



QUANTIM

Quantum Imaging IST-2000-26019



IMEDEA



Palma de Mallorca, Spain

<http://www.imedea.uib.es/PhysDept>

Image processing with Type I-TW-SHG

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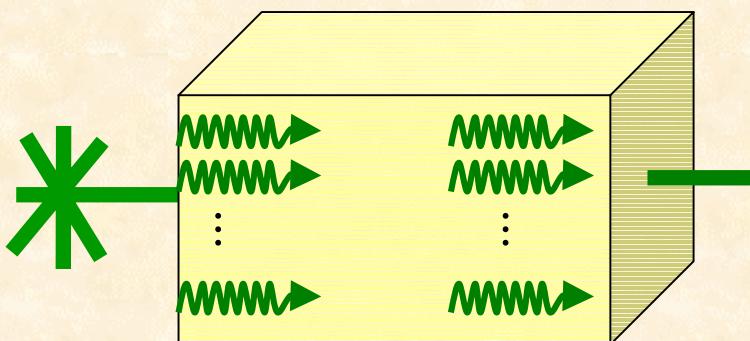


Project funded by the Future and Emerging Technologies arm of the IST Programme
FET-Open scheme

Propagation of the pump field through the nonlinear crystal

$\chi^{(2)}$ nonlinear crystal pumped
with a *strong* monochromatic
field at

Second Harmonic freq. (OPA)

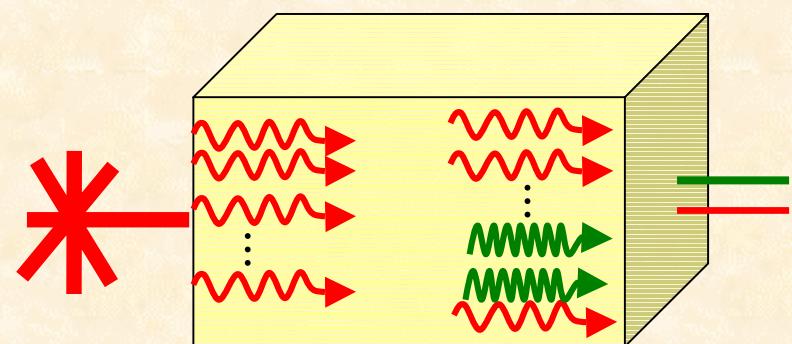


Pumping at 2ω

strong field at 2ω
weak field at ω

Generation of Fundamental field
only triggered by fluctuations
(Spontaneous emission)

Fundamental freq. (TW-SHG)



Pumping at ω

strong field at ω
strong field at 2ω

SHG: A strong homogeneous
field at frequency 2ω develops.

Behaviour in presence of an input signal

Three wave interaction:

$$H_{INT} = i\mathcal{V} \left(\sum_{K,k} \hat{A}_S^+(K) \hat{A}_F(k) \hat{A}_F(K-k) + \sum_{K,k} \hat{A}_S(K) \hat{A}_F^+(k) \hat{A}_F^+(K-k) \right).$$

Parametric approximation

- Strong homogeneous field at SH frequency: **Twin photon emission**

$$\hat{A}_S(k) \text{ in } H_{INT} \leftarrow \langle \hat{A}_S \rangle \delta^{(3)}(k) \quad H_{INT}^{Eff} = i\mathcal{V}_I \left(\sum_k \hat{A}_F(k) \hat{A}_F(-k) + \sum_k \hat{A}_F^+(k) \hat{A}_F^+(-k) \right) \quad \text{OPA}$$

SHG

- Strong homogeneous field at FH frequency: **Frequency conversion**

$$\hat{A}_F(k) \text{ in } H_{INT} \leftarrow \langle \hat{A}_F \rangle \delta^{(3)}(k) \quad H_{INT}^{Eff} = i\mathcal{V}_I \left(\sum_k \hat{A}_S^+(k) \hat{A}_F(k) + \sum_k \hat{A}_S(k) \hat{A}_F^+(k) \right)$$

Quantum operators
associated with FH and SH fields

$$\hat{A}_F(z, q, \Omega) = \delta^{(2)}(q)\delta(\Omega)c_F(z) + \hat{a}_F(z, q, \Omega)$$

$$\hat{A}_S(z, q, \Omega) = \delta^{(2)}(q)\delta(\Omega)c_S(z) + \hat{a}_S(z, q, \Omega)$$

q : transverse wave number
 Ω : frequency offset

Strong monochr. fields
generated by the pump
inside the crystal

Quantum operators:
(Quantum noise
propag. of input signal)

Propagation equations

Classical eq. of nonlinear optics for the strong homogeneous fields

$$\partial_z c_F(z) = -2Kc_F^*(z)c_S(z)e^{-i\Delta k.z}$$

$$\partial_z c_S(z) = Kc_F^2(z)e^{i\Delta k.z}$$

$\Delta k = 2k_F - k_S$: Collinear
phase mismatch

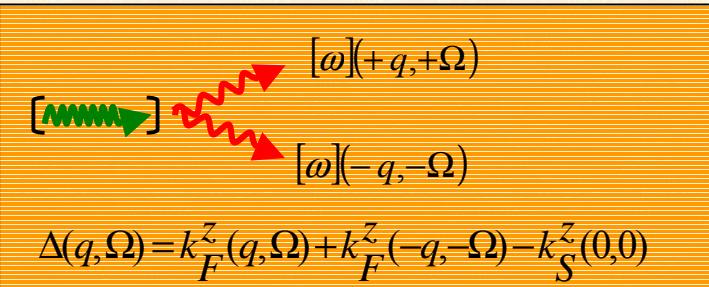
Zeroth order:

Propagation eq. for an input signal/quantum fluctuations

$$\partial_z \hat{a}_F(z, q, \Omega) = -2Kc_S(z)\hat{a}_F^+(z, -q, -\Omega)e^{-i\Delta(q, \Omega)z} \\ - 2Kc_F^*(z)\hat{a}_S(z, q, \Omega)e^{-iD(q, \Omega)z}$$

$$\partial_z \hat{a}_S(z, q, \Omega) = 2Kc_F(z)\hat{a}_F(z, q, \Omega)e^{iD(q, \Omega)z}$$

First order:



$$\Delta(q, \Omega) = k_F^z(q, \Omega) + k_F^z(-q, -\Omega) - k_S^z(0, 0)$$

$$D(q, \Omega) = k_F^z(q, \Omega) + k_S^z(0, 0) - k_S^z(q, \Omega)$$

INPUT OUTPUT TRANSFORMATION:

$$\hat{a}_{F,out}(q, \Omega) = u_F(q, \Omega)\hat{a}_{F,in}(q, \Omega) + v_F(q, \Omega)\hat{a}_{F,in}^+(-q, -\Omega)$$

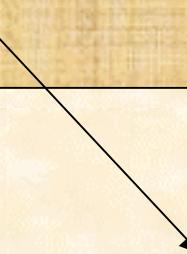
$$+ \mu_F(q, \Omega)\hat{a}_{S,in}(q, \Omega) + \nu_F(q, \Omega)\hat{a}_{S,in}^+(-q, -\Omega)$$

$$\hat{a}_{S,out}(q, \Omega) = u_S(q, \Omega)\hat{a}_{F,in}(q, \Omega) + v_S(q, \Omega)\hat{a}_{F,in}^+(-q, -\Omega)$$

$$+ \mu_S(q, \Omega)\hat{a}_{S,in}(q, \Omega) + \nu_S(q, \Omega)\hat{a}_{S,in}^+(-q, -\Omega)$$

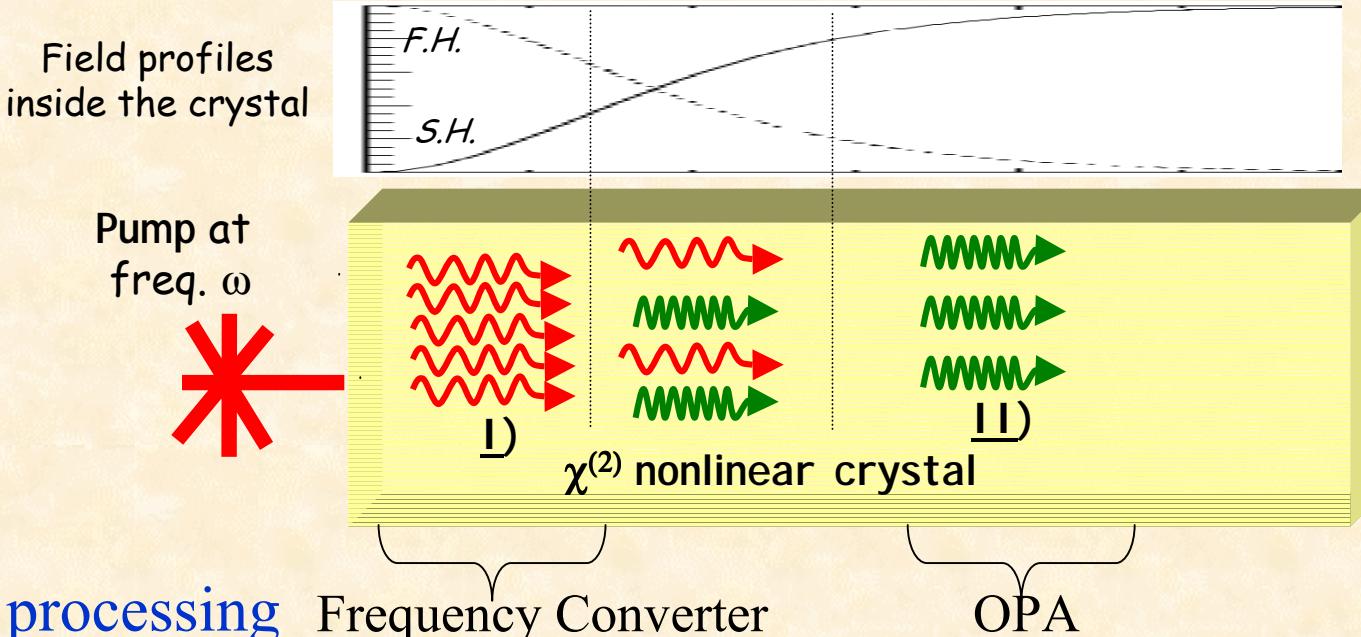


Operators associated with
the FH and SH outgoing waves
at frequency (q, Ω)



Operators associated with
the ingoing waves at
frequency (q, Ω) and $(-q, -\Omega)$

Perfect phase matching: $2k_F = k_S$



Injection of an image (2ω)



Frequency down converted version of the input image



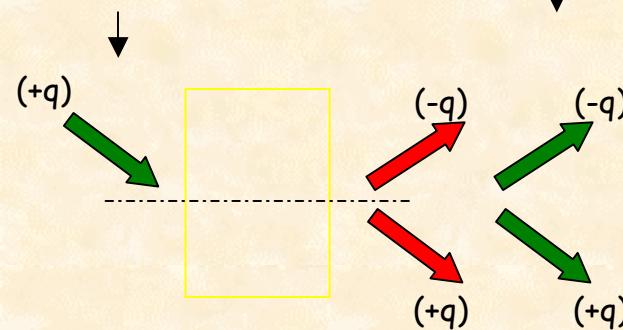
Amplification and duplication



Phase-insensitive configuration:

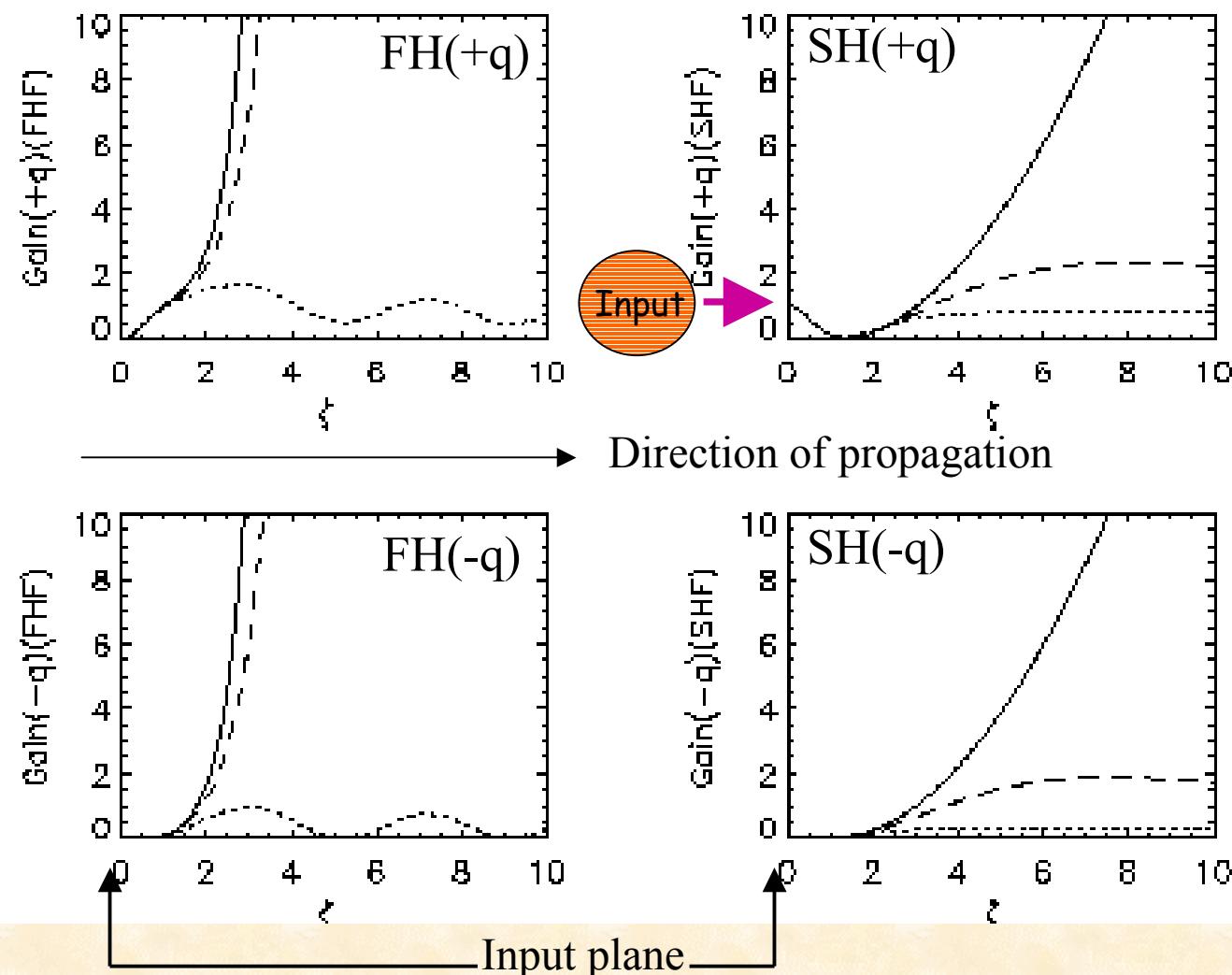
Input signal confined to one half of the input plane

Input signal: a coh. wave at 2ω with transv. wave vector $+q$



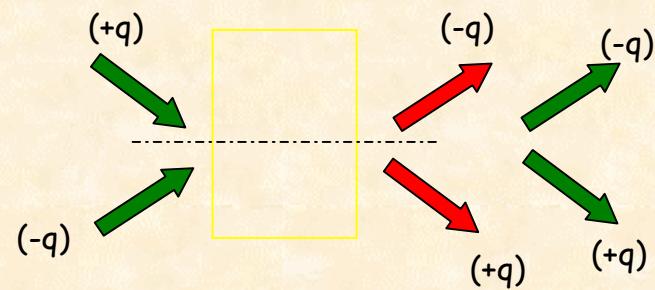
Output

Intensity of each output as a function of the propagation length



Phase-sensitive configuration: Input signal symmetric /optical axis

Input signal: two coh. waves with wave vector $+q$ and $-q$



SNR input
SNR output

Output intensity
Input intensity

output

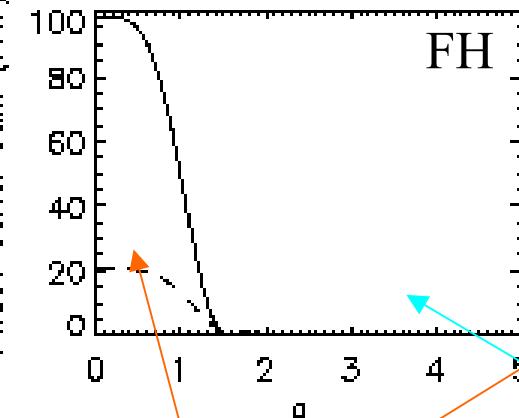
(-q)

(+q)

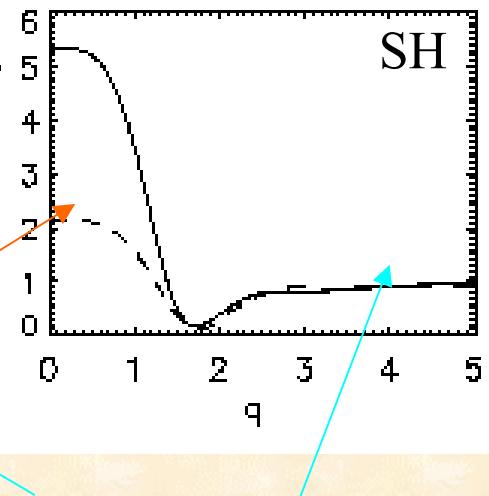
(+q)

(-q)

Phase sens. Gain (FH)



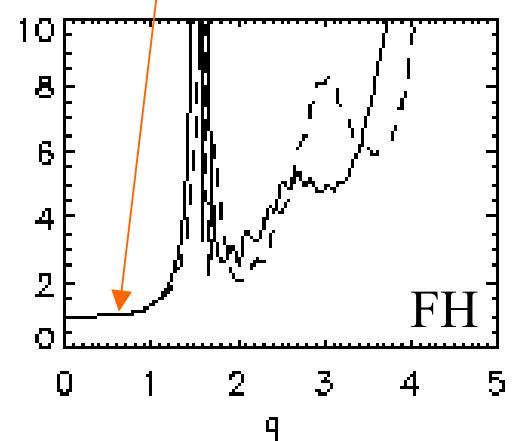
Phase sens. Gain (SHF)



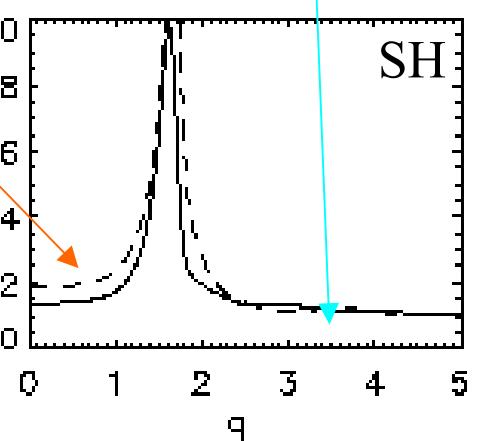
Domain of efficient freq. down conversion and amplification

The signal goes through the optical device without modification

Noise Figure (FH)



Noise Figure (SHF)

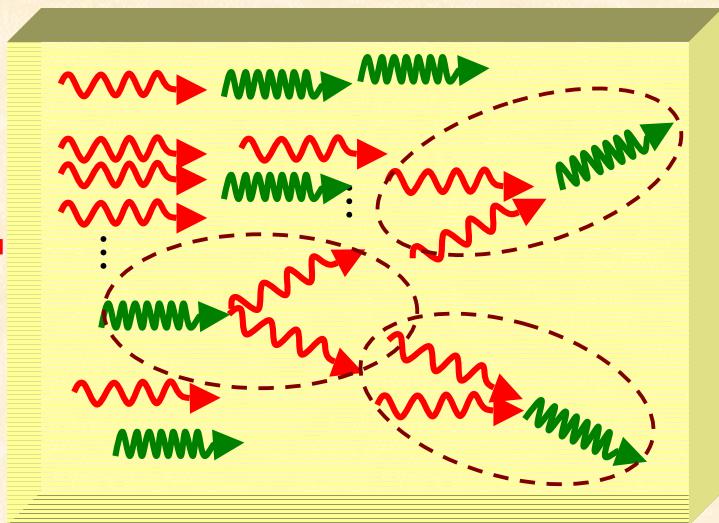


Work in Progress

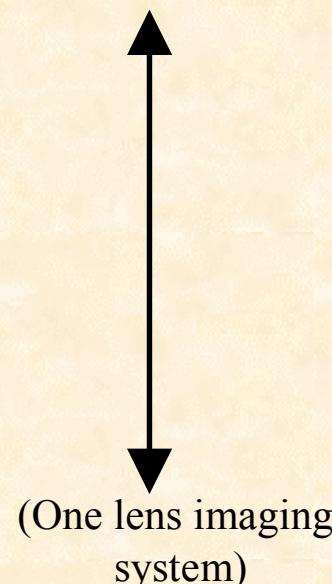
Spatial quantum correlations in the SHG fluorescence spectrum

Spontaneous emission: Fluorescence pattern

- at Fundamental
- at Second Harmonic



(Output plane)



$$\begin{array}{ll}
 N_F(q) & N_F(q) = \int_{-\tau/2}^{+\tau/2} dt \langle e_F^+(q, t) e_F(q, t) \rangle \\
 N_S(q) & \\
 \\
 \overline{N}_S(-q) & \text{Detection of the far field} \\
 \overline{N}_F(-q) & \text{fluorescence patterns} \\
 \end{array}$$

(Detection plane)

Results:

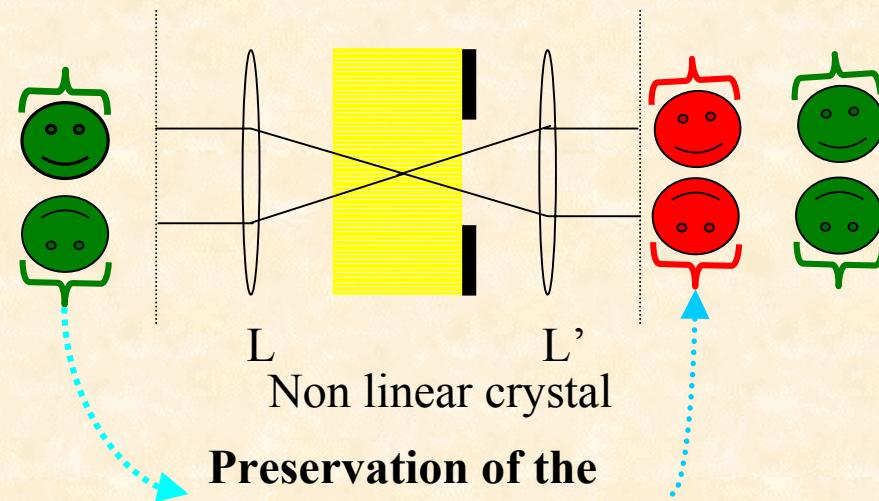
- * Strong (but not perfect) noise reduction in the intensity difference $N_F(q) - N_F(-q)$
- * Subpoissonian statistics of intensity difference $N_S(q) - N_S(-q)$
- * Superpoissonian statistics of cross intensity differences $N_F(q) - N_S(q)$ and $N_F(q) - N_S(-q)$

But Subpoissonian statistics of $N_F(q) - \lambda N_S(-q)$ for some values of λ
 whereas $N_F(q) - \lambda N_S(q)$ at best poissonian

Conclusion

SHG useful for quantum image processing

- allows frequency conversion of an optical signal before amplification
- possibility of noiseless operation



Outlook

Quantum image processing with TW - Type II – SHG

New possibilities due to the polarization degree of freedom.