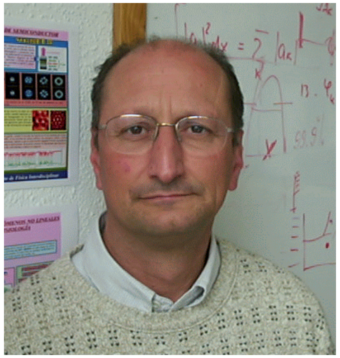
Growth Laws and Stable Droplets in Nonlinear Optics






Damià Gomila *Pere Colet* *Maxi San Miguel* *Gian-Luca Oppo*

Phys. Rev. Lett. 87, 194101 (2001)
IEEE J.Quantum Electr. (2002)

Vectorial Kerr Resonator

$$\partial_t E_\pm = -(1+i\eta\theta)E_\pm + i\nabla^2 E_\pm + E_0 + \frac{1}{4} i\eta \left[|E_\pm|^2 + \beta |E_\mp|^2 \right] E_\pm$$

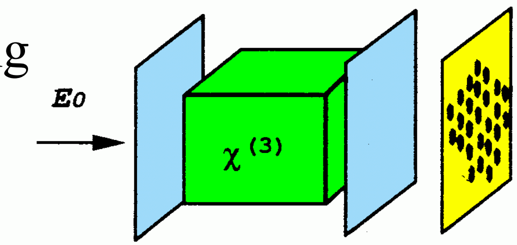
$E_\pm$: circularly polarized components

E_0 : $\hat{x}$-linearly polarized input

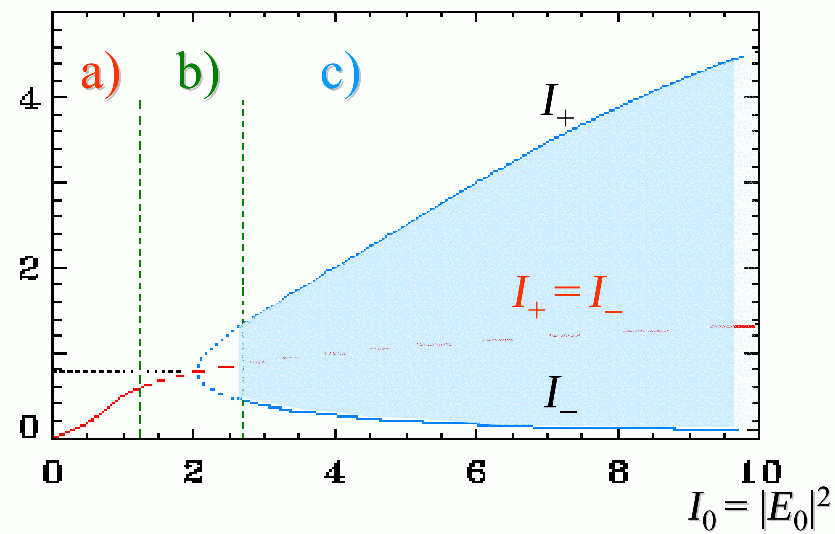
η = -1: self-defocusing

θ = 1: detuning

β =7 : coupling



Homogeneous solutions:



a) Symmetric homogeneous solution

b) Stripe patterns $\hat{y}$-polarized

c) Bistability between homogeneous elliptically polarized solutions

Stable Ising walls

Geddes et al. Opt. Comm. 111, 623 (1994)

Hoyuelos et al. Phys. Rev. E 58, 2992 (1998)

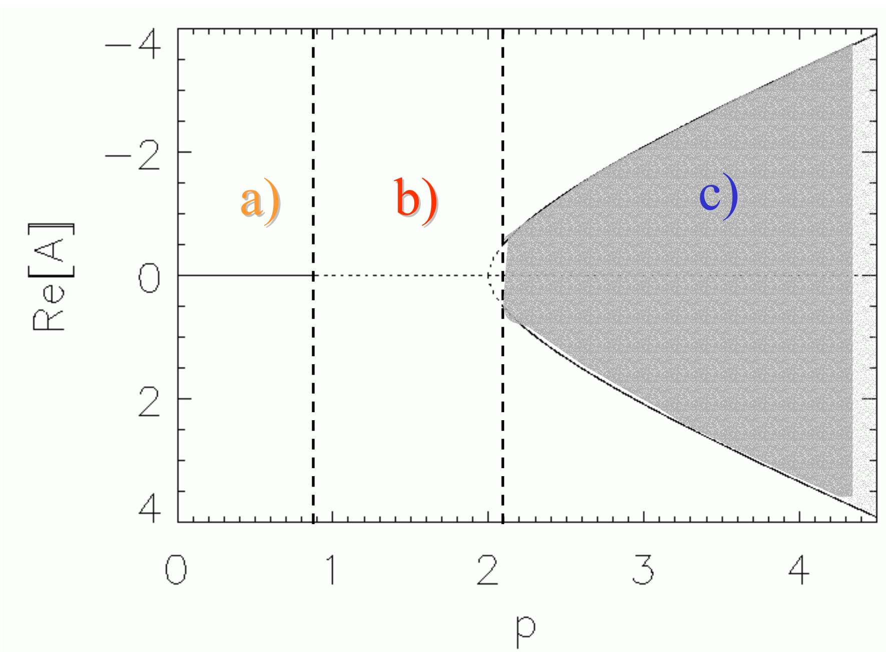
Parametrically Driven CGLE

$$\partial_t A = (\mu + i\nu)A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A + pA^*$$

$\alpha \sim \nu \sim p >> \mu, \beta$ $\rightarrow$ Pattern forming transition

P. Coullet and K. Emilsson, Physica D, 61, 119 (1992)

Homogeneous solutions:



a) Zero solution

b) Stripe and hexagonal Patterns

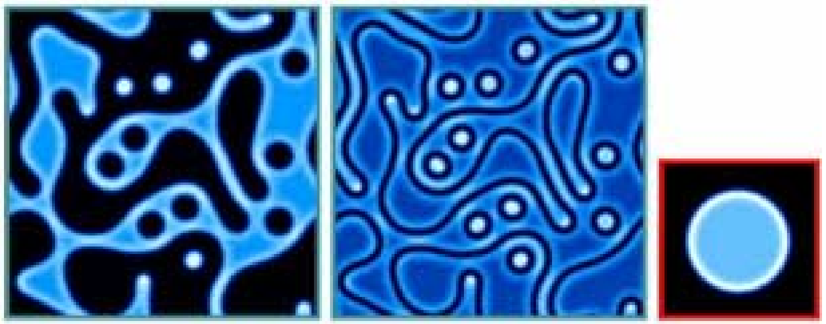
c) Bistability between homogeneous solutions (frequency locked solutions).

Stable Ising walls.

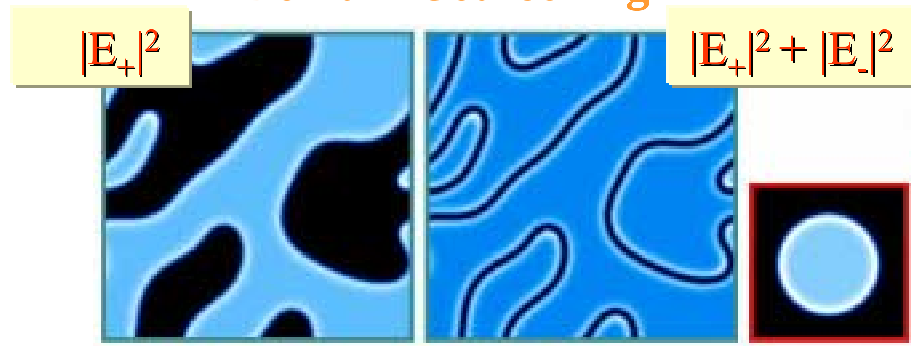
$\alpha = \nu = 2, \mu = \beta = 0$

Dynamics of Polarization Domain Walls in 2D

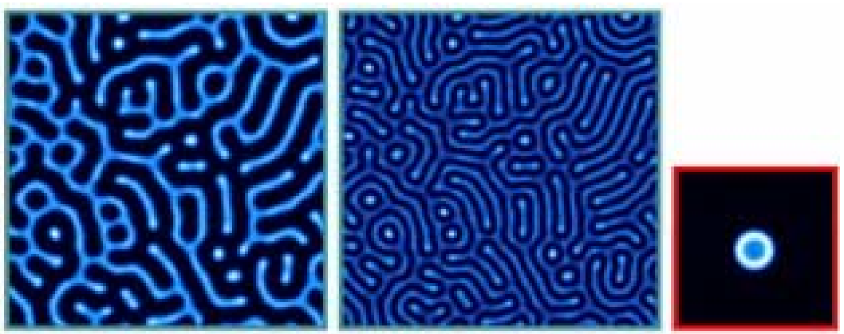
Localized Structures



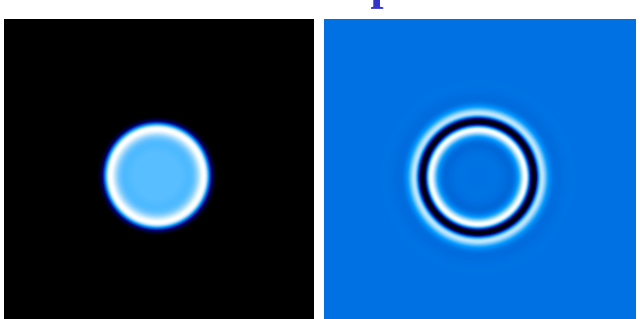
Domain Coarsening

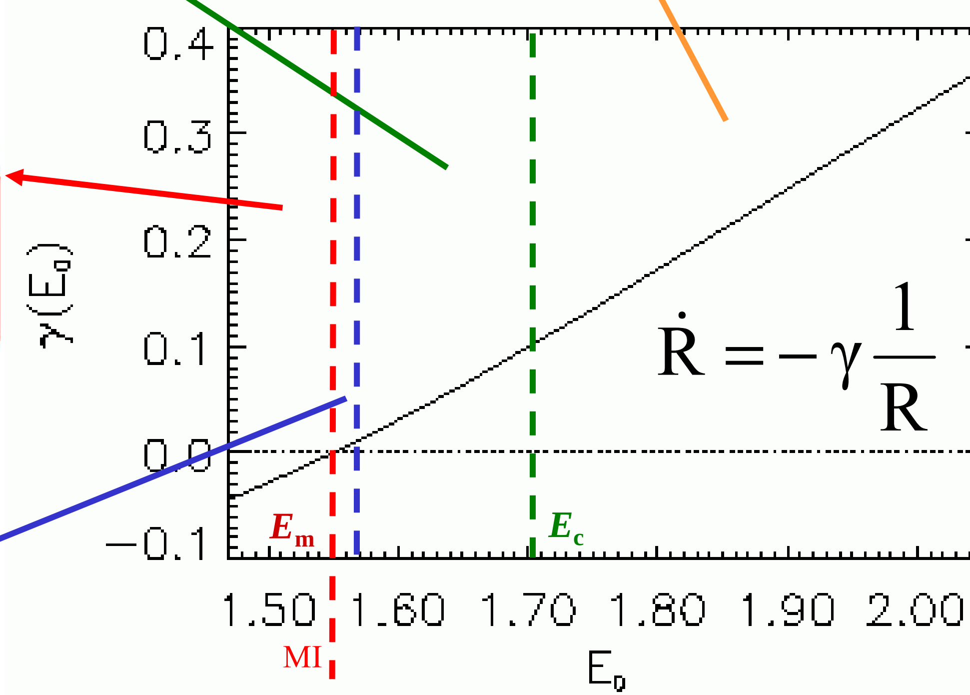


Labyrinthine Patterns



Stable Droplets

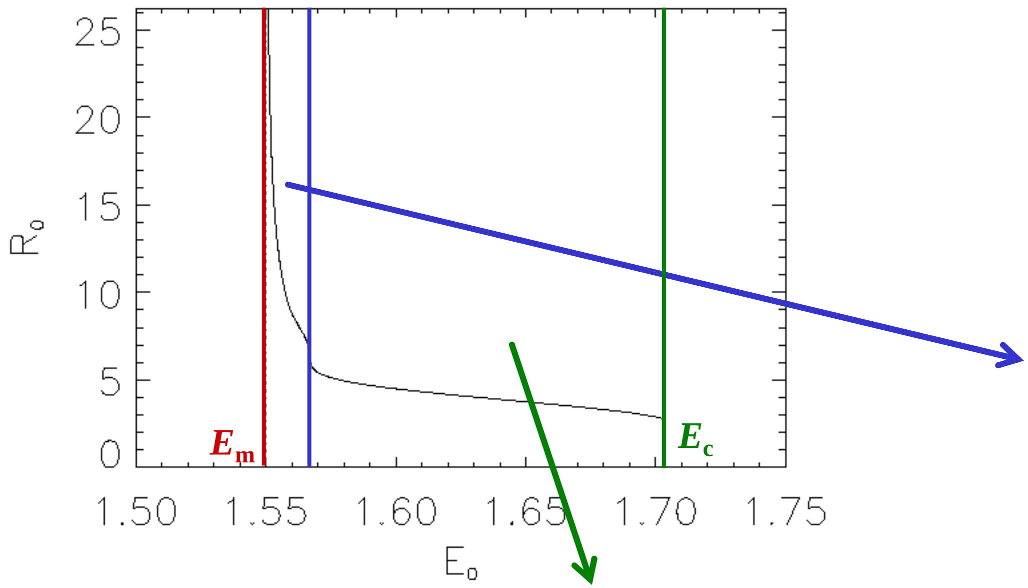




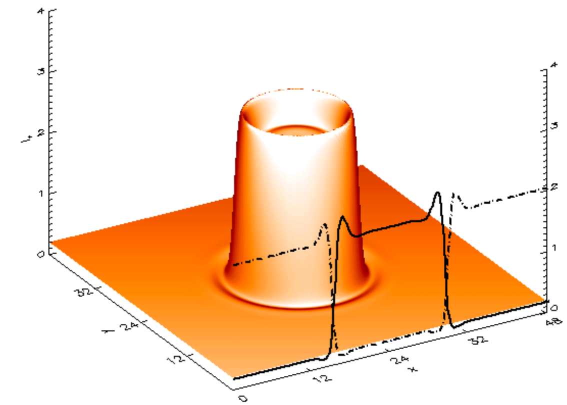
R. Gallego et al. PRE **61**, 2241-2244 (2000)

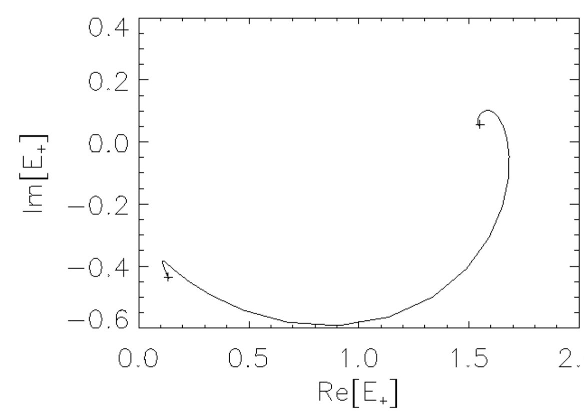
Localized Structures vs Stable droplets

Vectorial Kerr



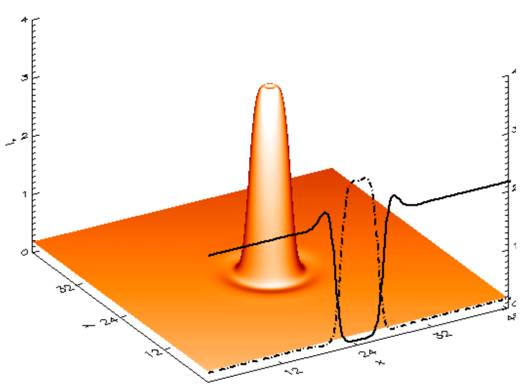
Stable Droplet

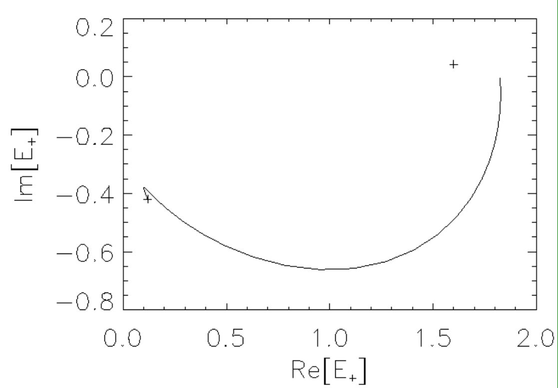




E_0=1.5524

Localized Structure





E_0=1.6

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Curvature Driven Dynamics

$$\partial_t \vec{\psi}(\vec{x}) = D\nabla^2 \vec{\psi}(\vec{x}) + \vec{W}(\vec{\psi}(\vec{x}), p)$$

- $\vec{\psi}$ is a real vector field

PCGLE

$$\vec{\psi} = \begin{pmatrix} \mathrm{Re}[A] \\ \mathrm{Im}[A] \end{pmatrix} \quad D = \begin{pmatrix} 1 & -\alpha \\ \alpha & 1 \end{pmatrix}$$

Vectorial Kerr

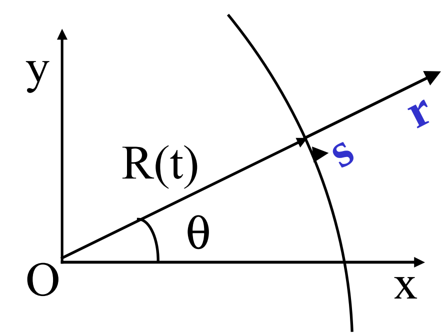
$$\vec{\psi} = \begin{pmatrix} \mathrm{Re}[E_+] \\ \mathrm{Im}[E_+] \\ \mathrm{Re}[E_-] \\ \mathrm{Im}[E_-] \end{pmatrix} \quad D = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- $\vec{W}(\vec{\psi})$ is a local nonlinear function of the fields.
- There exists a discrete symmetry ***Z*** such that there are two equivalent stable homogeneous solutions (PCGLE ***Z*: *A* → -*A*; Vectorial Kerr *Z*:** $E_{\pm} \rightarrow E_{\mp}$).
- Stable d=1 Ising walls $\vec{\psi}_0(x, p)$.

Local Coordinates r, s

$$\partial_t \rightarrow -\mathrm{v}\partial_r \qquad (\mathrm{v} \equiv \dot{\mathrm{R}})$$

$$\nabla^2 \rightarrow \partial_r^2 + \frac{\kappa}{1 + r\kappa}\partial_r + \frac{\kappa^2}{(1 + r\kappa)^2}\partial_\theta^2 \qquad \left(\kappa \equiv \frac{1}{\mathrm{R}}\right)$$



E. Meron, Phys. Rep. 218 (1992)

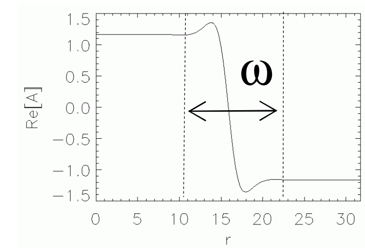
Curvature Driven Dynamics

$$D\partial_r^2\vec{\psi}(r)+(\mathrm{v}+\frac{\kappa}{1+r\kappa}D)\partial_r\vec{\psi}(r)+\vec{W}(\vec{\psi}(r),\mathrm{p})=0$$

Linearizing around the 1D profile: $\vec{\psi}(r)=\vec{\psi}_0+\vec{\psi}_1$

$$M\vec{\psi}_1=-(\mathrm{v}+\kappa D)\partial_r\vec{\psi}_0\ ,\quad M\equiv D\partial_r^2+\left.\frac{\delta\vec{W}}{\delta\vec{\psi}}\right|_{\vec{\psi}_0}$$

Gently curved fronts $\kappa\omega<<1$



$\vec{e}_0$ Goldstone Mode: $\vec{e}_0\equiv\partial_r\vec{\psi}_0\ ,\ M\vec{e}_0=0\,;\ M^{+}\vec{a}_0=0$

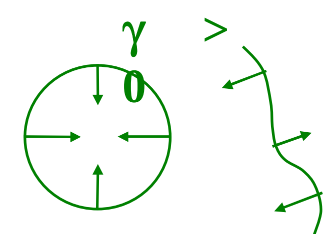
Solvability condition: $\mathrm{v}=-\gamma(\mathrm{p})\kappa$

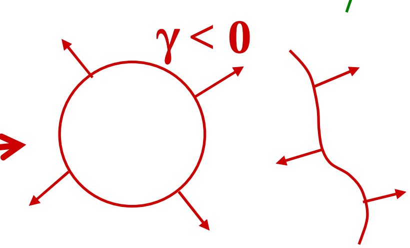
$$\gamma(\mathrm{p})\equiv\frac{\int_{-\infty}^{\infty}\vec{a}_0\cdot D\vec{e}_0\,dr}{\int_{-\infty}^{\infty}\vec{a}_0\cdot\vec{e}_0\,dr}$$

Circular domain: $\dot{\mathrm{R}}=-\gamma\frac{1}{\mathrm{R}}$

- $D=dI \Rightarrow \gamma=d>0\ ,\ \mathrm{v}=-d\kappa \quad \gamma>0$ Droplet shrinking
  $\vec{\psi}_1\propto\vec{e}_0$, no deformation

- $D=dI+C \Rightarrow \gamma=\gamma(\mathrm{p})\ ,\ \mathrm{v}=-\gamma\kappa$ $\begin{cases}\gamma>0 & \text{Droplet shrinking}\\ \gamma=0 & \text{Modulational instability}\\ \gamma<0 & \text{Droplet growth}\end{cases}$
  $\vec{\psi}_1\not\propto\vec{e}_0$, deformation $\propto\kappa$, C

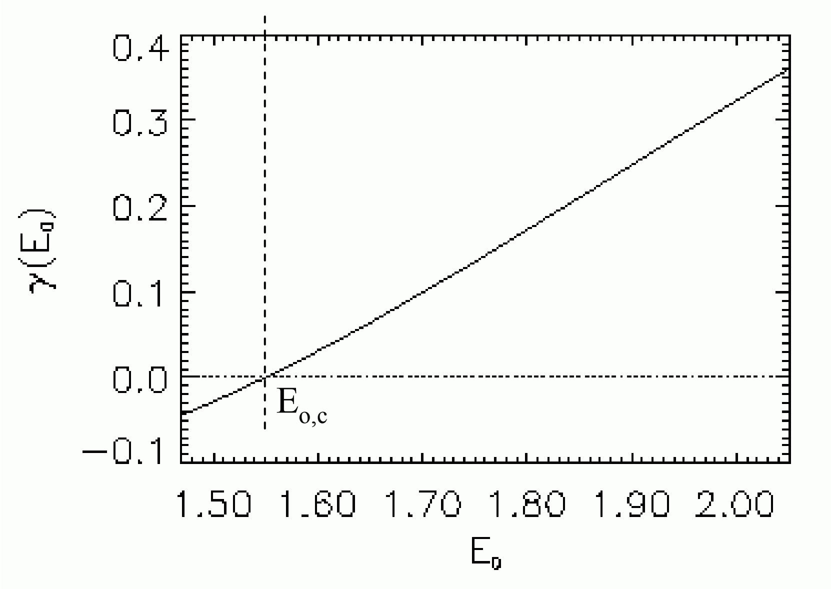




Curvature Driven Dynamics

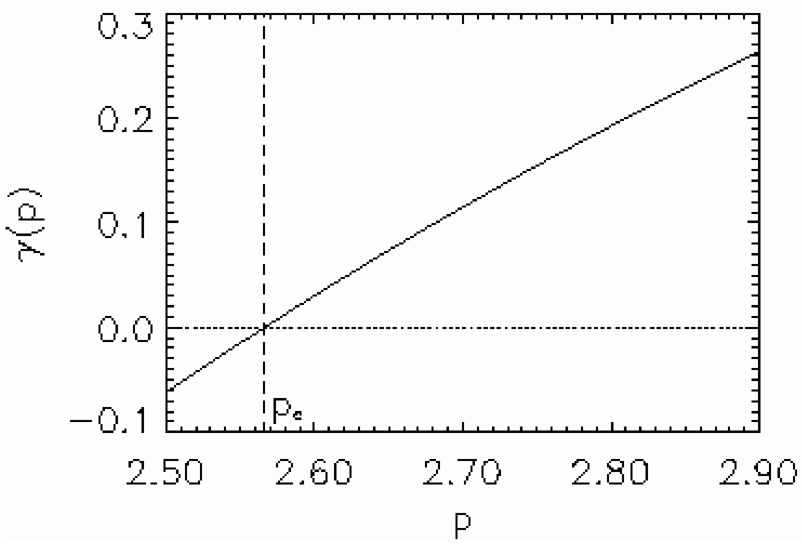
$$V = -\gamma\kappa$$

Vectorial Kerr



$E_{0,c} = 1.5498$

PCGLE



$p_c = 2.5663$

Amplitude Equation for the curvature, $\gamma = 0$

Radial symmetry $\Rightarrow$ $D\partial_r^2\vec{\psi}(r) + \left(-\frac{\dot{\kappa}}{\kappa^2} + \frac{\kappa}{1+r\kappa}D\right)\partial_r\vec{\psi}(r) + \vec{W}(\vec{\psi}(r),\mathrm{p}) = 0$

$$\mathrm{p} = \mathrm{p_c} + \varepsilon\mathrm{p}_1 \,,\; \gamma(\mathrm{p_c}) = 0 \,,\; \varepsilon \ll 1 \,,$$

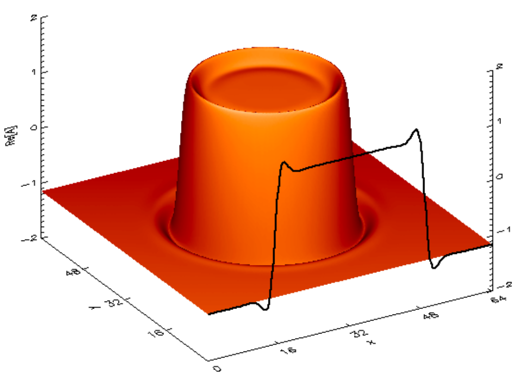
$$\vec{\psi} = \vec{\psi}_0 + \varepsilon^{1/2}\vec{\psi}_1 + \varepsilon\vec{\psi}_2 + \varepsilon^{3/2}\vec{\psi}_3 \,,$$

$$\kappa = \varepsilon^{1/2}\kappa_1 \,,\; \partial_t = \varepsilon^2\partial_T$$

$$\Rightarrow \quad O(\varepsilon^{3/2}) \quad \frac{\partial_T \kappa_1}{\kappa_1^2} = c_1\mathrm{p}_1\kappa_1 + c_3\kappa_1^3$$

$c_1 > 0 \,, c_3 < 0$

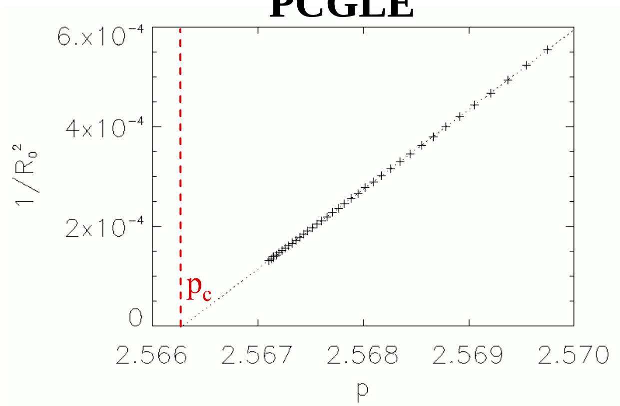
$$\dot{\mathrm{R}} = -c_1(\mathrm{p}-\mathrm{p_c})\frac{1}{\mathrm{R}} - c_3\frac{1}{\mathrm{R}^3} \quad \Rightarrow$$



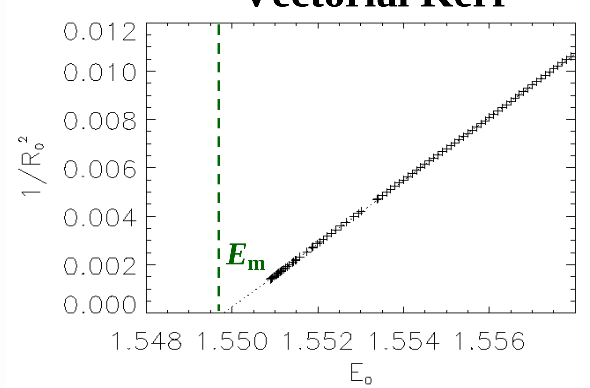
Stable Droplets

$$\mathrm{R}_0 = \frac{1}{\sqrt{\mathrm{p}-\mathrm{p_c}}}\sqrt{\frac{-c_3}{c_1}}$$

PCGLE



Vectorial Kerr



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Domain Growth

Thermodynamic Systems

• Well established asymptotic growth laws and mechanisms: R(t) ” t, $t^{1/2}$, $t^{1/3}$

• Selfsimilar evolution: Dynamical scaling C(**r**,t) ” C(**r**/R(t))

• Nonconserved dynamics: $t^{1/2}$ robust: Point defects (Z_N, ***Kaski et al. 1985***), 3d vortices (XY, ***Mondello et al. 1992***), chiral walls (anisotropic XY, ***Tutu et al. 1997***).

Nonequilibrium Systems:

• Pattern formation: Swift-Hohenberg d=2, R ” $t^{1/5}$ ***(Cross and Meiron 1995)***

• Hamiltonian vs. dissipative dynamics

• Nonpotential front motion and spiral dynamics ***(Gallego et al. 1998, 1999)***

• Localized structures.

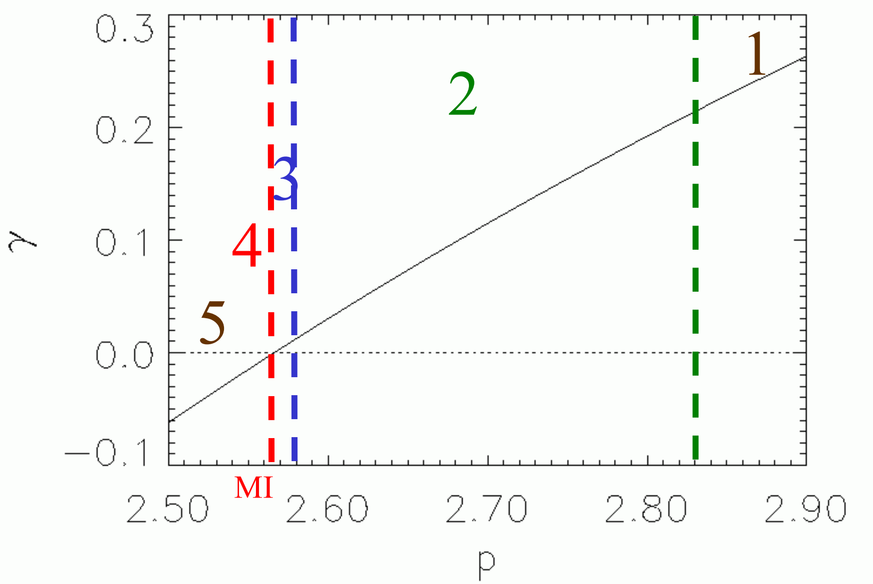
• Unstable domain walls.

Optical Systems:

• Asymptotic growth law?, Dominant mechanism?, Dynamical scaling?

Tlidi and Mandel (1998), Oppo et al. (1998), Peschel et al(1998)

Growth Laws

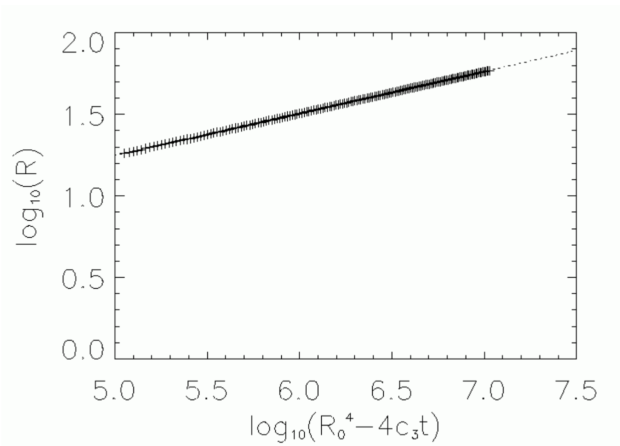


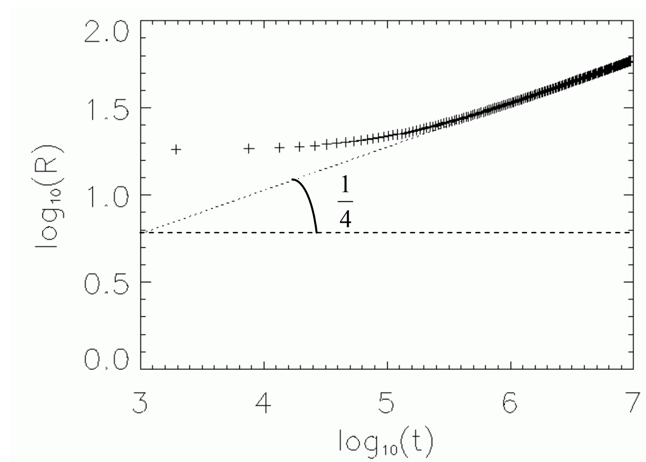
1, 5 → $\dot{R} = -\gamma \frac{1}{R}$, $R \approx t^{1/2}$ **shrinking, growth**

2 → $\dot{R} = -\gamma \frac{1}{R}$, $R \approx t^{-1/2}$ **until LS forms**

3 → $\dot{R} = -\gamma \left(\frac{1}{R} - \frac{R_0^2}{R^3} \right)$ **no power law**

4 → $\dot{R} = -c_3 \frac{1}{R^3}$, $R \approx t^{1/4}$ modulational instability, $\gamma = 0$





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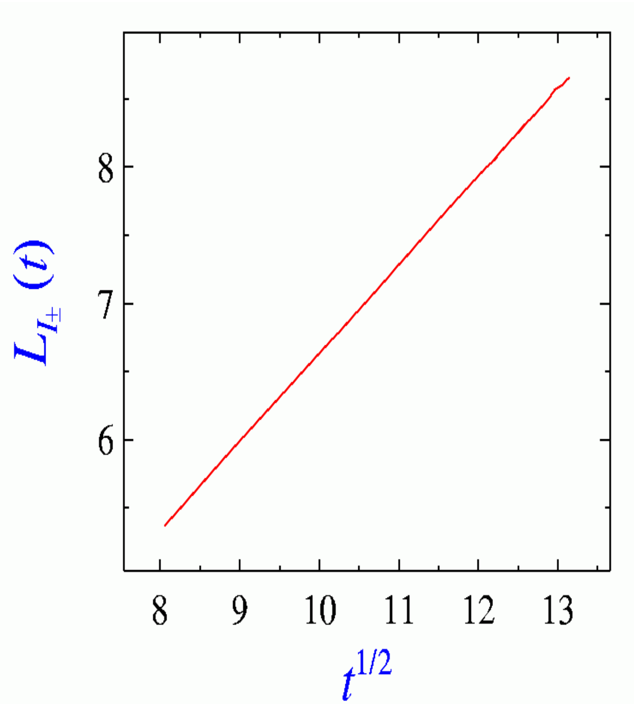
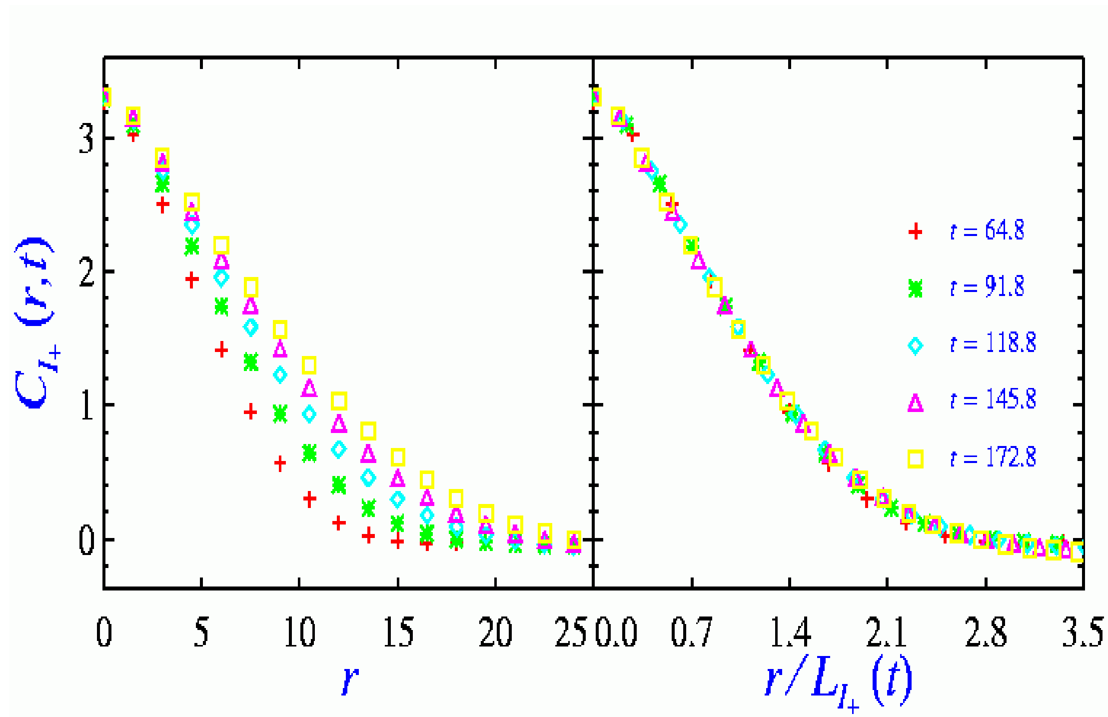
Growth law



Curvature driven

Dynamical scaling



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Dynamics of Arbitrary Domains

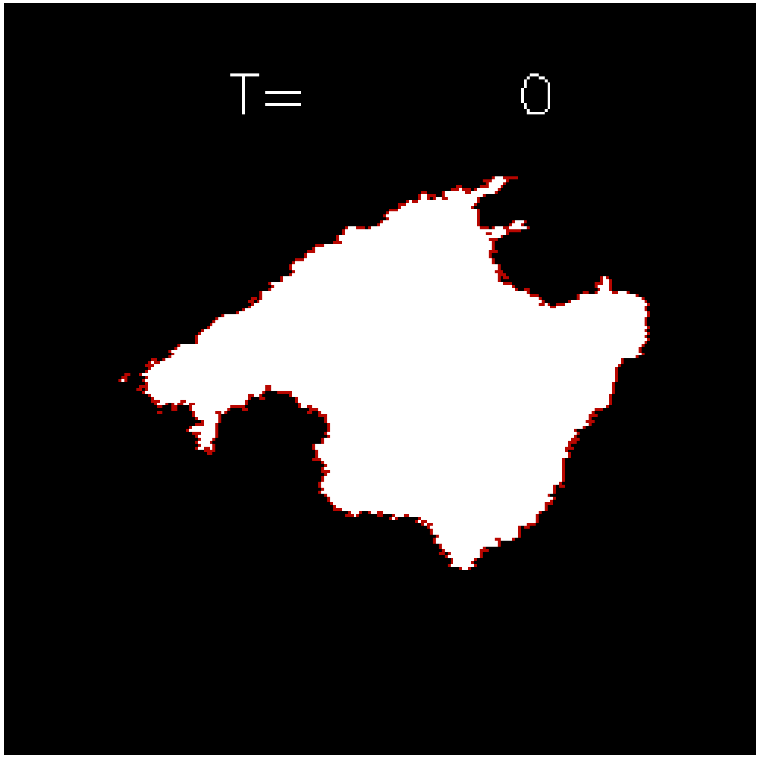
Normal front velocity: $v = -c_1(p - p_c)\kappa \; -c_2\kappa^2\partial^2_\theta\kappa \; -c_3\kappa^3$ $\quad c_1 > 0, \quad c_2, c_3 < 0$

Shrinking term $O(\kappa)$

Reduction of curvature differences $O(\kappa^2)$

Exploding term $O(\kappa^3)$

At the Modulational Instability $p = p_c$: $v = -c_2\kappa^2\partial^2_\theta\kappa \; -c_3\kappa^3$



D. Gomila et al,

Phys. Rev. Lett. 87, 194101 (2001);

IEEE J.Quantum Electr. (2002)