Fast Pulsing and Chaotic Itinerancy with a Drift in the Coherence Collapse of Semiconductor Lasers

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We report the first experimental observation of irregular picosecond light pulses within the coherence collapse of a semiconductor laser subject to delayed moderate optical feedback. This pulsing behavior agrees with the recent explanation of low frequency fluctuations as chaotic itinerancy with a drift. Theory and experiments show very good agreement.

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Delayed feedback-induced instabilities have been studied since the late 1970s in various dynamical systems. Besides these very early interests they are nowadays of particular interest because of their intrinsic high dimensionality and, related to that, their rich variety of dynamical phenomena [1]. Optical systems have played an important role for these investigations, and have boosted the interest in high-dimensional nonlinear dynamics [2–4].

A very popular delay system is the semiconductor laser subject to external optical feedback, because of its high sensitivity to external signals [5]. Semiconductor lasers show a sudden increase in their spectral linewidth from about 100 MHz to typically several tens of GHz for delay times not much smaller than the relaxation oscillation period and for moderate feedback levels. This phenomenon has been called coherence collapse [6], and has attracted a lot of research (e.g., [7–11]).

One dynamical phenomenon within the coherence collapse regime frequently attributed to is the so-called low frequency fluctuations phenomenon (LFF). It refers to fluctuations in the emitted light intensity with distinctly lower frequencies in comparison to the underlying relaxation oscillation frequencies and mode beating frequencies [12–14]. Recently, LFF has been explained as chaotic itinerancy with a drift [15]. This theory predicts erratic picosecond pulsing of the output power. In this paper we give experimental evidence confirming these predictions. We do this by comparing measured intensity pulses with pulse trains obtained by numerical modeling. The very good agreement supports the recent identification of the essential physical mechanism leading to the LFF behavior.

The dynamics of a single-mode semiconductor laser subject to moderate amounts of optical feedback from an external reflection is modeled by the following delay-differential rate equations [5]:

$$\dot{E}(t) = \frac{1}{2}(1 + i\alpha)\frac{\xi n(t)E(t)}{1 + \epsilon P(t)} + \gamma E(t-\tau)e^{-i\omega_0\tau}, \quad (1a)$$

$$\dot{n}(t) = (p-1)J_{\mathrm{th}} - \frac{n(t)}{T_1} - \left(\Gamma_0 + \frac{\xi n(t)}{1 + \epsilon P(t)}\right)P(t). \quad (1b)$$

Here we have written the complex optical field as $\mathcal{E}(t) = E(t)\exp(i\omega_0 t) = \sqrt{P(t)}\exp(i\omega_0 t)$, where ω_0 is the optical angular frequency of the stand-alone (solitary) laser, $P(t)$ is the photon number, and $n(t)$ is the excess number of electron-hole pairs with respect to the solitary value N_0. The parameters in (1a) and (1b) have their usual meaning: ξ is the bulk differential gain, ϵ accounts for gain saturation, α is the linewidth enhancement factor, Γ_0 is the inverse photon lifetime, pJ_{th} is the electrical pump current (J_{th} is its value at threshold), and T_1 is the electron-hole pair lifetime. The feedback is accounted for via the delay time τ and the feedback rate γ. The dimensionless effective feedback strength is defined as $C \equiv \gamma\tau\sqrt{1+\alpha^2}$.

The key to understanding the dynamics of this system is the interference between the laser field and the delayed field (returning from the external cavity). If the laser is replaced by a passive resonator, the resulting eigenfrequencies will have constant spacing and show alternating constructive (even multiples of π) and destructive (odd multiples of π) interference. Because semiconductor lasers exhibit gain as well as phase-amplitude coupling (α parameter) the fixed points, i.e., the eigenmodes, have more complicated properties. From Eqs. (1a) and (1b) it is found that each fixed point is defined by a monochromatic field and a constant carrier number. Each of them denotes a specific combination of phase matching between the phase $\varphi(t)$ and the phase $\varphi(t-\tau)$ (leading to a frequency $\omega_0 + \Delta\omega_s$), the associated carrier number n_s, and the resulting photon number P_s. In view of the important role of the phase matching, the proper dynamical variables are the round-trip phase difference $\eta(t) \equiv \varphi(t) - \varphi(t-\tau)$, $P(t)$, and $n(t)$. The fixed points lie on an ellipse in the $(\Delta\omega, n)$ plane and their loci are defined by [15]

$$\Delta\omega_s\tau = -C\sin[(\omega_0 + \Delta\omega_s)\tau + \arctan\alpha], \quad (2a)$$

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$$(\gamma\tau)^2 = \left(\Delta\omega_s\tau - \frac{\alpha\tau}{2}\xi n_s\right)^2 + \left(\frac{\tau}{2}\xi n_s\right)^2. \quad (2b)$$

In Fig. 1(a) we show such an ellipse. The filled diamond at zero frequency shift denotes the solitary laser state. The upper half of the fixed points on the ellipse, denoted by stars, all exhibit a saddle-node instability and can be regarded as the destructive interference solutions in the passive case. They are often called antimodes [14]. The other fixed points, denoted by open circles, can be regarded as the constructive interference solutions in the passive case. These are called compound cavity modes and are possibly subject to Hopf instabilities, indicating the birth of sustained relaxation oscillations [15].

From Fig. 1(a) we make the following two observations: First, note that the fixed point with maximum gain (most negative value n_s) also has the maximum frequency shift with respect to the solitary laser state. This means that the solitary laser, confronted with delayed optical feedback, will have to change its frequency considerably to profit maximally from the feedback. The fact that the maximum gain mode is not closest to the solitary laser state is caused by the α parameter. Second, the frequency spacing between the fixed points changes when we approach the maximum gain mode: Around the solitary laser frequency, i.e., around $\Delta\omega_s = 0$, the antimodes are located almost symmetrically in between the compound-cavity modes with respect to frequency. When the frequency shifts get larger, the antimodes move closer to the compound-cavity modes.

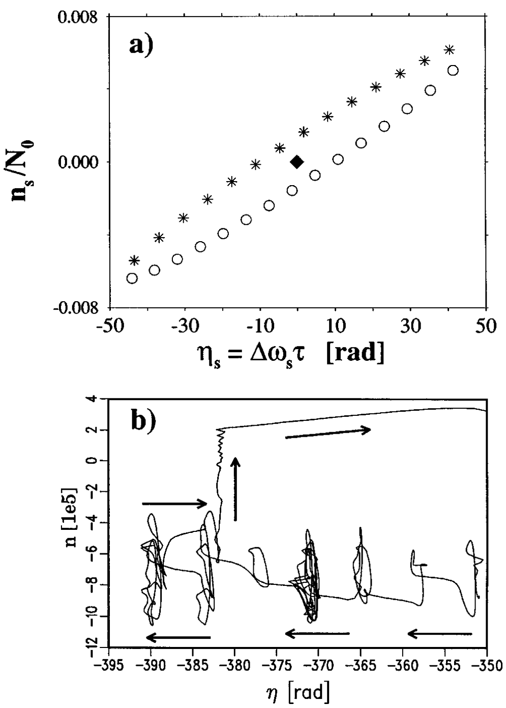

FIG. 1. (a) An ellipse of fixed points. Here we used feedback strength $C = 46$ so that the number of fixed points is not too large and the basic features of the ellipse can be recognized. $p = 1.5$ and $\alpha = 5$. (b) Calculated signature of LFF. Plotted is the trajectory in inversion versus phase-difference space. The arrows indicate the sequence of events. After visiting attractor ruins of the compound-cavity modes a fatal encounter with an antimode around $\eta = -382$ rad causes a sharp increase in inversion (power dropout). $C = 510$, $p = 1.02$, $\alpha = 5$.

Changing the feedback strength (or the pump strength) can give rise to excited relaxation oscillations, bringing the system into coherence collapse [6,9,10]. The reported routes to chaos in the coherence collapse [7,8] are governed by the interplay between the compound cavity modes and their excited relaxation oscillations.

Sano [14] was the first to recognize the role of the antimodes in LFF and proposed that LFF is a form of chaotic itinerancy [3] among the attractor ruins of the destabilized compound cavity modes, where each dropout is associated with the trajectory approaching too close to one of the (many) antimodes. It was shown by Van Tartwijk, Levine, and Lenstra [15] that on each attractor ruin the power is organized in irregular pulses. They also reported the necessity of these erratic power pulsations: Only in the "dark" periods between pulses (with a width of a small fraction of the delay time) can the system switch to a neighboring attractor ruin. Because of the amplitude-phase coupling, this visiting of attractor ruins shows a definite direction towards maximum gain. In Fig. 1(b) we show an example of this signature of LFF. The system visits attractor ruins on its way to maximum gain, interrupted by a fatal encounter with an antimode. The attractor ruins of the Hopf-type fixed points are the "scars" in Fig. 1(b). Clearly, the η value of the fixed points can still be recognized. The carrier number, however, is changing rapidly on the attractor ruins, in accordance with the erratic pulsing.

In traditional chaotic itinerancy the switching between the attractor ruins is random in direction. Here we see something new—chaotic itinerancy with a drift [15]. By organizing its power in pulses the system has found a way to obey both the feedback induced round-trip phase condition and the maximum gain drive.

In the following we present for the first time experimental results demonstrating the erratic pulsing of the light intensity in this scenario.

The experiments have been performed using a GaAs/GaAlAs laser diode (HLP 1400) emitting light at $\lambda = 835$ nm. This type of laser diode with its parameters and dynamical properties is the most studied in experiments on external optical feedback. In fact, most of the model calculations and experimental investigations concerning the coherence collapse and, in particular, the LFF regime have been done for this type of laser diode. Figure 2 shows the experimental setup of the laser with feedback. On the right hand side of the laser diode (LD) we have depicted the external cavity. It is realized by a microscope objective (MO) which collimates the beam and a high reflecting gold mirror. The delay time of the external cavity is chosen to be 3.6 ns. Additionally, a

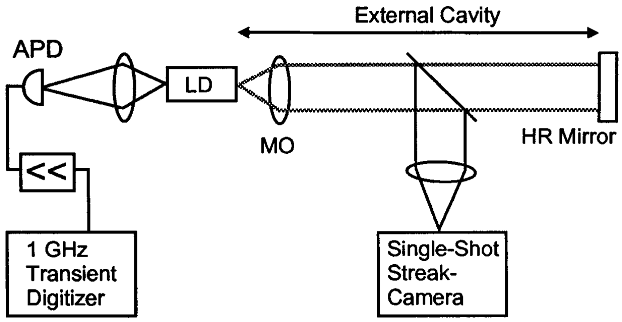

FIG. 2. Experimental setup.

50:50 beam splitter within the cavity is used to couple half of the intensity of the cavity to a single-shot streak camera in order to study the intensity dynamics on a sub-ns time scale. Therefore, we focus the light on the input slit of a Hamamatsu C1587 streak camera with a M1952 single-shot unit. This streak camera enables us to obtain time traces with a temporal window between 1 and 6.6 ns with a temporal resolution of $\sim$20 ps.

On the left hand side of the laser diode we use the emitted light for further signal detection. The time averaged laser intensity is measured by a slow *p-i-n* photodiode. The LFF fluctuations of the laser intensity on a ns time scale are measured by a fast avalanche photodiode. Its electrical output is amplified and analyzed in the time domain by a fast digital signal analyzer (bandwidth 1 GHz) and in the frequency domain by an electrical spectrum analyzer. Therefore, our setup allows us to register simultaneously the fast intensity dynamics using the single-shot streak camera as well as the LFF behavior on longer time scales. Figure 3(a) shows a time series obtained by the fast photodiode and the digital signal analyzer for an injection current of $I = 51.0$ mA. This is 1.03 times the threshold current of the laser diode without feedback. We recognize the well-known LFF-type behavior with fast intensity breakdowns within less than 10 ns and a slower relaxation $T_{\mathrm{relax}} \sim 40$ ns corresponding to about 10 cavity round trips. The breakdown events which appear irregularly in time have typical temporal spacings in between 50 ns and 1 μs depending on the injection current. Up to now these time series have often been used for the investigation of LFF, e.g., a type-II intermittency scenario has been deduced from the LFF distribution and the scaling with the injection current [13]. However, these signals are obtained by low-pass filtering the signal with a cutoff frequency of 1 GHz, hiding any faster dynamics. The dynamical time scales of the laser are associated with the relaxation oscillation frequency of typically a few GHz. Therefore, we have performed single-shot streak camera measurements which provide the appropriate time resolution to detect even the fast intensity dynamics of the laser.

Figure 3(b) shows a 6.6 ns time trace of the power for the same operation conditions as in Fig. 3(a), the noise floor being subtracted. The streak camera measurements show that main features of the LFF dynamics remain hidden in conventional measurements. An irregular pulsing behavior of the laser diode is observed as has been predicted by theory [15]. The relative modulation depth of the pulses is nearly 100%, the width of the pulses $\Delta T_p \sim 50-200$ ps and the distance of two pulses $\Delta t_{p\text{-}p} = 200-1000$ ps . Even in the presence of shot noise, which is estimated to be less than 10%, we can clearly recognize the pulses which are irregular, in both intensity and their temporal position. In between two pulses the intensity of the laser is nearly zero. The intensity of the single pulses varies strongly for different single shots which we interpret to be due to the temporal position within the LFF. This pulsing is observed for the whole LFF regime and even for higher currents where the power dropouts are not observable anymore.

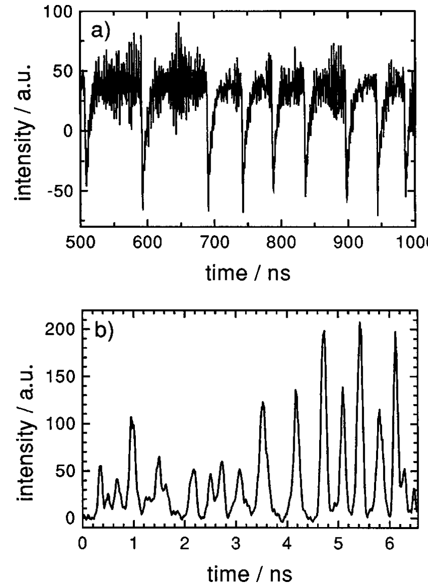

FIG. 3. Experimental time traces: (a) photodiode signal and (b) streak camera.

In order to compare the experimental results quantitatively to those resulting from the modeling, we have solved Eqs. (1a) and (1b) using a sixth-order Runge-Kutta method. We also included nonlinear gain, which in many cases appears to be indispensable for realistic results, since it can considerably postpone the birth of sustained relaxation oscillations [16]. We have found, however, that in the LFF regime nonlinear gain does not make a significant quantitative difference, since most relaxation oscillations are already heavily excited. The parameters used in the simulations are standard for the HLP1400 laser [16]: $\alpha = 4.5$, $T_1 = 1.1$ ns, $\Gamma_0 = 86$ ps^{-1}, $\xi = 2.556 \times 10^4$ s^{-1}, $\epsilon = 1.3 \times 10^{-7}$, and $J_{\mathrm{th}} = 49.5$ mA. From the feedback-induced threshold reduction ($\approx$6%), the feedback rate is estimated to be $\gamma \approx 50$ ns^{-1}. Including spontaneous emission noise does not change the main features of these simulations.

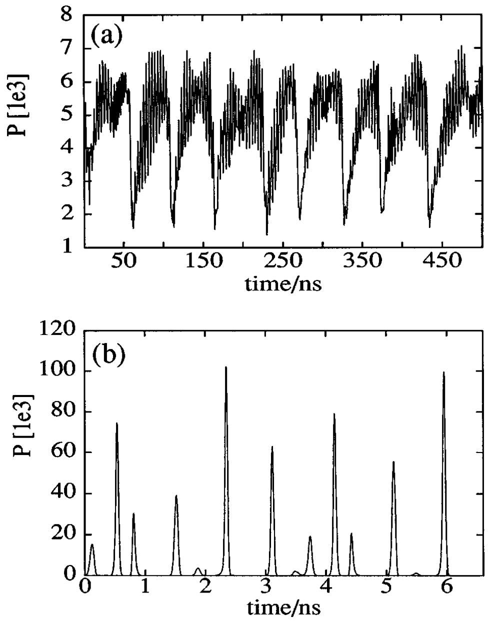

FIG. 4. Numerical simulation results: (a) averaged power and (b) the actual power organized in irregular pulses.

Figure 4 shows the temporal behavior of the emitted power for a pump current of 51 mA. To simulate the detection bandwidth of 1 GHz, we averaged the actual power over 1 ns, as shown in Fig. 4(a). When only averaged over 20 ps, i.e., the time resolution of our streak camera measurements, the power is behaving much more erratically, as can be seen in Fig. 4(b). The temporal behavior for both types of averaging agrees well with the experimental results shown in Fig. 3(b). Typical theoretical pulse widths are of the order of 74 ps, and the time between two pulses is about 100–1000 ps.

In conclusion, we have reported the first observation of irregular picosecond light pulses in the coherence collapse of a semiconductor laser underlying the low frequency fluctuations phenomenon. The pulsing behavior is a direct consequence of chaotic itinerancy with a drift, which is a manifestation of high-dimensional chaos in delayed feedback systems. In between two pulses the system may switch from one attractor ruin of a destabilized compound cavity mode to a next one, with a preferred direction towards maximum gain. These results show both the deterministic mechanism of the delay dynamics of semiconductor lasers within the coherence collapse and the possible complex phenomena in multiattractor systems and delay systems in general.

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[1] K. Ikeda and K. Matsumoto, Physica (Amsterdam) **29D**, 223 (1987).
[2] K. Ikeda, K. Otsuka, and K. Matsumoto, Prog. Theor. Phys. Suppl. **99**, 295 (1989).
[3] K. Otsuka, Phys. Rev. Lett. **65**, 329 (1990).
[4] I. Fischer, O. Hess, W. Elsässer, and E. O. Göbel, Phys. Rev. Lett. **73**, 2188 (1994).
[5] R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **16**, 347 (1980).
[6] D. Lenstra, B. H. Verbeek, and A. J. den Boef, IEEE J. Quantum Electron. **21**, 674 (1985).
[7] T. Mukai and K. Otsuka, Phys. Rev. Lett. **55**, 1711 (1985).
[8] J. Mørk, J. Mark, and B. Tromborg, Phys. Rev. Lett. **65**, 1999 (1990).
[9] G. C. Dente, P. S. Durkin, K. A. Wilson, and C. E. Moeller, IEEE J. Quantum Electron. **24**, 2441 (1988).
[10] J. Mørk, B. Tromborg, and J. Mark, IEEE J. Quantum Electron. **28**, 93 (1992).
[11] H. Li, J. Ye, and J. G. McInerney, IEEE J. Quantum Electron. **29**, 2421 (1993).
[12] J. Mørk, B. Tromborg, and P. L. Christiansen, IEEE J. Quantum Electron. **24**, 123 (1988).
[13] J. Sacher, W. Elsässer, and E. O. Göbel, Phys. Rev. Lett. **63**, 2224 (1989).
[14] T. Sano, Phys. Rev. A **50**, 2719 (1994).
[15] G. H. M. van Tartwijk, A. M. Levine, and D. Lenstra, IEEE Sel. Top. Quantum Electron. **1**, 466 (1995).
[16] M. P. van Exter, W. A. Hamel, J. P. Woerdman, and B. R. P. Zeijlmans, IEEE J. Quantum Electron. **28**, 1470 (1992).