Coupled Nonlinear Thermoelectric Transport in N-QD-SC Junctions

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Collaborators: **Rosa Lopez, David Sanchez**

IFISC

CSIC   Universitat de les Illes Balears

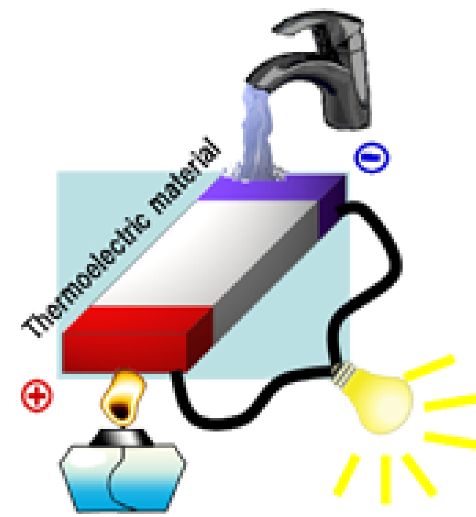
- Thermoelectric Effects
  - Nanoscale thermoelectricity
  - Nonlinear thermoelectric transport

- Superconducting Material
  - Fundamental and practical interests
  - Thermoelectric effects in superconductors

- Normal – Quantum Dot – Superconductor Hybrid Junctions

Hwang, Lopez, and Sanchez, Phys. Rev. B **91**, 104518 (2015).

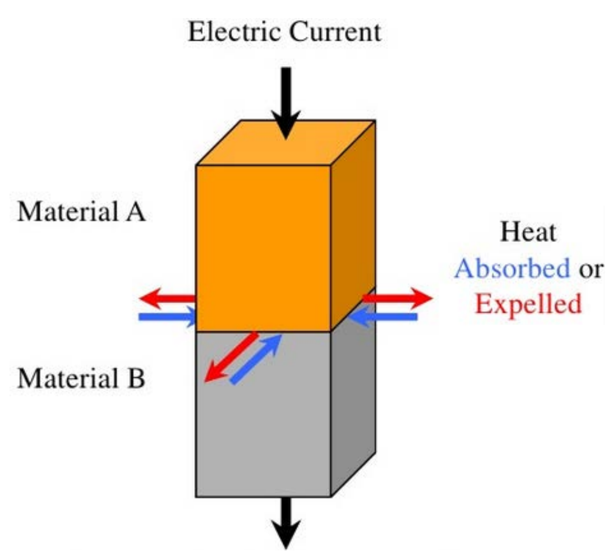
Seebeck Effect



$$I = G\Delta V + L\Delta T$$

Thermoelectric Effect

$$\Delta T \iff \Delta V$$



$$J = R\Delta V + K\Delta T$$

Peltier Effect

$$\mu_L = E_F + \frac{e\Delta V}{2}$$

μ_L, T_L	Quantum System	μ_R, T_R

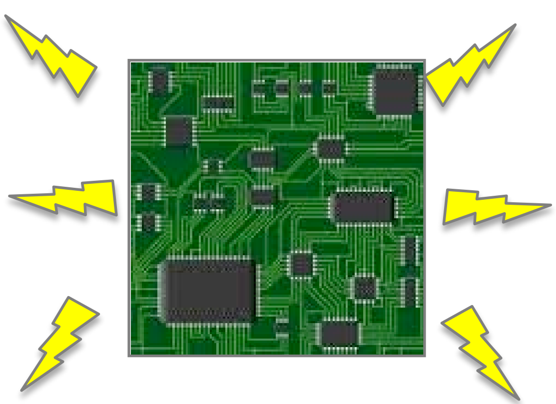
$$\mu_R = E_F - \frac{e\Delta V}{2}$$

Heat-to-Electricity Conversion : Seebeck Coefficient

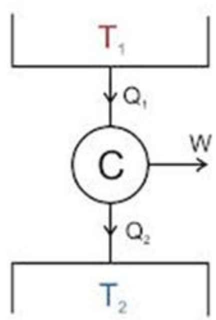
$$I = G\Delta V + L\Delta T$$

$$S = -\frac{\Delta V}{\Delta T}\bigg|_{I=0} = \frac{L}{G}$$

Control the circuit heat generation



High-efficiency heat engine



Transformative energy conversion technology

- Breakdown of reciprocity relations

  - Matthews, Battista, Sanchez, Samuelsson, and Linke, Phys. Rev. B **90**, 165428 (2014).
  - Hwang, Sanchez, Lee, and Lopez, New J. Phys. **15**, 105012 (2013).

- Rectification, Departures from Wiedemann-Franz Law, Nonlinear Seebeck and Peltier effects

  - D. Sanchez and R. Lopez, Phys. Rev. Lett. **110**, 026804 (2013).
  - R. Lopez and D. Sanchez, Phys. Rev. B **88**, 045129 (2013).

- Spin-polarized charge and heat currents in topological insulators

  - Hwang, Lopez, Lee, and Sanchez, Phys. Rev. B **90**, 115301 (2014).

nature physics ARTICLES
PUBLISHED ONLINE: 26 JANUARY 2015 | DOI: 10.1038/NPHYS3227

The Higgs mode in disordered superconductors close to a quantum phase transition

Daniel Sherman[1,2†], Uwe S. Pracht[2], Boris Gorshunov[2,3,4], Shachaf Poran[1], John Jesudasan[5], Madhavi Chand[5], Pratap Raychaudhuri[5], Mason Swanson[6], Nandini Trivedi[6], Assa Auerbach[7], Marc Scheffler[2], Aviad Frydman[1*] and Martin Dressel[2]

nature physics REVIEW ARTICLES
PUBLISHED ONLINE: 2 APRIL 2015 | DOI: 10.1038/NPHYS3242

Superconducting spintronics

Jacob Linder[1*] and Jason W. A. Robinson[2*]

REVIEW
12 FEBRUARY 2015 | VOL 518 | NATURE | 179
doi:10.1038/nature14165

From quantum matter to high-temperature superconductivity in copper oxides

B. Keimer[1], S. A. Kivelson[2], M. R. Norman[3], S. Uchida[4] & J. Zaanen[5]

Long-range Cooper pair splitter with high entanglement production rate

Wei Chen[1], D. N. Shi[1] & D. Y. Xing[2,3]

5 January 2015

NATURE | NEWS

Superconductivity record breaks under pressure

Everyday compound reported to conduct electricity without resistance at a record-high temperature, outstripping more exotic materials.

Edwin Cartlidge

12 December 2014

ARTICLE
Received 4 Aug 2014 | Accepted 4 Feb 2015 | Published 11 Mar 2015
DOI: 10.1038/ncomms7518

Berry phases and the intrinsic thermal Hall effect in high-temperature cuprate superconductors

Vladimir Cvetkovic[1] & Oskar Vafek[1,2]

REVIEWS OF MODERN PHYSICS, VOLUME 76, JULY 2004

Nobel Lecture: On superconductivity and superfluidity (what I have and have not managed to do) as well as on the "physical minimum" at the beginning of the XXI century*

Vitaly L. Ginzburg†

P.N. Lebedev Physics Institute, Russian Academy of Sciences, 119991 Moscow, Russian Federation

(Published 2 December 2004)

ON THE THERMOELECTRIC PHENOMENA IN SUPERCONDUCTORS

By V. L. GINSBURG,

Lebedev Physical Institute, Academy of Sciences of the USSR

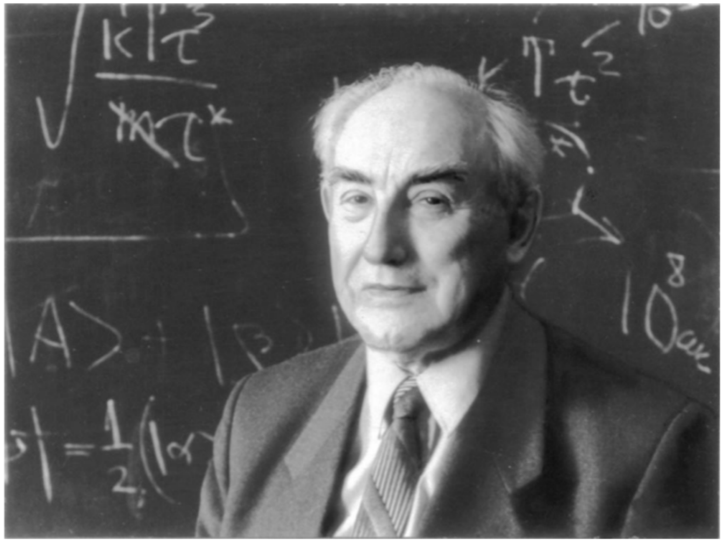
(Received November 23, 1943)

Thermoelectric properties of superconductors are discussed. A normal current j^n should appear in superconductors having temperature gradients; in isotropic superconductors this current is compensated by a superconducting current j^s and therefore cannot be observed. In superconducting crystals, on the contrary, the density of the resulting current $j = j^s + j^n$ does not vanish and, generally speaking, their magnetic field should enable one to detect the thermal current.

REVIEWS OF MODERN PHYSICS, VOLUME 76, JULY 2004

IV. THERMOELECTRIC PHENOMENA IN THE SUPERCONDUCTING STATE

The first attempt to observe thermoelectric phenomena and, specifically, thermoelectric current or thermal electromotive force in a nonuniformly heated circuit of two superconductors, to my knowledge, was made by Meissner (1927). He arrived at the conclusion that the thermoelectric effect is completely absent from superconductors. When I took an interest in this problem in 1943, this viewpoint was generally accepted [see, for instance, Burton *et al.* (1940) and especially the first and later editions of the book *Superconductivity* by Shoenberg (1965)]. However, I have encountered this assertion more recently as well. Nonetheless, this conclusion is erroneous, as I pointed out (Ginsburg, 1944b) as far back as 1944 (see Fig. 6).



The point is that the superconducting state can carry, apart from a superconducting current $\mathbf{j}_s$, a normal current $\mathbf{j}_n$ as well. This normal current is carried by "normal electrons," i.e., electron- or hole-type quasiparticles present in the metal in both the normal and superconducting states. In the superconducting state, the density of such normal quasiparticles depends strongly on the temperature and, generally, tends to zero as $T\rightarrow 0$.

Model Hamiltonian

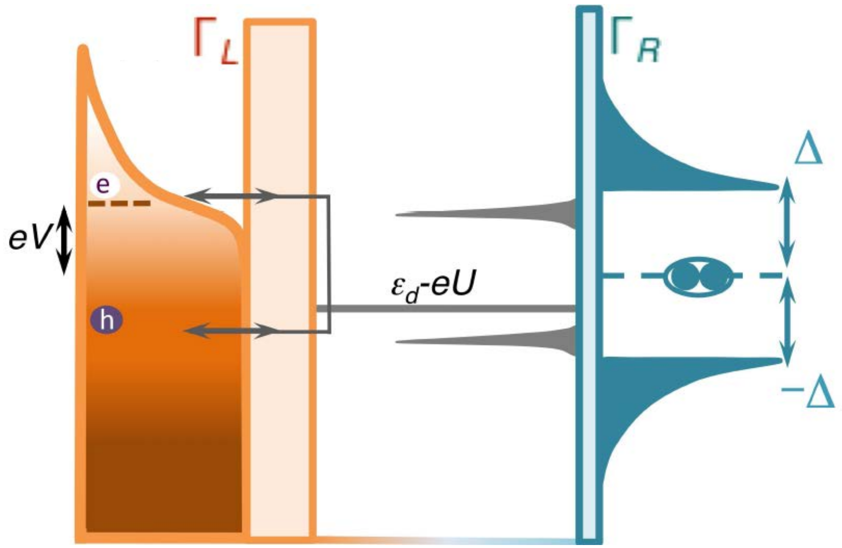
$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_D + \mathcal{H}_T$$

$$\mathcal{H}_L = \sum_{k\sigma} \varepsilon_{Lk} c^\dagger_{Lk\sigma} c_{Lk\sigma}$$

$$\mathcal{H}_D = \sum_\sigma (\varepsilon_d - eU) d^\dagger_\sigma d_\sigma$$

$$\mathcal{H}_R = \sum_{p\sigma} \varepsilon_{Rp} c^\dagger_{Rp\sigma} c_{Rp\sigma} + \sum_p \left[\Delta c^\dagger_{R,-p\uparrow} c^\dagger_{Rp\downarrow} + H.c.\right]$$

$$\mathcal{H}_T = \sum_{k\sigma} t_L c^\dagger_{Lk\sigma} d_\sigma + \sum_{p\sigma} t_R e^{\frac{i}{\hbar} eV_R t} c^\dagger_{Rp\sigma} d_\sigma + H.c.$$



Current

$$I = -e\langle \dot{N}_L(t)\rangle = -(ie/\hbar)\langle[\mathcal{H}, N_L]\rangle$$

$$N_L = \sum_{k\sigma} c^\dagger_{Lk\sigma} c_{Lk\sigma}$$

PHYSICAL REVIEW B **91**, 125401 (2015)

Finite-frequency noise in a quantum dot with normal and superconducting leads

Stephanie Droste,[1] Janine Splettstoesser,[2] and Michele Governale[1]

PHYSICAL REVIEW B **89**, 045422 (2014)

Nonlocal spectroscopy of Andreev bound states

J. Schindele,[*] A. Baumgartner, R. Maurand, M. Weiss, and C. Schönenberger

nature nanotechnology ARTICLES
PUBLISHED ONLINE: 15 DECEMBER 2013 | DOI: 10.1038/NNANO.2013.267

Spin-resolved Andreev levels and parity crossings in hybrid superconductor–semiconductor nanostructures

Eduardo J. H. Lee[1], Xiaocheng Jiang[2], Manuel Houzet[1], Ramón Aguado[3], Charles M. Lieber[2] and Silvano De Franceschi[1*]

nature physics LETTERS
PUBLISHED ONLINE: 14 NOVEMBER 2010 | DOI: 10.1038/NPHYS1811

Andreev bound states in supercurrent-carrying carbon nanotubes revealed

J-D. Pillet[1], C. H. L. Quay[1†], P. Morfin[2], C. Bena[3,4], A. Levy Yeyati[5] and P. Joyez[1*]

PRL **104**, 076805 (2010) PHYSICAL REVIEW LETTERS week ending 19 FEBRUARY 2010

Tunneling Spectroscopy of Andreev Energy Levels in a Quantum Dot Coupled to a Superconductor

R. S. Deacon,[1,*] Y. Tanaka,[2] A. Oiwa,[1,3,4] R. Sakano,[1] K. Yoshida,[3] K. Shibata,[5] K. Hirakawa,[4,5,6] and S. Tarucha[1,3,6,7]

PHYSICAL REVIEW B **81**, 121308(R) (2010)

Kondo-enhanced Andreev transport in single self-assembled InAs quantum dots contacted with normal and superconducting leads

R. S. Deacon,[1,*] Y. Tanaka,[2] A. Oiwa,[1,3,4] R. Sakano,[1] K. Yoshida,[3] K. Shibata,[5] K. Hirakawa,[4,5,6] and S. Tarucha[1,3,6,7]

PRL **104**, 246804 (2010) PHYSICAL REVIEW LETTERS week ending 18 JUNE 2010

Ferromagnetic Proximity Effect in a Ferromagnet–Quantum-Dot–Superconductor Device

L. Hofstetter,[1] A. Geresdi,[2] M. Aagesen,[3] J. Nygård,[3] C. Schönenberger,[1] and S. Csonka[1,2,*]

VOLUME 87, NUMBER 17 PHYSICAL REVIEW LETTERS 22 OCTOBER 2001

Excess Kondo Resonance in a Quantum Dot Device with Normal and Superconducting Leads: The Physics of Andreev-Normal Co-tunneling

Qing-feng Sun,[1] Hong Guo,[1] and Tsung-han Lin[2]

Andreev and Quasiparticle Current

$$I = I_A + I_Q$$

$$I_A = \frac{2e}{h}\int d\varepsilon\, T_A(\varepsilon)\big[f_L(\varepsilon - eV) - f_L(\varepsilon + eV)\big]$$

$$I_Q = \frac{2e}{h}\int d\varepsilon\, T_Q(\varepsilon)\big[f_L(\varepsilon - eV) - f_R(\varepsilon)\big]$$

Andreev and Quasiparticle Transmission

$$T_A(\varepsilon) = \Gamma_L^2 |G_{12}^r(\varepsilon)|^2$$

$$\widetilde{\Gamma}_R \;=\; \Gamma_R \Theta(|\varepsilon| - \Delta)|\varepsilon|/\sqrt{\varepsilon^2 - \Delta^2}$$

$$T_Q(\varepsilon) = \Gamma_L \widetilde{\Gamma}_R\Big(|G_{11}^r|^2 + |G_{12}^r|^2 - \frac{2\Delta}{|\varepsilon|}\mathrm{Re}\big[G_{11}^r (G_{12}^r)^*\big]\Big)$$

Nambu Space Green's Functions

Transmission functions depend on U

$$G_{11}^r(\varepsilon) = \left[\varepsilon - \varepsilon_d + eU + \frac{i\Gamma_L}{2} + \frac{i\Gamma_R}{2}\frac{|\varepsilon|}{\sqrt{\varepsilon^2 - \Delta^2}} + \frac{\Gamma_R^2 \Delta^2}{4(\varepsilon^2 - \Delta^2)} A^r(\varepsilon)\right]^{-1}$$

$$G_{12}^r(\varepsilon) = G_{11}^r(\varepsilon)\frac{i\Gamma_R \Delta}{2\sqrt{\varepsilon^2 - \Delta^2}} A^r(\varepsilon) \qquad A^r(\varepsilon) = \left[\varepsilon + \varepsilon_d - eU + \frac{i\Gamma_L}{2} + \frac{i\Gamma_R}{2}\frac{|\varepsilon|}{\sqrt{\varepsilon^2 - \Delta^2}}\right]^{-1}$$

PRL 110, 026804 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Scattering Theory of Nonlinear Thermoelectric Transport

David Sánchez[1,2] and Rosa López[1,2]
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[2]*Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain*
(Received 7 August 2012; revised manuscript received 12 December 2012; published 11 January 2013)

PHYSICAL REVIEW B **88**, 045129 (2013)

Nonlinear heat transport in mesoscopic conductors: Rectification, Peltier effect, and Wiedemann-Franz law

Rosa López and David Sánchez
Institut de Física Interdisciplinària i de Sistemes Complexos IFISC (UIB-CSIC), E-07122 Palma de Mallorca, Spain and Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain
(Received 20 December 2012; published 25 July 2013)

PHYSICAL REVIEW B, VOLUME 64, 104508

Nonlinear transport theory for hybrid normal-superconducting devices

Jian Wang,[1] Yadong Wei,[1] Hong Guo,[2] Qing-feng Sun,[3] and Tsung-han Lin[3]
[1]*Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China*
[2]*Center for the Physics of Materials and Department of Physics, McGill University, Montreal, PQ, Canada H3A 2T8*
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(Received 13 March 2000; revised manuscript received 17 April 2001; published 21 August 2001)

$$I = G_0 V + G_1 V^2 + L_0 \theta + L_1 \theta^2 + M_1 V \theta$$

Nonlinear transport coefficients $\boldsymbol{G_1}$, $\boldsymbol{L_1}$, $\boldsymbol{M_1}$ depend on the electrostatic potential $\boldsymbol{U}$

Cross thermoelectric coupling $\boldsymbol{M_1}$ plays an important role in subgap transport

Self-consistent Determination of U : Poisson's Equation

$$\delta U(\mathbf{r}) = \sum_\alpha \left[u_\alpha(\mathbf{r})V_\alpha + z_\alpha(\mathbf{r})\theta_\alpha\right]$$

$$\nabla^2 \delta U(\mathbf{r}) = -4\pi\delta\rho$$

Characteristic Potentials

$$u_\alpha = (\partial U/\partial V_\alpha)_{\text{eq}} \qquad z_\alpha = (\partial U/\partial \theta_\alpha)_{\text{eq}}$$

Charge Density Distribution

$$\delta\rho = \rho - \rho_{\text{eq}} = i\int d\varepsilon [G^<_{11}(\varepsilon) - G^<_{11,\text{eq}}(\varepsilon)]$$

$$G^<_{11}(\varepsilon) = \frac{i\Gamma_L}{2\pi}\left[|G^r_{11}|^2 f_L(\varepsilon - eV) + |G^r_{12}|^2 f_L(\varepsilon + eV)\right] + \frac{i\widetilde{\Gamma}_R}{2\pi} f_R(\varepsilon)\left(|G^r_{11}|^2 + |G^r_{12}|^2 - \frac{2\Delta}{|\varepsilon|}\text{Re}[G^r_{11}(G^r_{12})^*]\right)$$

PRL 110, 026804 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Scattering Theory of Nonlinear Thermoelectric Transport

David Sánchez[1,2] and Rosa López[1,2]
[1]Institut de Física Interdisciplinària i de Sistemes Complexos IFISC (UIB-CSIC), E-07122 Palma de Mallorca, Spain
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(Received 7 August 2012; revised manuscript received 12 December 2012; published 11 January 2013)

Charge Density Distribution

$$\delta\rho = \rho_{\rm inj} + \rho_{\rm scr} = \sum_\alpha (D_\alpha V_\alpha + \widetilde{D}_\alpha \theta_\alpha) - \Pi \delta U$$

Particle and Entropic Injectivities

$$D_\alpha = (\partial\rho/\partial V_\alpha)_{\rm eq} \qquad \widetilde{D}_\alpha = (\partial\rho/\partial\theta_\alpha)_{\rm eq}$$

Lindhard Function : Screening Effect

$$\Pi = -(\delta\rho/\delta U)_{\rm eq}$$

Particle Injectivity

$$\nu_\alpha^p(E,\sigma) = (2\pi i)^{-1} \sum_\beta \mathrm{Tr}[s_{\beta\alpha}^\dagger \frac{ds_{\beta\alpha}}{dE}]$$

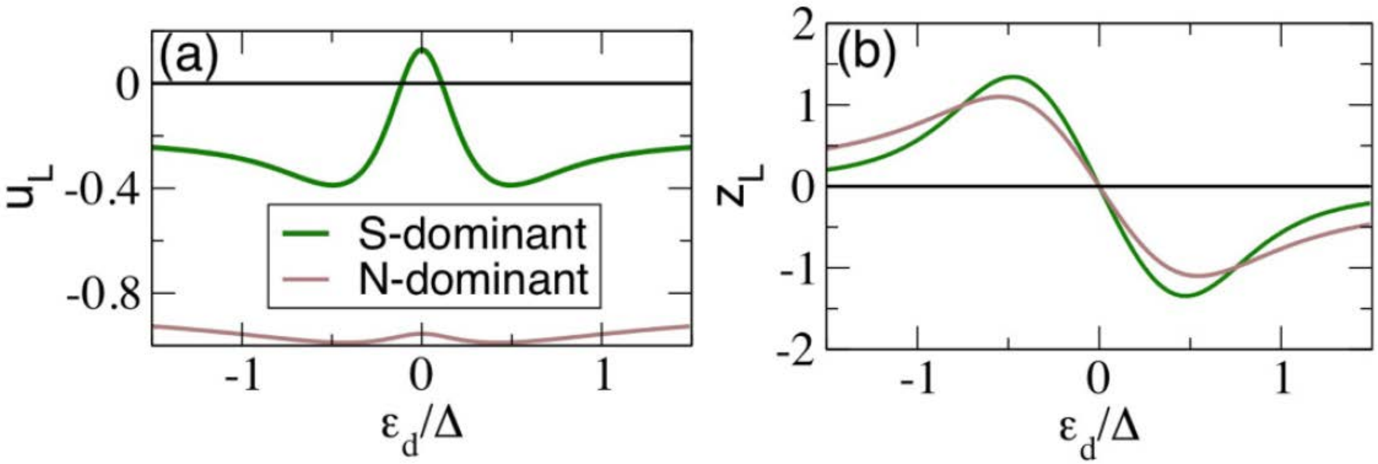
Entropic Injectivity

$$\nu_\alpha^e(E,\sigma) = (2\pi i)^{-1} \sum_\beta \mathrm{Tr}[\frac{E-E_F}{T} s_{\beta\alpha}^\dagger \frac{ds_{\beta\alpha}}{dE}]$$

Capacitance Model

$\delta\rho = C\delta U$

Phys. Rev. B **65**, 045321 (2002).



$$\delta U = \sum_\alpha [u_\alpha V_\alpha + z_\alpha \theta_\alpha]$$

$$u_L = \frac{-e\Gamma_L}{C+\Pi}\int \frac{d\varepsilon}{2\pi}\left(-\partial_\varepsilon f\right)\left(|G^r_{11}|^2 - |G^r_{12}|^2\right)_{\text{eq}}$$

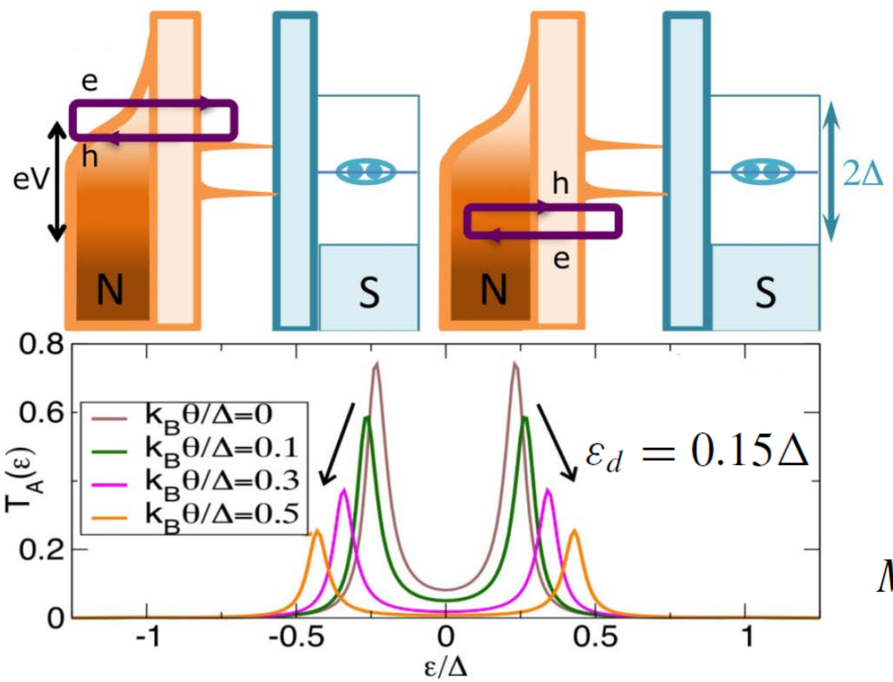
$$z_L = \frac{-\Gamma_L}{C+\Pi}\int \frac{d\varepsilon}{2\pi}\frac{\varepsilon - E_F}{T}\left(-\partial_\varepsilon f\right)\left(|G^r_{11}|^2 + |G^r_{12}|^2\right)_{\text{eq}}$$

$$\Pi^p = -\frac{\delta\rho^p}{\delta U}\bigg|_{\text{eq}} = \int \frac{d\varepsilon}{2\pi} f_{\text{eq}}(\varepsilon)\left[\Gamma_L \frac{\delta\left|G^r_{11}(\varepsilon)\right|^2}{\delta U} + \tilde{\Gamma}_R\left(\frac{\delta\left|G^r_{11}(\varepsilon)\right|^2}{\delta U} - \frac{\Delta}{|\varepsilon|}\frac{\delta}{\delta U} G^r_{11}\left[G^r_{12}\right]^*\right)\right]_{\text{eq}}$$

$$\Pi^h = -\frac{\delta\rho^h}{\delta U}\bigg|_{\text{eq}} = \int \frac{d\varepsilon}{2\pi} f_{\text{eq}}(\varepsilon)\left[\Gamma_L \frac{\delta\left|G^r_{12}(\varepsilon)\right|^2}{\delta U} + \tilde{\Gamma}_R\left(\frac{\delta\left|G^r_{12}(\varepsilon)\right|^2}{\delta U} - \frac{\Delta}{|\varepsilon|}\frac{\delta}{\delta U} G^r_{12}\left[G^r_{11}\right]^*\right)\right]_{\text{eq}}$$

$$I_A = \frac{2e}{h}\int d\varepsilon\, T_A(\varepsilon)\left[f_L(\varepsilon - eV) - f_L(\varepsilon + eV)\right]$$

$$V = 0 \qquad f_L(\varepsilon) - [1 - f_L(-\varepsilon)] = 0 \implies I_A = 0$$



$$I_A = G_0 V + G_1 V^2 + M_1 V\theta$$

$$G_0 = \frac{4e^2}{h}\int d\varepsilon(-\partial_\varepsilon f)T_{A,\mathrm{eq}}$$

$$G_1 = \frac{4e^2}{h}\int d\varepsilon(-\partial_\varepsilon f)u_L \left.\frac{dT_A}{dU}\right|_{\mathrm{eq}}$$

$$M_1 = \frac{4e^2}{h}\int d\varepsilon(-\partial_\varepsilon f)\left[z_L\frac{dT_A}{dU} + \frac{\varepsilon - E_F}{T}\frac{\partial T_A}{\partial \varepsilon}\right]_{\mathrm{eq}}$$

Hwang, Lopez, and Sanchez, Phys. Rev. B **91**, 104518 (2015).

PHYSICAL REVIEW B **91**, 104518 (2015)

Cross thermoelectric coupling in normal-superconductor quantum dots

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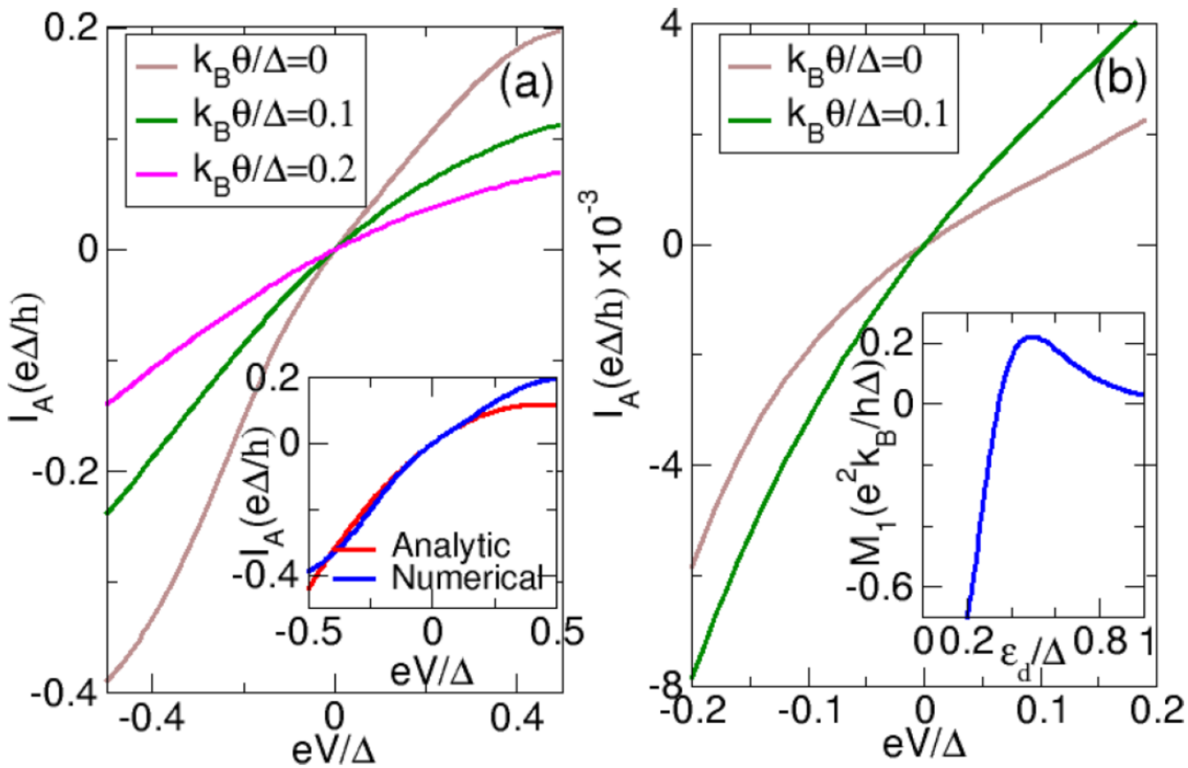
(Received 12 December 2014; revised manuscript received 8 March 2015; published 23 March 2015)

$$I_A = G_0 V + G_1 V^2 + M_1 V \theta$$

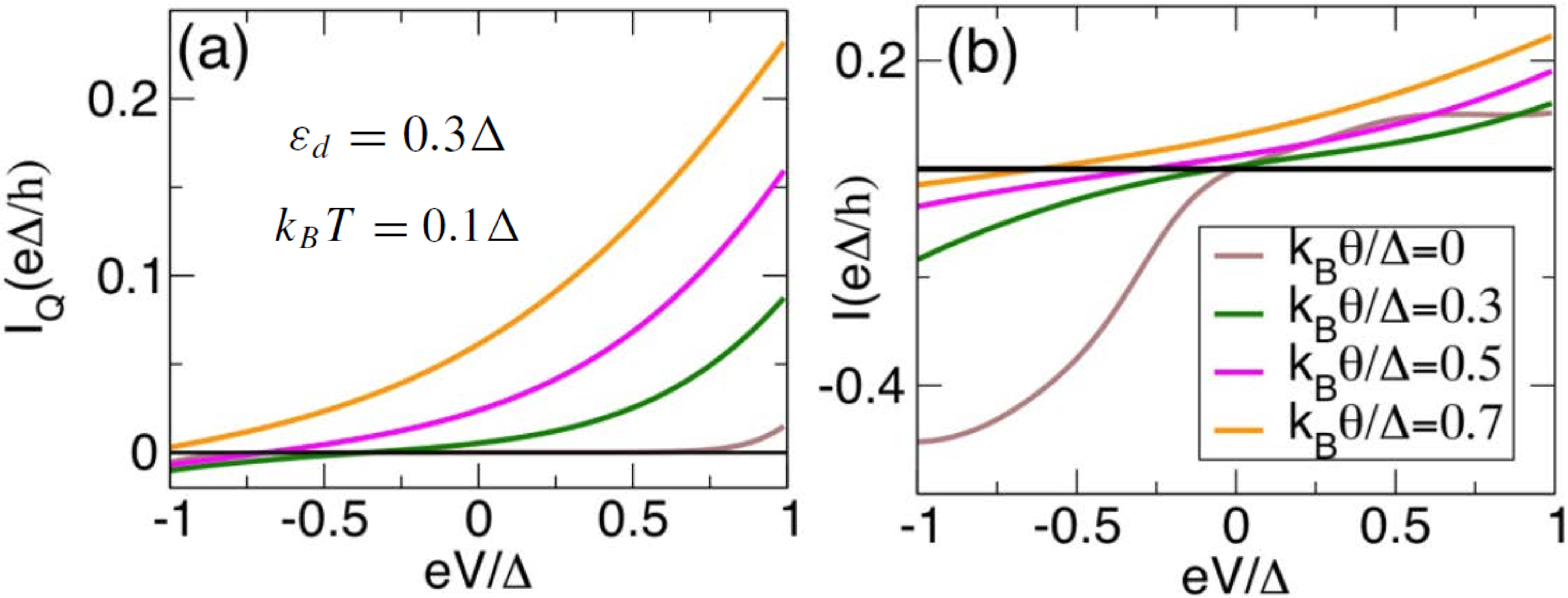
(a) $\varepsilon_d = 0.2\Delta$

(b) $\varepsilon_d = 0.7\Delta$

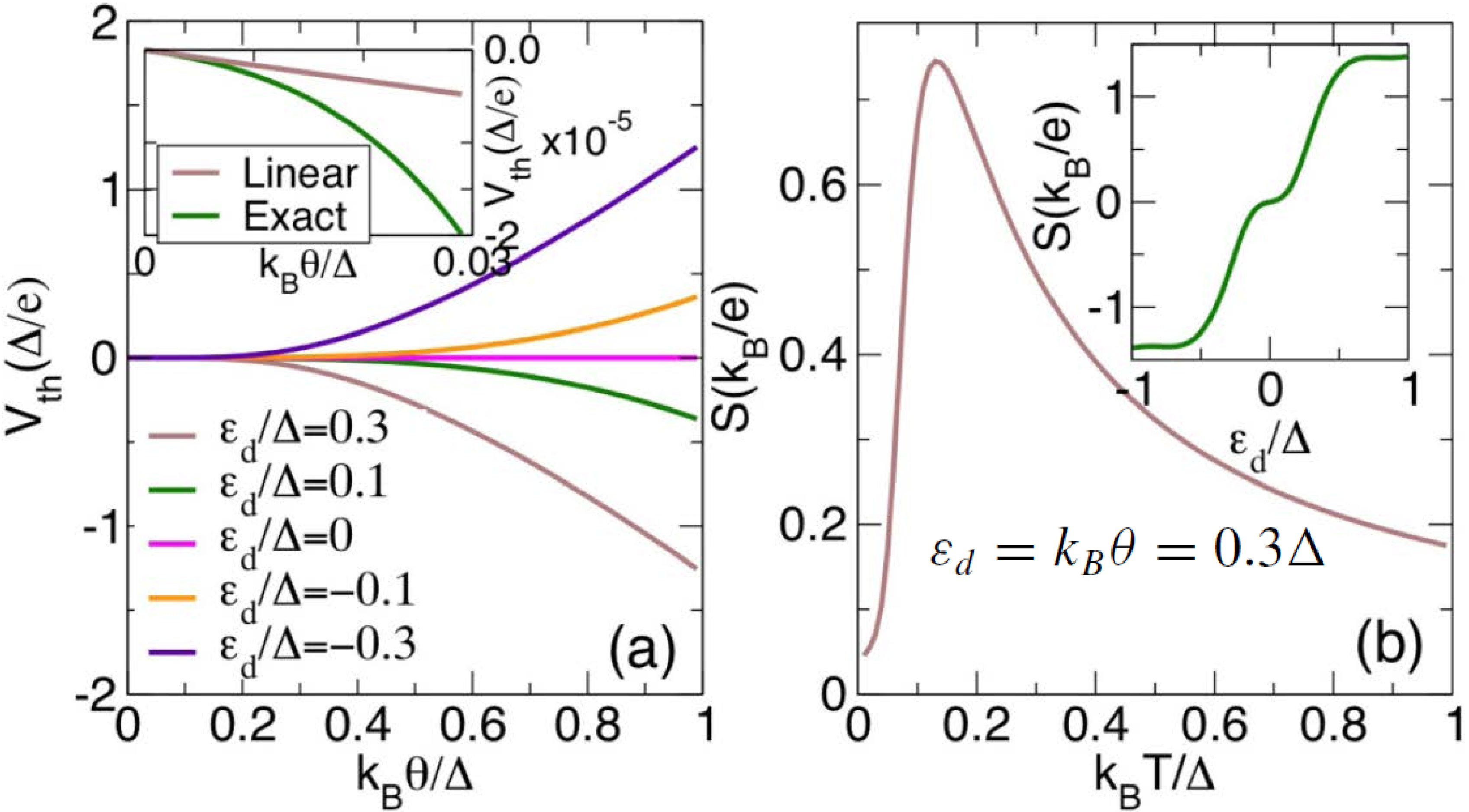
$k_B T = 0.1\Delta$



(a) Quasiparticle Current (b) Total Current



Hwang, Lopez, and Sanchez, Phys. Rev. B **91**, 104518 (2015).



Hwang, Lopez, and Sanchez, Phys. Rev. B **91**, 104518 (2015).

Inset of (b)

$k_BT = 0.1\Delta$

$k_B\theta = 0.3\Delta$

Conclusion

- NS hybrid junctions are poor thermoelectric devices at low bias due to particle-hole symmetry

- Andreev processes cancel linear Seebeck effects and a nonlinear treatment of thermopower is called for

- Electric current through N-QD-S can be manipulated with a thermal bias using a unique cross coupling that arises only in the nonlinear regime of transport

- High thermovoltages can be created due to quasiparticle tunneling for moderate thermal gradient

- Observed thermoelectric effect can appear with all relevant energy scales well below the superconducting gap