to relying on GA evolutionary rules. Additionally, AIA only needs to solve the assignment problem once per iteration, whereas parallel optimization mechanism in GA requires solving the problem for the size of the population, which is more time-consuming. Although SAB is also based on an approximation mechanism, AIA directly approximates the relationship between O-D demands and link flows according to the equilibrium assignment. SAB utilizes Taylor expansion for approximation, which is time-consuming because of sensitivity analysis of the lower-level equilibrium problem. It needs to be noted that the accuracy of AIA is highly dependent on the characteristics of the assignment problem. When the assignment problem involves scenarios of high complexity, the AIA algorithm may no longer be applicable. For example, link costs are simultaneously influenced by their own flow and the flow of other links.

In the SF case, the optimal value obtained from AIA is inferior to that of GA, as AIA is a heuristic algorithm that can only yield a Nash-Cournot solution (Friesz and Harker 1985), not the same as the solution of the original bi-level programming model.

5.2. Combined travel versus no combined travel

To investigate if the combined travel matters for the capacity of multimodal transport networks, we execute this group of experiments. Without loss of generality, we evaluate the network capacity under different logit parameters. The network capacity without combined travel is obtained based on the SF supernetwork with no transfer links.

As shown in Figure 4(a), comparison of the capacity under the same logit parameters indicates that the transport network capacity will be underestimated if combined travel is not considered. This is because combined travel provides options for additional travel demands. To further explore the impact of combined travel on travellers when the network is at capacity, we compare the average generalized costs of the shortest path (ASP) with and without considering the combined travel. ASP is the average value of the shortest paths of all O-Ds in the network. Figure 4(b) shows that combined travel can reduce the costs of individuals when the transport network is at capacity.

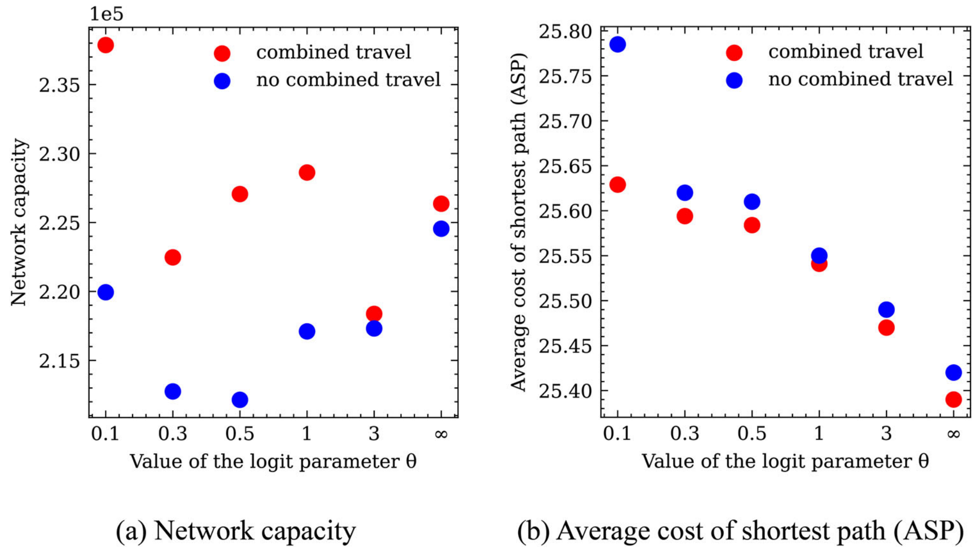


Figure 4. Combined travel versus no combined travel.

5.3. Capacity under changes in parameters

Transfer cost and public transport frequency are important factors affecting combination travel in multimodal transport networks (Liao et al. 2020), and thus these factors impact the capacity of multimodal transport networks as well. The impact by the SF network is tested by using the AIA algorithm, with the results shown in Figures 5 and 6.

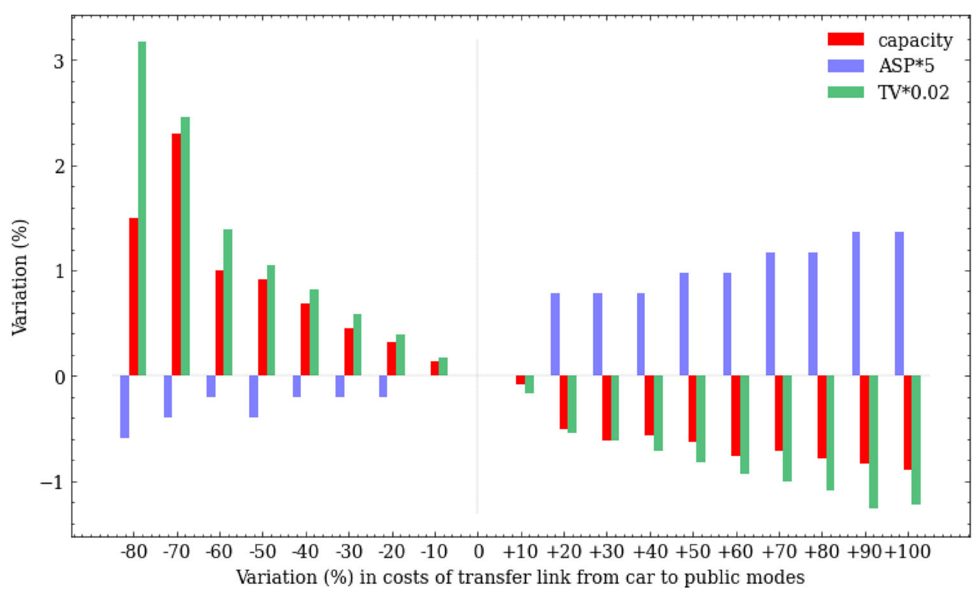


Figure 5. Capacity under changes in transfer cost.

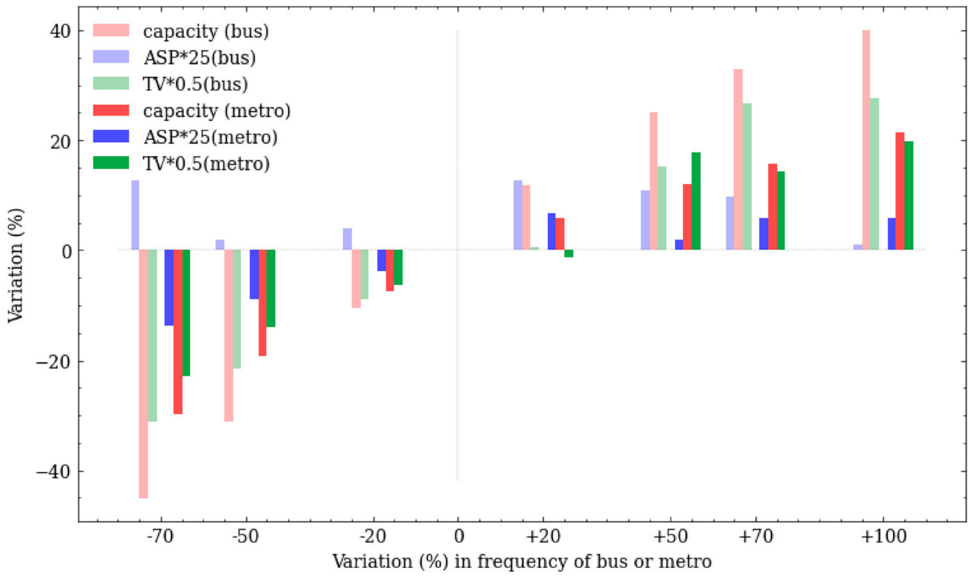


Figure 6. Capacity under changes in the frequency of bus or metro.

The ASP is used to measure individual travel costs. The transfer volume (TV) is used to refer to the sum of volume from private cars to public transit (bus and metro). A large TV indicates a high share of public transport modes.

As shown in Figure 5, red bars indicate that the capacity of example 2 deteriorates with increased transfer cost. Green bars show that reducing transfer costs promotes the increase of public transport share, but blue bars indicate that travel costs may increase at the same time. However, there is not a strict correlation. In this SF network example, the 70% reduction in transfer cost might be used as a reference for network design, as this is the point at which the network capacity reaches the maximum.

As shown in Figure 6, red bars indicate that the capacity improves significantly with the increased frequency of public transport. Comparing the light red and dark red bars, it is found that capacity is more sensitive to variations in the bus frequency than to that of the metro. Because there are more bus stops than metro stations in SF, the bus subnetwork provides more options for travel volume than the metro subnetwork. The blue bars demonstrate that increasing frequency does not always reduce travel costs. The reason is that higher frequency promotes more demand and increases the cost caused by congestion. Further comparison of the light blue bars with the dark blue bars reveals that increasing metro frequency reduces more travel costs than increasing bus frequency. This is because a metro has a larger vehicle capacity than a bus, which helps to reduce the cost caused by in-vehicle crowding. The green bars show that the share of public transport increases with the frequency of buses and metro, but the trend is no longer significant after it reaches a certain value (in this case, zero). This implies that there may be an optimal frequency for a high share of public transport.

6. Discussion

6.1. Shortcomings of the proposed model

In the proposed model, deviations from real-world scenarios may arise due to several assumptions. For instance, it neglects flow interactions between various transportation modes (e.g. cars and buses sharing the same road link) by assuming that these modes operate in separate networks. Furthermore, it assumes that all travellers make choices by following the logit model and walk at a constant speed, with traveller heterogeneity neglected. Additionally, there are no constraints on travel generation and attraction, real-world urban land use is not taken into account.

Further research is needed to define and quantify the capacity of transport network. For instance, it is imperative to account for the distinction between long-distance and short-distance trips, as they consume varying levels of transportation resources. The inclusion of trip distance as a factor in network capacity calculations remains an area of ongoing exploration.

These assumptions and considerations provide a foundation for future research to improve the model's applicability and adaptability in various urban settings.

6.2. AIA and iterative estimation-assignment algorithm

The basic idea used in AIA is fundamentally the same as the existing estimation-assignment algorithm (Hall and Willumsen 1980). Both algorithms use the ratio of O-D demands and link flow to estimate changes in the assignment. However, there are two differences:

(1) The O-D demands (see algorithm: AIA) is taken as the initial input in our method instead of ratio matrix in other research (Yang, Bell, and Meng 2000; Yang et al. 1992). Since O-D demands are the focus in our model, the ratio matrix is just an intermediate variable.
(2) The dial algorithm-based assignment (see algorithm: The Dial-based MSA.) is applied to calculating the ratio matrix. Since the link-based assignment solution is unique, the choice of multiple path solutions is avoided (Yang et al. 1992).

In summary, these differences are tailored to the specific characteristics of our model, allowing a more effective solution of the proposed model.

7. Conclusions

This paper presents an approach to evaluate the capacity of multimodal transport networks while taking combined travel into account. Combined travel is expressed in the logit-based SUE model of a multimodal supernetwork, and this model is used to obtain constraints for the capacity model. Numerical examples demonstrate that the model is effective and the proposed AIA method performs best among existing algorithms.

This method outputs information on O-D pairs (structure and demand) and links utilization when the network reaches capacity. The former provides a reference for urban land use planning, and the latter can be used to identify bottleneck links. For future research, we will explore the practicality of this method on a real multimodal transport network with more modes (such as bicycles and emerging transportation modes) incorporated. We will also discuss optimal network design strategies for maximizing network capacity.

Disclosure statement

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