The most energetic passive state

collaboration with Antonio Acin, Karen Hovhannisyan, Marti Perarnau, Paul Skrzypzcyk, Jordi Tura

Quantum Battery $H = \sum_{i=0}^{d-1} E_i |i\rangle\langle i|$

$\rho \cong$ 

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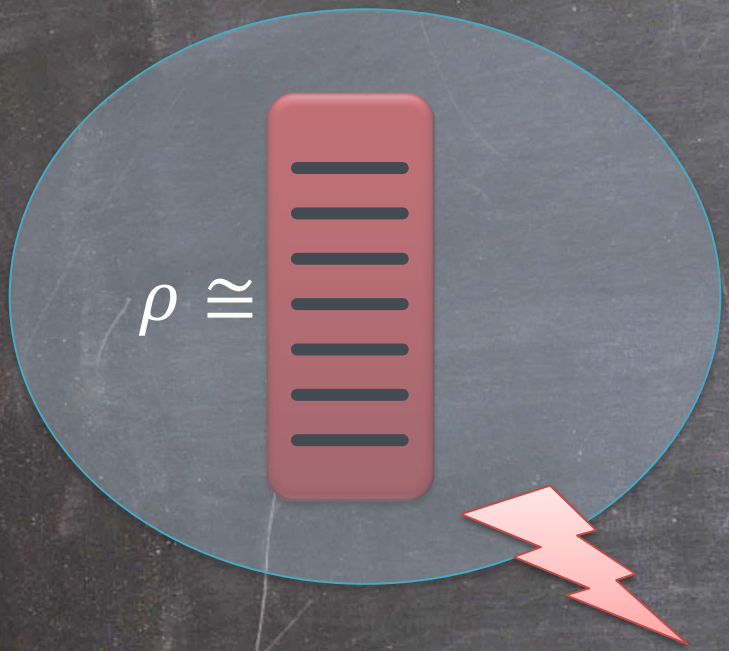
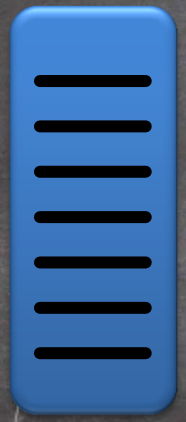


$U\rho U^{\dagger}$

Quantum Battery $H = \sum_{i=0}^{d-1} E_i |i\rangle\langle i|$

$\rho' \cong$ 

All possible average energy extracted, i.e.

$Tr[H(\rho' - U\rho' U^\dagger)] \leq 0$

State is now passive:

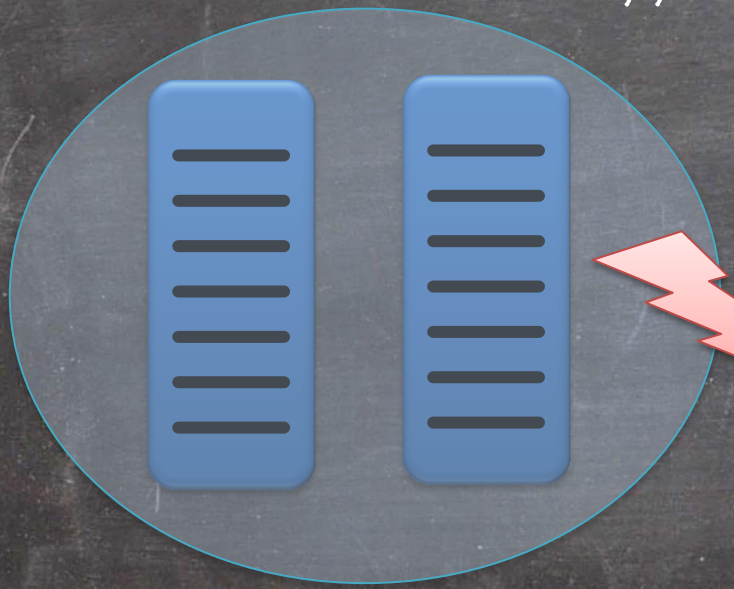
$$\rho' = \rho_p = \sum_{i=0}^{d-1} p_i |i\rangle\langle i| \text{ with } p_i \geq p_{i+1}$$

W. Pusz and S. L. Woronowicz, Commun. Math. Phys. 58, 273 (1978)
A. Lenard, J. Stat. Phys. 19, 575 (1978)

Beware:

$$\rho_p{}^{\otimes 2} = \sum_{i=0}^{d-1}\sum_{j=0}^{d-1} p_i\, p_j |ij\rangle\langle ij|$$

Is not necessarily passive itself!



"hidden" activatable work

$$W_{max} \leq \max_{\sigma, s.t.\ S(\sigma)=S(\rho)} Tr\left[H\left(\rho_p - \sigma\right)\right]$$

Relation to entanglement discussed in:
K.V. Hovhannisyan, M. Perarnau-Llobet, MH, and A. Acin, Phys. Rev. Lett. 111, 240401 (2013)

Unique completely passive state:

Minimum energy for a given entropy

$$\tau(\beta) = \frac{1}{Z} \sum_{i=0}^{d-1} e^{-\beta E_i} |i\rangle\langle i|$$

How much work can be hidden?

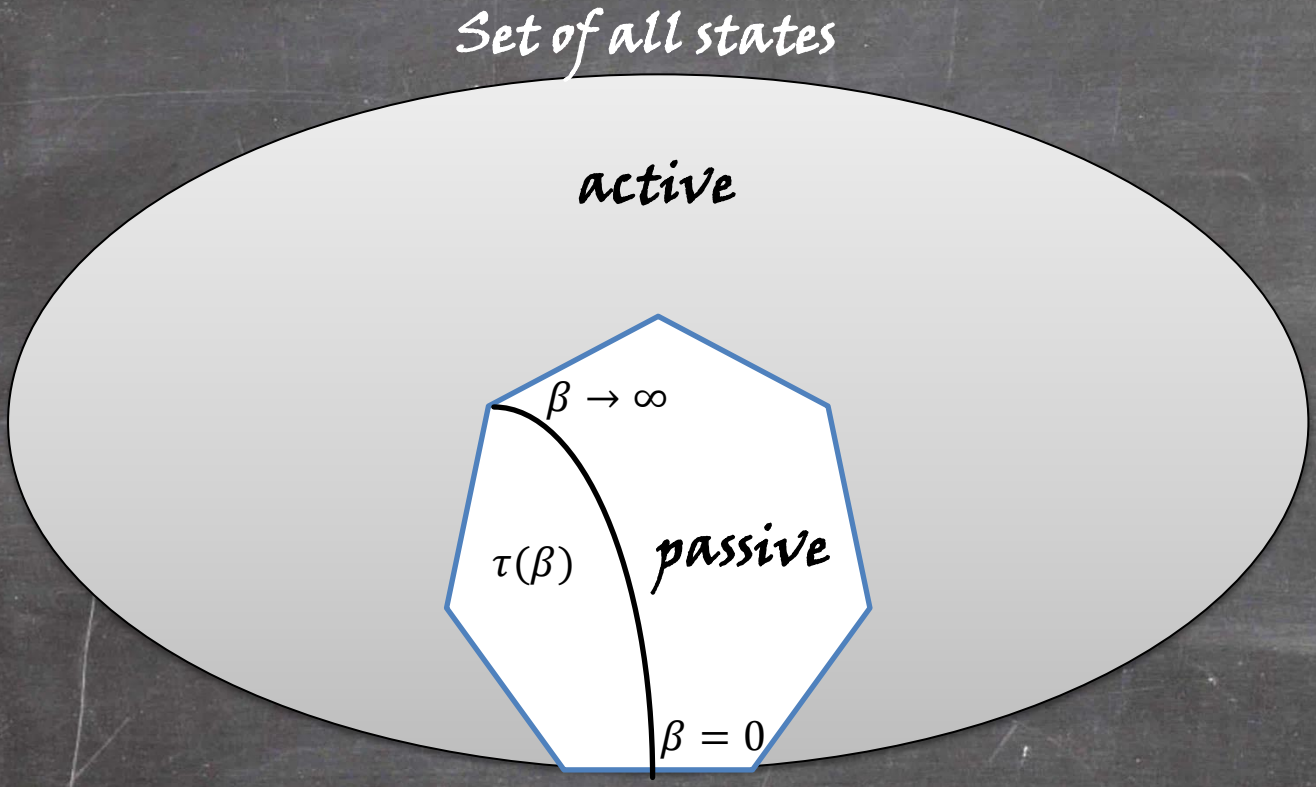
What is the most energetic passive state?

Polytope: $$\rho_p = \sum_{i=1}^{d} p_i \omega_i$$

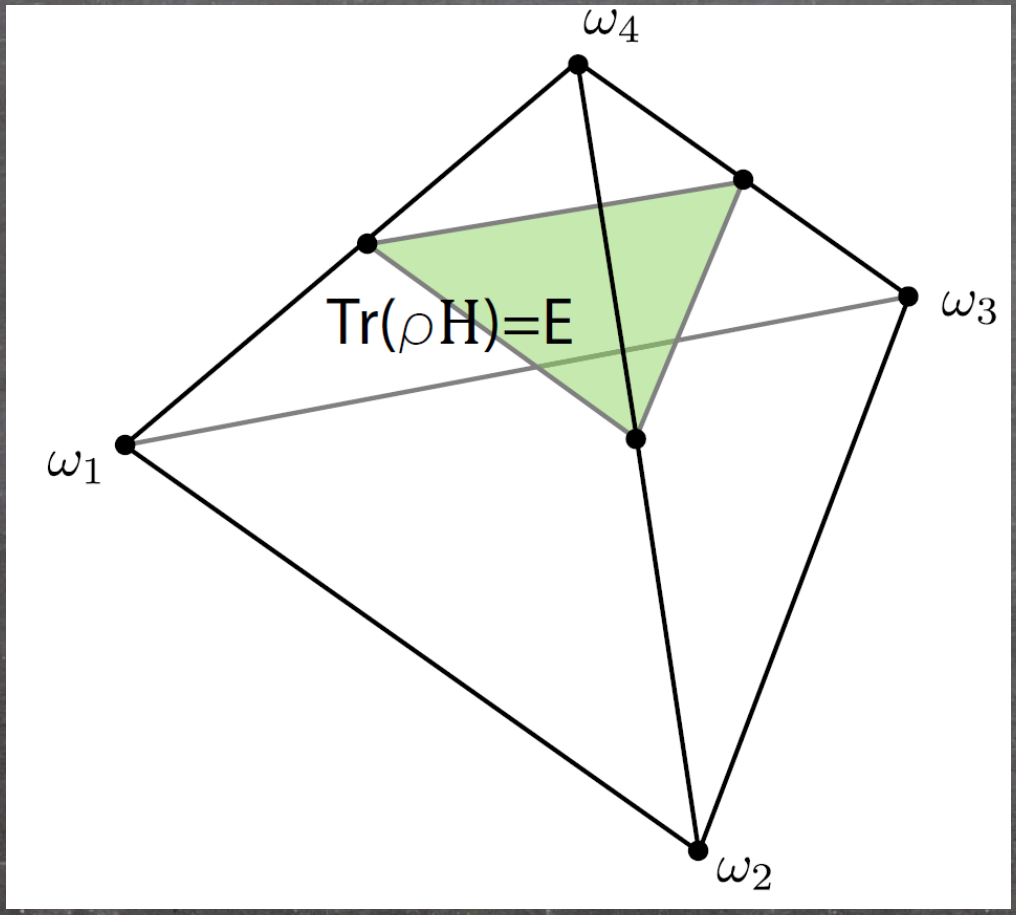
With vertices:

$$\omega_k = \frac{1}{k} \sum_{i=0}^{k-1} |i\rangle\langle i|$$

M. Perarnau-Llobet, K. V. Hovhannisyan, MH, P. Skrzypczyk, J. Tura, A. Acín, arXiv:1502.07311


Set of all states
active
$\beta \to \infty$
$\tau(\beta)$
passive
$\beta = 0$

Minimize entropy for a given energy:

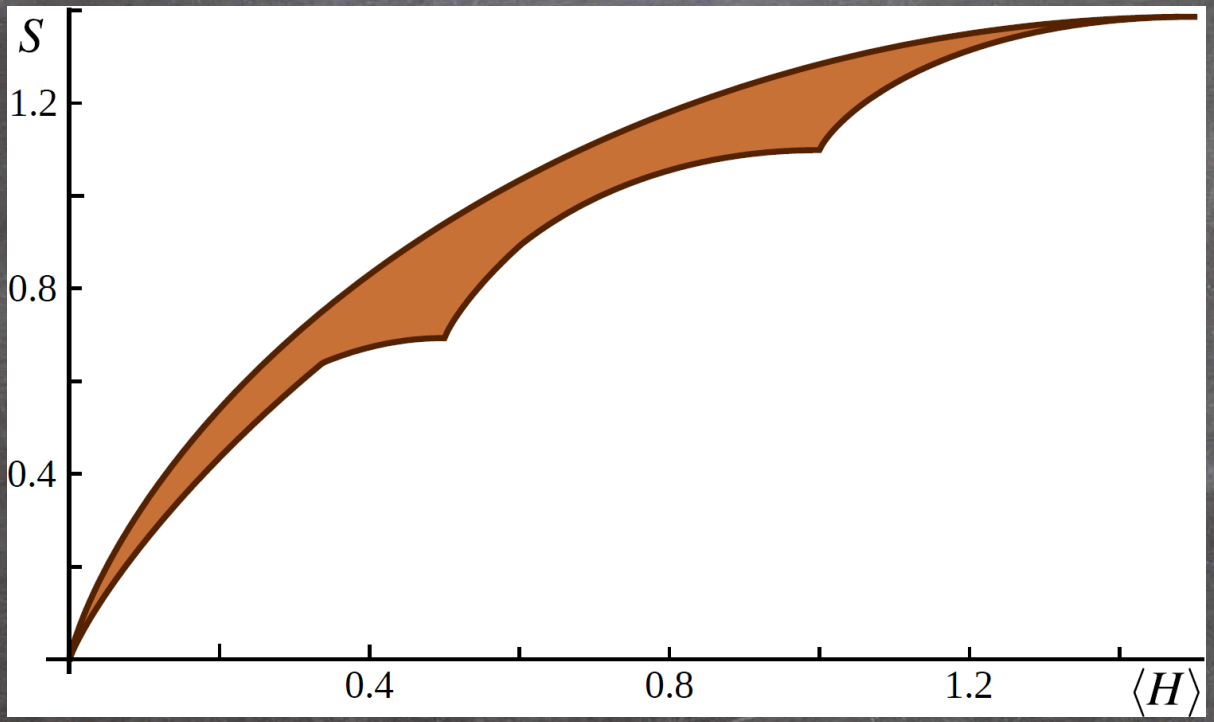


(equivalent to maximizing energy for given entropy)

Most energetic passive states are edges!

$$\rho_{p-max} = p\omega_i + (1-p)\omega_j$$

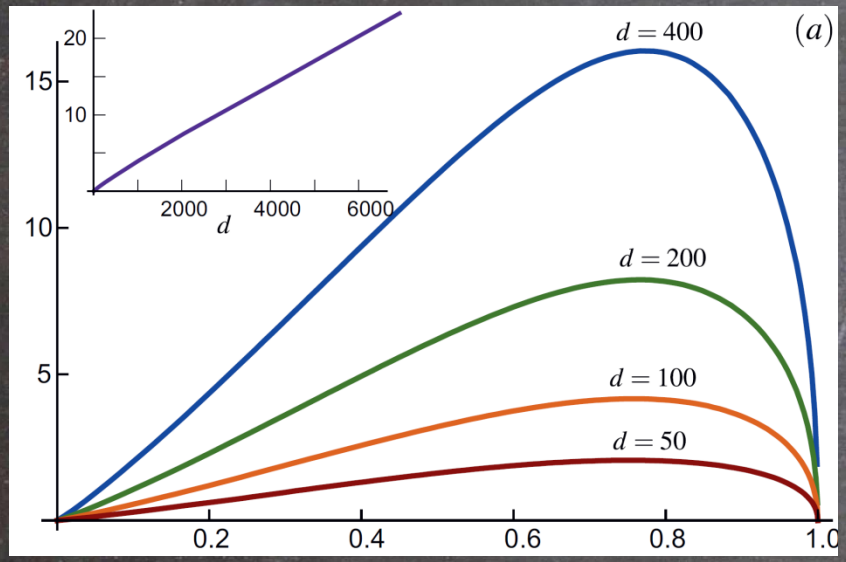
Let's take a look at d=4 (equally spaced)



Physics?

Density of states

subexponential



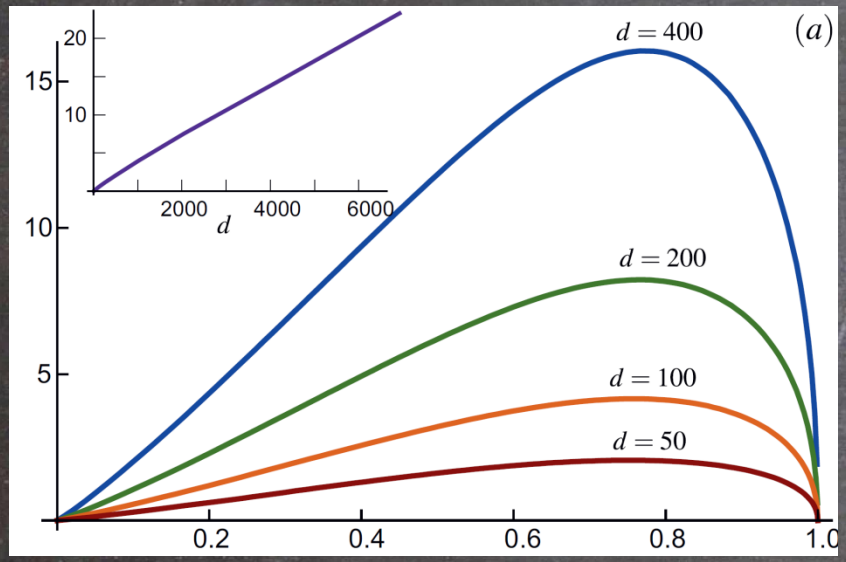
$S / \ln d$

Physics?

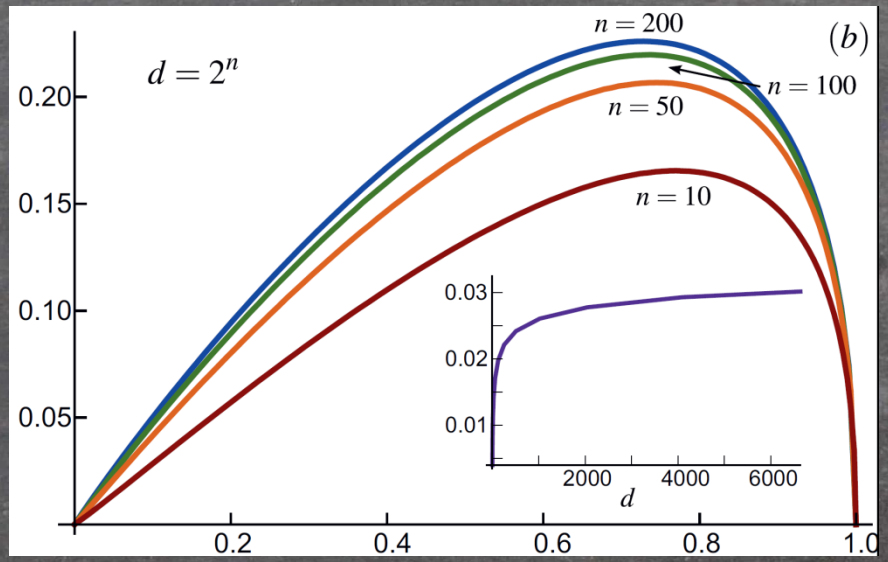
Density of states

subexponential

exponential



$S/\ln d$



$S/\ln d$

Conclusions:

a) There is an analytic form of the most energetic passive state

b) Provides lower bound for extractable work

c) Most physical passive states are "close" to thermal states

M. Perarnau-Llobet, K. V. Hovhannisyan, MH, P. Skrzypczyk, J. Tura, A. Acín, arXiv:1502.07311

Thanks for the attention.

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