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Estimation of delay times from a delayed optical feedback laser experiment

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Abstract. – We estimate delay times from high-dimensional chaotic time series experimentally obtained from a fast optical time-delayed feedback system. The experiment consists of a semiconductor laser, where the instabilities are induced by an external T-shaped cavity introducing two delay times into the laser. The delay times are determined by a filling factor analysis and found to give a better estimate than those obtained by autocorrelation functions. Finally, the possibility of this method for the reconstruction of the system's differential equations is discussed.

The characterization of nonlinear dynamical behavior and the identification of the underlying deterministic time-evolution laws from experimental time series has turned out to be one of the key problems in the study of nonlinear dynamical systems. For dynamical systems with a low number of degrees of freedom, embedding techniques [1] have been exceptionally successful for the computation of chaotic indicators (dimensions, Lyapunov exponents, entropies), and for the modelling of these systems (for an overview see, *e.g.* [2]). However, severe problems arise for systems which exhibit a number of dynamical degrees of freedom distinctly larger than $N \sim 5$. A prominent class of dynamical systems that can have a large number of dynamical degrees of freedom are nonlinear systems with a time-delayed feedback. The study of such systems with time-delayed feedback were initiated by Ikeda *et al.* [3]. The *Ikeda scenario* turned out to be a paradigm for the dynamical behavior of delayed-feedback systems under the variation of control parameters. The key features of the Ikeda scenario are the occurrence of multistability of periodic or chaotic attractors and the onset of high-dimensional deterministic chaos via attractor merging. Recently, a semiconductor laser system has been

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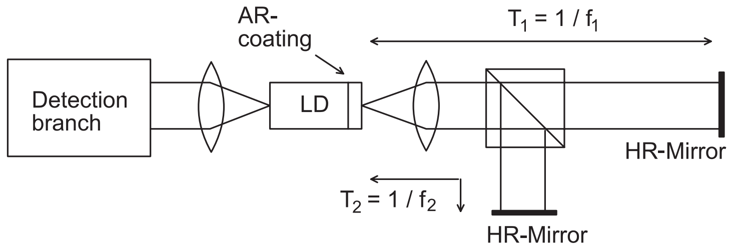

Fig. 1. – Experimental setup: semiconductor laser coupled to a T-shaped external cavity. The T-shaped cavity consists of two cavity arms introducing two delay times T_1 and T_2 with $T_1 = 2T_2$.

demonstrated to experimentally show the characteristic phenomena of this scenario. The low- and intermediate-dimension regime of the time series could be characterized via an improved correlation dimension calculation method [4]. However, the characterization and modelling of high-dimensional states ($N > 5$) with the help of embedding techniques is a fundamental problem due to the requirements on the amount of data and its precision. Therefore, in the light of embedding techniques, the high-dimensional dynamics observed in the experimental laser system could be considered as nondeterministic.

Recently, a novel technique for the identification of time-delay systems has been introduced [5]-[7]. Distinct progress has been achieved along these lines with the help of the ACE-algorithm, allowing for the first time the identification of multiple delay times [8], as well as with the help of embedding techniques in order to identify nonscalar time-delay systems from a scalar measurement only [9]. The technique relies on the basis that an N-dimensional time-delay system can be identified by analyzing an N-dimensional time series. This identification can also be accomplished in the case of high-dimensional chaotic as well as transient motion. The method allows the determination of the delay time and the estimation of the time-delay differential equation from experimental time series. So far, the method has been only applied to time series obtained via the numerical integration of a time-delayed differential equation and to time series taken from an electronic oscillator. In both cases the underlying time-delayed differential equations were known *a priori*. In this paper, this method is used for the first time to estimate delay times from the time-series of an experiment, in particular a semiconductor laser in an external cavity, where the governing delay rate equations are of a less simple form.

The setup of the experiment exhibiting the delay-induced, high-dimensional chaotic dynamics is depicted in fig. 1. We have employed a Fabry-Pérot–type, edge-emitting semiconductor laser based on the GaAs / AlGaAs material system (HLP1400) emitting at $\lambda = 830$ nm. The facet facing the external cavity is antireflection (AR) coated with a residual reflectivity of $R_\mathrm{r} \sim 10^{-4}$ in order to get a better coupling of the laser to the cavity and to avoid further instabilities like the coherence collapse (see, *e.g.*, [10]). The laser has been driven with constant injection currents below the threshold of the solitary laser diode, but above the threshold of the laser with external cavity. The cavity consists of the uncoated facet of the laser diode (LD) and two external high reflecting gold mirrors ($R \approx 98\%$). A beamsplitter divides the light intensity equally between the two cavity arms with the external mirrors. The lengths of the two arms have been chosen as $l_1 = 3.000(2)$ m and $l_2 = 1.500(2)$ m corresponding to round trip frequencies of 50.00 MHz and 100.0 MHz, respectively.

The intensity dynamics of the laser has been detected by a fast Si-avalanche photodiode. Its electrical output signal is amplified and recorded using a fast digital oscilloscope with a bandwidth of 1 GHz and a sampling rate of 2 GSamples / s. The length of the time-series has been 32 678 data points measured with a 8-bit resolution. This special T-shaped cavity configuration has been chosen, because it exhibits typical delay-induced instabilities related to the Ikeda scenario, showing the whole scenario from stable emission up to high-dimensional chaos [4]. For the analysis, we concentrate on time series from the high-dimensional chaotic regime for which a correlation dimension analysis fails. They can be found for injection currents $J > 1.5 J_{\mathrm{th}}$, where J_{th} is the threshold current. For the analysis six time series are taken, obtained for three different injection currents, all of them from the high-dimensional regime.

In the following we describe the application of the identification procedure for time-delay systems [5]-[7] to the experimental time series. In the case of the experimental laser system as described above a scalar time-series, the intensity $I(t)$ of the laser signal, is available for the analysis, allowing the identification of a scalar time-delay system. Therefore, the aim is to detect nonlinear correlations of the variables $(\dot{I}, I, I_{\tau_0})$ in the form of a scalar time-delay differential equation

$$\dot{I} = h_r(I, I_{\tau_0}), \qquad I_{\tau_0} = I(t - \tau_0), \tag{1}$$

with a yet unknown function h_r and a yet unknown delay time τ_0. It has been observed that a scalar ansatz already yields important information, *i.e.* the delay time, even if the system investigated is nonscalar [7]. The same is expected to be valid in the case of delay systems with multiple delay times as is the case in the present analysis. The ansatz (1) corresponds to the hypothesis that the dynamics of the investigated system is governed by a one-dimensional localized nonlinearity [11], [12] together with an infinite-dimensional linear subsystem, where the linear subsystem only allows for a uniform, dispersionless transport of signals with a single delay time τ_0.

First, one has to estimate the time derivative $\dot{I}(t)$ from the time-series. Obviously, the estimation of time derivatives from experimental time-series is sensitive towards additional noise and a special care has to be taken for that purpose. With the help of a spectral analysis of the time-varying intensity we were able to confirm that the signal does not contain a significant amount of high-frequency noise ($f > 1.0$ GHz). Therefore, we assume that the time-dependent intensity measured with a sampling rate 2.0 GSamples/s allows the estimation of the time-continuous, high-dimensional chaotic signal with the help of adequate interpolating procedures. Here, we estimate the time-continuous signal with the help of a cubic-spline interpolation. The interpolated signal has a sampling rate of 10.0 GSamples/s. The time derivative was estimated from the interpolated signal with the help of a parabolic approximation.

Next, we have to estimate the yet unknown delay time τ_0 from the data. To this end, we perform a filling factor analysis, with the advantages 1) not to rely on any parametrization, 2) not to require a large amount of data, and 3) not to be sensitive towards additional noise [7]. The basic idea is the following: If the variables $(\dot{I}, I, I_{\tau_0})$ are correlated via eq. (1), the variables $(\dot{I}, I, I_{\tau_0})$ in a three-dimensional space have to be confined to a two-dimensional surface. This is detected with the help of the τ-dependent filling factor, which we compute from the variables $(\dot{I}, I, I_\tau)$ by covering the data in a three-dimensional space with a cube, whose axes are oriented along the $(\dot{I}, I, I_\tau)$-axis and whose volume is minimal. Then, each side of the cube is divided into P equally sized parts and the cube is cut along those lines yielding P^3 cubes of equal size. Finally, we count the number of cubes, when varying τ, which are visited by the data points $(\dot{I}, I, I_\tau)$ at least one time, relative to the total number P^3 of cubes. For random noise the filling factor is one, whereas for a value of τ, where the dynamics is enclosed in a surface, we expect a local minimum to appear in the τ-dependent filling factor indicating the delay time. In the case of the time-dependent intensity $I(t)$ of the laser experiment the filling factor has been computed for several values of P, with $3 \le P \le 12$. In fig. 2, we show the τ-dependent filling factor for 4 different values of P. For a small value of τ the filling factor is small,

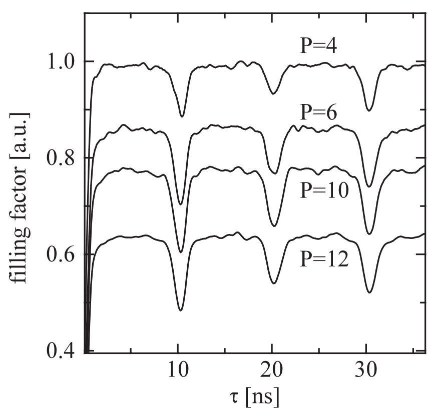

Fig. 2

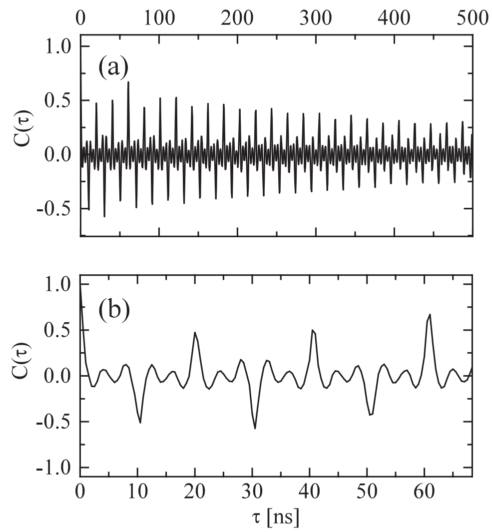

Fig. 3

Fig. 2. – Filling factor analysis of the time-dependent intensity: the τ-dependence of the filling factor is shown for several values of P, where P^3 is the number of equally sized cubes, which are used to cover the trajectory $(\dot{I}, I, I_\tau)$ in a three-dimensional space. For the analysis we used a time series of interpolated data (sampling rate of the measurement: 2 GHz; sampling rate of the interpolated data: 10 GHz) with 163 390 data points.

Fig. 3. – (a) Time-averaged correlation function of the time-varying intensity; (b) enlargement of the correlation function in the vicinity of the delay times $\tau_1 = (10.00 \pm 0.01)$ ns and $\tau_2 = (20.00 \pm 0.02)$ ns.

indicating the local correlations in time. With increasing τ the filling factor also increases due to the high-dimensional chaotic nature of the dynamics until it reaches a plateau. For $\tau \approx \tau_1 = 10.0$ ns and $\tau \approx \tau_2 = 20.0$ ns and its integer multiples the filling factor shows pronounced local minima indicating the nonlocal correlations in time due to the underlying time-delay system.

It is well established that the delay times can, in most cases, also be estimated with the help of autocorrelation functions [13]. But, by definition, the autocorrelation function can only detect linear correlations of the variables (I, I_τ), leading to an overestimation of the delay time due to a finite reaction time of the system [14], or even a failure of the detection. The correlation function of a measured time series is shown in fig. 3. The delay times τ_1 and τ_2 are indicated by the correlation function as local maxima and minima, which alternate in sign. Note that the correlation function is sensitive to *any* correlation in the form $I \propto I_\tau$, not only to the correlations in the form $I \propto f(I_\tau)$, where f is a nonlinear function, induced by a time delay. Therefore, the detection of the delay time is somewhat open to interpretation, since additional local extrema appear in the correlation function indicating the dominance of different dynamical frequencies of the external cavity. In table I we now compare the determined delay time estimated by the filling factor and the autocorrelation function method. To this end a statistical mean of the estimates taken from six time series has been performed, allowing us to considerably decrease the errors in the case of the estimation of τ_1, while in the case of τ_2, the error could not be decreased by statistical averaging since the estimation of τ_2 is subject to larger fluctuations.

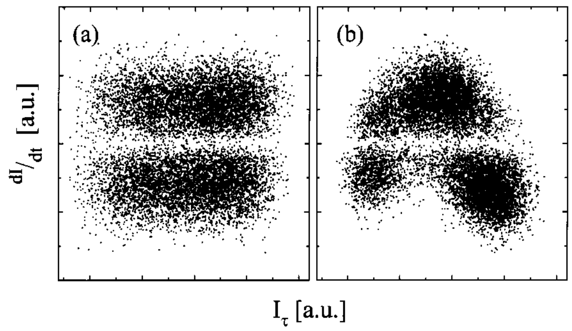

Fig. 4. – Intersections of the 3-D-trajectory $(\dot{I}, I, I_\tau)$ with the plane $I = 0$ for (a) $\tau = 5.0$ ns, and (b) $\tau = 10.0$ ns.

TABLE I. – *Comparison of the estimation of delay times.*

	τ_1 [ns]	τ_2 [ns]
Experiment	(10.00 ± 0.01)	(20.00 ± 0.02)
Autocorrelation	(10.4 ± 0.1)	(20.4 ± 0.5)
Filling factor	(10.1 ± 0.2)	(20.3 ± 0.3)

As expected, the estimates with the help of the autocorrelation function are systematically too large. We find that the estimates with the help of the filling factor give better results for the estimation of τ_1. The deviations of the value of τ_2 might be due to a) the coincidence of the first-order minimum of τ_2 and the second-order minimum of τ_1; b) the fact that the identification is performed in the space $(I, I_\tau, \dot{I})$ and not in a higher-dimensional space. In fig. 4, we visualize the correlations of the variables $(\dot{I}, I_\tau)$ for $I = 0$. For $\tau = 5.0$ ns, the points appear totally decorrelated. For $\tau = 10.0$ ns, a correlation of the form $\dot{I} \propto f(I_\tau)$, where f is a noninvertible function, is observed. This is the source of the local minimum in the filling factor and a characteristic property of nonlinear time-delay systems exhibiting chaotic dynamics. The correlation of the form $\dot{I} \propto f(I_\tau)$ cannot be detected by the autocorrelation function and therefore this is the reason for its shortcomings to estimate the delay time. On the other hand, the filling factor analysis can detect these correlations and for that reason turns out to be a successful tool to estimate the delay times in the laser experiment.

Finally, we shortly comment on the identification of the scalar time-delay differential equation according to eq. (1). To this end, we computed a model for the system with the help of a locally linear fit, which we subsequently used for the forecasting of the dynamics. The iteration of the model yielded trajectories which were qualitatively different from the measured dynamics $I(t)$. Therefore, we infer that the dynamics is not governed by a scalar time-delay system with a single delay time, while the delay times can still be well estimated; this corresponds to a situation already encountered in the case of a two-dimensional analogue of the Mackey-Glass equation [7].

Future research concentrates on the measurement of an additional observable from the experiment, which allows for the identification of a nonscalar time-delay system. A successful identification of the time-delay–induced dynamics would be useful for the estimation of internal parameters of the laser, which are otherwise not accessible in a single experiment under the nonlinear dynamical conditions.

In conclusion, we have presented the analysis of high-dimensional chaotic time-series, obtained by a semiconductor laser experiment. In the experiment the instability is induced by an external cavity, which essentially introduces two time delays into the system. The dimensionality of the data is too large in order to estimate the chaotic indicators (Lyapunov exponents, attractor dimensions) from the time-series and the data have to be considered as nonaccessible to the time-series analysis methods of nonlinear dynamics. Here, we choose an alternative approach: In the spirit of a recently proposed identification method for time-delay systems we used a filling factor analysis to estimate the delay times from the experimental data. We find that the filling factor analysis can be a simple and powerful tool for the estimation of delay times from experimental data.

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