Dephasing and heat currents in flux qubits

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Phase-sensitivity of heat currents in superconducting junctions (flux qubits)


- Introduction
  - Heat currents in superconducting junctions ⇒ Phase-dependence
  - Superconducting qubits: Delft qubit and fluxonium
- Sensitivity of heat currents to the qubit state
- Relate heat current sensitivity to dephasing time
  - Temperature gradients: decoherence due to qubit-state sensitive heat currents (example Delft qubit)
  - Heat currents as a phenomenological tool to study dephasing due to quasiparticle tunneling (example fluxonium)
(Heat) current through a superconducting junction

Josephson current carried by Cooper pairs forming the superconducting condensate:

\[ I = I_0 \sin \varphi \]

Heat current carried by quasiparticles:

\[ \dot{Q}^\ell = \frac{d}{dt} \langle H_\ell - \mu_\ell N_\ell \rangle = -\frac{i}{\hbar} \langle [H, H_\ell - \mu_\ell N_\ell] \rangle \]

⇒ Phase dependent through Andreev reflection processes.
(Phase-dependent quasiparticle tunneling.)

Phase-dependent heat current carried by quasiparticles

Weak tunneling - perturbative treatment

\[
\dot{Q}(T_1, T_2) = \frac{2}{e^2 R} \int_{|\Delta_{\text{max}}|}^{\infty} d\omega \omega \frac{\omega^2 - |\Delta(T_1)\Delta(T_2)| \cos(\varphi)}{\sqrt{\omega^2 - |\Delta(T_1)|^2} \sqrt{\omega^2 - |\Delta(T_2)|^2}} [f_1(\omega) - f_2(\omega)]
\]

- Pure quasiparticle contribution + interference part
- Temperature dependence enters via gaps and Fermi functions
- Divergent for small temperature gradients

Non-perturbative result, dominated by a weakly bound Andreev state

\[
\dot{Q}(T, \delta T) = - \left[ \kappa_0 - \kappa_1 \sin^2 \left( \frac{\varphi}{2} \right) \ln \left( \sin^2 \left( \frac{\varphi}{2} \right) \right) + \kappa_2 \sin^2 \left( \frac{\varphi}{2} \right) \right] \delta T
\]


Measurement of phase-dependent heat currents

- Use the tunability of the superconducting phase in a SQUID.
- Measurement of temperature differences.
  ⇒ Flux-dependent heat current proves Maki’s result.
- Heat currents are coherently tunable.

Flux qubit (Delft design)

Fluxoid quantization

\[ \varphi_a - \varphi_b + \varphi_c = -2\pi \frac{\Phi}{\Phi_0} \]

Charging + Josephson energy

\[ H = -4E_C \frac{\partial^2}{\partial \varphi^2} + U(\varphi) \]

- Sweet spot at \( f = \frac{\Phi}{\Phi_0} = \frac{1}{2} \): States are degenerate and placed at \( \pm \varphi^* \).
- Small changes in the flux lead to shifts of the minima: \( \delta \varphi \propto \delta f \)
- Approximate well-localized states \( |\psi_L\rangle \) and \( |\psi_R\rangle \) by oscillator states.

Improving the performance: fluxonium

Effective fluxonium Hamiltonian

\[ H = -4E_C \frac{\partial^2}{\partial \varphi^2} + \frac{1}{2} E_L \varphi^2 - E_J \cos (\varphi + 2\pi f) \]

with fluxoid quantization \( \varphi = M\varphi_L = -\varphi_b - 2\pi f \)

and effective superinductance

\[ E_L = \frac{E_J}{\beta M} = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_{\text{eff}}} \ll E_J \]

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Qubit arms at different temperatures

Different "quasiparticle baths"

- Quasiparticle coherence length $\ll$ arm length.

Different temperatures

- Heating due to operation.
- Model for non-equilibrium distribution of quasiparticles.

Heat current into different arms:

$$\dot{Q}^l = i \frac{\hbar}{\hbar} \langle [H, H_l - \mu_l N_l] \rangle = \dot{Q}_{qp} - \dot{Q}_{int} (\Delta \varphi)$$
Sensitivity of the heat currents to the qubit state

Expectation value of the heat currents in the two qubit states:

\[
\dot{Q}_L/R = \langle \psi_{L/R} | \dot{Q}_{qp} + \dot{Q}_{\text{int}}(\varphi) | \psi_{L/R} \rangle \\
\propto \dot{Q}_{qp} + \dot{Q}_{\text{int}}(\varphi_{L/R})
\]

- Microscopic details of the junctions in \( \dot{Q} \), qubit properties in \( |\psi_{L/R}\rangle \).

For the Delft qubit:

Sensitivity of the heat current detected in "reservoir" \( \ell \)

\[
s_\ell = \frac{\dot{Q}_R^\ell - \dot{Q}_L^\ell}{\dot{Q}_R^\ell + \dot{Q}_L^\ell}
\]

\[ T_1 = 0.1T_{\text{crit}} \]
\[ T_2 = 0.1T_{\text{crit}} \]
\[ T_3 = 0.3T_{\text{crit}} \]
Sensitivity of the heat currents to the qubit state

- Heat currents are indeed sensitive to the qubit state!
- Approximately linear dependence of $s_\ell$ on the flux $\delta f$.
- No sensitivity at the sweet spot (at $f = 1/2$ qubit states are degenerate and have $\varphi_{R/L} = \pm \varphi^*$).
- What is the impact on / relation to the phase coherence of the qubit?

For the Delft qubit:

Sensitivity of the heat current detected in "reservoir" $\ell$

$$s_\ell = \frac{\dot{Q}_R^\ell - \dot{Q}_L^\ell}{\dot{Q}_R^\ell + \dot{Q}_L^\ell}$$

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Toy model for the qubit in contact with quasiparticle baths

State/Phase-dependent coupling takes account for heat current sensitivity!

\[
H_{\text{toy}} = -\frac{\varepsilon}{2}T^3 - \frac{w}{2}T^1 + \sum_{l=1,3} \sum_{k,\sigma} (\varepsilon_l, k - \mu_l) a_{l,k,\sigma}^\dagger a_{l,k,\sigma} \\
+ \sum_{k,q,\sigma} \left[ a_{1,k,\sigma}^\dagger a_{3,q,\sigma} (V_0 T^0 + V_3 T^3) + \text{H.c.} \right].
\]

Reproduce the macroscopic thermal current with:

\[
\dot{Q}_{L/R}^{\text{toy}} = \frac{\pi}{\hbar} \int_{-\infty}^{\infty} d\omega \omega (|V_0|^2 \pm |V_3|^2) N_1(\omega) N_2(\omega) [f_1(\omega) - f_2(\omega)]
\]

\[\Rightarrow \text{Extract } V_0(\varphi_L, \varphi_R) \text{ and } V_3(\varphi_L, \varphi_R)!\]
Time-evolution of the qubit-state pseudo-spin

Qubit reduced density matrix:

\[ S = \begin{pmatrix}
\rho_{LR} + \rho_{RL} \\
i(\rho_{LR} - \rho_{RL}) \\
\rho_{LL} - \rho_{RR}
\end{pmatrix} \]

Pauli rate equation:

\[ \dot{S}(t) = S(t) \times \begin{pmatrix}
(w \approx 0) \\
0 \\
\epsilon
\end{pmatrix} - \frac{1}{\tau_\phi} \begin{pmatrix}
S_1(t) \\
S_2(t)
\end{pmatrix} \]

The phase-sensitive heat current constitutes a measurement process!

\[ \rho_{LL}(t) \approx \rho_{LL}(0) \]
\[ \rho_{RR}(t) \approx \rho_{RR}(0) \]
\[ \rho_{LR}(t) \approx \rho_{LR}(0)e^{-i\epsilon t}e^{-t/\tau_\phi} \]
\[ \rho_{RL}(t) \approx \rho_{RL}(0)e^{+i\epsilon t}e^{-t/\tau_\phi} \]

With the dephasing time \(\tau_\phi\)

- Temperature **gradients** leading to heat currents cause decoherence.
- With the study of heat conductances we are able to capture (a part of) the dephasing mechanism due to quasiparticle tunneling.
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Dephasing of the Delft qubit due to a temperature gradient

- Delft qubit with two different temperatures $T_1 \neq T_2$.
- Large temperature gradient, $\Delta(T_1) \neq \Delta(T_2)$

\[ \frac{1}{\tau_\phi} = \frac{4\pi |V_3|^2 N_1^0 N_2^0}{\hbar} \int |\Delta| d\omega \omega^2 \frac{[1 - f_1(\omega)]f_2(\omega) + [1 - f_2(\omega)]f_1(\omega)}{\sqrt{\omega^2 - |\Delta(T_1)|^2} \sqrt{\omega^2 - |\Delta(T_2)|^2}} \]

- Zero dephasing for vanishing phase-sensitivity, $V_3 \approx 0$.
- No dephasing at the sweet spot, where $s = 0$.
- Compare dephasing time to response to temperature gradient

\[ d = \frac{\tau_\phi}{|\Delta_{\text{max}}|} \left( \dot{Q}_L - \dot{Q}_R \right) \]
Dephasing times of the Delft qubit

\[ d = \frac{\tau_\phi \left( \dot{Q}_L - \dot{Q}_R \right)}{|\Delta_{\text{max}}|} \]

Independent of microscopic details of the device!

For large \( \delta T \), we have \( d \approx 1 \)

Qubit dephases after the time \( \tau_\phi \), which it takes to transfer \( \Delta \) by the heat current difference!
Dephasing times of the Delft qubit

\[ d = \frac{\tau_\phi \left( \dot{Q}_L - \dot{Q}_R \right)}{|\Delta_{\text{max}}|} \]

Independent of microscopic details of the device!

For large \( \delta T \), we have \( d \approx 1 \)

Qubit dephases after the time \( \tau_\phi \), which it takes to transfer \( \Delta \) by the heat current difference!

\[ \tau_\phi \approx \frac{\Delta}{e^2 R n_{\text{qp}}} \]

\[ \tau_\phi \approx 1 \text{ns for } \delta T >> T_{\text{min}} \]

\[ \tau_\phi \approx 1 \mu\text{s for } T_{\text{min}} \approx T_{\text{max}} \leq 0.1 T_c \]
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Dephasing of Fluxonium due to quasiparticles

- Small accidental temperature gradients – linear response.
- Heat **conductance** determines the dephasing.
- Heat conductance contains information about quasiparticle occupation and phase-dependent quasiparticle tunneling.
- Phenomenological approach to dephasing!
  J. Leppäkangas and M. Marthaler, Phys. Rev. B 85, 144503 (2012);
- Direct access to the **phase-dependent weak bound state**, dominant at small $T, \delta T$.

Response to small temperature gradients – linear response

$$\dot{Q}(\varphi, T, \delta T) = - \left( \kappa_0 - \kappa_1 \sin^2 \frac{\varphi}{2} \ln \left( \frac{\sin^2 \frac{\varphi}{2}}{2} \right) + \kappa_2 \sin^2 \frac{\varphi}{2} \right) \delta T$$

Dependence on the specific realization of the superinductance

Role of the superinductance for the dephasing? \((E_L = E_J / \beta M)\)

Total heat current into one "reservoir":

\[
\dot{Q} = \dot{Q}(-\varphi_b, T, \delta T) + \dot{Q}(\varphi_M, T_M, \delta T / M),
\]

Sensitivity does not depend on \(M\) or \(\beta\) separately:

- Only second part of \(\dot{Q}\) can depend on \(M\) or \(\beta\).
- \(\varphi_M = -(\varphi_b + 2\pi f) / M\) is very small \(\Rightarrow\) only \(\kappa_M^0\) contributes.
- \(\kappa_0^M\) is independent of the qubit state.
- Contribution to \(s = \dot{Q}_L - \dot{Q}_R\):
  \(\kappa_0^M \delta T / M + \kappa_0^b \delta T\) is independent of \(M\) or \(\beta\) and only depends on \(E_L\).

Response to small temperature gradients – linear response

\[
\dot{Q}(\varphi, T, \delta T) = -\left(\kappa_0 - \kappa_1 \sin^2 \frac{\varphi}{2} \ln \left(\sin^2 \frac{\varphi}{2}\right) + \kappa_2 \sin^2 \frac{\varphi}{2}\right) \delta T
\]

Sensitivity/Dephasing in the fluxonium limit $E_L \ll E_J$

**Sensitivity suppressed with $E_L/E_J$**

$$s \approx 2 \frac{\kappa_1}{\kappa_0} \left( \frac{\pi E_L}{E_J} \right)^2 \ln \left( \frac{\pi E_L}{2E_J} \right) \delta f$$

- Allows for large number of operations with $\tau_\phi \propto s^{-1}$.
- for $k_B T \leq 0.15 \Delta$, more than $10^4$ operations.

**Response to small temperature gradients – linear response**

$$\dot{Q}(\varphi, T, \delta T) = - \left( \kappa_0 - \kappa_1 \sin^2 \frac{\varphi}{2} \ln \left( \sin^2 \frac{\varphi}{2} \right) + \kappa_2 \sin^2 \frac{\varphi}{2} \right) \delta T$$

Conclusions

- Heat currents through superconducting junctions are phase-dependent.
- Heat currents in superconducting qubits are sensitive to the qubit state.
- Temperature gradients lead to dephasing in flux qubits due to quasiparticles.
- Dephasing times can become of the order of the actual, known limitations (Delft qubit).
- Study of the heat conductance for vanishing $\delta T$ is a powerful and insightful phenomenological approach to dephasing due to quasiparticle tunneling.
- Fluxonium is well protected against this mechanism due to the superinductance!