QUANTUM FLUCTUATIONS IN TYPE I OPO ABOVE THRESHOLD

R. Zambrini

P. Colet

M. San Miguel

I·M·E·D·E·A
Institut Mediterrani d'Estudis Avançats

S. Barnett

The University of Strathclyde in Glasgow

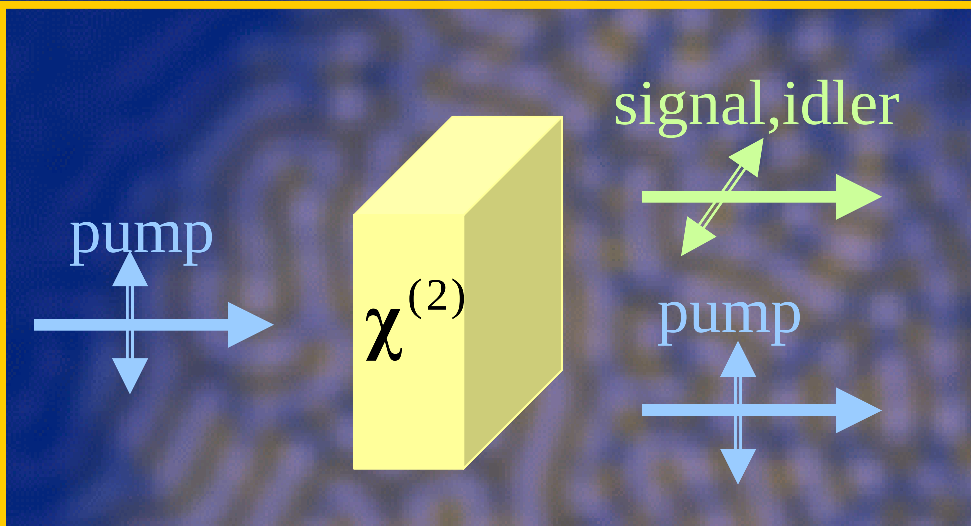
AIM: study QUANTUM NON-LINEAR correlations in optical patterns in type I OPO

When linearizing is acceptable?

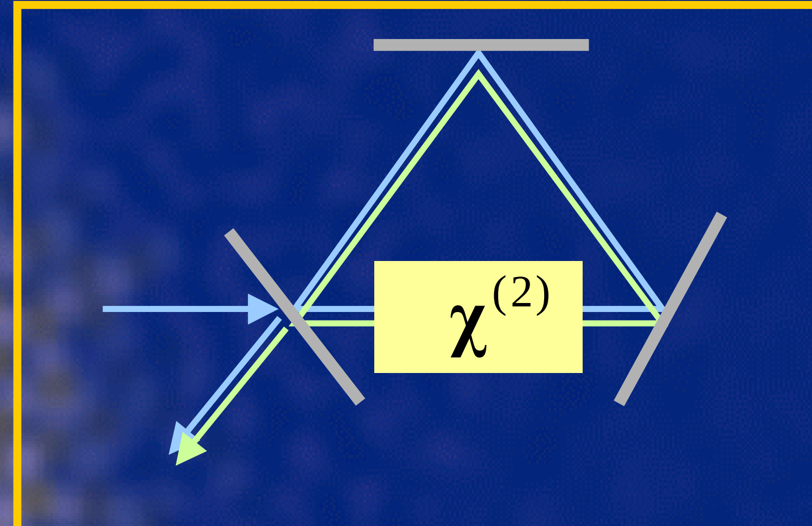
- ✓ STABLE STEADY STATE and “small” fluctuations
- ✗ NEAR THRESHOLD: critical fluctuations
- ✗ CONVECTIVE REGIME (walk-off): macroscopic amplified fluctuations
- ✗ PATTERN (walk-off): PHASE-AMPLITUDE and MULTIMODE COUPLINGS
- ✗ Spatial structures breaking translational invariance (large phase fluctuations)

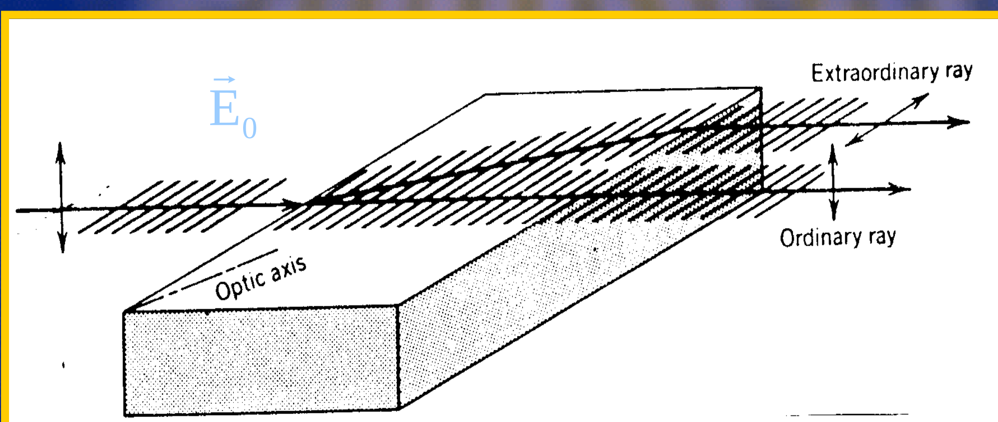
Model

Type I phase matching



OPO





$\vec{A}_1$

$\vec{A}_0$

- birefringence
- walk-off effect
- advection velocity

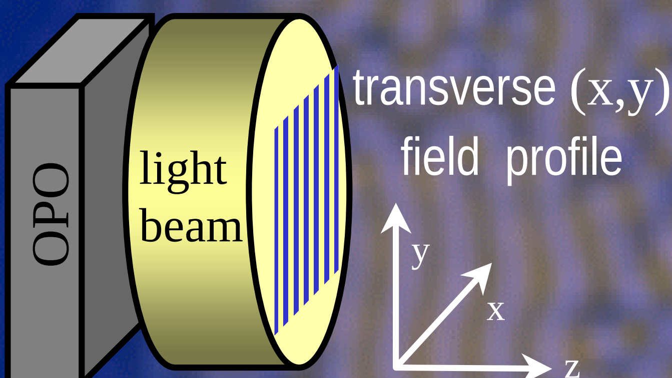
$$\nu \partial_y A_1(x, y)$$

Classical equations

$$\partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + ia_0\nabla^2 A_0 + 2igA_1^2\right] + \sqrt{\varepsilon_0}\,\xi_0(\mathbf{r},t)$$

$$\partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + \nu\partial_y A_1 + ia_1\nabla^2 A_1 + igA_1^* A_0\right] + \sqrt{\varepsilon_1}\,\xi_1(\mathbf{r},t)$$

paraxial, slowly varying, single mode and mean field approximations



∇^2	transverse Laplacian	$\mathbf{r}=(x,y)$	
Γ_i	cavity decay rates	Δ_i	cavity detunings
a_i	diffraction coeff.	E_0	pump amplitude
g	nonlinear coeff.	ξ_i	Gaussian white noise

- signal detuning $\Delta_1 < 0$
- input > signal generation threshold

PATTERN FORMATION

OUTLINE

PART 1 Type I OPO with walk-off

- Instability regimes.
- Two schemes featuring nonlinear Langevin equations.
- Quantum fluctuations and correlations at threshold and in the convective regime.
- Symmetry breaking effects on twin beams.

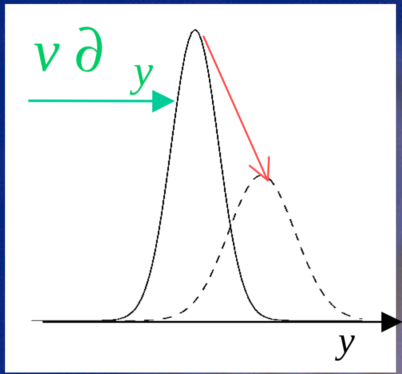
PART 2 Type I OPO without walk-off

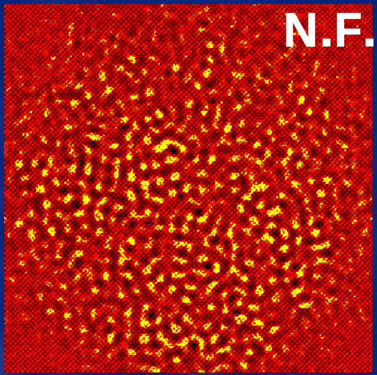
- Quantum correlations below and at threshold
- Twin beams in multimode patterns.
- Twin beams in disordered structures.

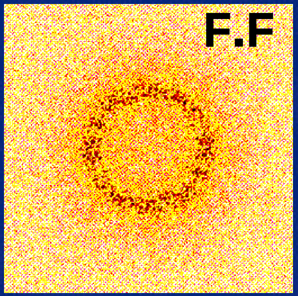
Stability in presence of walk-off: 3 dynamical regimes

DOPO:

N.F. F.F

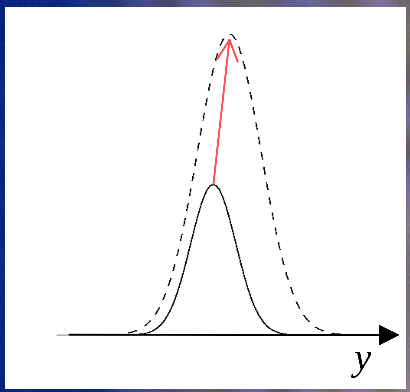


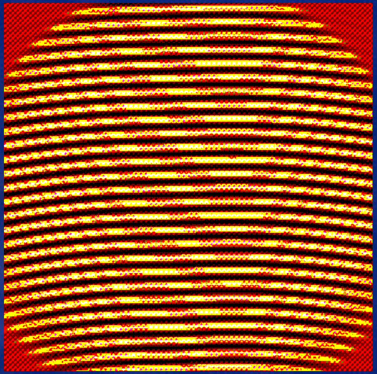


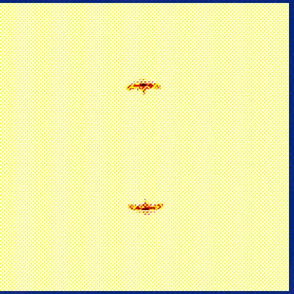


Absolutely Stable

Perturbation decays.
Regime of Quantum Images

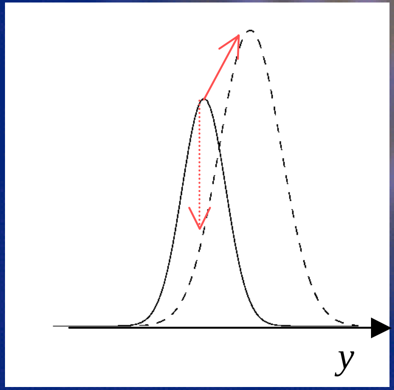


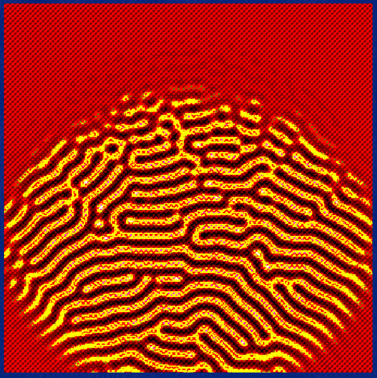


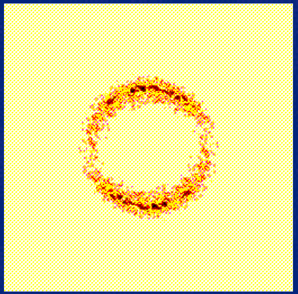


Absolutely unstable

Perturbation spreads faster than advection velocity







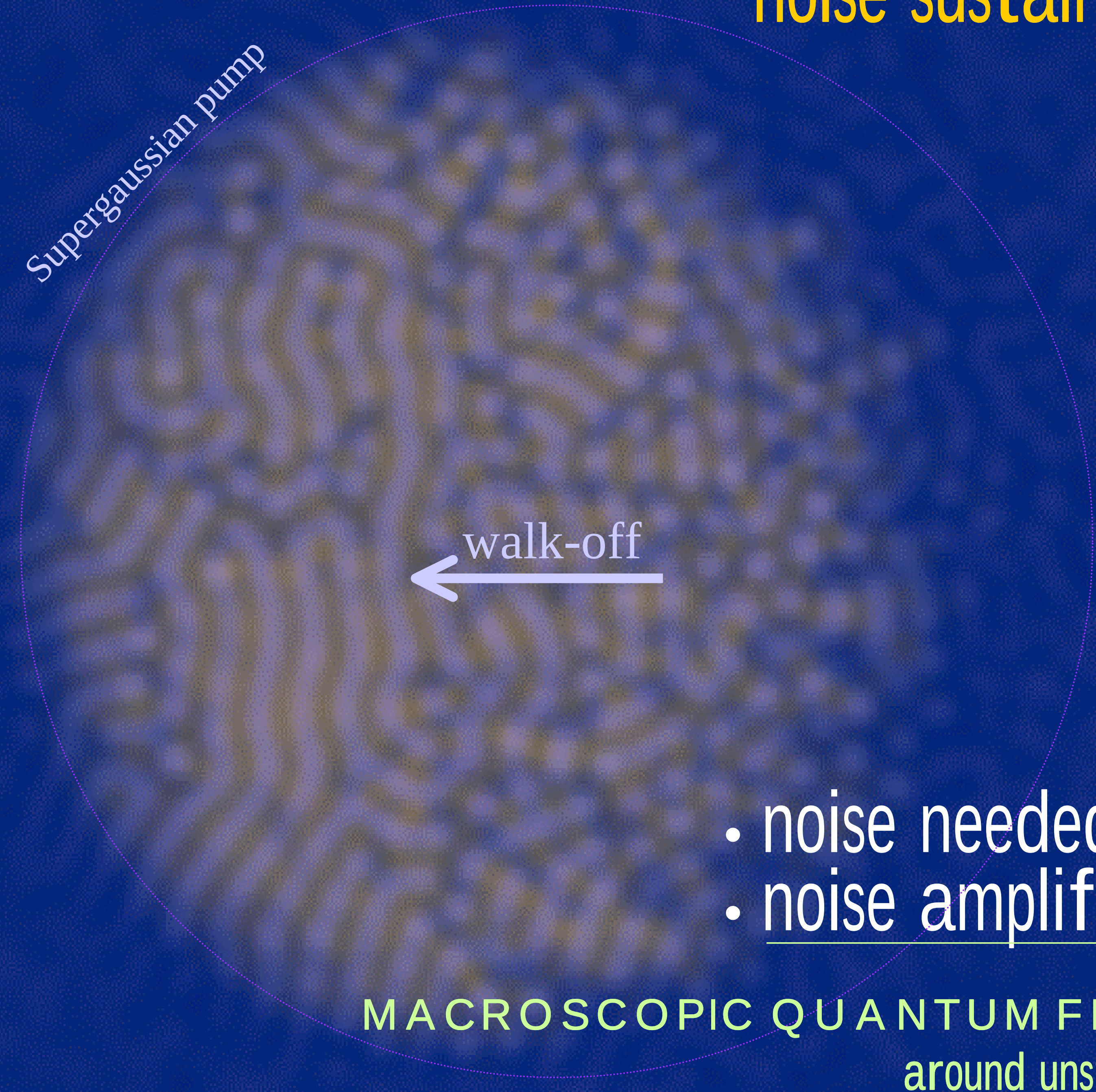
Convectively Unstable

Perturbation spreads slower than advection velocity

NOISE SUSTAINED PATTERNS

*M.Santagiustina & al., PRL, **79**, 3633 (1997)*

noise sustained pattern



- noise needed at all times
- noise amplification

MACROSCOPIC QUANTUM FLUCTUATIONS

around unstable reference state

Quantum description of the convective regime

Hamiltonian

$$\hat{H}_f = \hbar\int d^2\vec{x}\left[\gamma_0\hat{A}_0^+\left(\Delta_0 - a_0\nabla^2\right)\hat{A}_0 + \gamma_1\hat{A}_1^+\left(\Delta_1 - a_1\nabla^2 + i\nu\partial_y\right)\hat{A}_1\right]$$

$$\hat{H}_{ext} = i\hbar\int d^2x\left(E_0\hat{A}_0^+ - E_0^*\hat{A}_0\right)$$

$$\hat{H}_{int} = i\hbar\frac{g}{2}\int d^2x\left[\hat{A}_0\left(\hat{A}_1^+\right)^2 - \hat{A}_0^+\hat{A}_1^2\right]$$

Heisenberg equations

$$\partial_t\hat{A}_0 = -\gamma_0\left(1 + i\Delta_0 - ia_0\nabla^2\right)\hat{A}_0 + E_0 - \frac{g}{2}\hat{A}_1^2 + \hat{F}_0$$

$$\partial_t\hat{A}_1 = -\gamma_1\left(1 + i\Delta_1 - ia_1\nabla^2 - \nu\partial_y\right)\hat{A}_1 + g\hat{A}_0\hat{A}_1^+ + \hat{F}_1$$

$$\left[\hat{F}_j(\vec{x},t), \hat{F}_k^+(\vec{x}',t')\right] = \delta_{jk}\,2\gamma_j\delta(\vec{x}-\vec{x}')\delta(t-t')$$

Problems in phase space description: ***W*, *P*, P_+, *Q***

Linearization not useful: fluctuations amplified from an unstable state

Two proposals of **NONLINEAR** approximations:

- time dependent parametric approximation (*W*-representation)
- Langevin equations in the *Q*-representation

OUTLINE

PART 1 Type I O P O with walk-off

- Instability regimes.
- Two schemes featuring nonlinear Langevin equations.
- Quantum fluctuations and correlations at threshold and in the convective regime.
- Symmetry breaking effects on twin beams.

PART 2 Type I O P O without walk-off

- Quantum correlations below and at threshold
- Twin beams in multimode patterns.
- Twin beams in disordered structures.

Time dependent parametric approximation

Convective regime:

- amplification of quantum noise in signal field
- macroscopic stable pump

approximation

- $\hat{A}_0 \Rightarrow A_0$ classical macroscopic pump field
- Then $\hat{H}^{eff}$ quadratic Hamiltonian for the operator $\hat{A}_1$
- Functional Fokker-Planck equation for W
- partial differential Langevin equation for the signal

$$\partial_t \alpha_1 = -\gamma_1\left(1 + i\Delta_1 - ia_1\nabla^2 - \nu\partial_y\right)\alpha_1 + gA_0\alpha_1^* + \sqrt{\gamma_1}\,\xi(\vec{x},t)$$

Complex white Gaussian noise

Heisenberg equation for the pump $\hat{A}_0$

- $F_0 = 0$
- $\hat{A}_1^2 \Rightarrow \alpha_1^2$ c-number field

$$\partial_t A_0 = -\gamma_0\left(1 + i\Delta_0 - ia_0\nabla^2\right)A_0 + E_0 - \frac{g}{2}\alpha_1^2$$

Mean value = expectation value of Heisenberg equation

Q-representation

Functional derivative equation for $Q(\alpha_0, \alpha_1)$

$$\frac{\partial Q}{\partial t} = \left[-\left(\frac{\delta}{\delta\alpha_i} V_i + c.c. \right) - \frac{g}{2}\left(\alpha_0 \frac{\delta^2}{\delta\alpha_1^2} + c.c. \right) + 2\gamma_i \frac{\delta^2}{\delta\alpha_i \delta\alpha_i^*} \right] Q$$

$$\left. \begin{aligned} V_0 &= -\gamma_0 \left(1 + i\Delta_0 - ia_0 \nabla^2\right)\alpha_0 + E_0 - \frac{g}{2}\alpha_1^2 \\ V_1 &= -\gamma_1 \left(1 + i\Delta_1 - \nu\partial_y - ia_1 \nabla^2\right)\alpha_1 + g\alpha_0 \alpha_1^* \end{aligned} \right\} \text{drift}$$

$$|\alpha_0| < 2\frac{\gamma_1}{g} = 2\left|\alpha_0^{thr}\right|$$

D>0

partial differential Langevin equations

$$\partial_t \alpha_0 = V_0(\alpha_0, \alpha_1) + \sqrt{2\gamma_0}\,\xi_0(\vec{x}, t)$$

$$\partial_t \alpha_1 = V_1(\alpha_0, \alpha_1) + \sqrt{2\gamma_1}\,\xi_1(\vec{x}, t)$$

Phase sensitive multiplicative noise

$$\left\langle \xi_0(\vec{x},t)\xi_0^*(\vec{x}',t') \right\rangle = \delta(\vec{x}-\vec{x}')\delta(t-t')$$

$$\left\langle \xi_1(\vec{x},t)\xi_1^*(\vec{x}',t') \right\rangle = \delta(\vec{x}-\vec{x}')\delta(t-t')$$

$$\left\langle \xi_1(\vec{x},t)\xi_1(\vec{x}',t') \right\rangle = -\frac{g}{2\gamma_1}\alpha_0(\vec{x},t)\delta(\vec{x}-\vec{x}')\delta(t-t')$$

OUTLINE

PART 1 Type I O P O with walk-off

- Instability regimes.
- Two schemes featuring nonlinear Langevin equations.
- Quantum fluctuations and correlations at threshold and in the convective regime (T D P A).
- Symmetry breaking effects on twin beams.

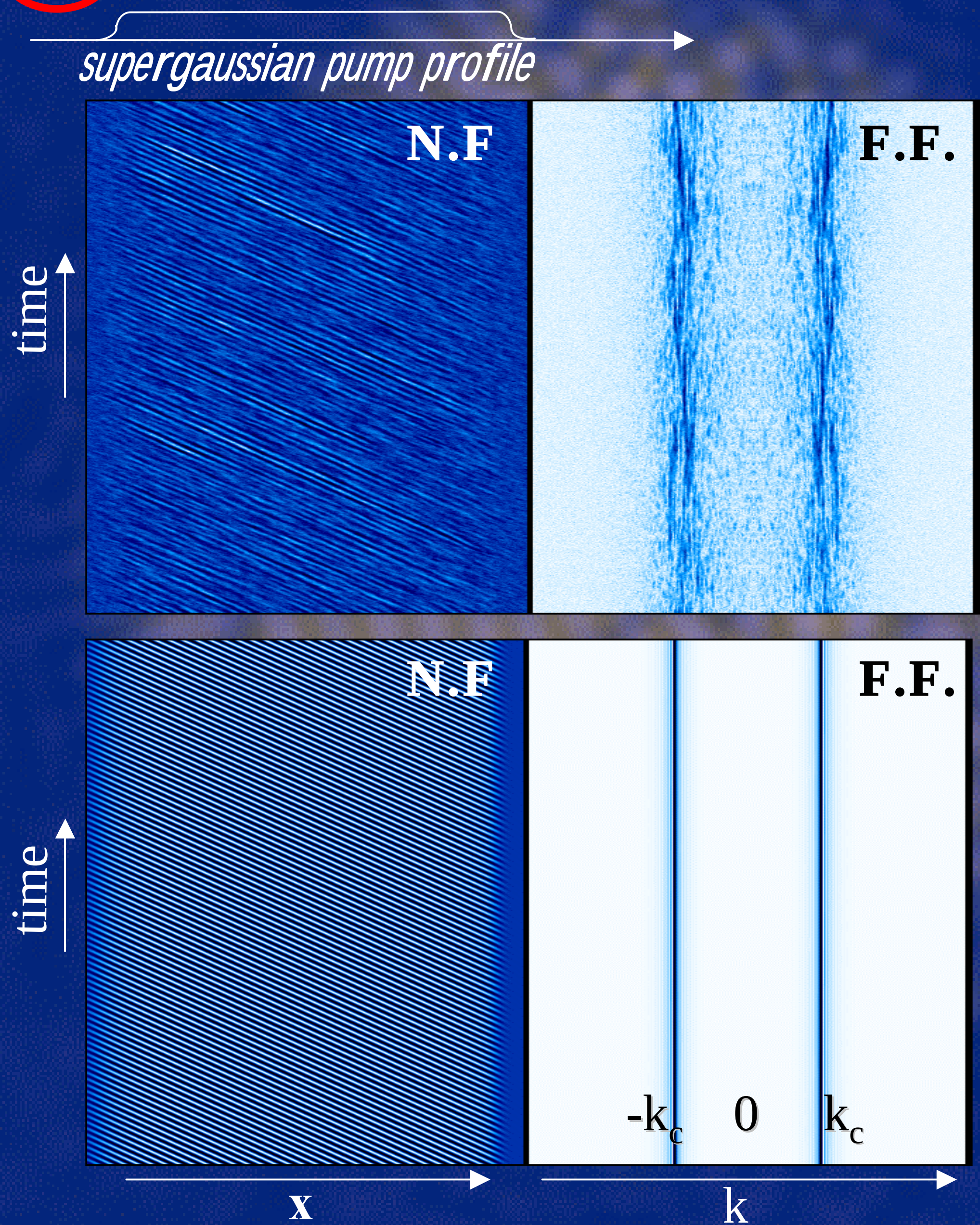
PART 2 Type I O P O without walk-off

- Quantum correlations below and at threshold
- Twin beams in multimode patterns.
- Twin beams in disordered structures.

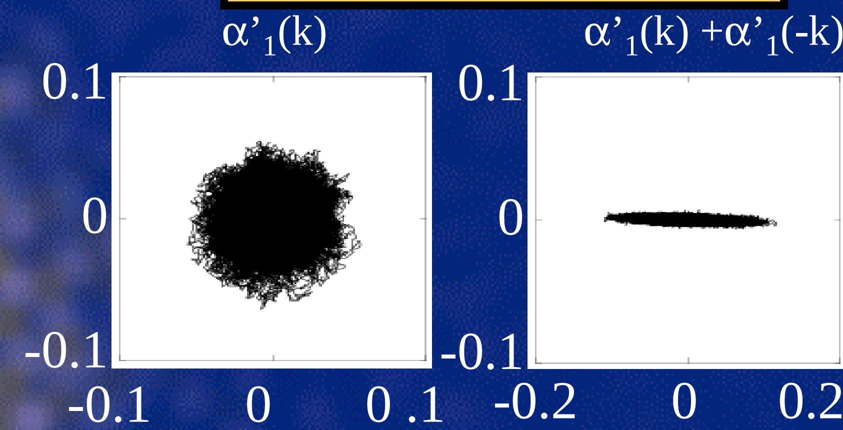
Phase space trajectories (TDPA)

1D

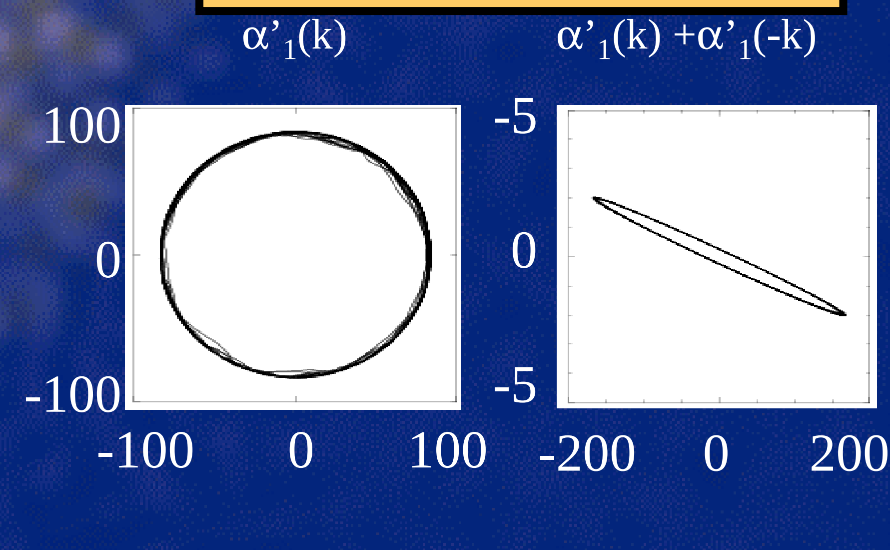
$$\alpha_1(k) = \alpha'_1(k)\, e^{i\omega(k)t}$$



Below threshold



Absolutely unstable regime

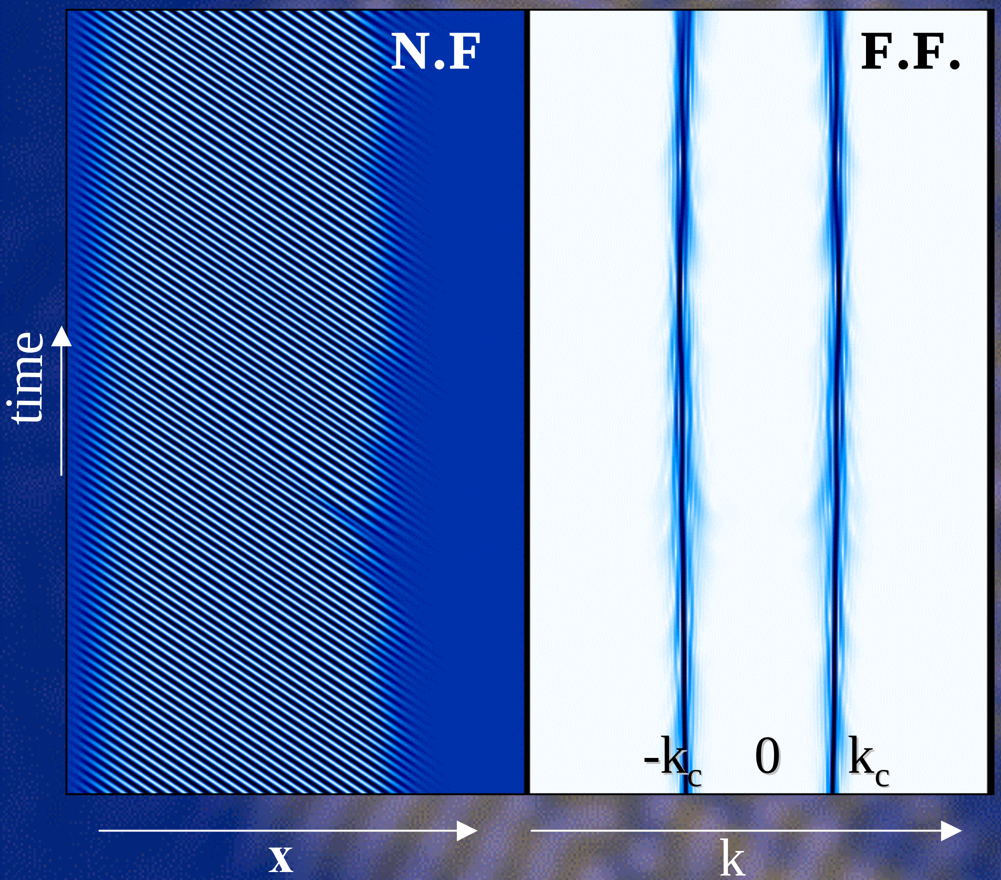


Phase space trajectories

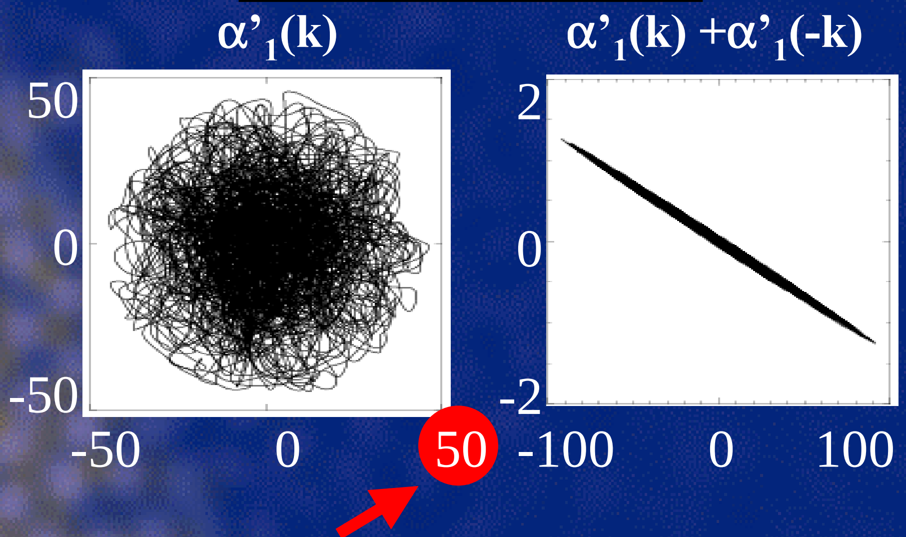
$$\alpha_1(k) = \alpha'_1(k)\, e^{i\omega(k)t}$$

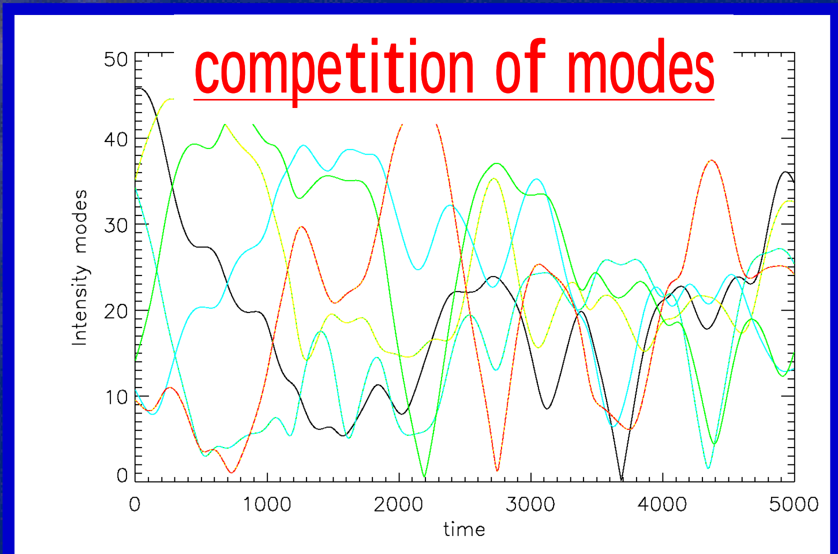
1D

supergaussian pump profile



Convective regime

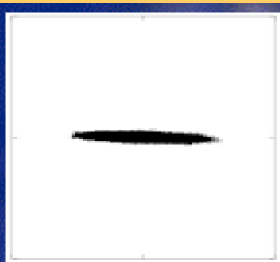




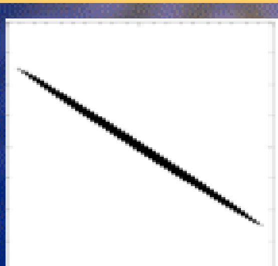
From TRAJECTORIES to probability DISTRIBUTIONS

$W(\alpha_1(k)+\alpha_1(-k))$

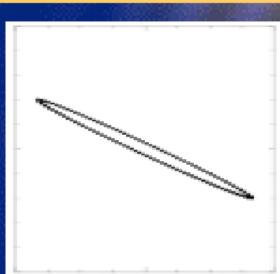
Below threshold

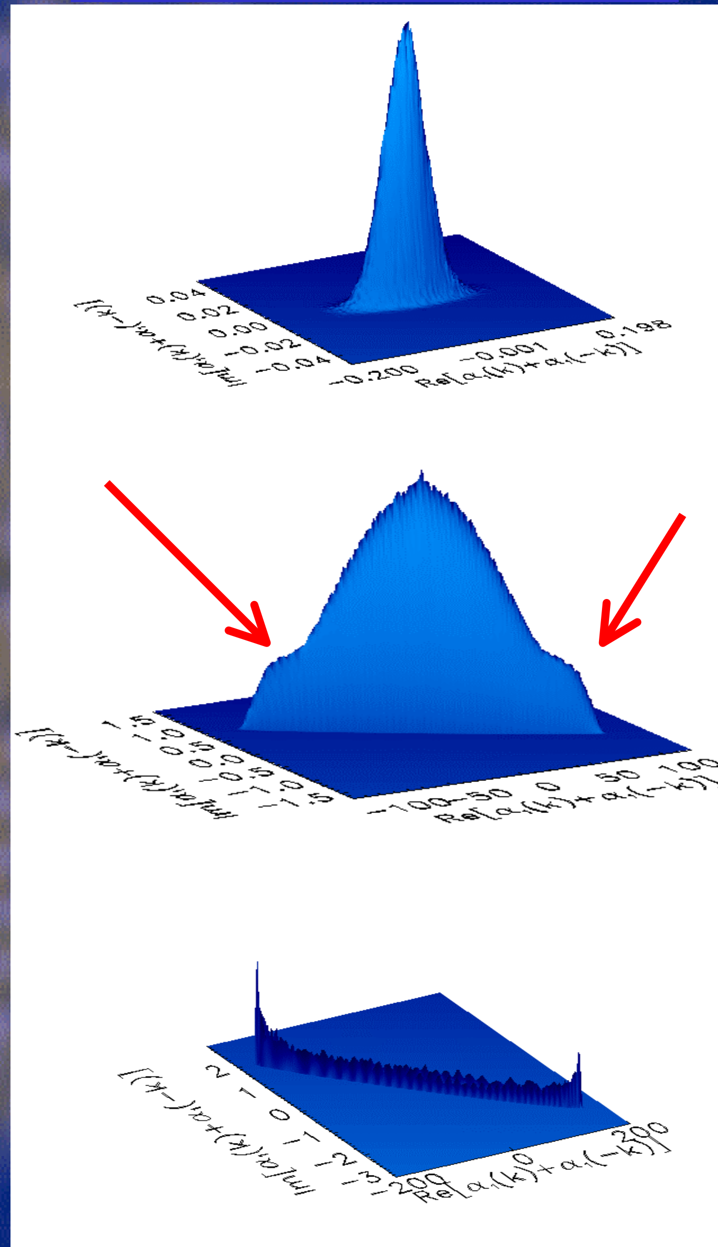


Convective regime



Absolutely unstable regime





precursors of peaks above threshold

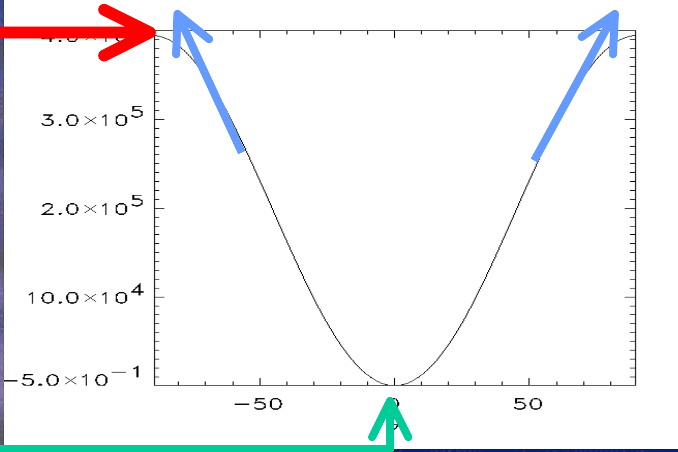
NON GAUSSIAN DISTRIBUTION

nonlinear regime

NONLINEAR QUANTUM CORRELATIONS AT THRESHOLD

• Squeezing in quadratures of FF modes

$$\frac{\left\langle :\left(\mathrm{Re}\left\{\left[\alpha_1(k_c)-\alpha_1(-k_c)\right]e^{i\theta}\right\}\right)^2 :\right\rangle}{\mathrm{S.N.}}$$



• nonlinear model
finite value of variance

• at threshold: θ=0, 50% squeezing

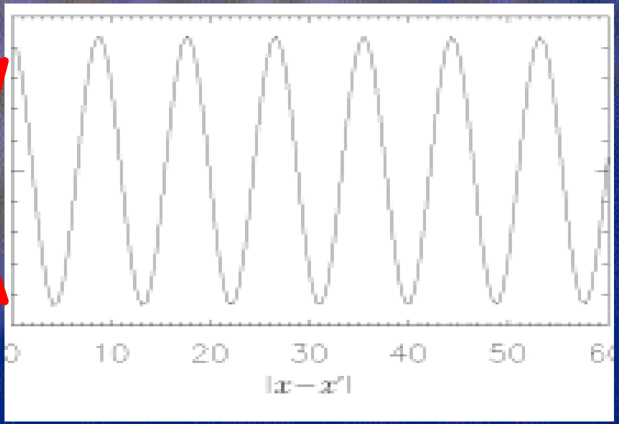
• **Linear** model: divergence of fluctuations in unsqueezed quadratures ($\sim A_0/(A_0-1)$)

• Near field correlations in quadratures

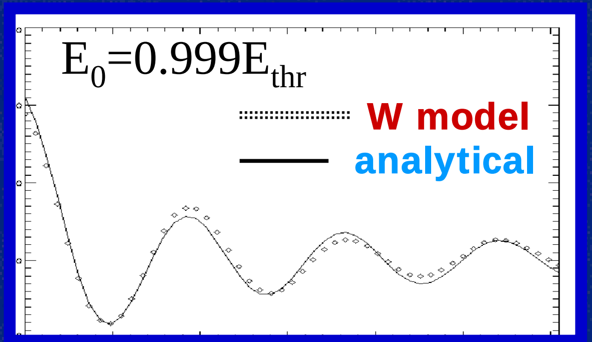
$$\left\langle : \mathrm{Re}\left(\alpha_1(\vec{x})\right)\mathrm{Re}\left(\alpha_1(\vec{x}')\right) : \right\rangle$$

• Finite amplitude of oscillations

• Critical behavior



good agreement with linear analytical results below thr.



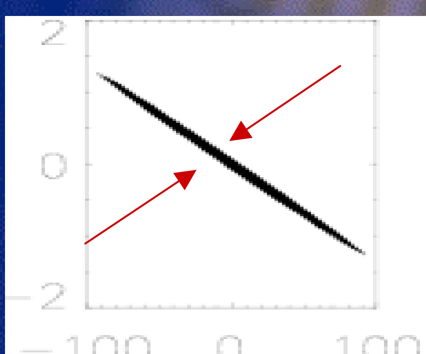
NONLINEAR QUANTUM CORRELATIONS in CONVECTIVE REGIME ($1 < E_0/E_{thr} < 1.035$)

very near to threshold ($E_0/E_{thr} \approx 1.001$) Gaussian fluctuations:

- squeezing in quadratures below S.N. $\langle : (\mathrm{Re}[\alpha_1(k_c) - \alpha_1(-k_c)])^2 : \rangle = -0.5\,\mathrm{S.N.}$
- twin beams perfect intensity correlations $\langle : (\delta\hat{N}_1(k_c) - \delta\hat{N}_1(-k_c))^2 : \rangle \cong -0.5\,\mathrm{S.N.}$

$E_0/E_{thr} \approx 1.01$ **NONLINEAR** MACROSCOPIC FLUCTUATIONS

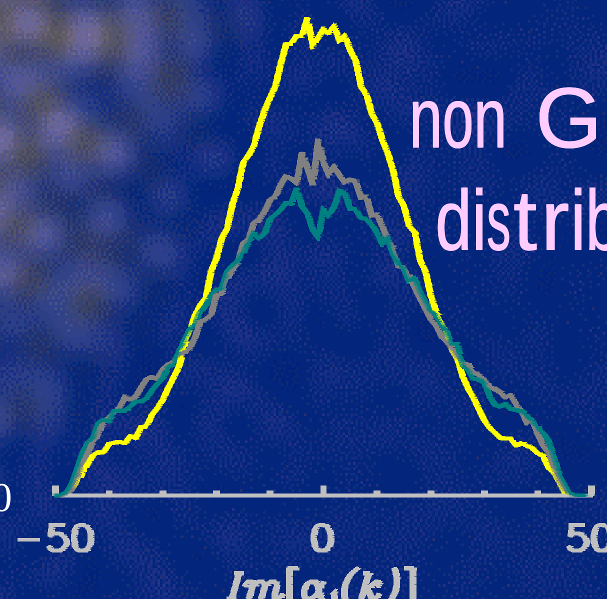
Squeezing of macroscopic fluctuations, but not below S.N.



classical correlations

$: (\mathrm{Re}\{[\alpha_1(k_c) - \alpha_1(-k_c)]e^{i\theta}\})^2 : \rangle > 0$

$\langle : (\delta\hat{N}_1(k_c) - \delta\hat{N}_1(-k_c))^2 : \rangle > 0$



non Gaussian distributions

OUTLINE

PART 1 Type I O P O with walk-off

- Instability regimes.
- Two schemes featuring nonlinear Langevin equations.
- Quantum fluctuations and correlations at threshold and in the convective regime.
- Symmetry breaking effects on twin beams

PART 2 Type I O P O without walk-off

- Quantum correlations below and at threshold
- Twin beams in multimode patterns.
- Twin beams in disordered structures.

SYMMETRY BREAKING EFFECTS

Transverse walk-off - conserves momentum (translational invariance)

- breaks the FF reflection symmetry $k \longleftrightarrow -k$

questions:

Spatial twin beams in presence of walk-off? $N_1(k_c) = N_1(-k_c)$?

Absolutely stable regime with walk-off (flat pump)

Linearized problem around the steady state below thr.

$$H^L_{int} = i\hbar \frac{g}{2} \int dk \left[A_0^{st} \hat{A}_1^+(k) \hat{A}_1^+(-k) - h.c. \right]$$

$$\left[H^L_{int}, N_1(k_c) - N_1(-k_c) \right] = 0$$

Twin beams in the linear regime

Non-linear 3 MODES interaction

$$H^L_{int} = i\hbar \frac{g}{2} \left[\hat{A}_0(0) \hat{A}_1^+(k_c) \hat{A}_1^+(-k_c) - h.c. \right]$$

$$\left[H^{(3)}_{int}, N_1(k_c) - N_1(-k_c) \right] = 0$$

Momentum conservation: twin beams

Non-linear 5 MODES interaction

$$H^{(5)}_{int}\left(\hat{A}_0(0), \hat{A}_1(\pm k_c), \hat{A}_0(\pm 2k_c) \right)$$

$$\left[H^{(5)}_{int}, N_1(k_c) - N_1(-k_c) \right] \neq 0$$

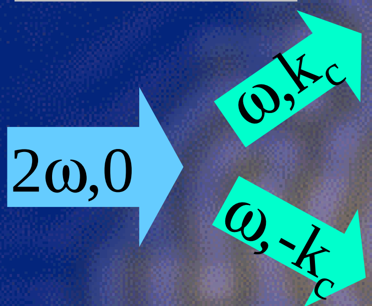
$$\text{if} \quad A_i(k) = A_i(-k) \quad \text{then} \quad [\,,\,] = 0$$

Dependence on $A_i(k)$ and $A_i(-k)$: different for the broken symmetry!

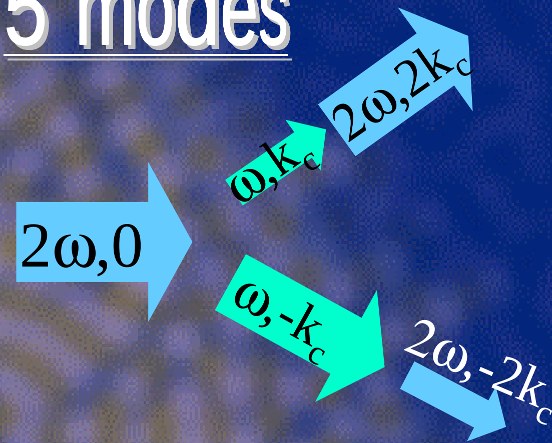
walk-off → $N_1(k_c) - N_1(-k_c)$ not a constant of motion

Numerical result: $N_1(k_c) \gtrless N_1(-k_c)$ depending on the sign of walk-off

3 modes



5 modes

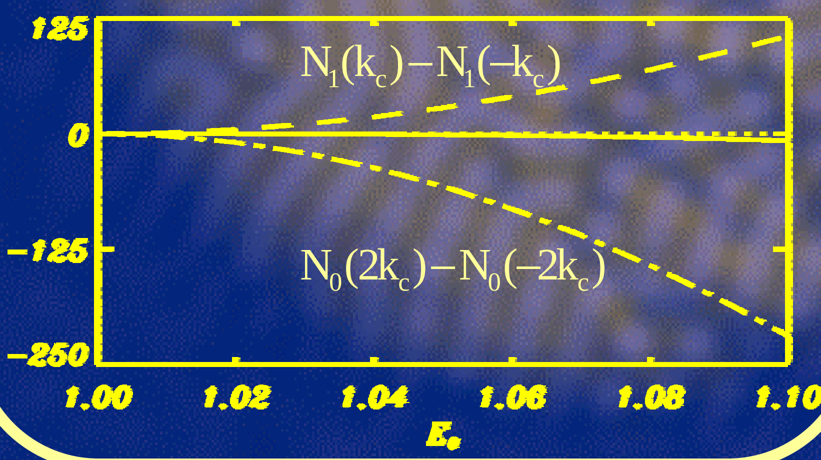


If $N_1(k_c) < N_1(-k_c)$

then $N_0(2k_c) > N_0(-2k_c)$

Conservation of the total transversal momentum

$$\int dk\left[N_0(k) + N_1(k)\right]k$$



Twin beams correlations in non equivalent beams:

one beam is more intense and more noisy,

but the intensity difference is sub-Poissonian

$$\frac{\left\langle :(\delta\hat{N}_1(k_c) - \delta\hat{N}_1(-k_c))^2: \right\rangle}{S.N.(k_c)} \cong -0.5$$

OUTLINE

PART 1 Type I O P O with walk-off

- Instability regimes.
- Two schemes featuring nonlinear Langevin equations.
- Quantum fluctuations and correlations at threshold and in the convective regime.
- Symmetry breaking effects on twin beams.

PART 2 Type I O P O without walk-off

- Quantum correlations below and at threshold
- Twin beams in multimode patterns.
- Twin beams in disordered structures.

Type I OPO with vanishing walk-off

Model:

Q-representation $|\alpha_0| < 2\frac{\gamma_1}{g} = 2|\alpha_0^{thr}|$ ⟹ D> 0

large range of validity !

partial differential Langevin equations

$$\partial_t \alpha_0 = -\left[(1 + i\Delta_0) - i\nabla^2\right]\alpha_0 + E_0 - \tfrac{1}{2}\alpha_1^2 + \sqrt{\frac{2}{\sqrt{a}}}\,\frac{g}{\gamma}\,\xi_0(x,t)$$

$$\partial_t \alpha_1 = -\left[(1 + i\Delta_1) - 2i\nabla^2\right]\alpha_1 + \alpha_0\alpha_1^* + \sqrt{\frac{2}{\sqrt{a}}}\,\frac{g}{\gamma}\,\xi_1(x,t)$$

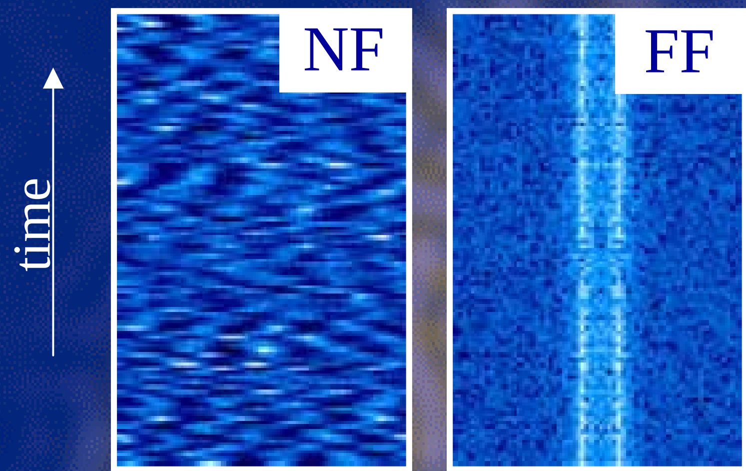
vanishing walk-off !

phase sensitive multiplicative noise depending on α_0

AIM: study of the FF spatial twin beams correlations from below threshold to $1.5E_{thr}$

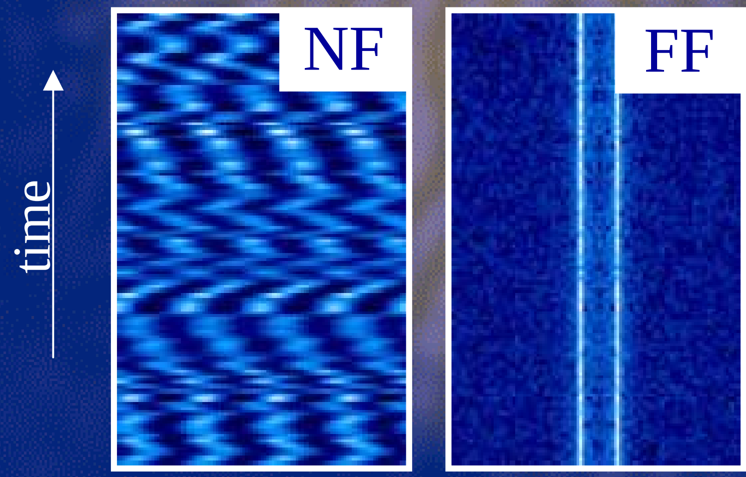
– stripe patterns(harmonics), frozen chaos –

Below threshold ($0.999E_{thr}$)



Quadrature and twin beams correlations below threshold can be analytically calculated within linear approximation.

At threshold (E_{thr})



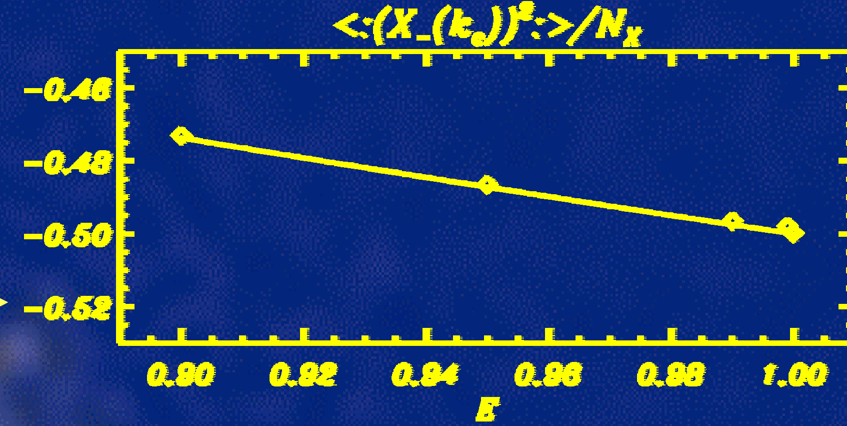
Linear approximations fail in predicting correlations of undamped quantities.

NONLINEAR calculations of critical fluctuations correlations.

• Quadrature correlations

Squeezed quadrature

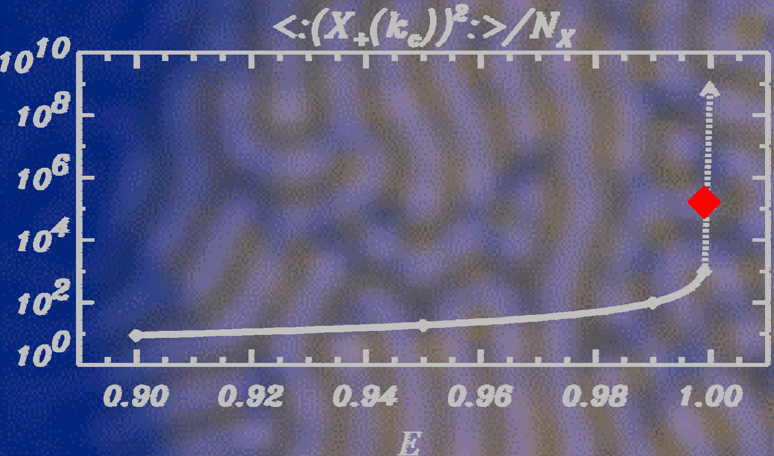
Agreement with analytical linear results



Unsqueezed quadrature:agreement below threshold

Divergence at threshold in linear treatment.

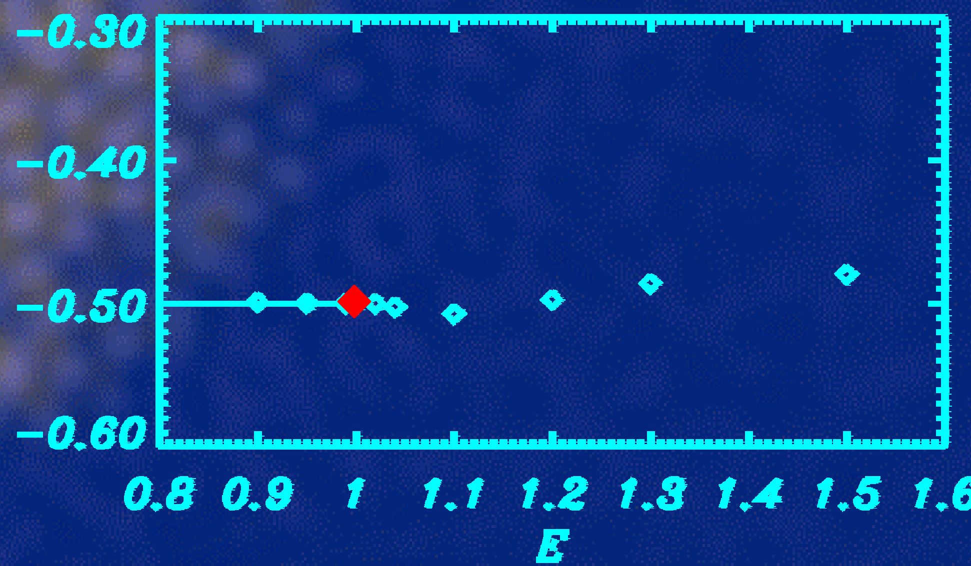
Nonlinear saturation at threshold: finite variance!



• Intensity correlations

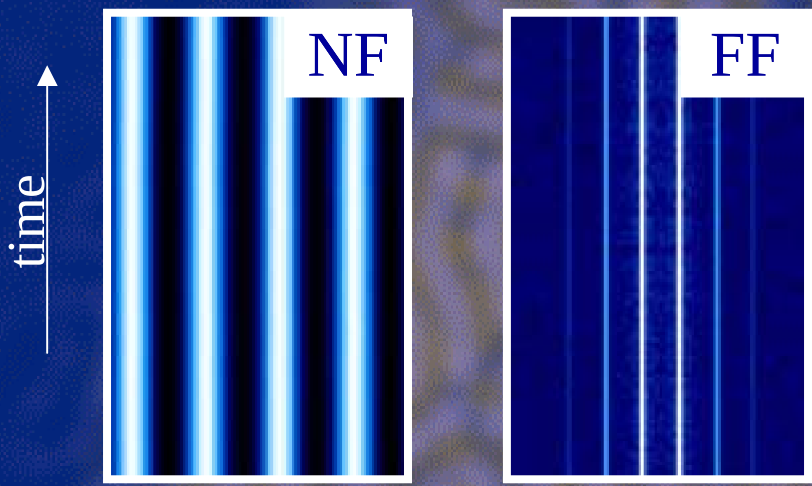
$$\frac{\left\langle :(\delta\hat{N}_1(k_c)-\delta\hat{N}_1(-k_c))^2: \right\rangle}{S.N.(k_c)} \cong -0.5$$

Twin beams correlations crossing the threshold



Above threshold ($1.1E_{thr}$)

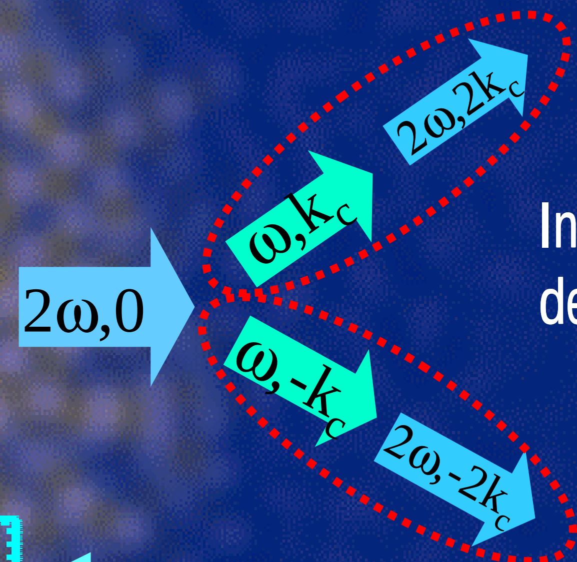
Stationary rolls



Nonlinear processes of harmonics generation.

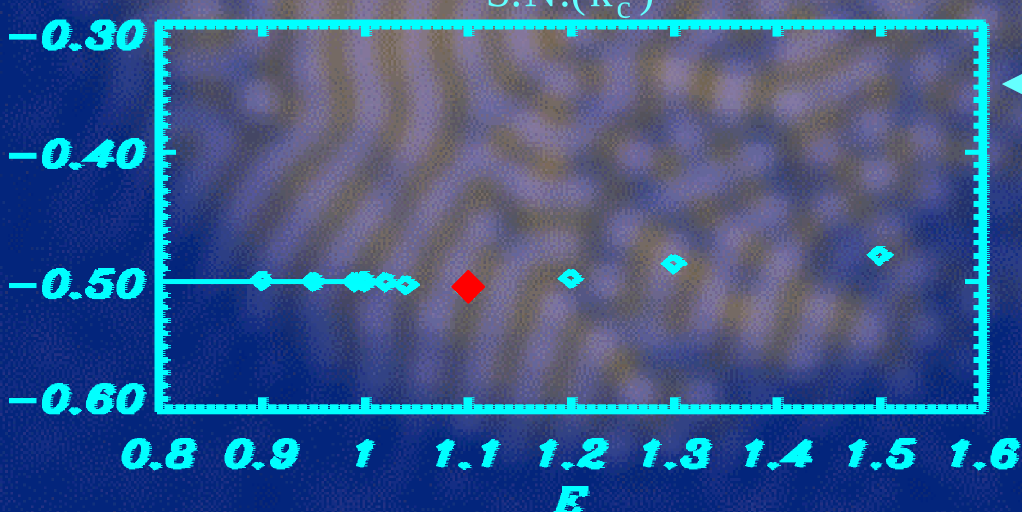
Twin beams in multimode interaction?

No momentum conservation constraint.



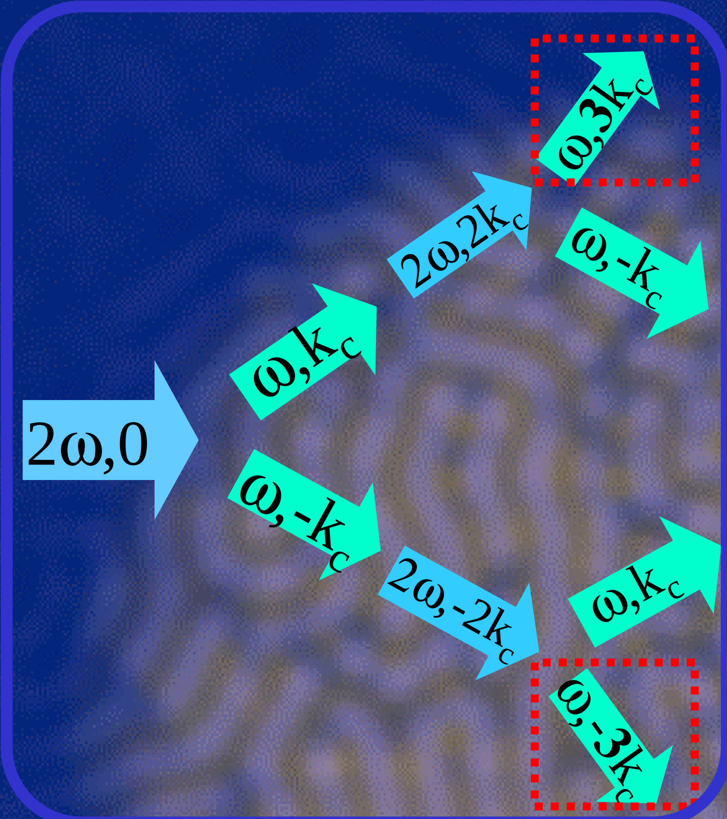
Incoherent depletion?

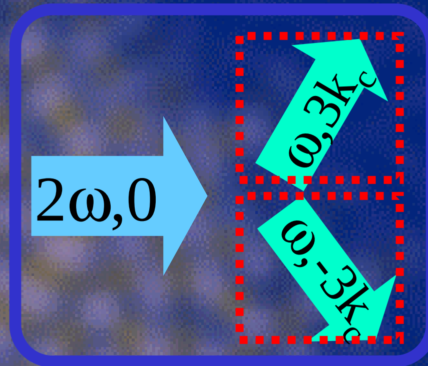
$$\frac{\left\langle :(\delta\hat{N}_1(k_c)-\delta\hat{N}_1(-k_c))^2: \right\rangle}{S.N.(k_c)}$$



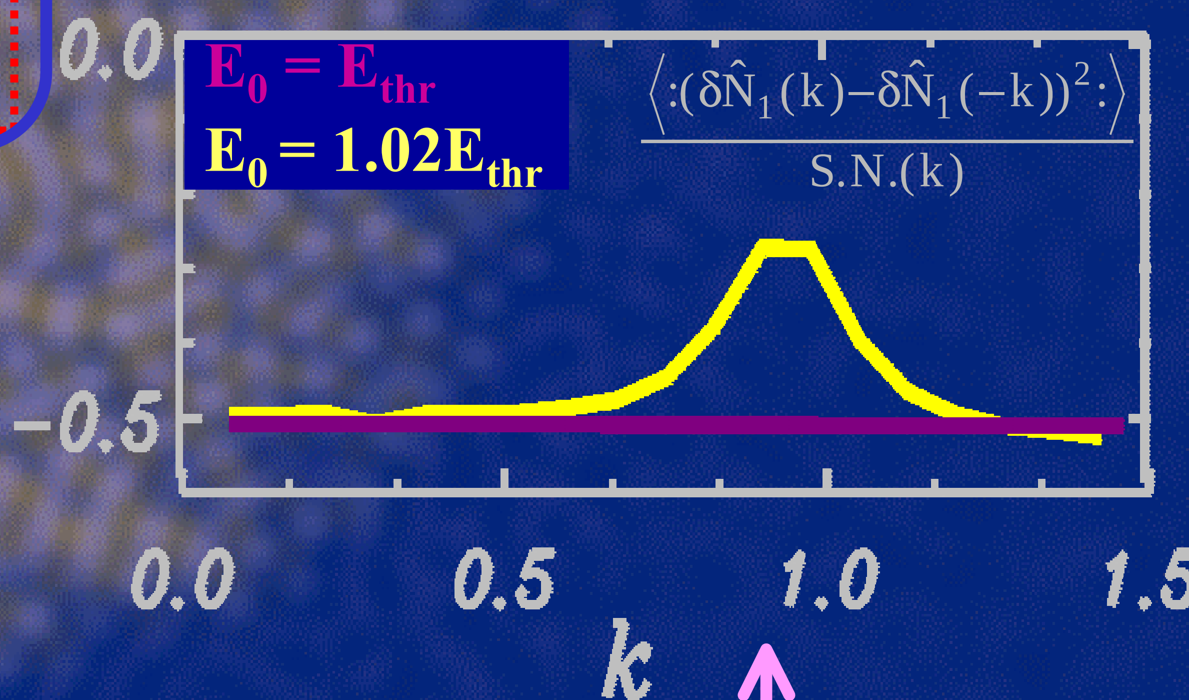
Twin beams correlations between the critical modes survive in multimode interactions!

Secondary processes



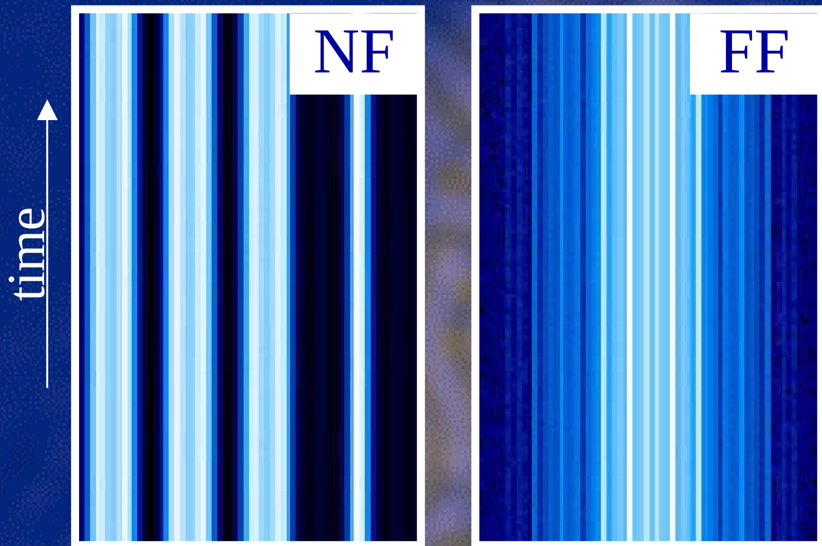


Generation of the spatial third harmonic in the signal

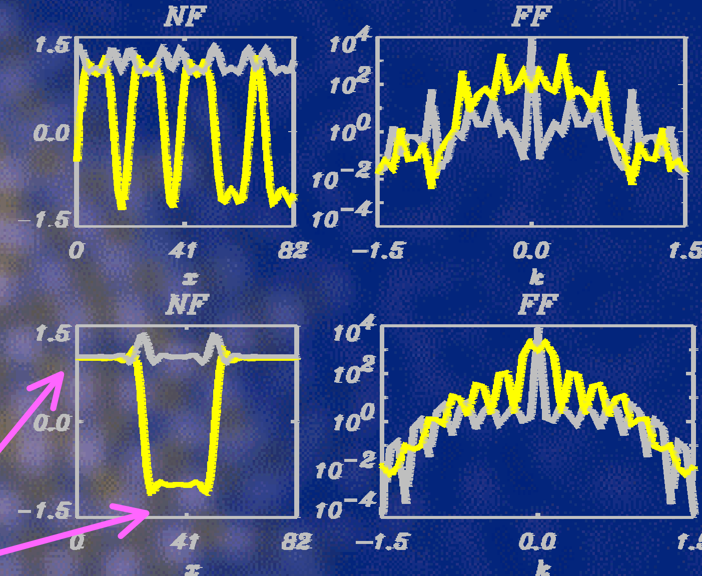


Reduction of the twin beams correlations for the third harmonic $\pm 3k_c$

Further above threshold ($1.5E_{thr}$)



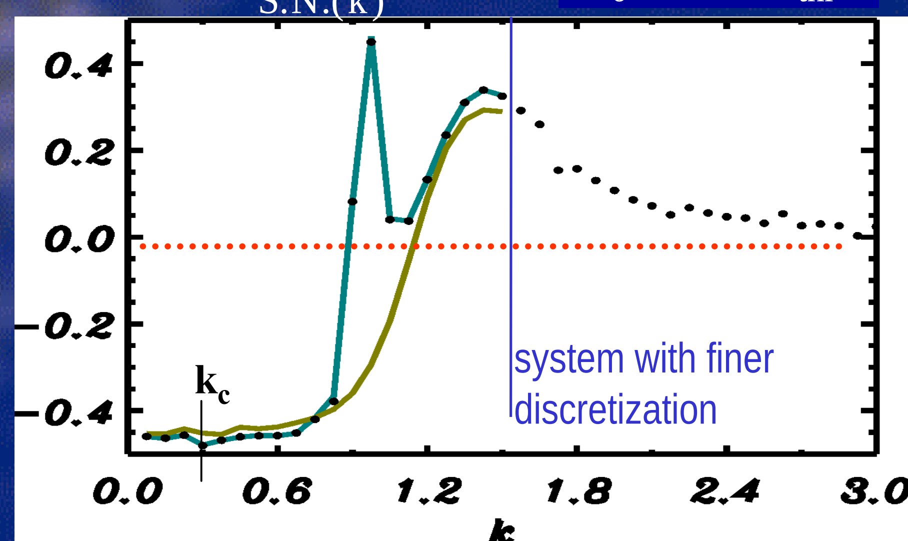
Spatial profile of signal and pump (2initial conditions)



2 homogeneous stable solutions
connected by fronts with oscillatory tails

$$\frac{\left\langle :(\delta\hat{N}_1(k)-\delta\hat{N}_1(-k))^2: \right\rangle}{S.N.(k)}$$

$E_0 = 1.05E_{thr}$



Twin beams in a disordered structure

- bandwidth
- dependence on selected spatial structure (ex: from rolls, step)

Summary

• Proposal of two new methods (TDPA and Q-representation) to describe nonlinear quantum fluctuations in OPO.

Type I OPO with walk-off (TDPA method) :

• Quantum correlations of critical fluctuations at threshold.

• Convective regime, patterns sustained by quantum noise:

- nongaussian Wigner distribution;

- intensity and quadrature correlations degraded when fluctuations are macroscopic: classical correlations.

• Symmetry breaking effects due to walk-off in multimode patterns:

one mode is more intense and fluctuates more than the opposite mode, but twin beams correlations are preserved.

Summary (continue)

Type I O P O without walk-off (Q-representation method) :

• Spatial twin beams quantum correlations in a wide parameter region (50% above threshold)

– at threshold for all k (momentum conservation);

– in multimode stripe patterns for all k. (secondary processes = no momentum conservation constraint).

– in disordered structures (frozen chaos) for a bandwidth of k.

Influence of the selected spatial structure on the spatial spectrum of squeezing.

References:

• “Macroscopic quantum fluctuations in noise-sustained optical patterns”
Roberta Zambrini, Stephen M. Barnett, Pere Colet and Maxi San Miguel
Phys. Rev. A **65**, 023813 (2002). (Erratum **65**, 049901 (2002)).

• “Twin beams, non linearity and walk-off in Optical Parametric Oscillators”
Roberta Zambrini and Maxi San Miguel
Phys. Rev. A **66**, 023807 (2002).

• “Non-classical behavior in multimode and disordered transverse structures in OPO. Use of the Q representation.”
Roberta Zambrini, Stephen M. Barnett, Pere Colet, and Maxi San Miguel
Eur. Phys. J. D **22**, 461 (2003).

Use of Q representation method in SHG:

• “Quantum properties of transverse pattern formation in second-harmonic generation”
Morten Bache, Pierre Scotto, Roberta Zambrini, Maxi San Miguel and Mark Saffman,
Phys. Rev. A 66, 013809 (2002).