IMEDEA

QUANTUM CORRELATIONS IN VECTORIAL PATTERNS IN A KERR-CAVITY

R.Zambrini[1,2], M.Hoyuelos[1,2,3], A.Gatti[2], L.Lugiato[2], P.Colet[1], M.San Miguel[1], A.Sinatra[2]

1 Instituto Mediterráneo de Estudios Avanzados, IMEDEA(CSIC-UIB),07071 Palma de Mallorca,Spain. http://www.imedea.uib.es/PhysDept

2 Dipartimento di Fisica dell'Università di Milano, Via Celoria 16, 20133 Milano, Italy

3 Departamento de Fisica, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, 7600 Mar Del Plata, Argentina

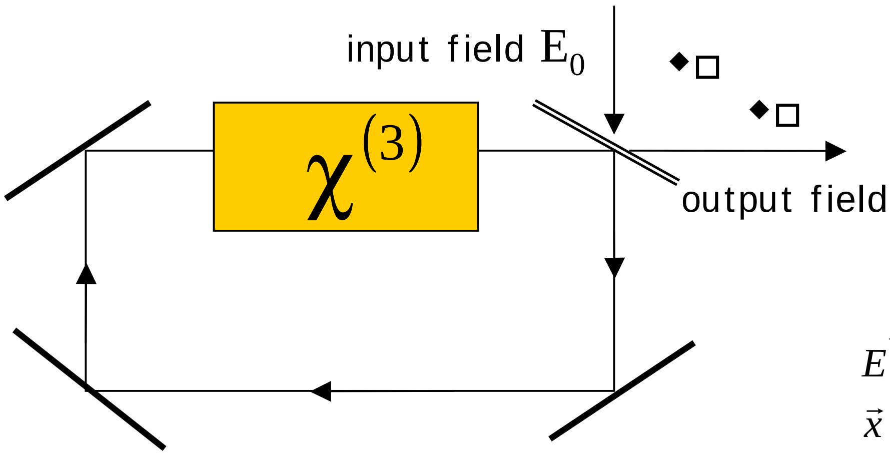
Acknowledgments: European Commission TMR network QSTRUCT

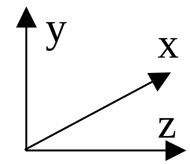
Abstract

We study quantum correlations among different components of the spatial spectrum of the light intensity field close to a pattern forming instability in a self-defocusing Kerr cavity. The instability is associated with the polarization of the light: for an X- polarized input a stripe-pattern arises in the Y-polarized field. We derive the linearized dynamical equations for the c-numbers associated in the Wigner representation with quantum fluctuations. We consider two models, a continuous one (including all transverse modes) and a simplified model with only three relevant modes.

We calculate the quantum correlations between the homogeneous input and the output field modes as well as the correlations between the different output field modes. In the continuous model we introduce a way to avoid instantaneous divergences of the output field. Finally, we discuss the applicability of this system for QND measurements.

Self-defocusing Kerr medium in a planar resonator





$$E^{\rightarrow}(\vec{x}, z, t) = \underbrace{\vec{E}(\vec{x}, t)}_{\text{field envelop}} e^{i(k_0 z - w_0 t)}$$

$$\vec{x} = (x, y)$$

Classical vectorial equation(1)

$$\frac{\partial \vec{E}}{\partial t} = -(1 - i\theta)\vec{E} + \vec{E}_0 + i\nabla^2 \vec{E} - ig\left[A\left(\vec{E}\cdot\vec{E}^*\right)\vec{E} + \frac{B}{2}\left(\vec{E}\cdot\vec{E}\right)\vec{E}^*\right]$$

- θ (cavity detuning), $\vec{E}_0$ (input field X - polarized),
- $g \propto \chi^{(3)}$, $A = 0.25$ and $B = 1.5$ (suscept. param.)
- ∇^2 transverse Laplacian
- time scaled with cavity decay rate and space with $\sqrt{a}$, a being the diffraction strength

(1) *J.B.Geddes, J.V.Moloney, E.M.Wright and First, Opt.Comm.* ***111,*** *623(1994)*

Classical results: pattern formation

Homogeneous X-polarized stationary solution E_s

$$E_0 = E_s\left[1 + i\left(|E_s|^2 - \theta\right)\right]$$

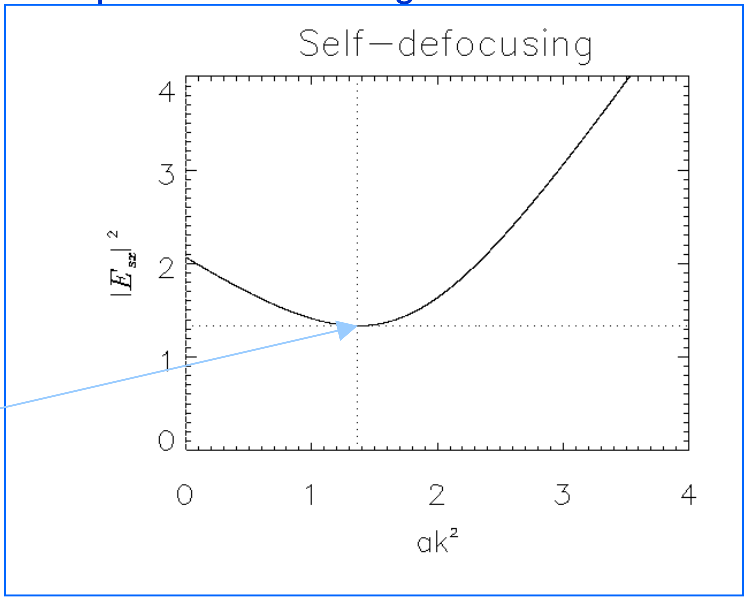
$|E_s| < |E_{th}|$ ➡ the homogeneous solution is stable

$|E_s| > |E_{th}|$ ➡ pattern formation in the Y-polarized field

Polarization Instability!!

No pattern formation in the scalar case

Marginal stability diagram for the X-polarized homogeneous solution



$\left(ak_c^2, |E_{th}|^2\right)$

In the threshold of spatial instability the pattern arises with critical wave number k_c.

critical values

$$\begin{cases} |E_{th}|^2 = \dfrac{2}{B} \\ ak_c^2 = \theta + 1 - \dfrac{2}{B} \end{cases}$$

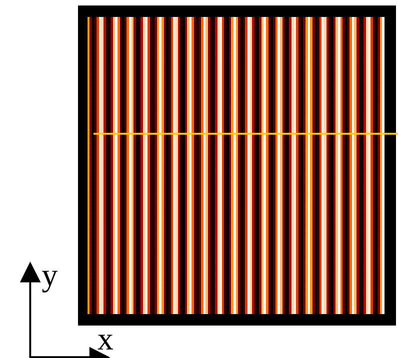
M.Hoyuelos, P.Colet, M.San Miguel and D.Walgraef, Phys.Rev.E, ***58,*** *2992 (1998)*

Cavity field above threshold

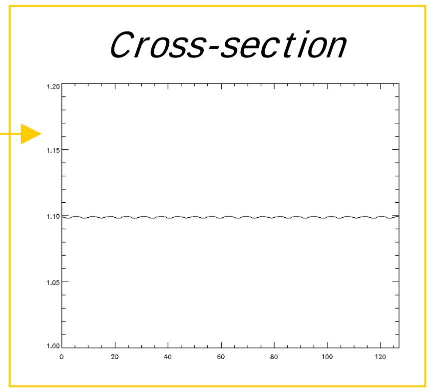
$E_0=1.13\ E_{th},\ \theta=1$

X-component

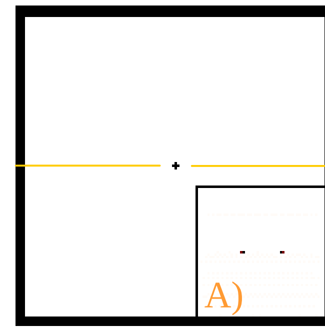
NEAR FIELD



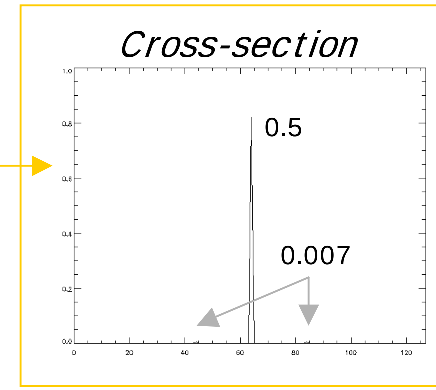
$Re(E_x)$



FAR FIELD

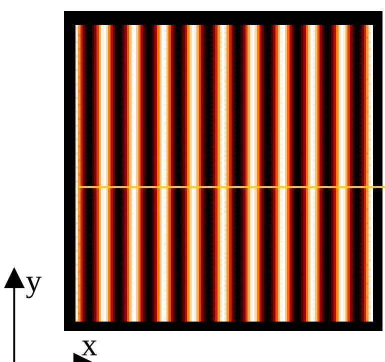


$|\tilde{E}_x(k_x,k_y)|$

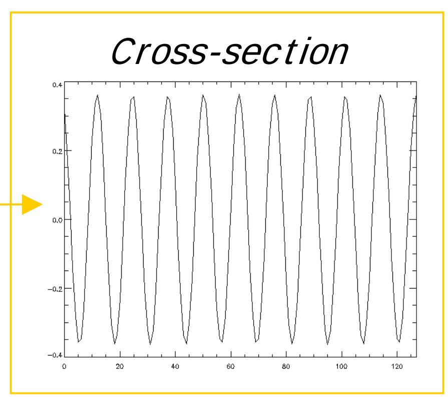


Y-component

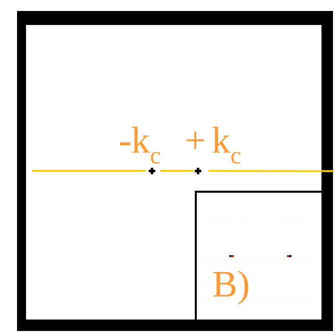
NEAR FIELD



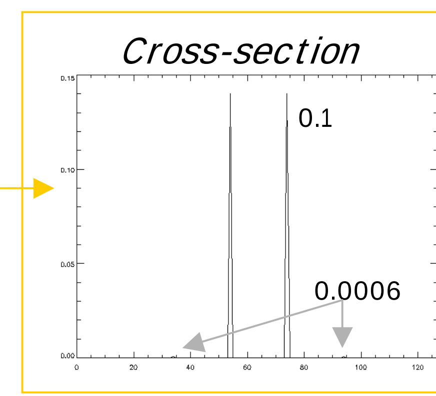
$Re(E_y)$



FAR FIELD



$|\tilde{E}_y(k_x,k_y)|$



A) and B) show the less intense modes in far field

Quantum formulation

CORRESPONDENCE: $\mathbf{E}_\pm$ classical field $\Leftrightarrow$ $\mathbf{A}_\pm$ operator field

± indicate circularly polarized components of field

MASTER EQUATION FOR THE REDUCED DENSITY OPERATOR OF THE FIELD

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + \Lambda\rho$$

Hamiltonian that describes reversible dynamics of the intracavity field:

$$H = H_0 + H_{est} + H_{\text{int}}$$

$$\begin{cases} H_0 = -\hbar K \int d^2\vec{x} \left[A_+^\dagger(\vec{x})\left(\theta + a\nabla^2\right)A_+(\vec{x}) + A_-^\dagger(\vec{x})\left(\theta + a\nabla^2\right)A_-(\vec{x}) \right] \\ H_{est} = i\hbar K \int d^2\vec{x} E_0 \left[A_+^\dagger(\vec{x}) - A_+(\vec{x}) + A_-^\dagger(\vec{x}) - A_-(\vec{x}) \right] \\ H_{\text{int}} = \hbar K g \int d^2\vec{x} \left\{ \frac{\alpha}{2}\left[A_+^{\dagger 2}(\vec{x})A_+^2(\vec{x}) + A_-^{\dagger 2}(\vec{x})A_-^2(\vec{x}) \right] + \beta\left[A_+^\dagger(\vec{x})A_-^\dagger(\vec{x})A_+(\vec{x})A_-(\vec{x}) \right] \right\} \end{cases}$$

Liouvillian Λ accounts for dissipation through the partially reflecting mirror

$$\Lambda\rho = K \sum_{i=+,-} \int d^2\vec{x} \left\{ \left[A_i(\vec{x}), \rho A_i^\dagger(\vec{x}) \right] + \left[A_i(\vec{x})\rho, A_i^\dagger(\vec{x}) \right] \right\}$$

K is the cavity linewidth, α and β ($\alpha+\beta=2$) are related to susceptibility and the other parameters are previously defined.

We consider a CLASSICAL COHERENT INPUT.

R.Zambrini, M.Hoyuelos, A.Gatti, L.Lugiato, P.Colet and M.San Miguel, unpublished

Linear model for the quantum fluctuations

operator $\mathbf{A}_\pm \Leftrightarrow$ complex numeric field $\alpha_\pm$

We consider the small quantum fluctuations around the classical mean value

$$\alpha_\pm = F_\pm + \Delta\alpha_\pm$$

F stationary mean fields
$\Delta\alpha$ cavity fluctuations

The functional $W(\Delta\alpha_+,\Delta\alpha_-)$ in the Wigner rapresentation

satisfies a FOKKER-PLANCK EQUATION, with positive diffusion matrix.

Langevin equations

$$\frac{\partial\Delta\alpha_\pm(\vec{x},t)}{\partial t} = \left[-(1-i\theta)+i\nabla^2 - i\left(2\alpha\,|F_\pm(\vec{x})|^2+\beta\,|F_\pm(\vec{x})|^2\right)\right]\Delta\alpha_\pm(\vec{x},t)$$

$$-i\alpha F_\pm^2(\vec{x})\,\Delta\alpha_\pm^*(\vec{x},t) - i\beta F_\pm(\vec{x})F_\mp^*(\vec{x})\,\Delta\alpha_\mp(\vec{x},t)$$

$$-i\beta F_\pm(\vec{x})F_\mp(\vec{x})\,\Delta\alpha_\mp^*(\vec{x},t)+\sqrt{2}\Delta\alpha_\pm^{in}(\vec{x},t)$$

$$\left\langle\Delta\alpha_i^{in}(\vec{x},t)\Delta\alpha_i^{in*}(\vec{x}',t')\right\rangle = \frac{1}{2}\delta_{ij}\delta(\vec{x}-\vec{x}')\delta(t-t') \qquad i=+,-$$

white noise (vacuum fluctuations)

Stationary and input fields scaled with $\sqrt{g}$, fluctuations with $\sqrt{a}$

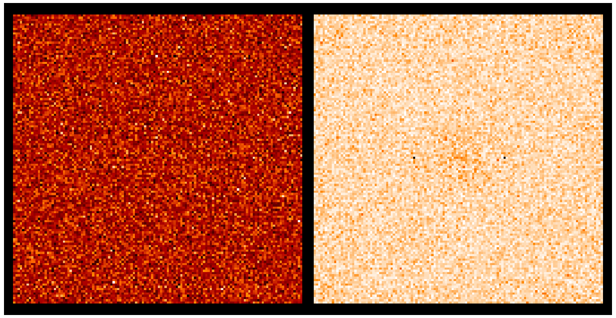
Quantum fluctuations

Numerical simulation of Langevin equation shows that:

- fluctuations of the X-polarized component are homogeneously distributed in space,

X

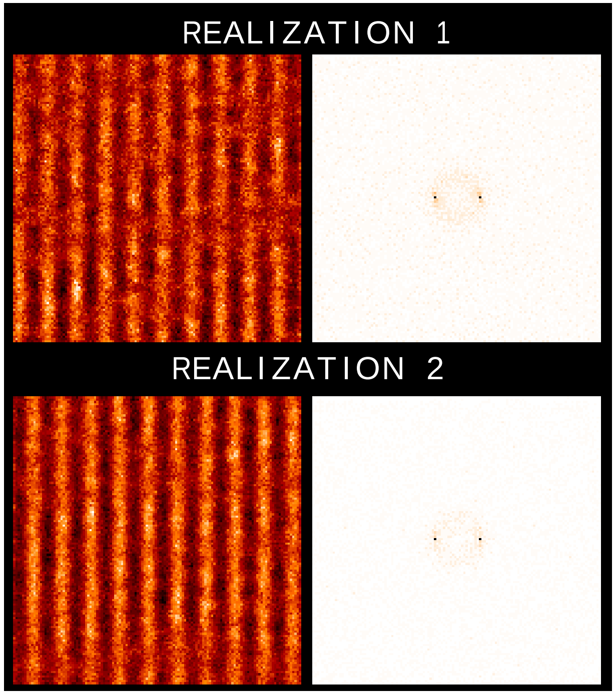
Near field | far field



- fluctuations of the Y-component show a stripe pattern, like in the stationary solution, but shifted to the left or the right by a quarter-period.

Y

Near field | far field

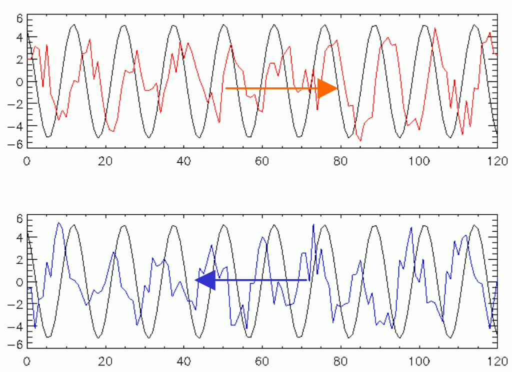


Cross-section of two realization of fluctuations

——— stationary solution

——— realization 1

——— realization 2



Goldstone mode

Ingredients:

translational simmetry in the plane (x,y)

+

$F_{\pm}(x,y)$ solution with breaks symmetry

Mathematics:

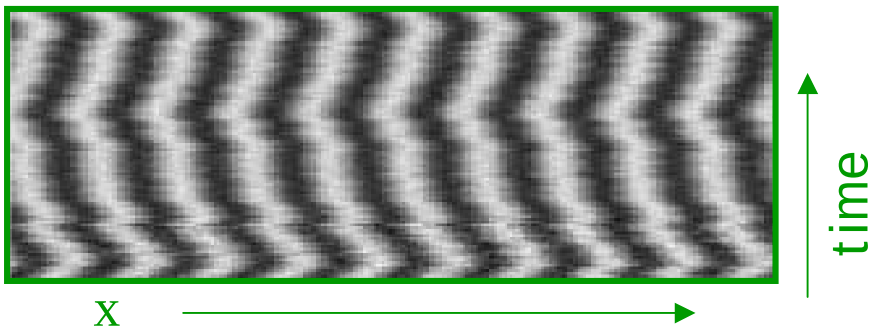
$\nabla F_{\pm}(x,y)$ is neutrally stable mode (Goldstone mode) of linearized problem around $F_{\pm}$

Consequences:

there are undamped fluctuations corresponding to a rigid motion of the pattern .

Simulation of the classical equations with noise for a d=1 system.

Notice the rigid motion of the transverse pattern in time



Correlations in large systems

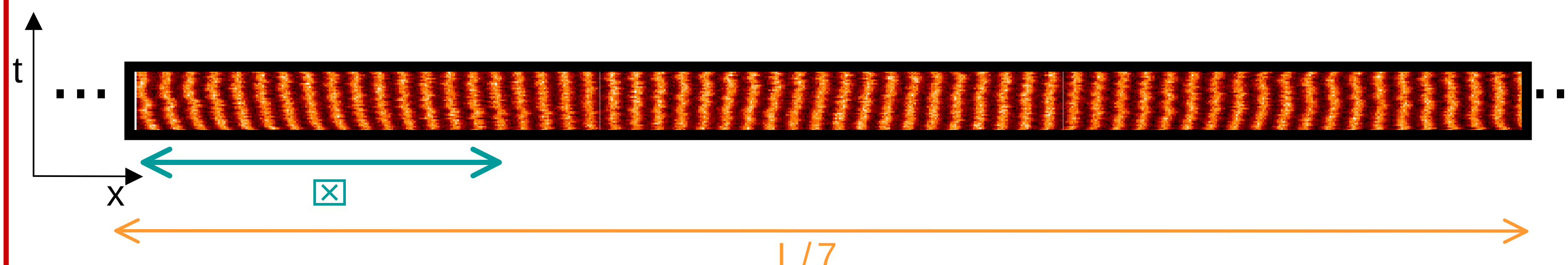
$E_y \approx A e^{i k_c x}$ with $A = R e^{i\psi}$

⊠: correlation length
L: system size

Small system: L < ⊠

•Rigid pattern motion associated with Goldstone mode x ✸ x_0+x ⟹ ⍓ ✸ $⍓_0$+⍓

Large system: L >> ⊠



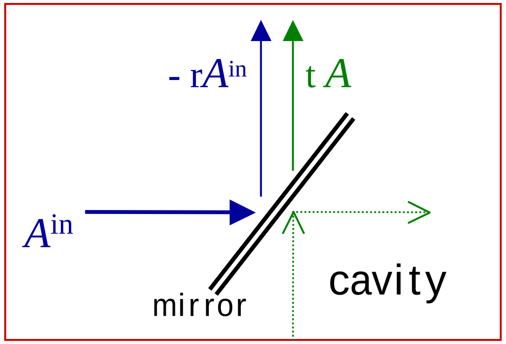
•Weakly damped longwavelength perturbations close to the Goldstone mode easily excited by noise

•Noise destroys long range order for $d<d_c$ phase fluctuations $<⍓_q> \sim q^{-2}$

•Pattern moves locally in different directions

Fields outside the cavity

We are interested in the fluctuations outside the cavity !



fields in cavity: $A_{\pm}(x,y,t)$ ⟹ fields outside: $A_{\pm}^{out}(x,y,t)$

input-output relations[3] for scaled operators fields:

$$A_{\pm}^{out} = \sqrt{2}A_{\pm} - A_{\pm}^{in}$$

(3) M.J.Collet and C.W.Gardiner, Phys.Rev. ***A*** *30, 1386 (1984)*

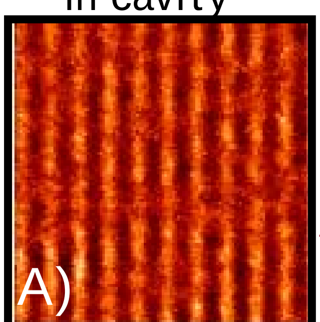
Relations for C-number fluctuations:

$$\Delta\alpha_{\pm}^{out}(\vec{x},t) = \sqrt{2}\Delta\alpha_{\pm}^{out}(\vec{x},t) - \Delta\alpha_{\pm}^{in}(\vec{x},t)$$

White noise representing fluctuations of the coherent input field. Instantaneous values are ill-defined !

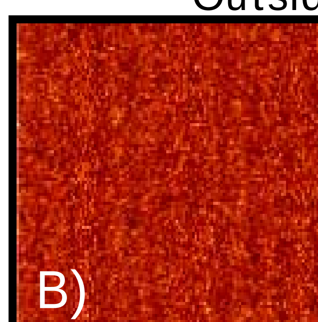
We average these fast fluctuating quantities in a small time window: $\int_{t}^{t+\Delta t} \Delta\alpha_{i}^{out}(\vec{x},t')\,dt'$

in cavity

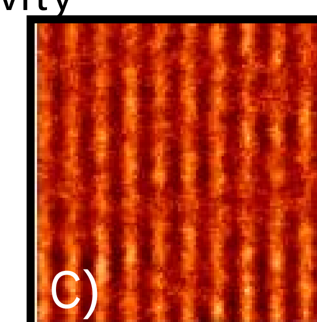


$\Delta\alpha_y$ (Δt= 0.005)

Outside the cavity



$\Delta\alpha_y^{out}$ (Δt= 0.005)



$\Delta\alpha_y^{out}$ (Δt= 0.5)

B) If Δt is too small the output fluct. are too noisy!

3 modes approximation

Simplest description ➤ 3 relevant modes:

Far field: 3 spots

X-component Y-component

- one homogeneous X-polarized
- two Y-polarized modes of wavenumber $\pm k_c$

$$H = H_0 + H_{est} + H_{int}$$

$$H_0 = -\hbar K\left[\theta \hat{a}_0^\dagger \hat{a}_0 + \theta_1\left(\hat{b}_1^\dagger \hat{b}_1 + \hat{b}_2^\dagger \hat{b}_2\right)\right] \qquad \theta_1 = \theta - a\left|k_c\right|^2$$

$$H_{ext} = i\hbar K E_0\left(\hat{a}_0^\dagger - \hat{a}_0\right)$$

$$H_{int} = \hbar g K\left[(\alpha-1)\left(\hat{a}_0^2 \hat{b}_1 \hat{b}_2 + H.C.\right) + \alpha\left(\hat{a}_0^\dagger \hat{b}_1^\dagger \hat{a}_0 \hat{b}_1 + \hat{a}_0^\dagger \hat{b}_2^\dagger \hat{a}_0 \hat{b}_2\right) - \tfrac{1}{2}\left(\hat{a}_0^{\dagger 2} \hat{a}_0^2 + \hat{b}_1^{\dagger 2} \hat{b}_1^2 + \hat{b}_2^{\dagger 2} \hat{b}_2^2\right)\right]$$

with same parameters that in Continuous Hamiltonian

We derive Langevin equations for the evolution of the 3 c-numbers associated with fluctuations of operators $\hat{a}_0, \hat{b}_0$ and $\hat{b}_1$

We compare results between the 3 modes and the continuous models

(4) *M.Hoyuelos,A.Sinatra,P.Colet,L.Lugiato and M.San Miguel Phys.Rev. A* ***59****, 1622 (1999)*

Spatial correlations

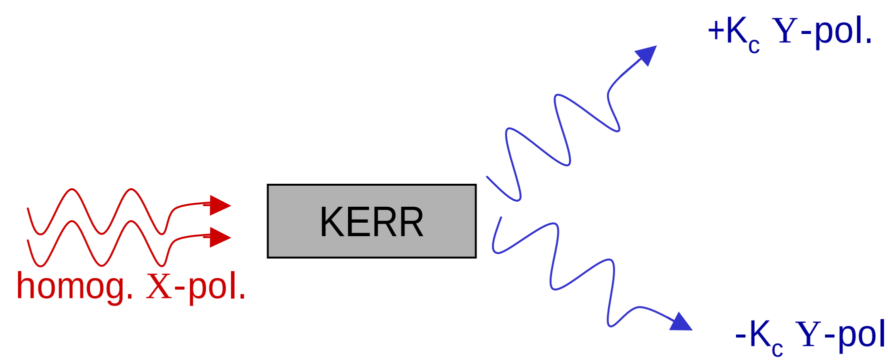
Classical results[5] :

§ anticorrelation between intensity fluctuations in the pump mode and Y-modes

§ correlation between two Y-modes + k_c and -k_c

PRINCIPAL MICROSCOPIC PROCESS:

simultaneous destruction of 2 pump photons and creation of 2 Y-pol. photons



Are there quantum features in the correlations between intensity fluctuations of the pump and the Y-polarized field, and between the two signals of opposite wave-number?

(5) *M.Hoyuelos,P.Colet and M.San Miguel, Phys.Rev. E* ***58****,74 (1998)*

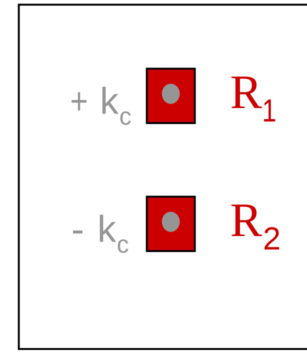
Correlation between k_c and $-k_c$

Correlation origin → conservation of transverse momentum in the 4 wave mixing process

definition

Operators of photons number per time unit over 2 regions $R_{1,2}$ of the FAR FIELD, out of cavity

$$N_{yR_j}^{out}(t) = \int_{R_j} d\vec{k} A_y^{\dagger out}(\vec{k},t) A_y^{out}(\vec{k},t) \qquad j = 1,2$$



Y-polarized far field

We calculate correlations of scaled photons number fluctuations.

§ Equal time correlations

Results:

<u>positive</u> correlation as in the semiclassical case

$$\left\langle \Delta n_1^{out}(t) \Delta n_2^{out}(t) \right\rangle_P > 0$$

where $< >_P$ means expectation value for operators in normal ordering.

§ Two times correlations

definitions

Squeezing spectrum

$$S_j(\omega) = \left\langle \delta N_{yR_j}^{out} \delta N_{yR_j}^{out} \right\rangle_\omega$$

with

$$\left\langle \delta N_{yR_j}^{out} \delta N_{yR_j}^{out} \right\rangle_\omega = \int dt \left\langle \delta N_{yR_j}^{out}(t) \delta N_{yR_j}^{out}(0) \right\rangle_{symm} e^{-i\omega t}$$

$$\left\langle W(t) Z(0) \right\rangle_{symm} = \frac{\left\langle W(t)Z(0) + Z(0)W(t) \right\rangle}{2}$$

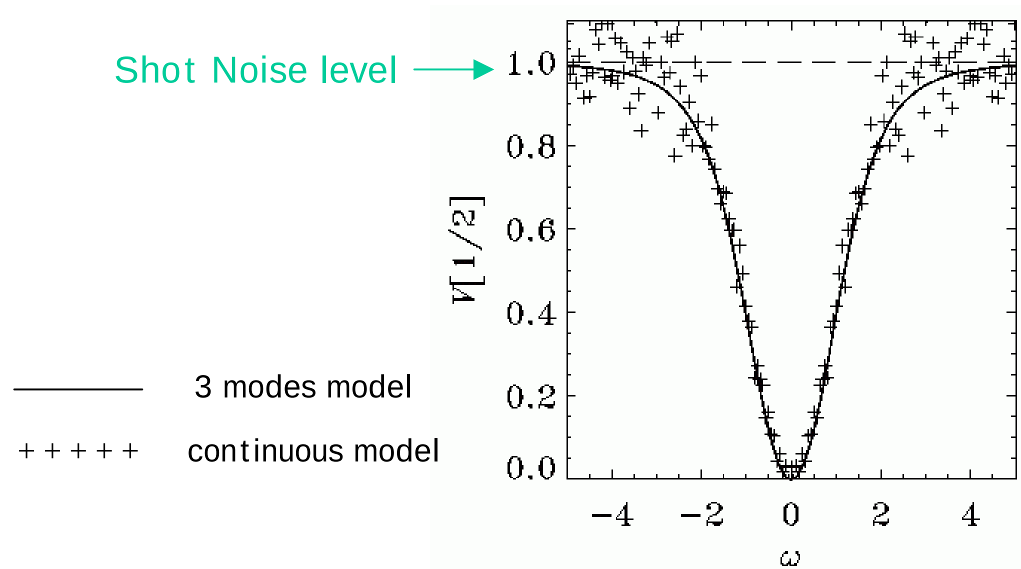
Conditional variance

$$V[1|2] = S_1 \left(1 - \frac{\left| \left\langle \delta N_1^{out} \delta N_2^{out} \right\rangle_\omega \right|^2}{S_1 S_2} \right)$$

Results:

The correlation between two modes is not classic! The variance is below the shot noise limit.

Agreement between results in 3 modes and continuous model



Parameters: E_0=1.3 and θ=1.7

QND measurement

A quantum measurement usually perturbs the measured quantity, adding a ‘back-action noise’. The idea behind quantum non-demolition (QND) measurements is to leave the observed quantity unperturbed, while adding the back-action noise into another (complementary) observable.

We want to check the conditions for using the Kerr-cavity as a QND device.

Idea: use polarization correlations

SIGNAL beam X-polarized input	METER beam Y-polarized input

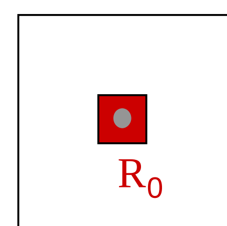
Measurement of the the signal with the smallest possible perturbation QND, taking advantage of the correlation between the pattern and the homogeneous mode fluctuations.

Correlation origin → conservation of transverse momentum in the 4 wave mixing process

definition
like the previous ones

$$N_{xR_0}^{out}(t) = \int_{R_0} d\vec{k} A_x^{\dagger out}(\vec{k},t) A_x^{out}(\vec{k},t)$$

$$N_{1+2}^{out}(t) = N_1^{out}(t) + N_2^{out}(t)$$



Homogeneous X-comp. in far field

QND MEASUREMENT satisfies: $V(0|1+2) < 1$ and $C_s + C_m > 1$

- C_s normalized correlation between input and output fluctuations of the homogeneous mode.
- C_m normalized correlation between fluctuations in the homogeneous input and in the output pattern

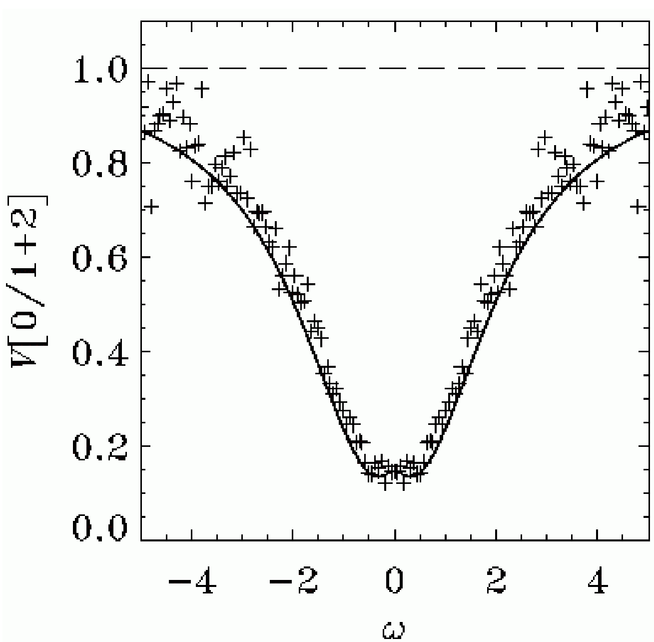
Results:

The correlation between two modes is quantum!

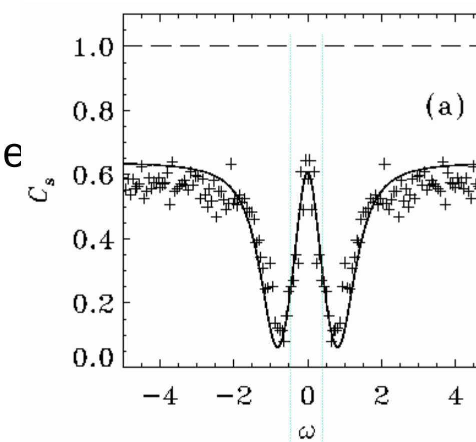
The variance is below the shot noise limit.

This cavity can used as QND device.

Correspondence between results in 3 modes and continuos mode

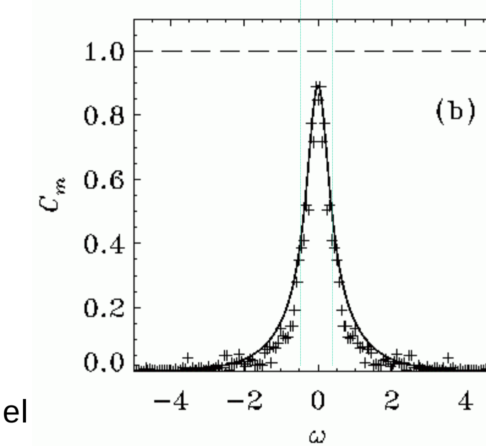


Shot Noise level



C_s+C_m>1

when $|\omega| < 0.3$



——— 3 modes model

+ + + + continuous model

Parameters: E_0=1.3 and θ=1.7