Work Package 4

Non-classical polarization properties of macroscopic light beams in type II OPO below threshold

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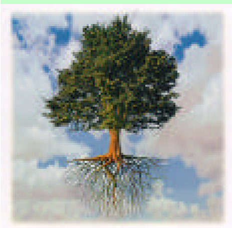

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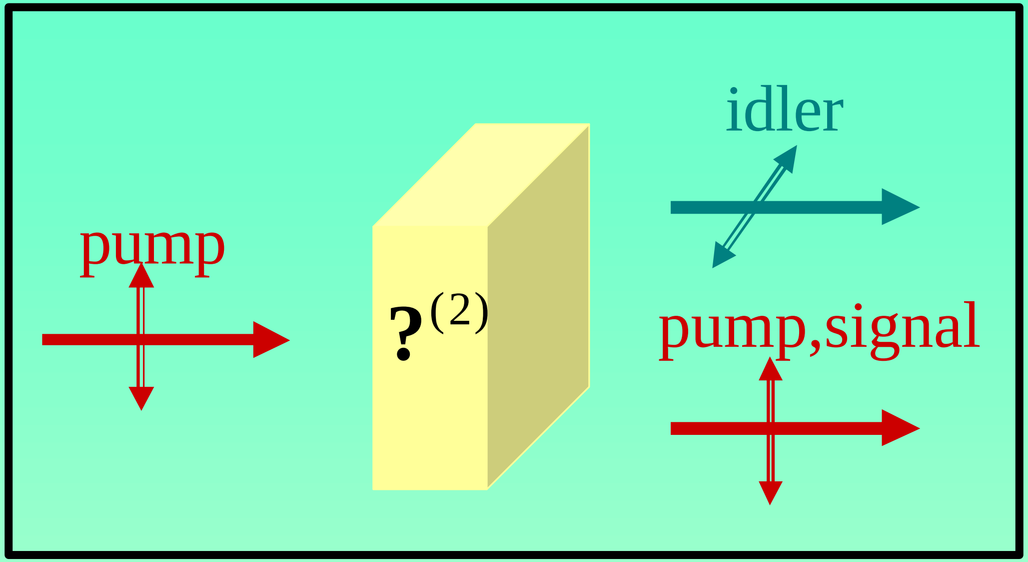
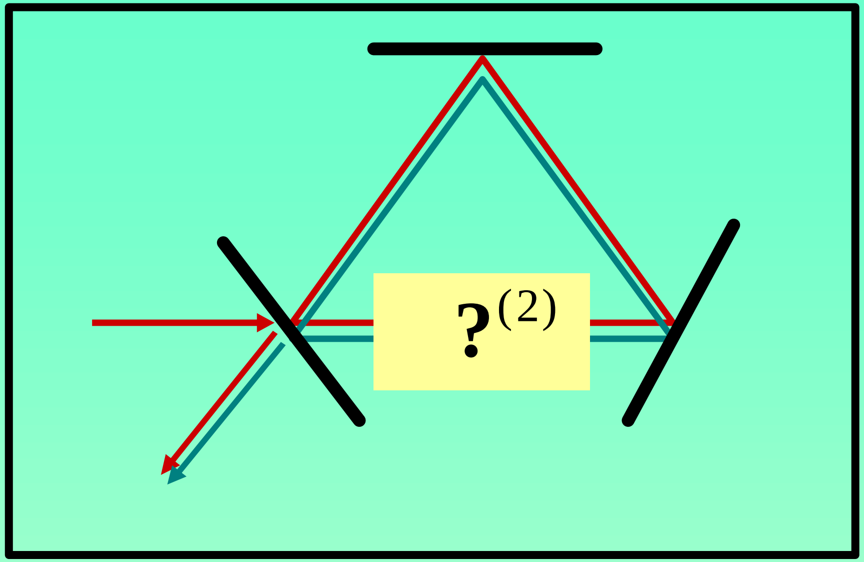
information society technologies

type II phase matching | **OPO**





below threshold ⟹ $\hat{\mathbf{A}}_0 \Rightarrow \mathbf{A}_0$

- linear Heisenberg equations

$$\partial_t \hat{\mathbf{A}}_1(\mathbf{x},\mathbf{y},\mathbf{t}) = \mathbf{L}_1(\hat{\mathbf{A}}_1,\hat{\mathbf{A}}_2;\mathbf{A}_0) + \sqrt{\gamma_1}\,\hat{\mathbf{A}}_1^{\mathbf{in}}$$

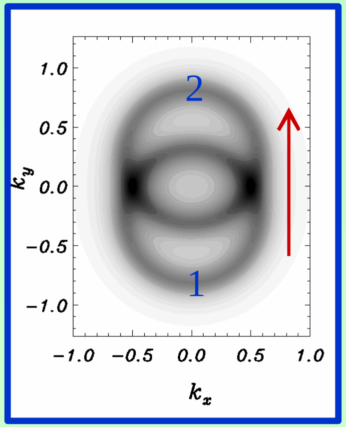
$$\partial_t \hat{\mathbf{A}}_2(\mathbf{x},\mathbf{y},\mathbf{t}) = \mathbf{L}_2(\hat{\mathbf{A}}_1,\hat{\mathbf{A}}_2;\mathbf{A}_0) + \sqrt{\gamma_2}\,\hat{\mathbf{A}}_2^{\mathbf{in}}$$

- input/output

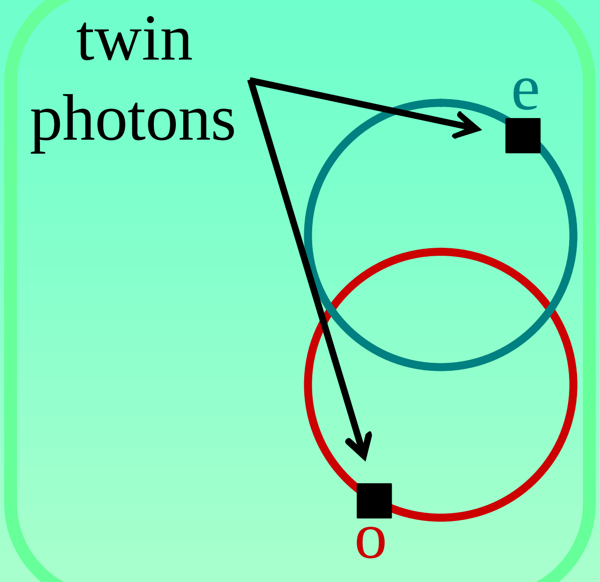
birefringence ⟹ **Transverse walk-off** $\partial_{\mathbf{y}}$

symmetry breaking

$k_y \longleftrightarrow -k_y$



Output far field correlations



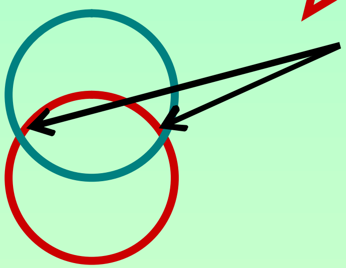
Quadrature correlations

✓ • spatial EPR entanglement between quadratures of type I OPO (Gatti & al.'99) ⇒ extended to type II OPO <u>neglecting</u> walk-off and considering **o** and **e** fields.

? • Spatial EPR entanglement in arbitrary <u>quadratures-polarization</u> components

? • Walk-off effects

Intensity correlations (Stokes operators)



✓ • Microscopic polarization entanglement in PDC (low photon rate)

• <u>Macroscopic</u> polarization entanglement in PDC (high photon rate)
Alessandra talk

? • <u>Macroscopic</u> polarization entanglement in type II OPO below threshold (twin beams). Walk-off effects.

Spatial EPR entanglement in quadrature-polarization components

• 1) Polarization selection (QG) with wave retarder, polarization rotator, linear polarizer

$$\hat{A}_{\Gamma\Theta}(\vec{k}) = \hat{A}_1(\vec{k})\cos\Theta + \hat{A}_2(\vec{k})e^{i\Gamma}\sin\Theta$$

• 2) Quadrature selection (Y) by homodyne detection

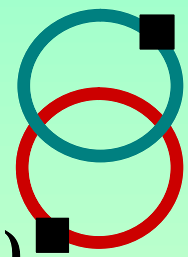
$$\hat{A}^{\Psi}_{\Gamma\Theta}(\vec{k}) = \hat{A}_{\Gamma\Theta}(\vec{k})e^{i\Psi} + \hat{A}^{+}_{\Gamma\Theta}(\vec{k})e^{-i\Psi}$$

Local FF properties of $\hat{\mathbf{A}}^{Y}_{GQ}(\vec{\mathbf{k}})$

•classical variance not homogenous

•walk-off ⟶ dependence on Θ

Spatial correlations



$\hat{\mathbf{A}}^{Y'}_{G'Q'}(\vec{\mathbf{k}}')$

$\hat{\mathbf{A}}^{Y}_{GQ}(\vec{\mathbf{k}})$

$V_g(\pm\vec{k}, \omega; \vec{\Phi})$

$\vec{\Phi} = [\Gamma, \Theta, \Psi, \Gamma', \Theta', \Psi']$

$\Theta = \Theta' + \pi/2, \ \Gamma = \Gamma' + \pi$

$\hat{X}_j \ \hat{Y}_j$ quadratures of two modes $j = 1,2$

$V^{-}(\hat{X}_1 | \hat{X}_2) < 1 \wedge V^{+}(\hat{Y}_1 | \hat{Y}_2) < 1$

$$V^{\pm}(\hat{A}|\hat{B}) = \min_g \frac{V(\hat{A} \pm g\hat{B})}{V(\hat{A}_{SN})}$$

EPR entanglement

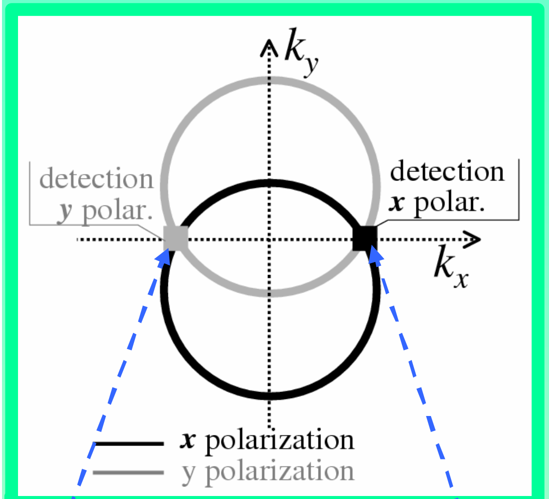
(g=1, inseparability**)**

Detection scheme unaffected by walk-off ($k_y=0$)

$$V_g(\pm\vec{k}_H, w; [G, Q, Y, G+p, Q+\frac{p}{2}, Y'])$$

$V_{g=1}$ independent on Q

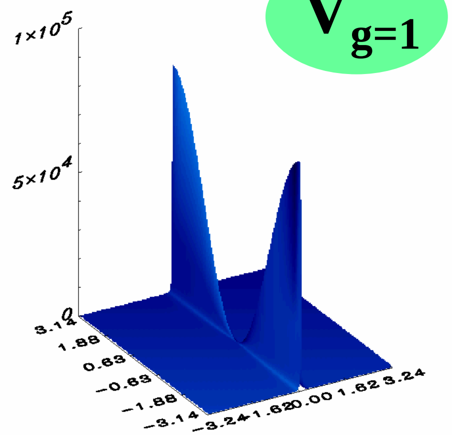
$Y + Y' + G$ (with G struck through)



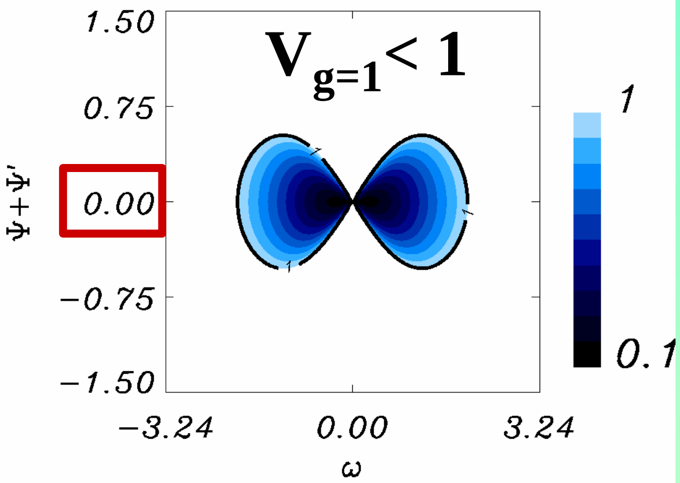
$+\vec{k}_H$ ⟷ $-\vec{k}_H$

reflection symmetry

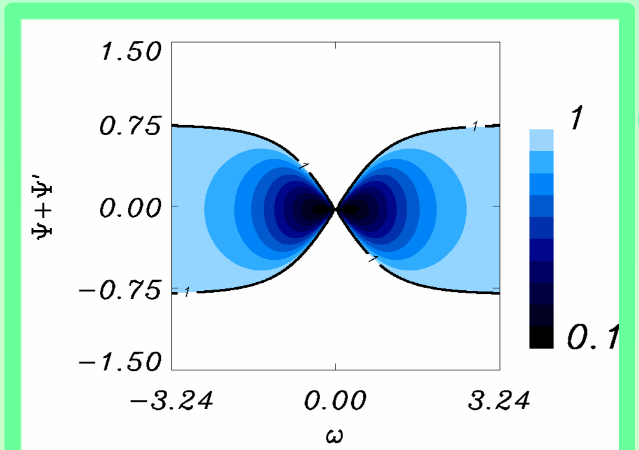
$V_{g=1}$



$V_{g=1} < 1$

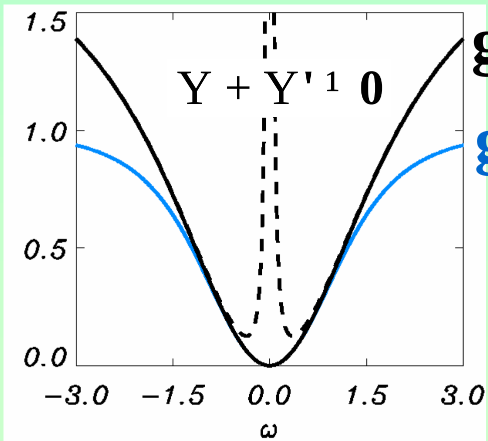


V < 1, EPR entanglement



$V_{g=\bar{g}}$

improvement of EPR at large w



g =1

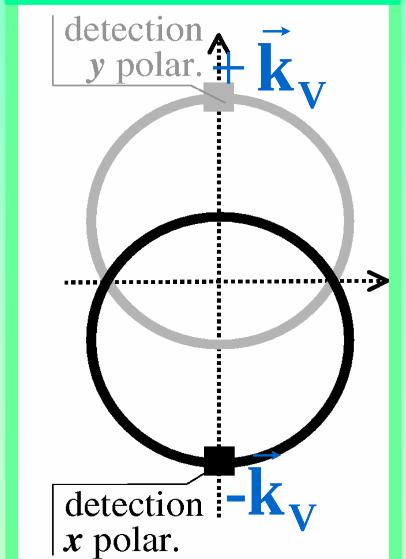
$g = \bar{g}$

Detection scheme in the walk-off direction

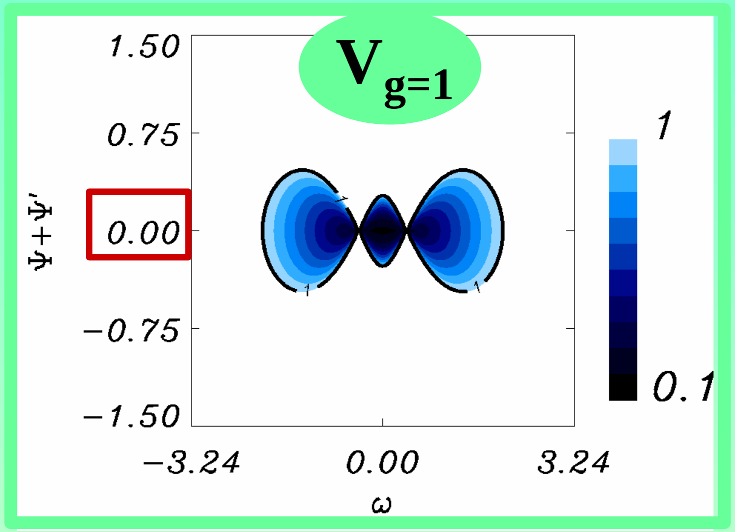
NO reflection symmetry $\bar{k} \not\leftrightarrow -\bar{k}$

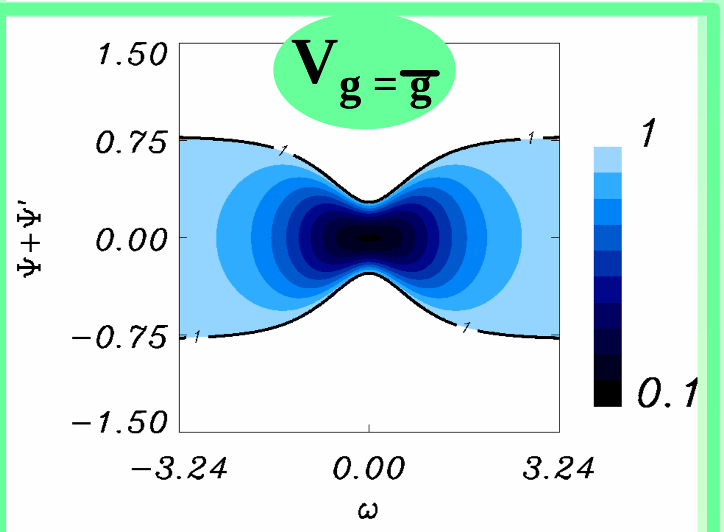
$$V_g(\pm\vec{k}_V, \omega; [\Gamma, \Theta, \Psi, \Gamma+\pi, \Theta+\frac{\pi}{2}, \Psi']) \text{ depends on } \Theta \begin{cases} \bullet\ a)\ \ \Theta = \pi/2 \\ \bullet\ b)\ \ \Theta = 0 \end{cases}$$

a) $\Theta = \pi/2$



vertical bright

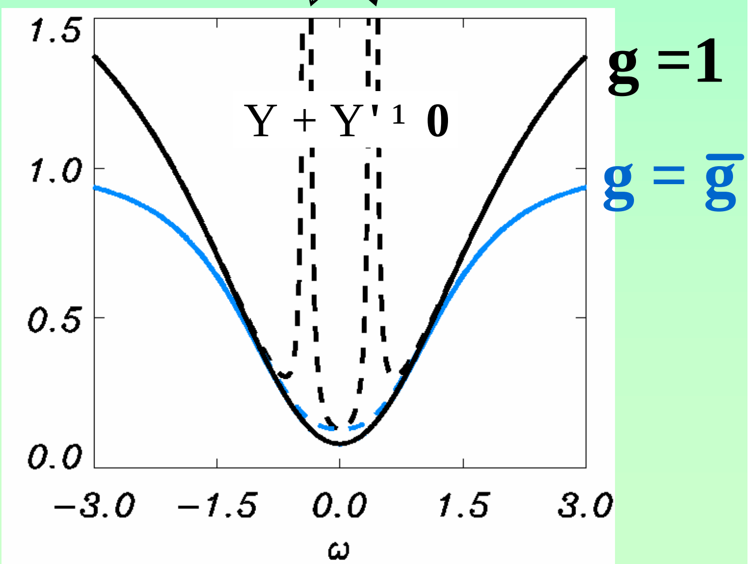




improvement in w and Y+Y'

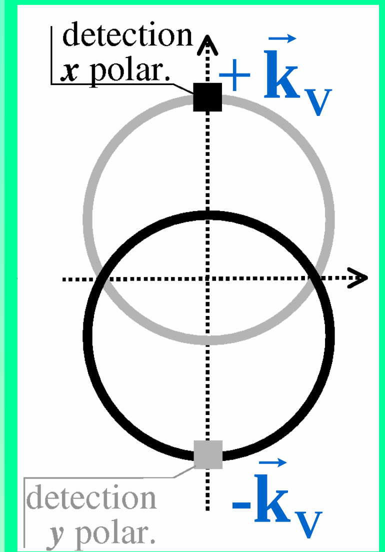
V < 1 , EPR entanglement

Hopf frequencies



Detection scheme in the walk-off direction *(b)*

b) $\Theta = 0$

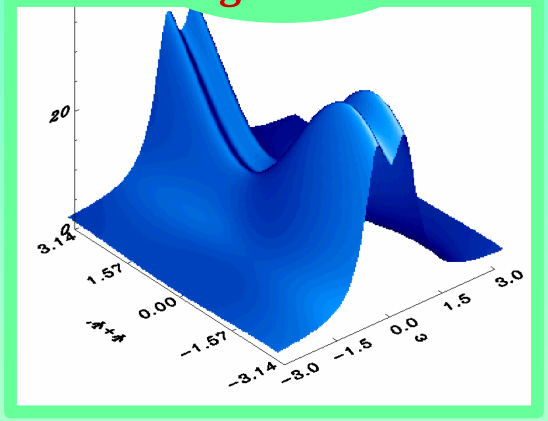


vertical dark

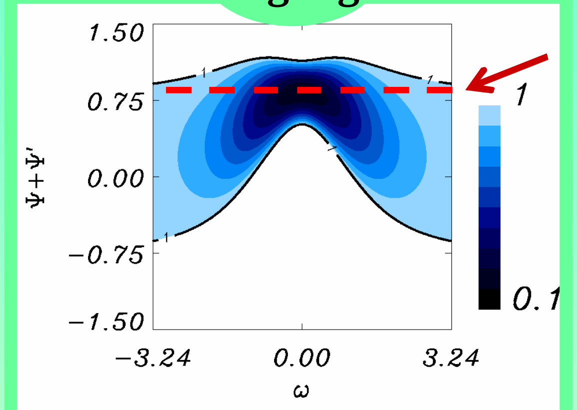
The selected modes are <u>not</u> the <u>critical</u> modes

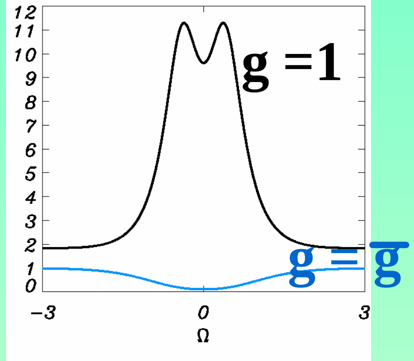
V < 1 , EPR entanglement

$\mathbf{V_{g=1} > 1}$



$\mathbf{V_{g=\bar{g}}}$

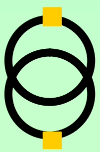




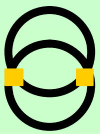
$\mathbf{V_{g=\bar{g}}}$

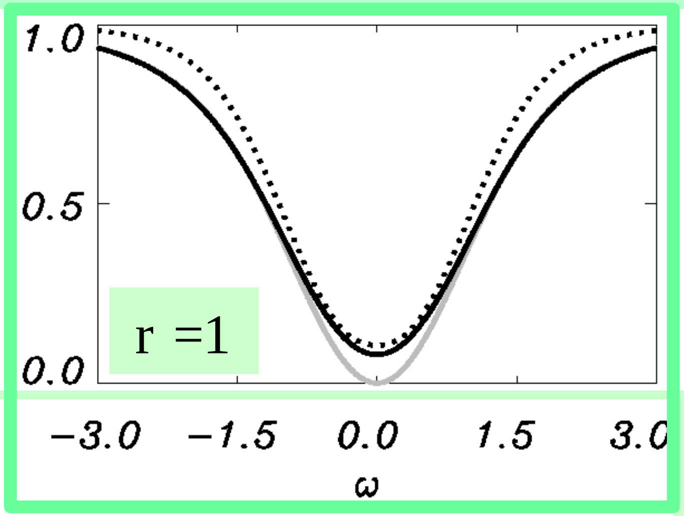
Q = p/2

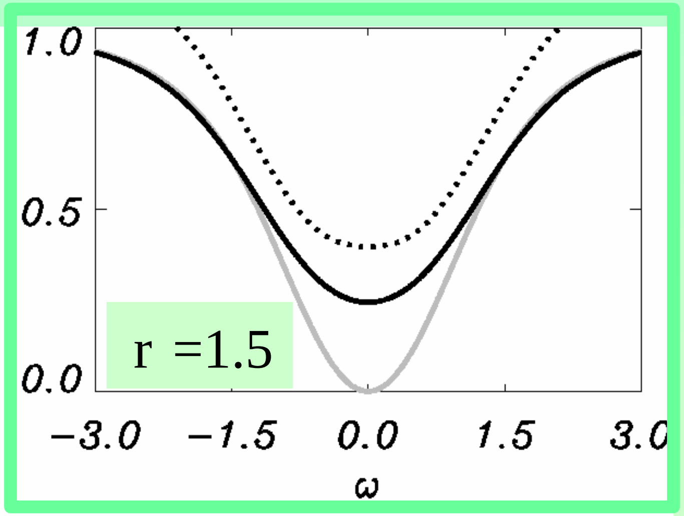
Q = 0



" Q







Polarization far field local properties

Stokes operators

$$\hat{S}_0(\vec{k},t) = \hat{A}_1^+(\vec{k},t)\hat{A}_1(\vec{k},t) + \hat{A}_2^+(\vec{k},t)\hat{A}_2(\vec{k},t) \qquad \uparrow + \rightarrow$$

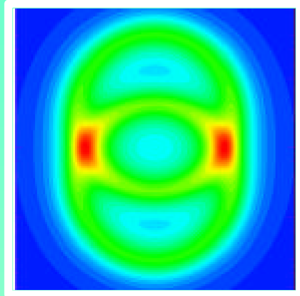
$$\hat{S}_1(\vec{k},t) = \hat{A}_1^+(\vec{k},t)\hat{A}_1(\vec{k},t) - \hat{A}_2^+(\vec{k},t)\hat{A}_2(\vec{k},t) \qquad \uparrow - \rightarrow$$

$$\hat{S}_2(\vec{k},t) = \hat{A}_1^+(\vec{k},t)\hat{A}_2(\vec{k},t) + \hat{A}_2^+(\vec{k},t)\hat{A}_1(\vec{k},t) \qquad \nwarrow - \nearrow$$

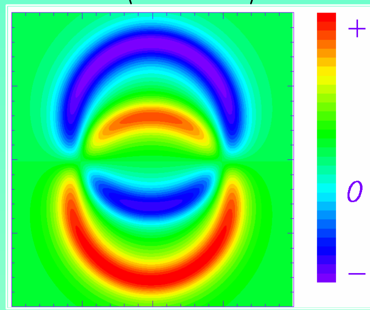
$$\hat{S}_3(\vec{k},t) = -i\left[\hat{A}_1^+(\vec{k},t)\hat{A}_2(\vec{k},t) - \hat{A}_2^+(\vec{k},t)\hat{A}_1(\vec{k},t)\right] \qquad \bigcirc - \bigcirc$$

$$D^2\hat{S}_1(\vec{k},t)D^2\hat{S}_2(\vec{k},t') \geq \frac{1}{4}\left|\left\langle\left[\hat{S}_1(\vec{k},t),\hat{S}_2(\vec{k},t')\right]\right\rangle\right|^2 = \left|\left\langle\hat{S}_3(\vec{k},t)\right\rangle\right|^2 \delta(t-t')$$

$\left\langle \hat{S}_0(\vec{k},t)\right\rangle$ = tot.intensity

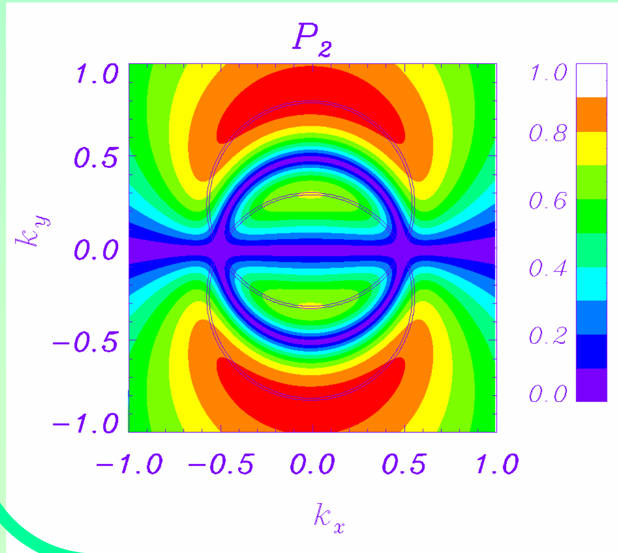


$\left\langle \hat{S}_1(\vec{k},t)\right\rangle$



$$\left\langle \hat{S}_2(\vec{k},t)\right\rangle = \left\langle \hat{S}_3(\vec{k},t)\right\rangle = 0$$

II order polarization degree $P_2 = \dfrac{\sum_{i=1}^{3}\left\langle \hat{S}_i(\vec{k},t)\right\rangle^2}{\left\langle \hat{S}_0(\vec{k},t)\right\rangle}$

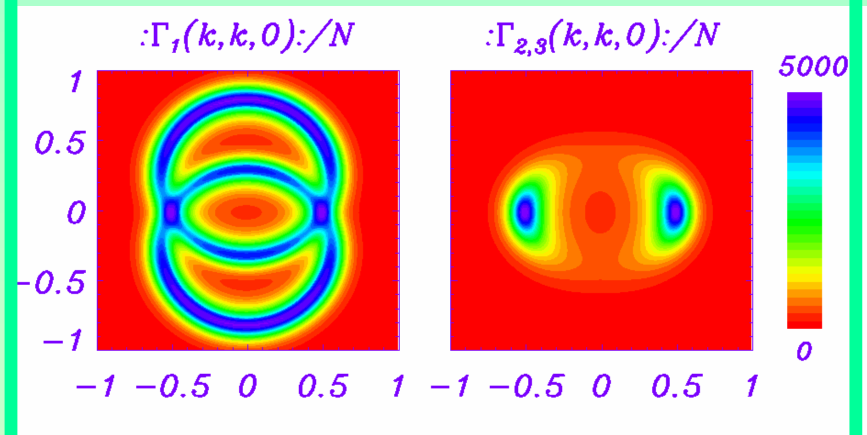


polarized

unpolarized

spectral varian ces $\Gamma_i(\vec{k},\vec{k}',\Omega) =$

$$\int e^{i\Omega t}\left\langle \hat{S}_i(\vec{k},t)\hat{S}_i(\vec{k}',0)\right\rangle - \left\langle \hat{S}_i(\vec{k},t)\right\rangle\left\langle \hat{S}_i(\vec{k}',0)\right\rangle dt$$



Classical variances, no squeezing, no hidden polarization

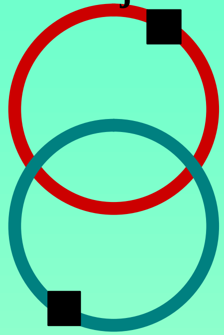
Polarization entanglement

Correlations between *opposite* far field points

$$D_1(\vec{k},\vec{k}',\Omega) =: \int e^{i\Omega t} \left\langle (\hat{S}_1(k,t)+\hat{S}_1(k',t))(\hat{S}_1(k,0)+\hat{S}_1(k',0)) \right\rangle dt$$

$$D_j(\vec{k},\vec{k}',\Omega) =: \int e^{i\Omega t} \left\langle (\hat{S}_j(k,t)-\hat{S}_j(k',t))(\hat{S}_j(k,0)-\hat{S}_j(k',0)) \right\rangle dt \quad (j=0,2,3)$$

$$\hat{\mathbf{S}}_j(+\vec{\mathbf{k}},\mathbf{t})$$



$$\hat{\mathbf{S}}_j(-\vec{\mathbf{k}},\mathbf{t})$$

$$\left\langle (\hat{S}_1(k,t)+\hat{S}_1(-k,t)) \right\rangle = \left\langle \hat{S}_j(k,t)-\hat{S}_j(-k,t) \right\rangle = 0 \quad (j=0,2,3)$$

$$\Delta^2(\hat{S}_1(\vec{k},t)+\hat{S}_1(-\vec{k},t'))\Delta^2(\hat{S}_2(\vec{k},t)-\hat{S}_2(-\vec{k},t')) \geq \left| \left\langle \hat{S}_3(\vec{k},t)-\hat{S}_3(-\vec{k},t') \right\rangle \right|^2 \delta(t-t') = 0$$

$$\mathbf{D}_0(\vec{\mathbf{k}},-\vec{\mathbf{k}},0) = \mathbf{D}_1(\vec{\mathbf{k}},-\vec{\mathbf{k}},0) = 0$$

perfect correlations

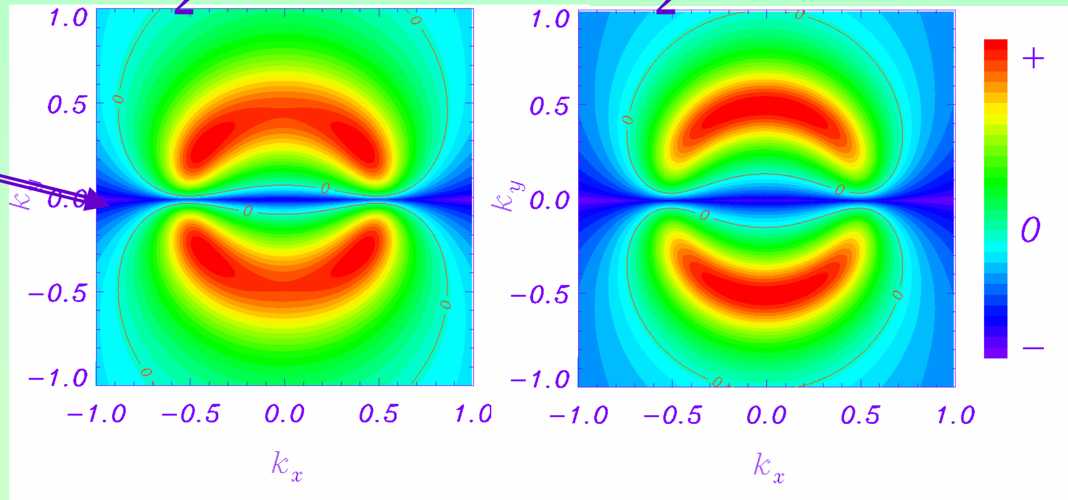
$$\mathbf{D}_2(\vec{\mathbf{k}},-\vec{\mathbf{k}},0) = \mathbf{D}_3(\vec{\mathbf{k}},-\vec{\mathbf{k}},0) \neq 0$$

IF k_y=0

$$\mathbf{D}_2(\mathbf{k}_x,-\mathbf{k}_x,0) = \mathbf{D}_3(\mathbf{k}_x,-\mathbf{k}_x,0) = 0$$

perfect correlations in ALL Stokes parameters!

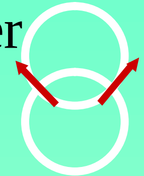
$:D_2:$ for r=1 $\quad$ $:D_2:$ for r=0.5



CONCLUSIONS

• 1) Spatial EPR entanglement in quadratures-polarization components **?**

When walk-off=0 or for k_y=0: almost complete noise suppression in the proper quadratures difference of any 'orthogonal' polarization components



• 2) Walk-off effects **?** The variance of the quadratures difference of orthogonal polarization components depends now on the reference polarization Θ, the EPR correlations are reduced and the quadrature angle depends on Θ.

• 1) Local properties of the polarization in the far field: noisy Stokes operators (>SN)

When walk-off=0 or for k_y=0 all the Stokes operators vanish on average (no limitation in precision of simultaneous measurements) and fluctuations are not sensitive to polarization optical elements.

• 2) Macroscopic polarization entanglement in OPO below threshold **?**

When walk-off=0 or for k_y=0 perfect correlations in all the Stokes parameters measured in opposite far field points.

• 3) Walk-off effects **?** The entanglement in the second and third Stokes parameters is lost in the far field regions affected by walk-off.