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Palma de Mallorca, Spain
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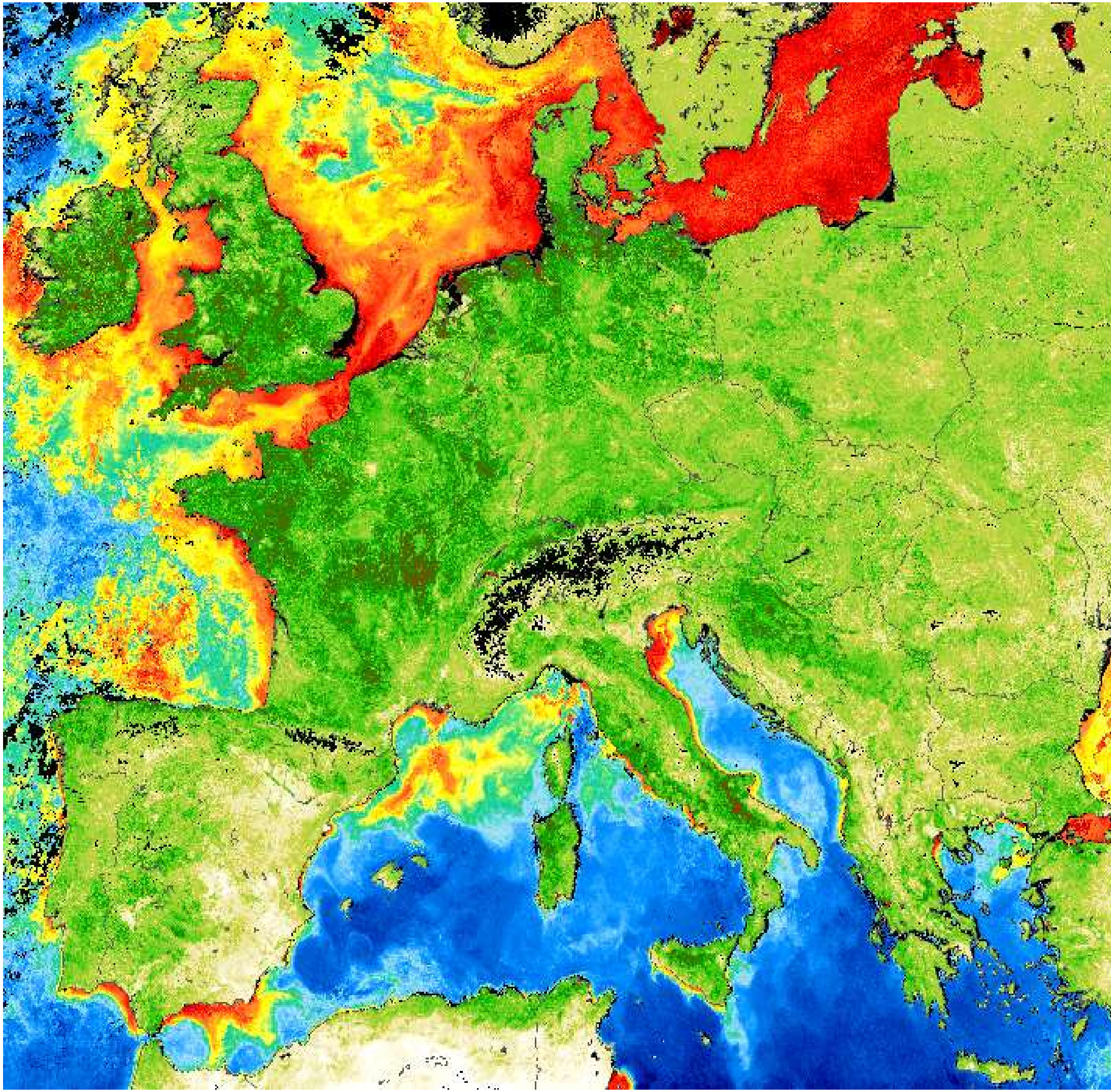
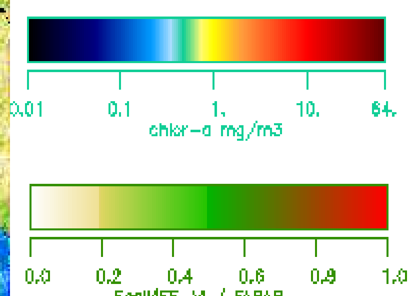
Transport and mixing in the Mediterranean sea by Finite Size Lyapunov Exponents calculation

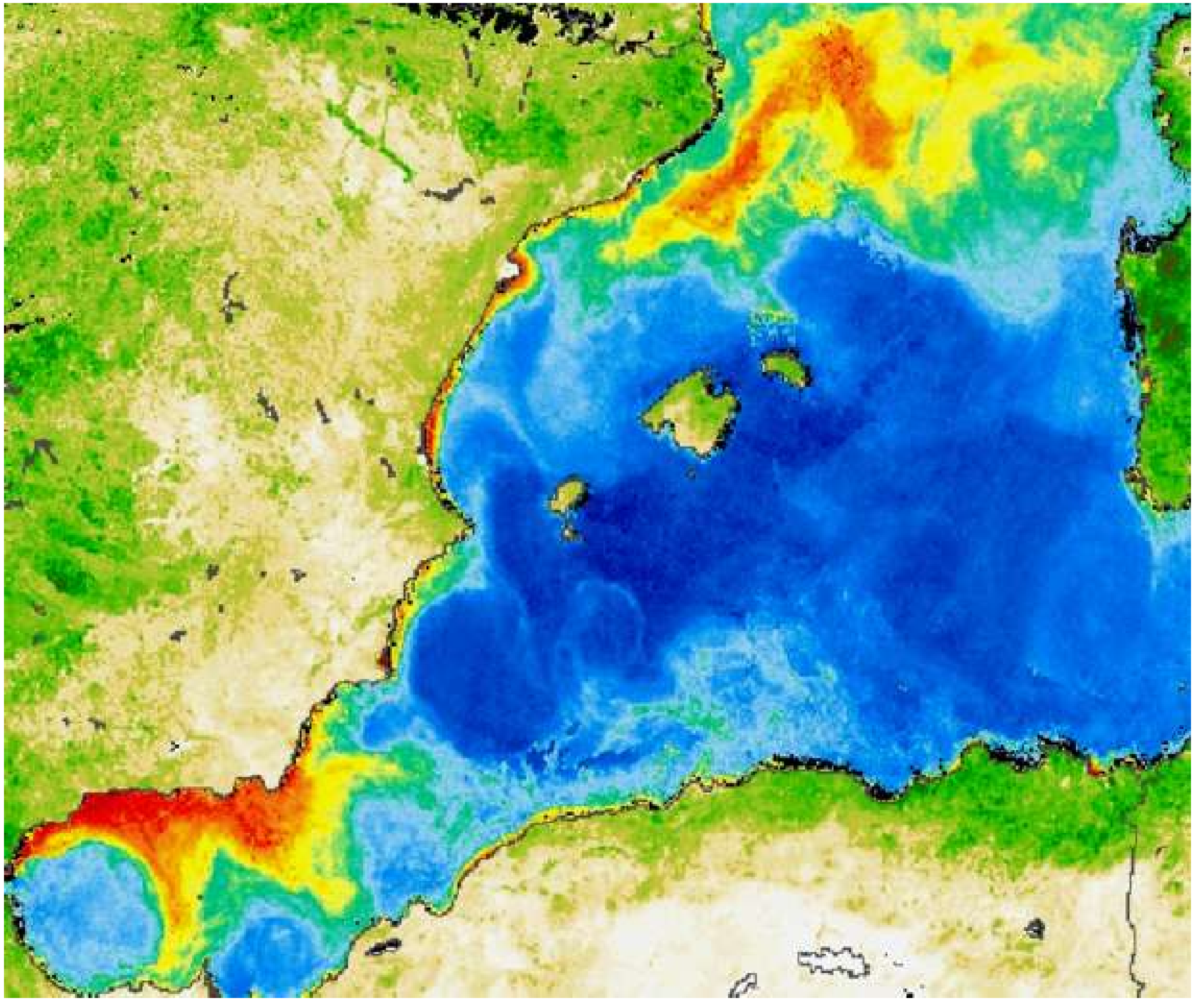
Work in collaboration with: Emilio Hernández-García, Cristóbal López

Chlorophyll seen from SeaWiFs

Monthly composite April 2000







Growing availability of new data

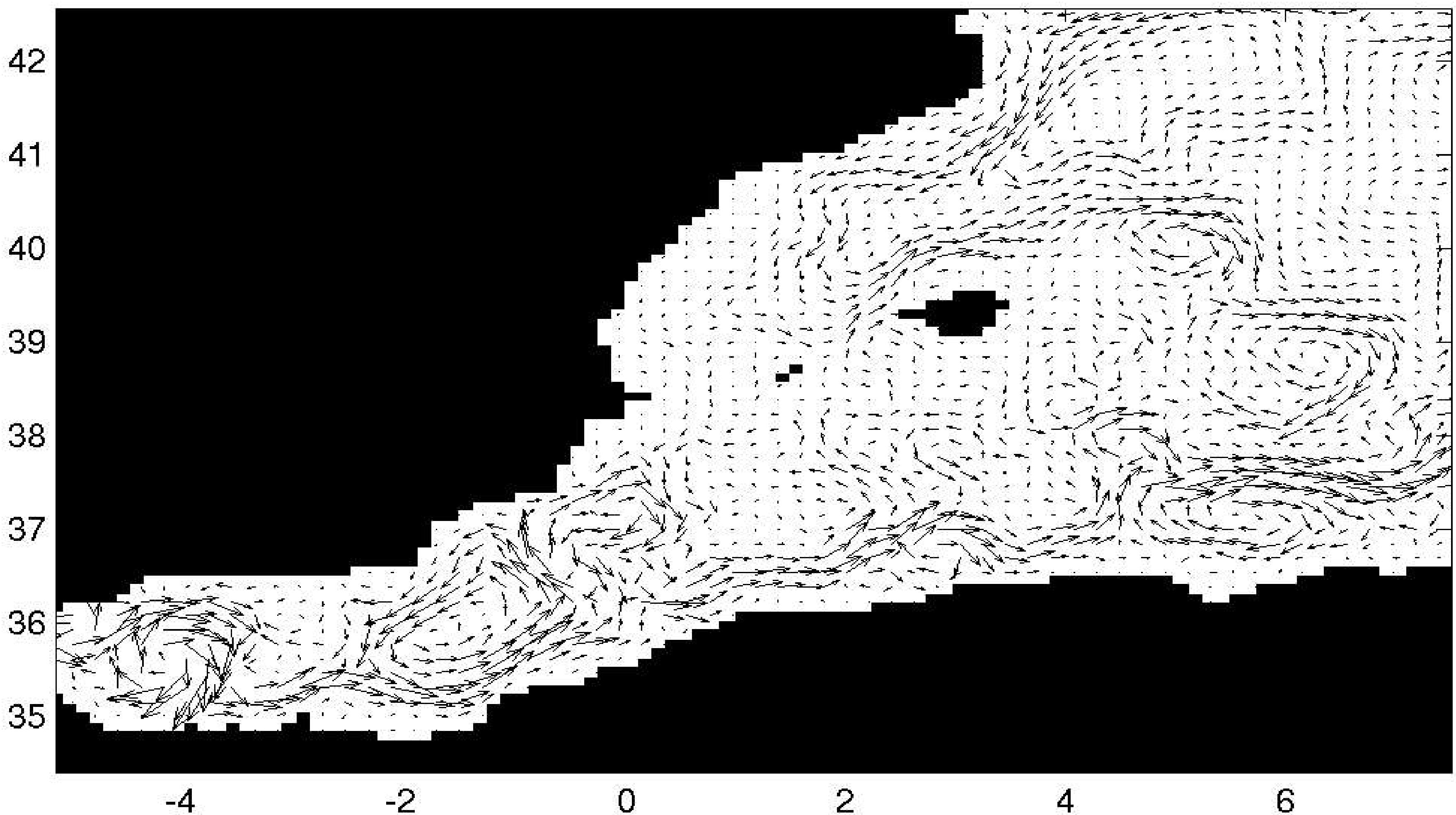
- Chlorophyll
- Temperature
- Salinity
- ...

Improved space and time resolution of the velocity field

- Realistic models (numerical simulations)
- Earth-based, satellite observations

DieCAST simulation data of Mediterranean velocity field

- Simulations of velocity, salinity, and temperature
- Space: solid grid with 1/8 deg. Of step (approx. 10 km) and 30 horizontal layers
- Coriolis terms, wind stress, heat, freshwater forcing



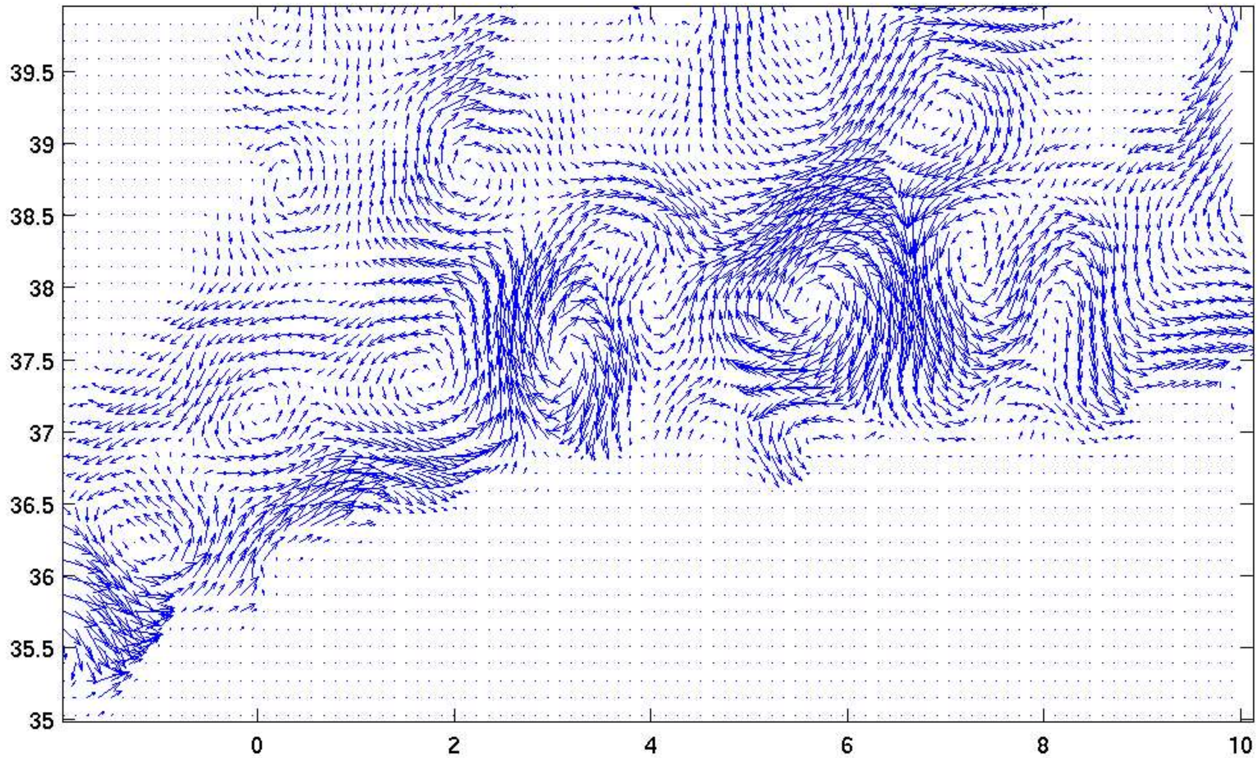
Real data (satellite + in situ measurements)

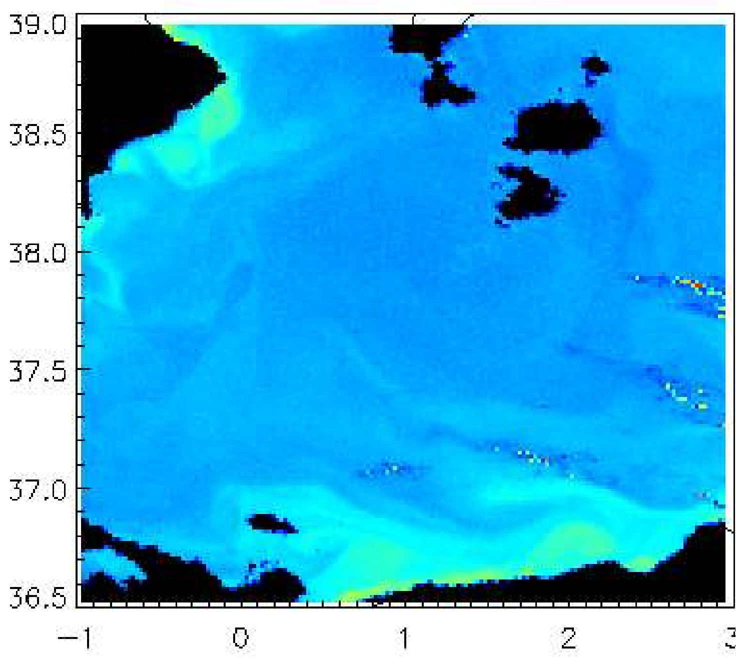
- Time resolution of 10 days
- High space resolution (few km)

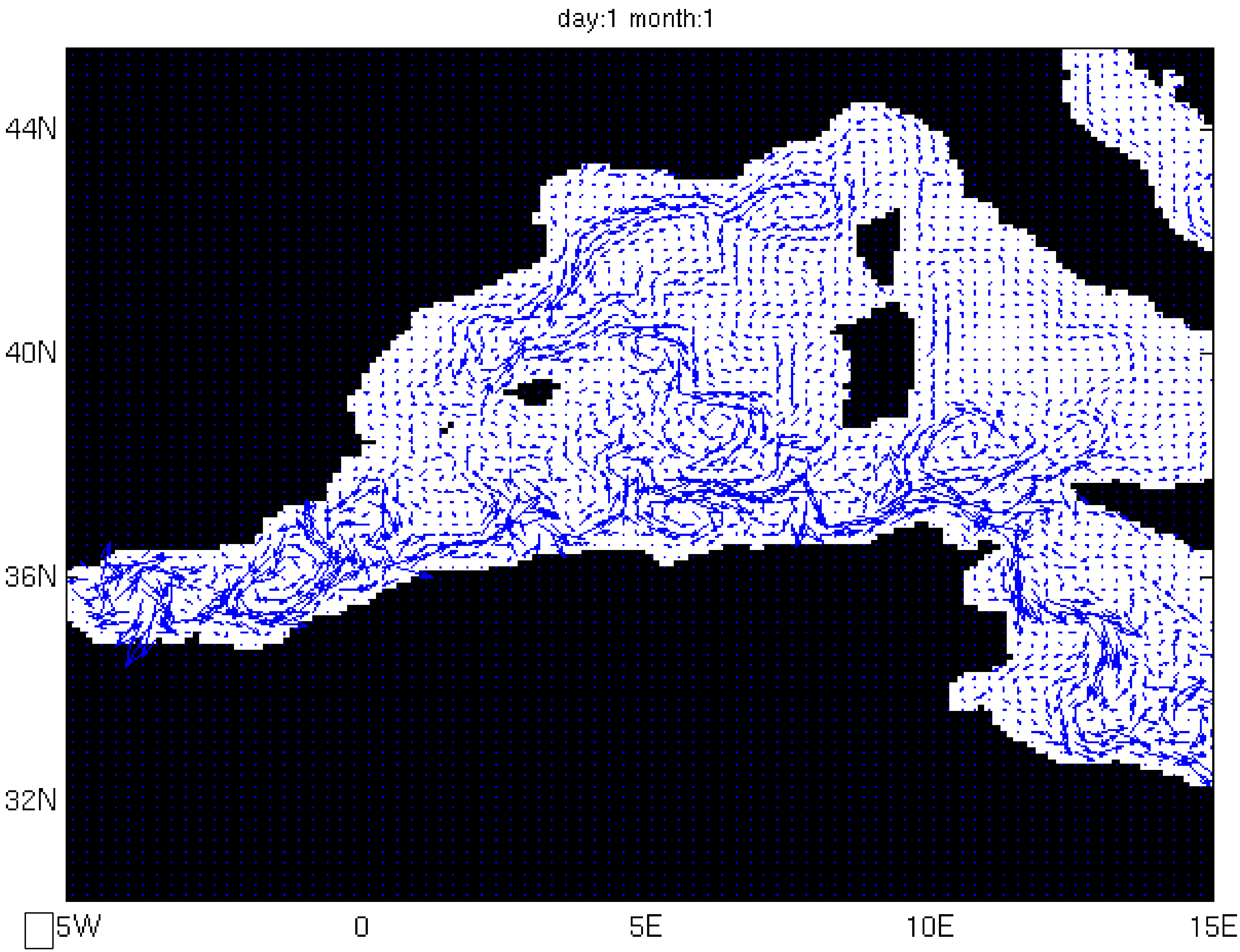
"Noise:"
- Cloud coverage (spectroscopic meas.)
- Satellite movement
- Geoid problem

Lot of interpolation and filtering

29 July 1997






day:1 month:1
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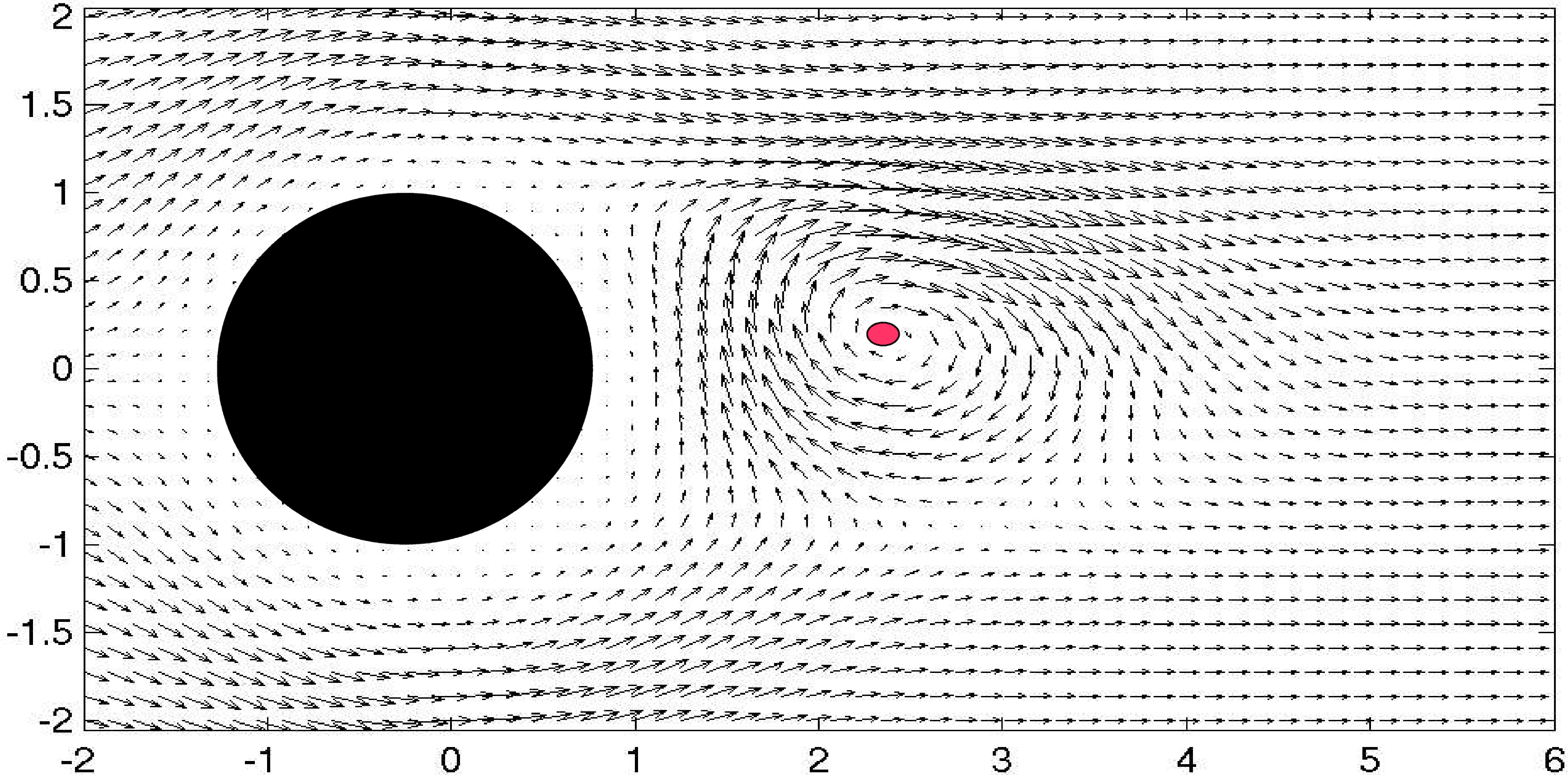
Aim: deduction of the phase portrait from the velocity field

$$\frac{dx}{dt} = u(x,t)$$

- Structure from raw data
- Template for geophysical processes

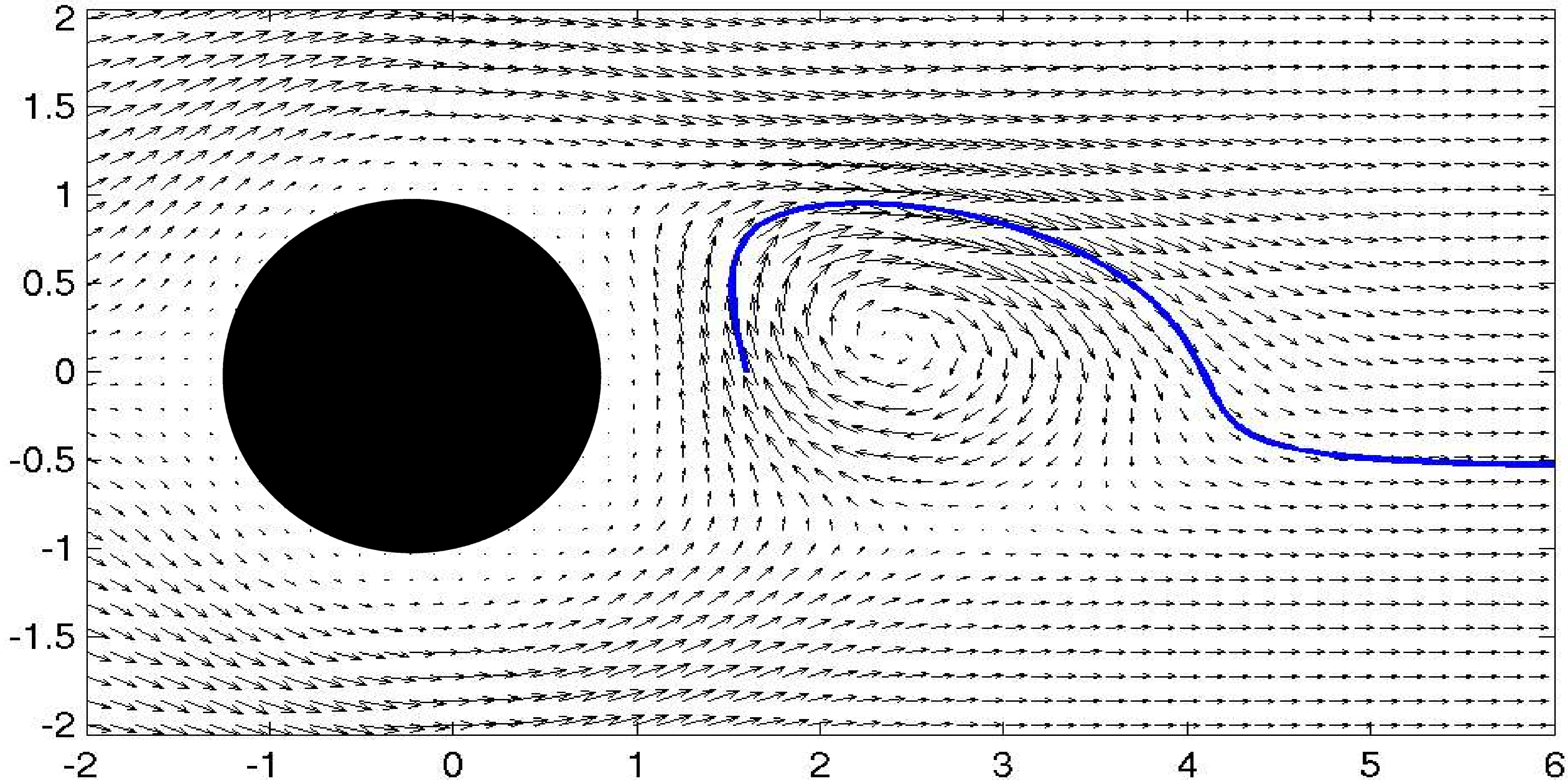
ODEs		Velocity field
Phase portrait	=>	Transport template
KAM tori, stable/unstable manifolds		Transport barriers and avenues
Chaotic sets		Mixing regions
.....		.....

Example 1: von Kármán velocity field

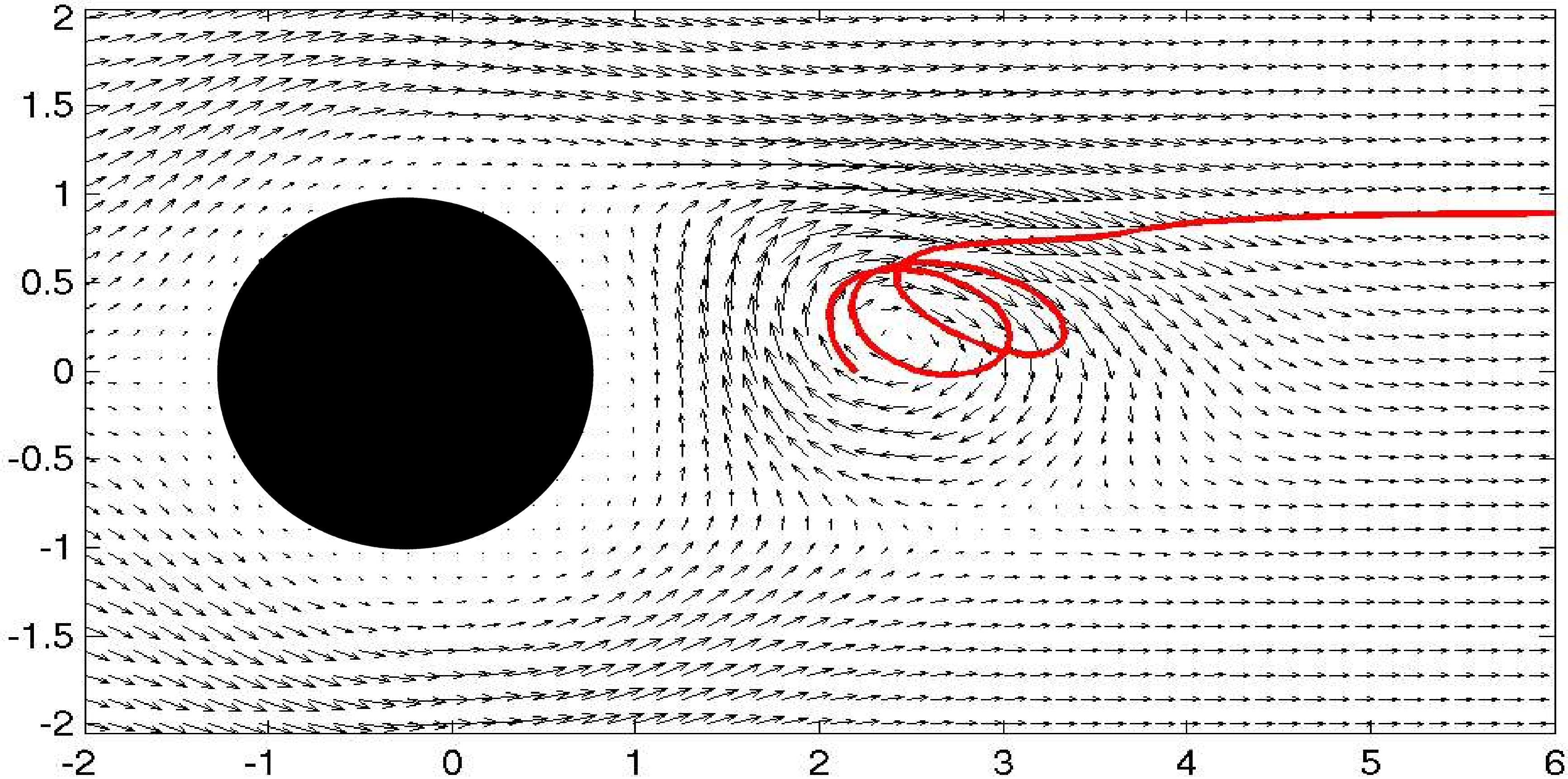


Eulerian analysis: the dynamics is controlled by an elliptic point ( ), few hyperbolic points, and invariant 1D tori (incompressible flow).

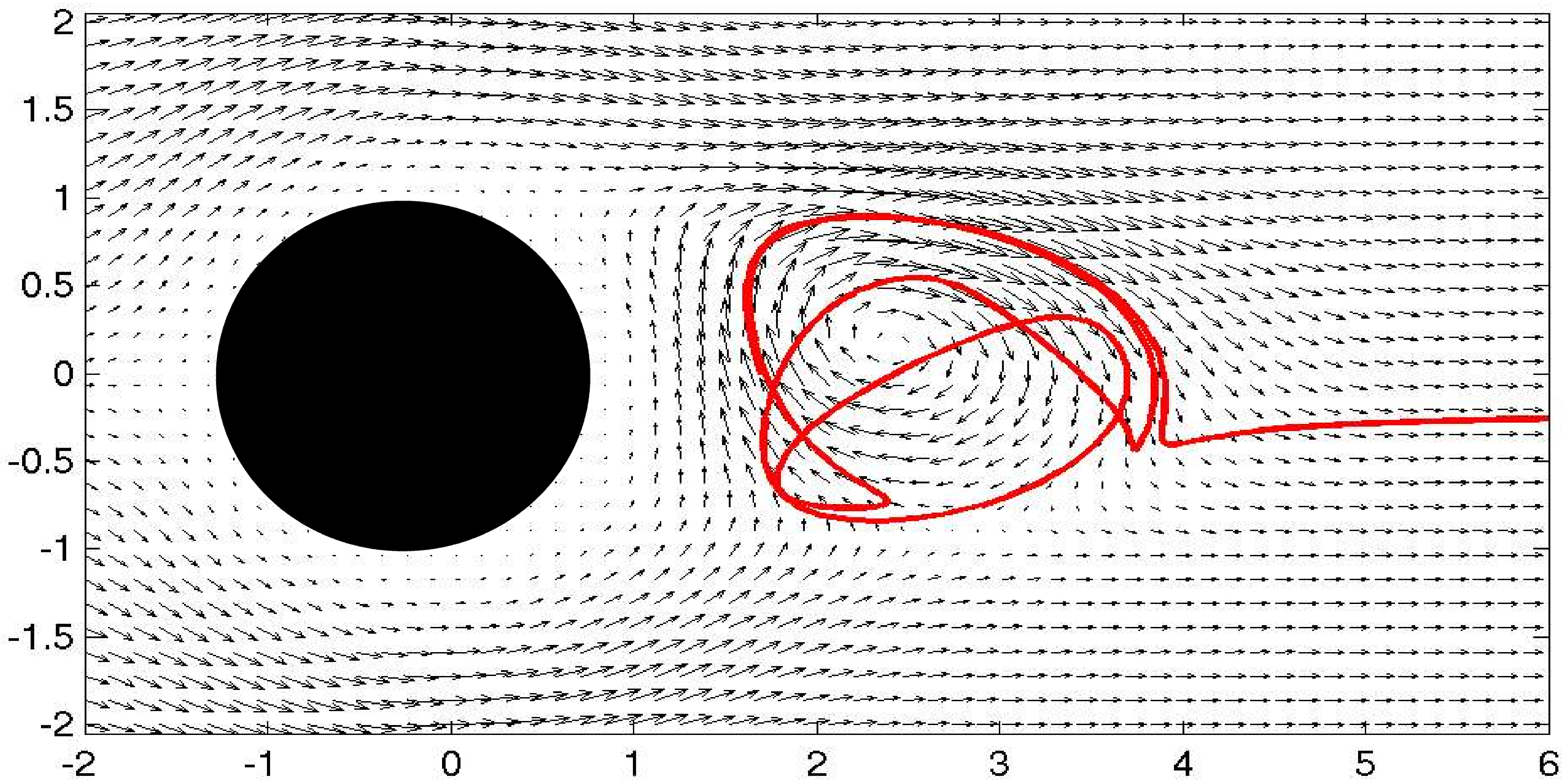
A trajectory consistent with the Eulerian analysis...



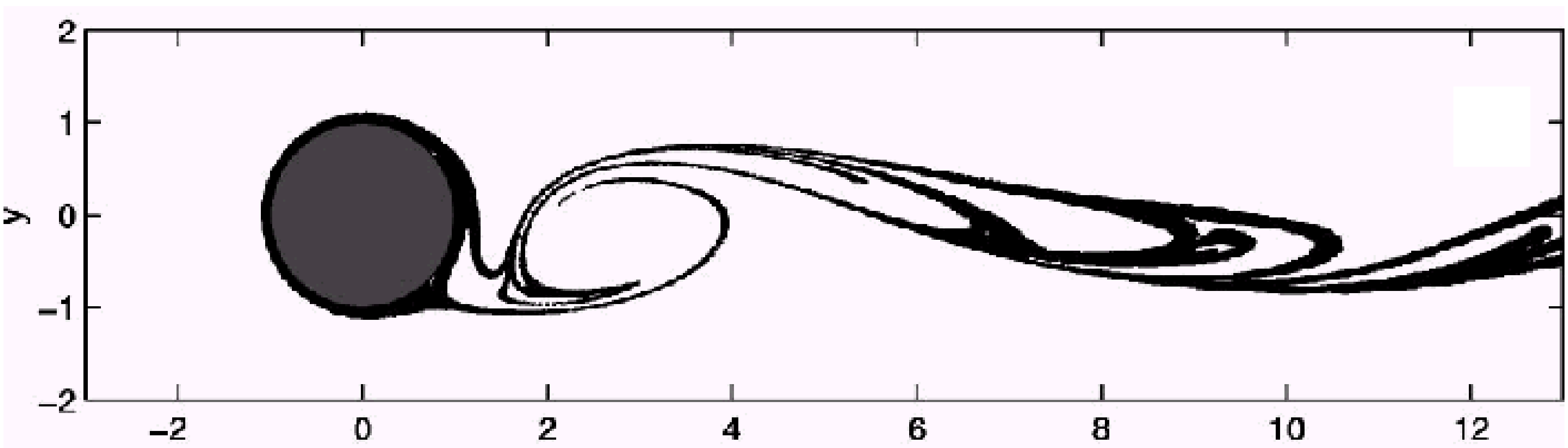
... but there are no tori that separate the phase space...



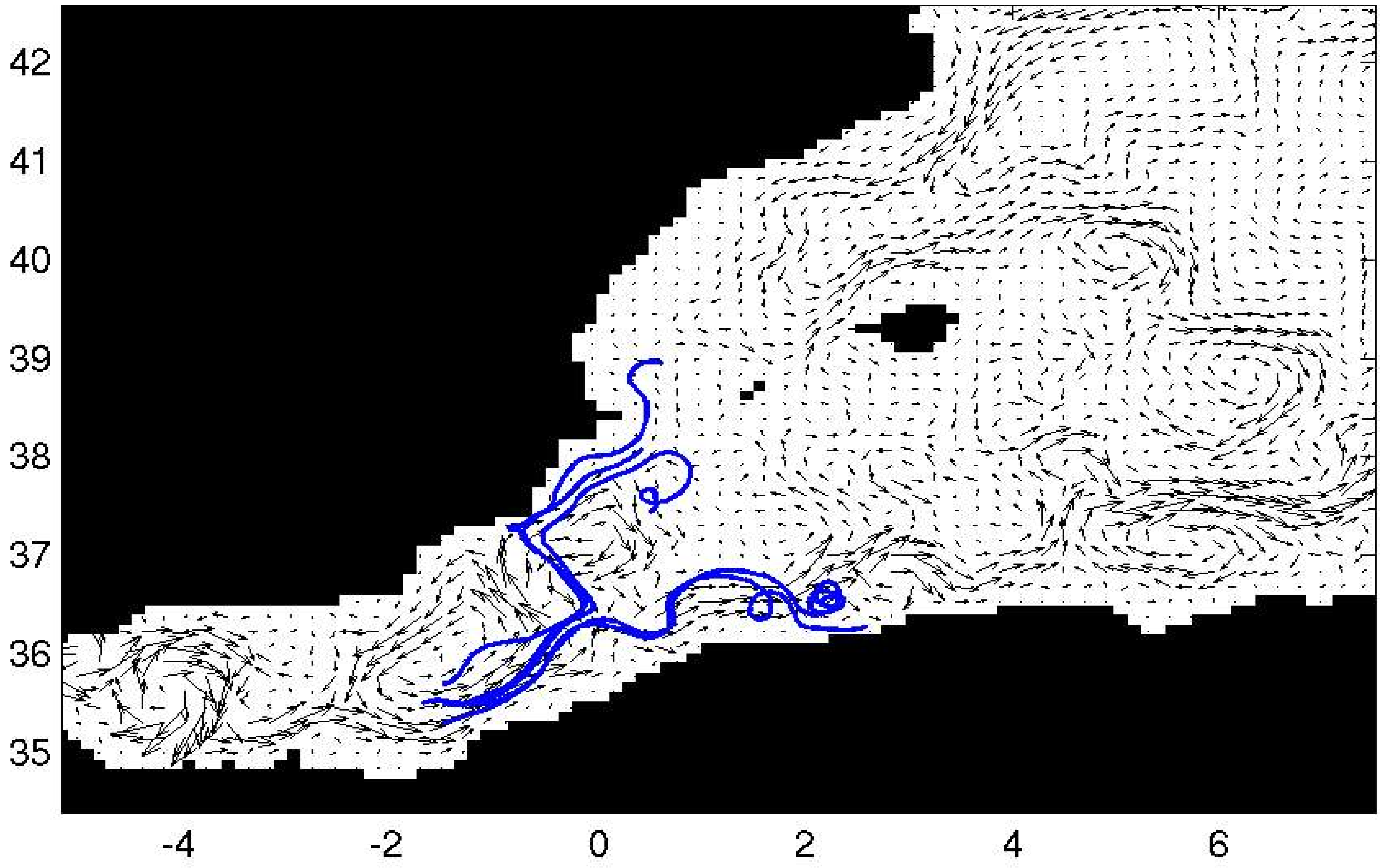
... and there are trajectories that suggest chaotic structures



In spite of the simple Eulerian field, complex structures
are unveiled by tracers



Tracers experiments in the Mediterranean sea (simulated data)



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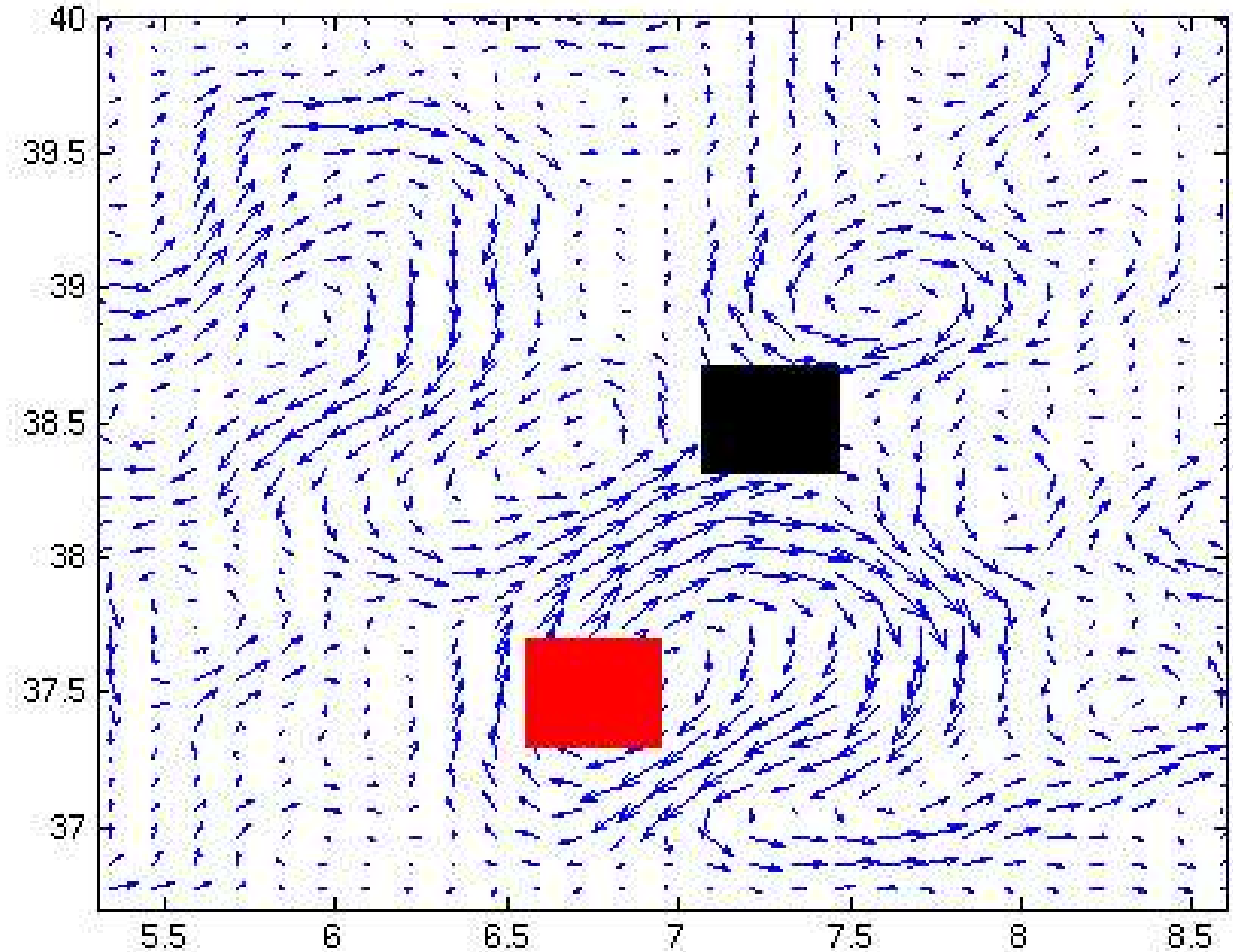

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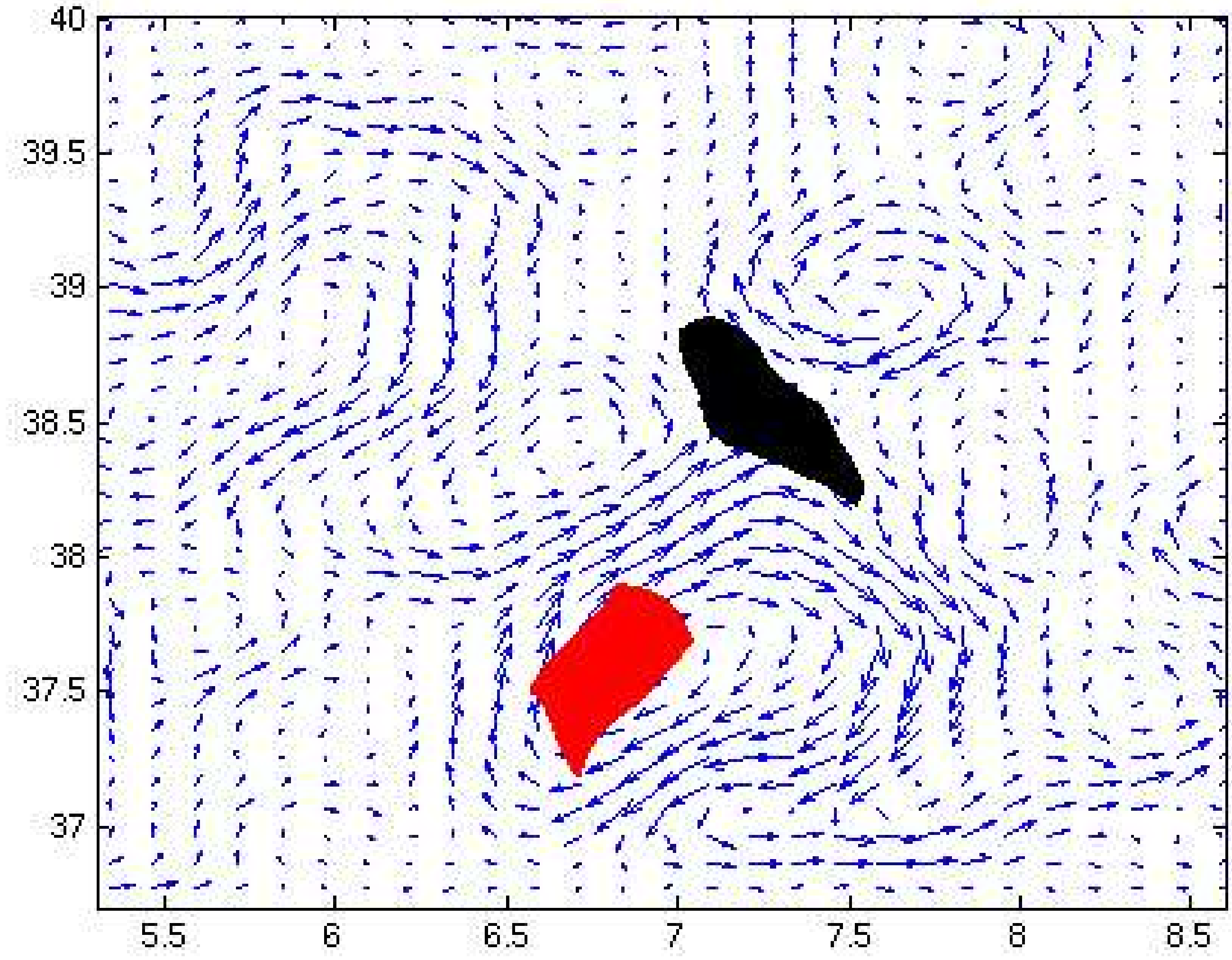

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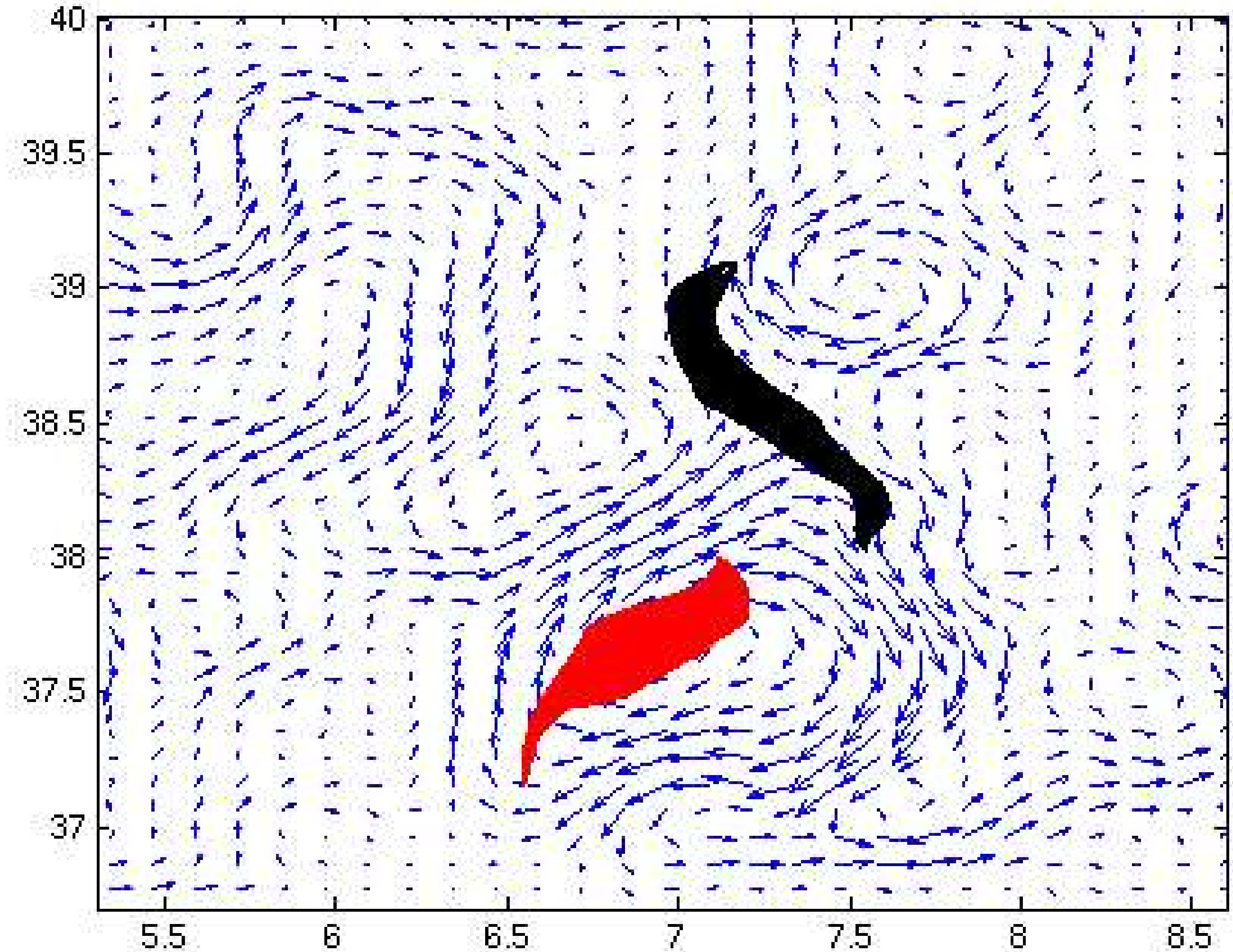

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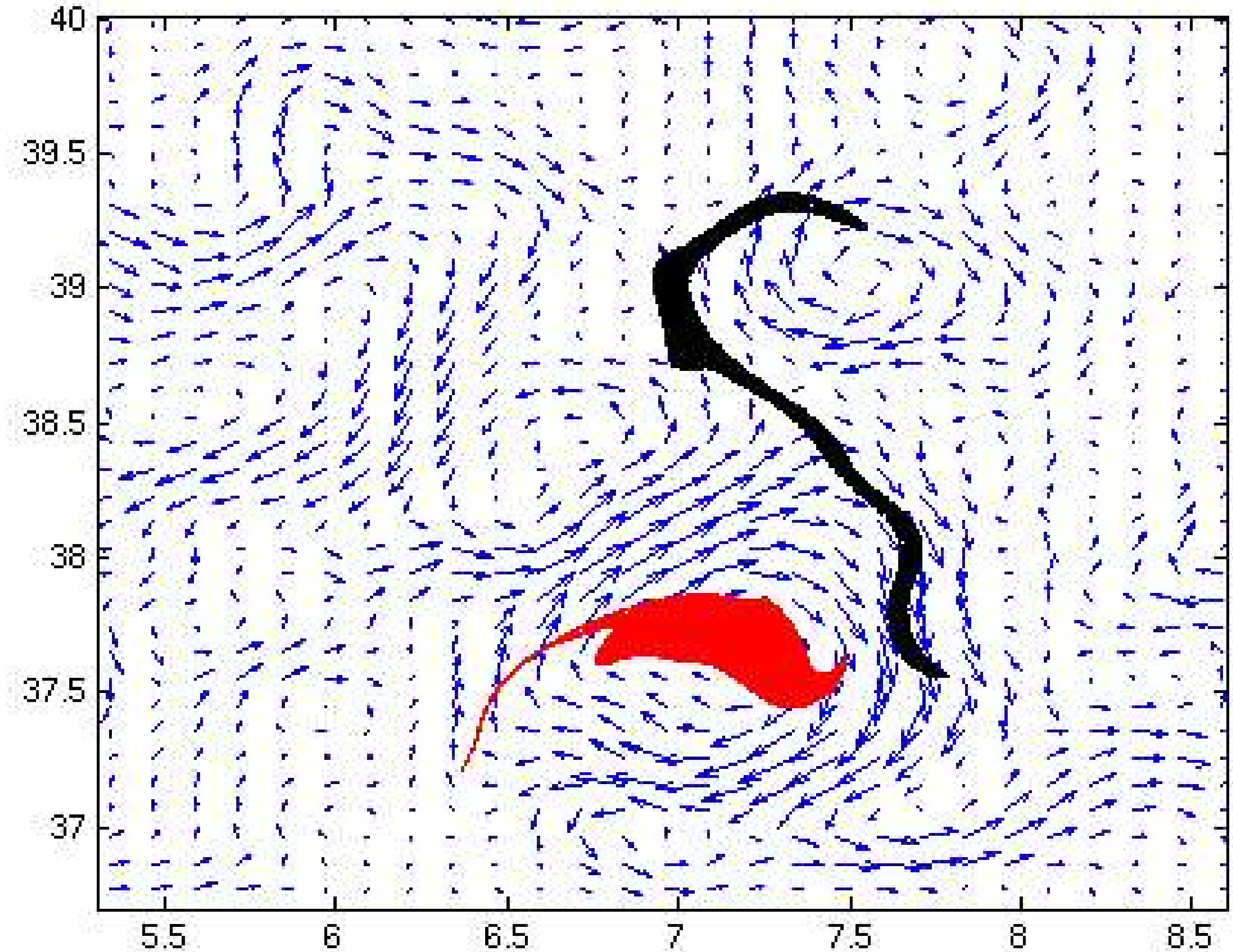

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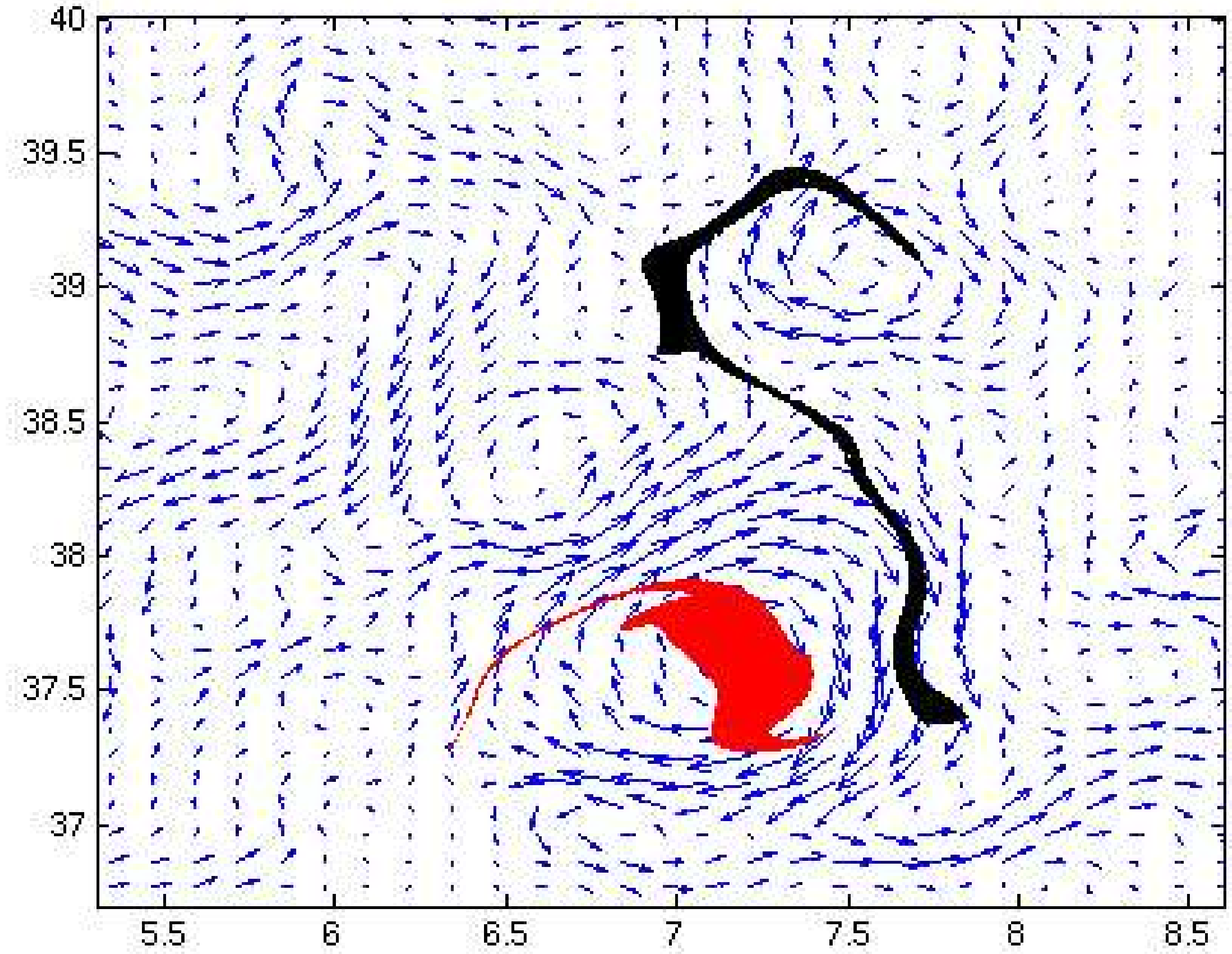

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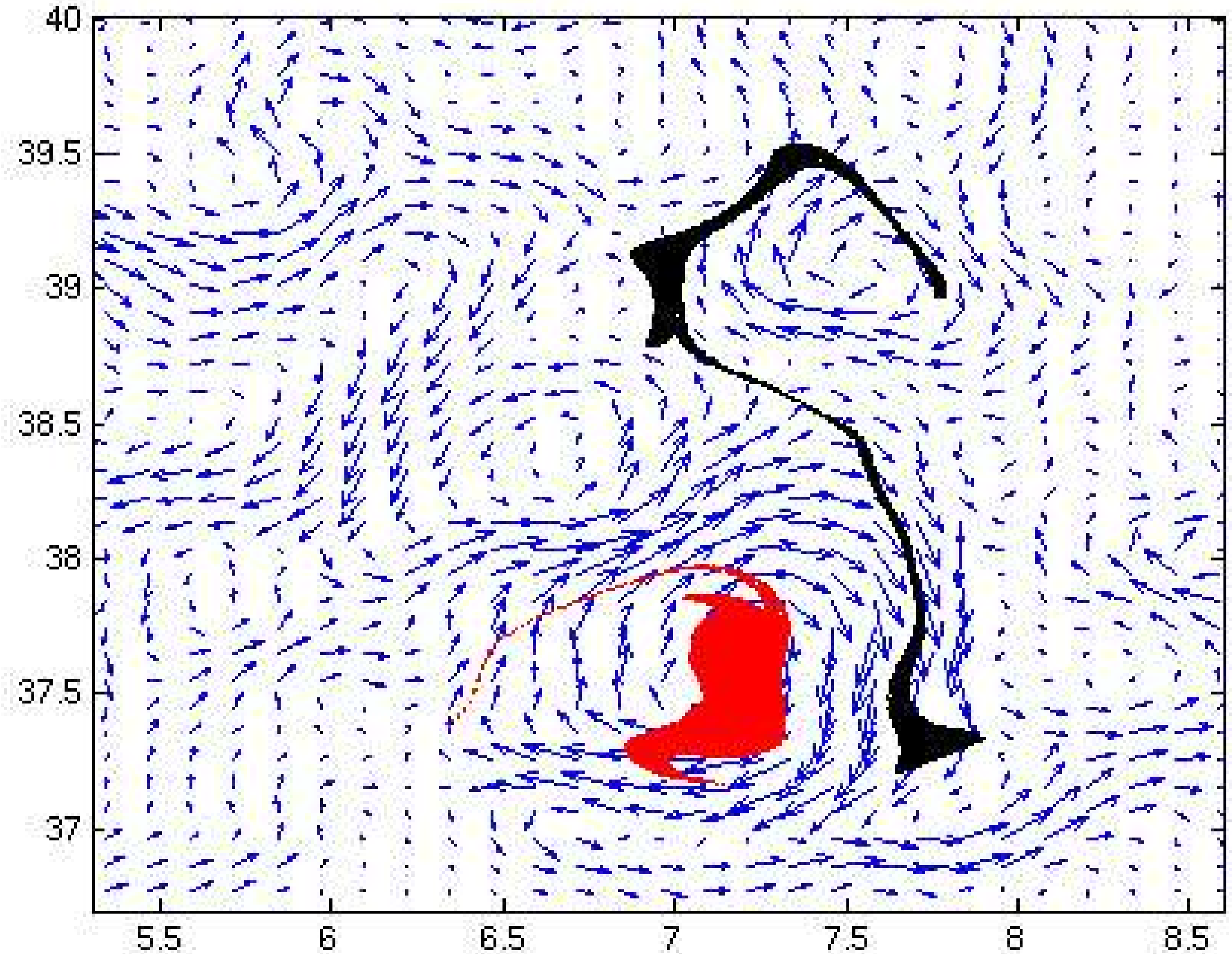

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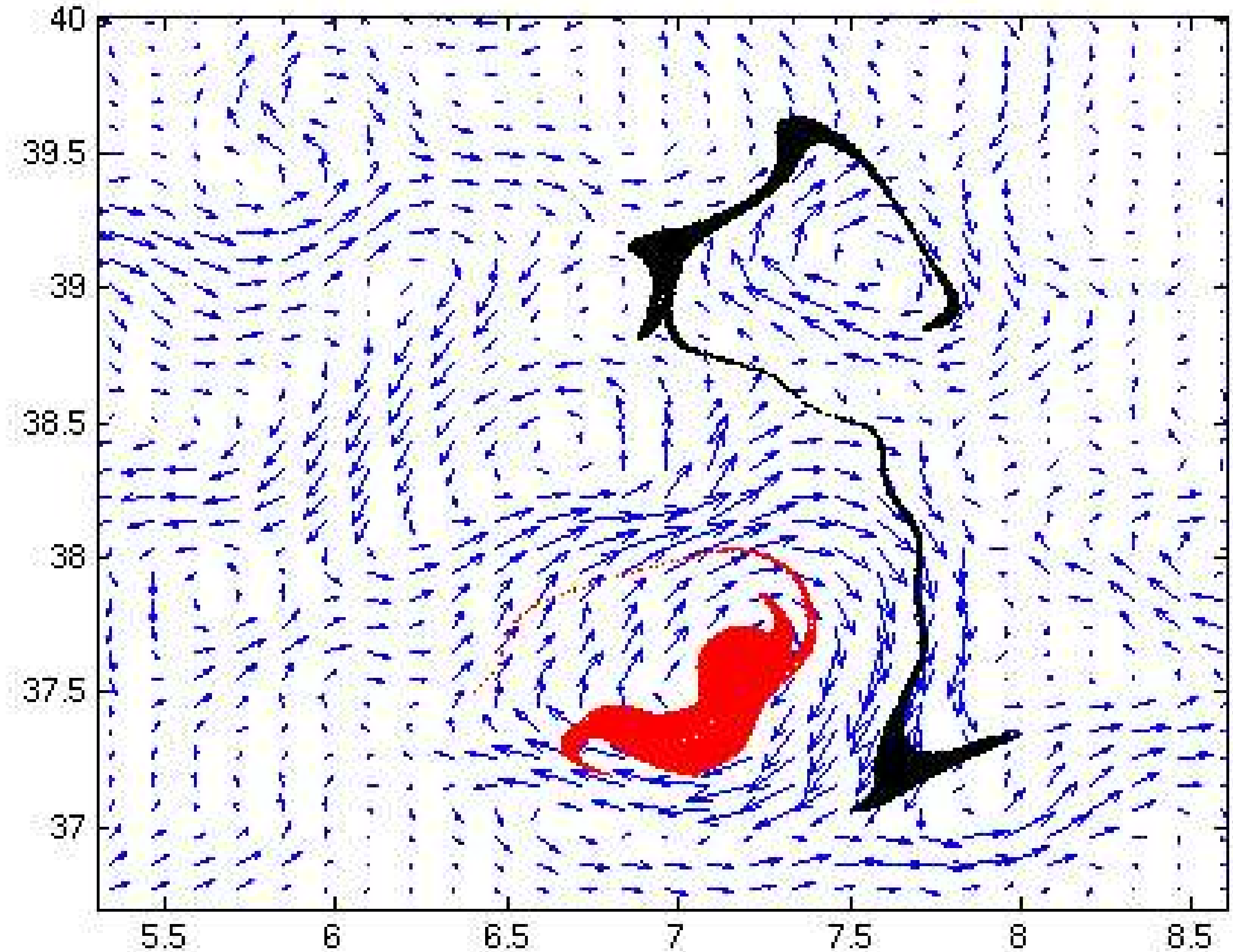

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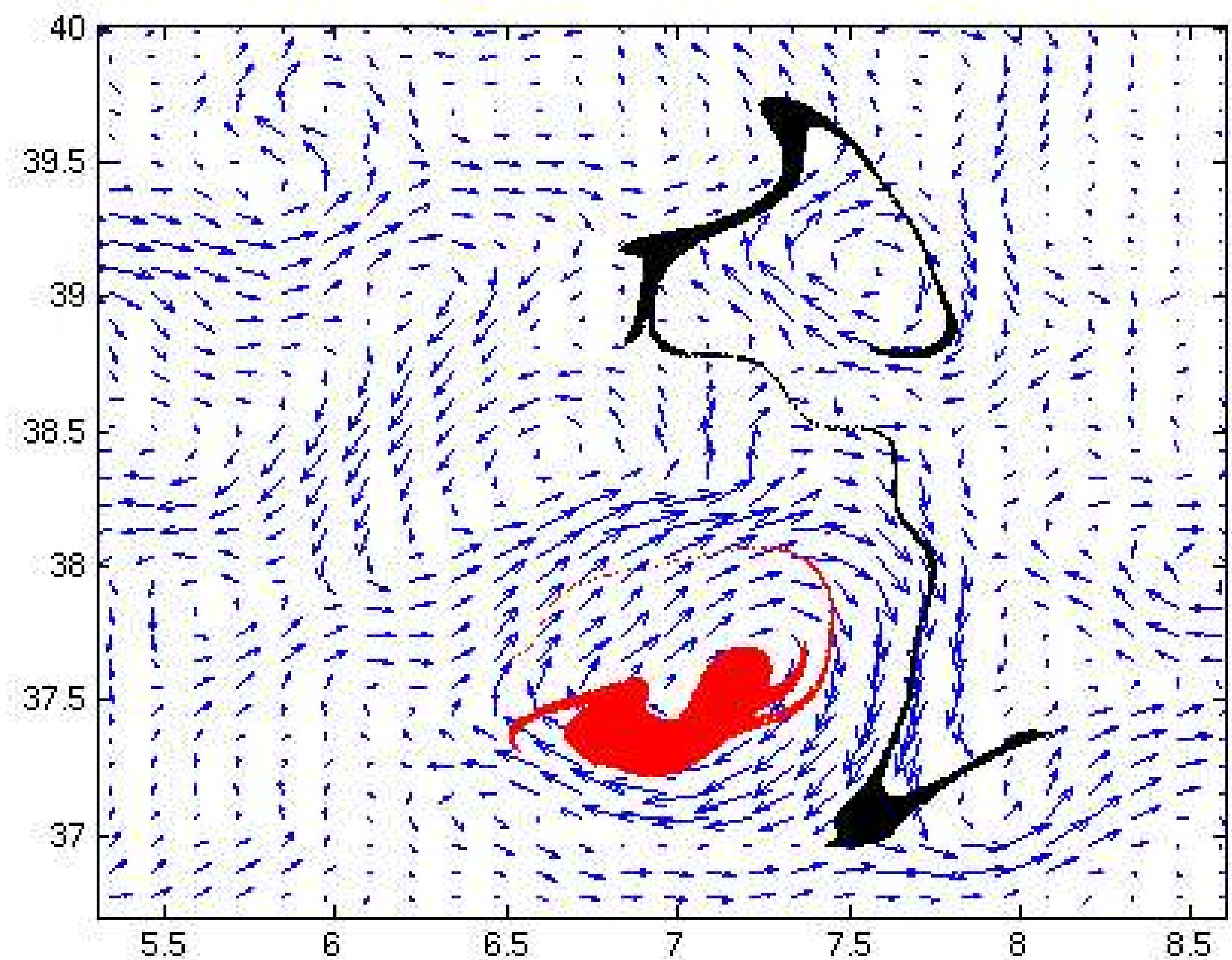

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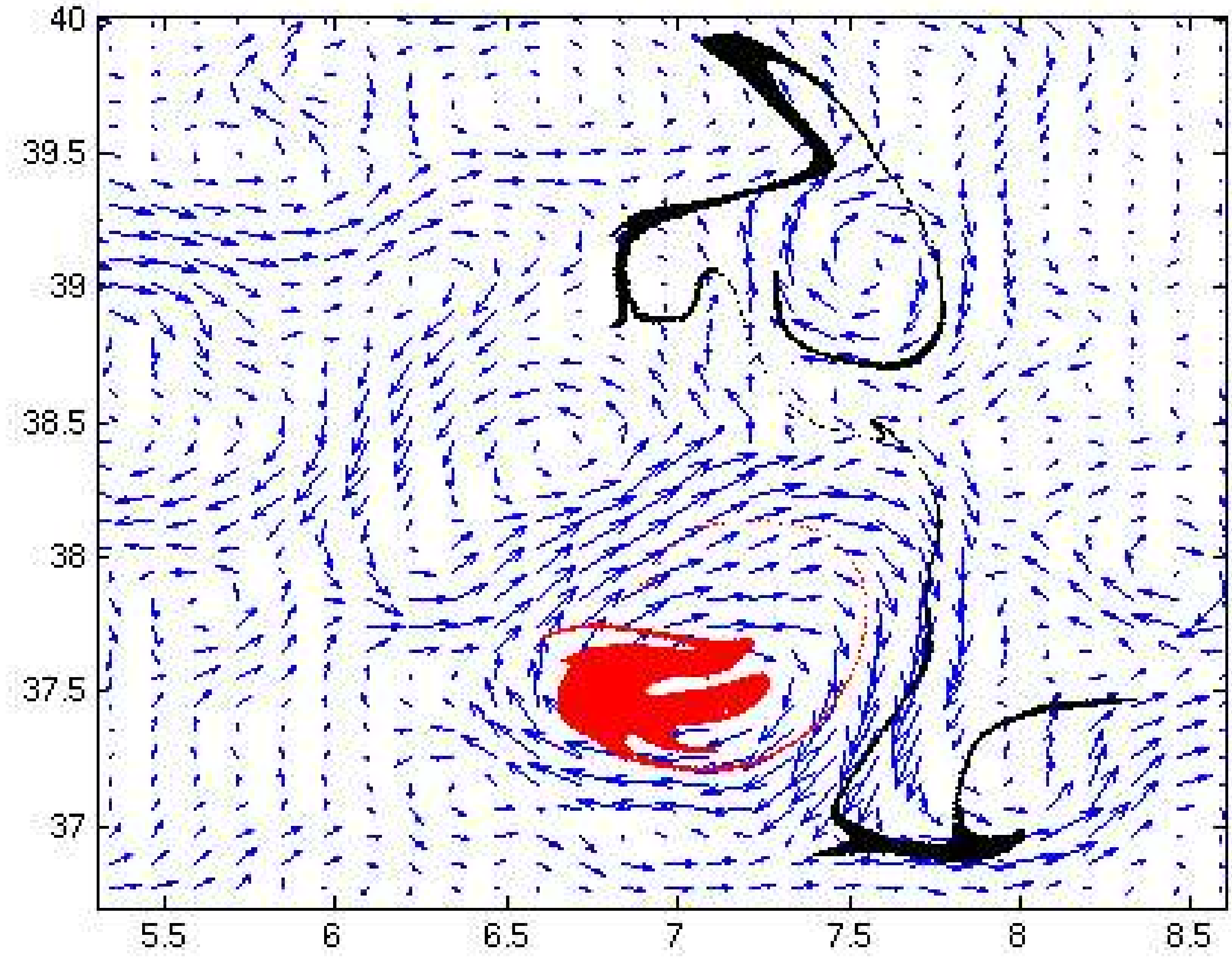

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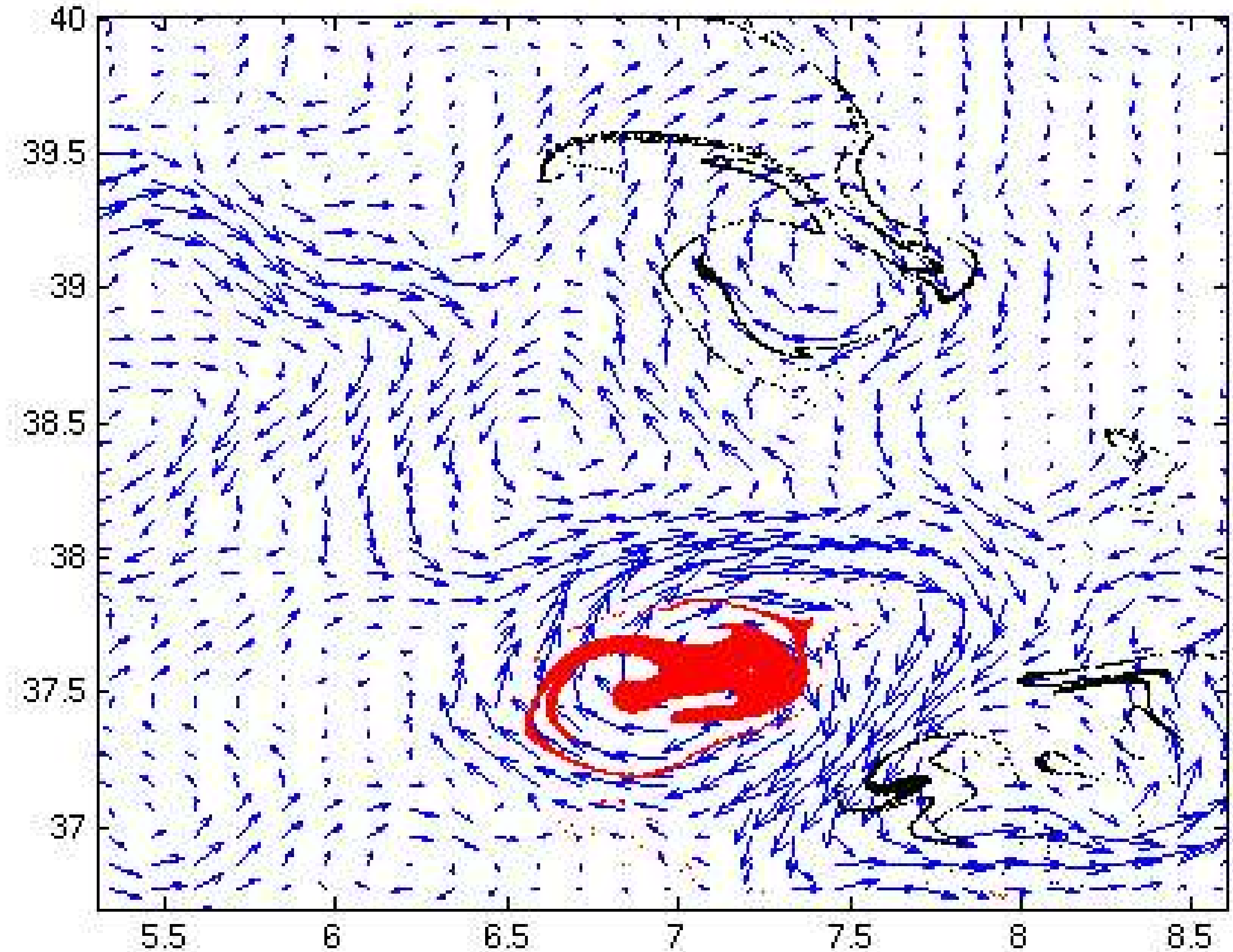
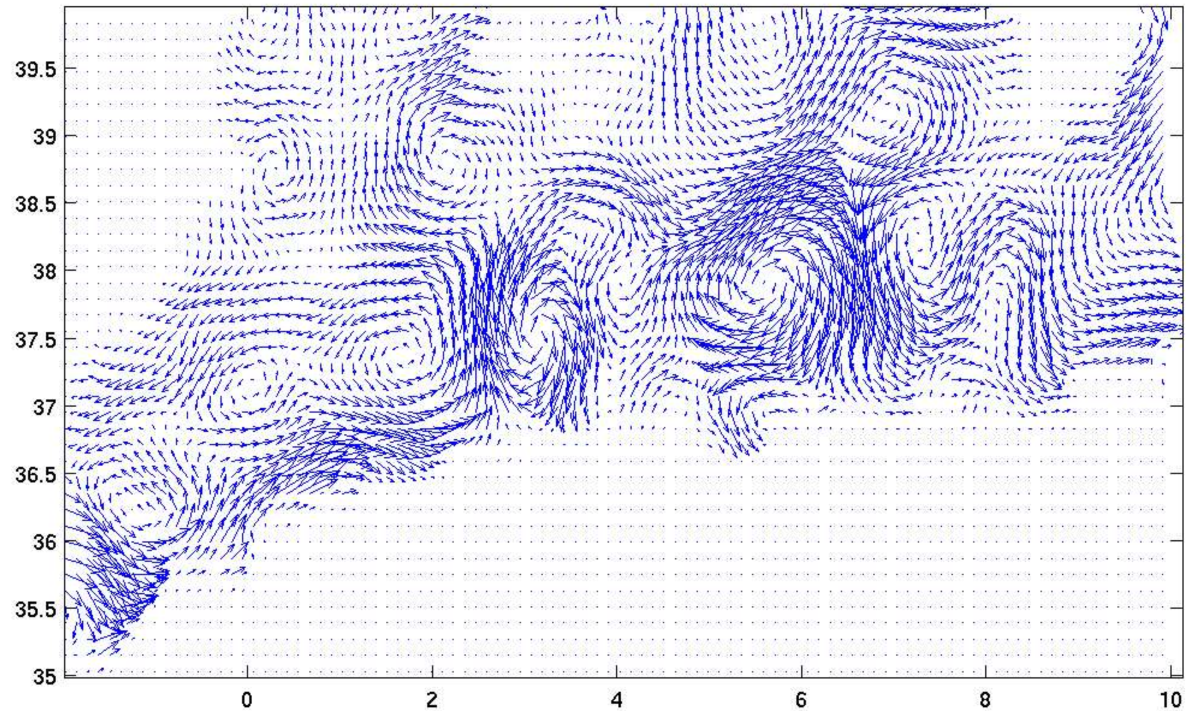
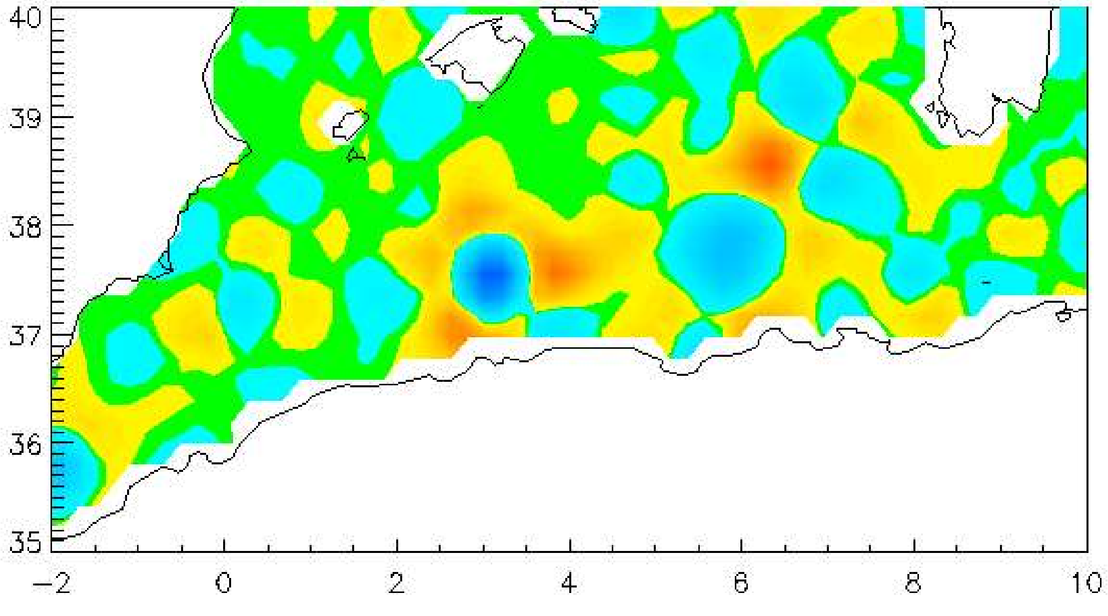


The Okubo-Weiss method

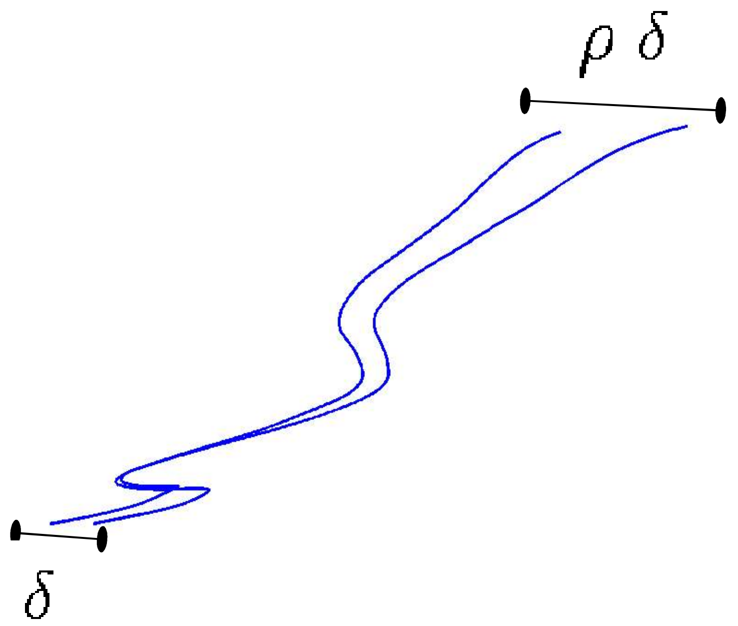
Separate the strain from
the vorticity in the **Eulerian**
velocity field





Finite Size Lyapunov Exponents (FSLEs)

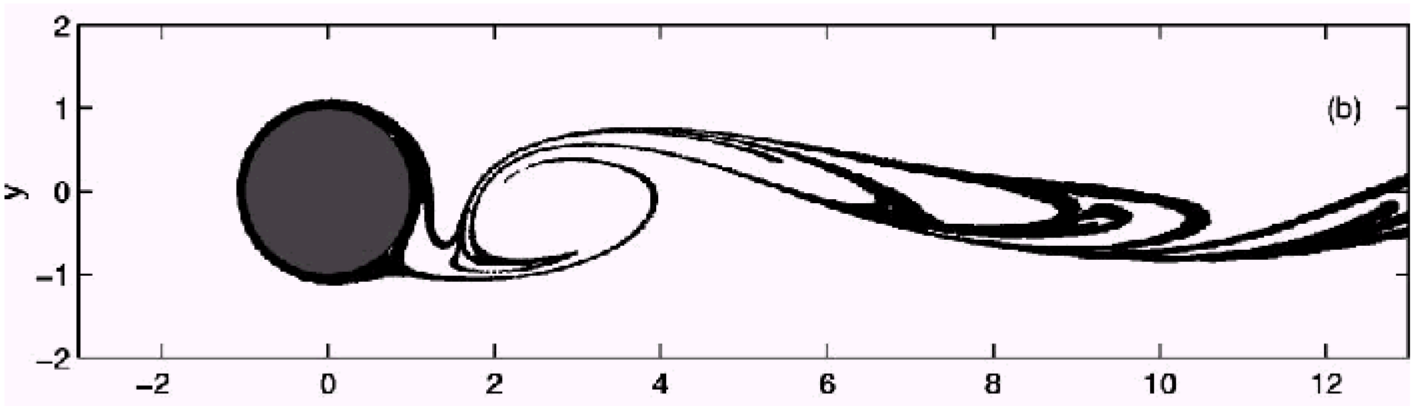
Aurell et al., Phys. Rev. Lett. **77**, 1262 (1996)
Boffetta et al., J. of Phys. A, **30**, 1 (1997) chao-dyn/9904049

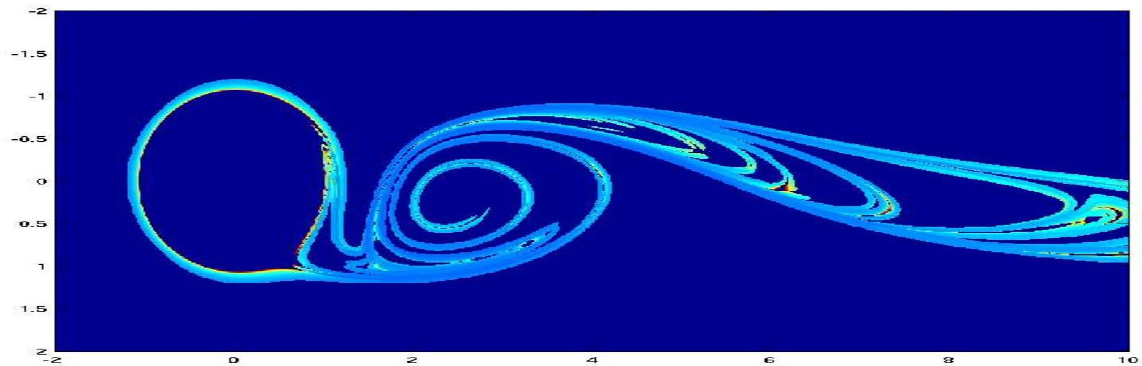


δ =Initial separation
ρ =amplification factor
τ = time needed for the perturbation to grow

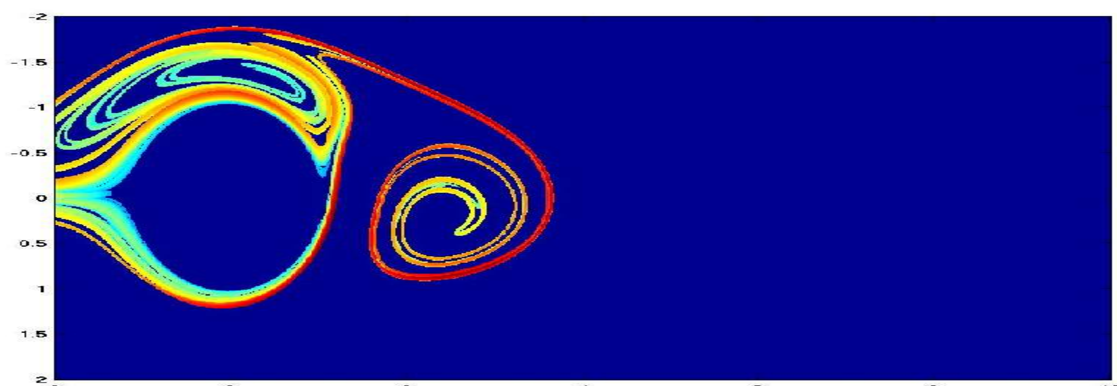
$$\lambda = \frac{\ln \rho}{\tau(\delta)}$$

The FSLEs can be used to detect stable and unstable manifolds of Lagrangian hyperbolic points



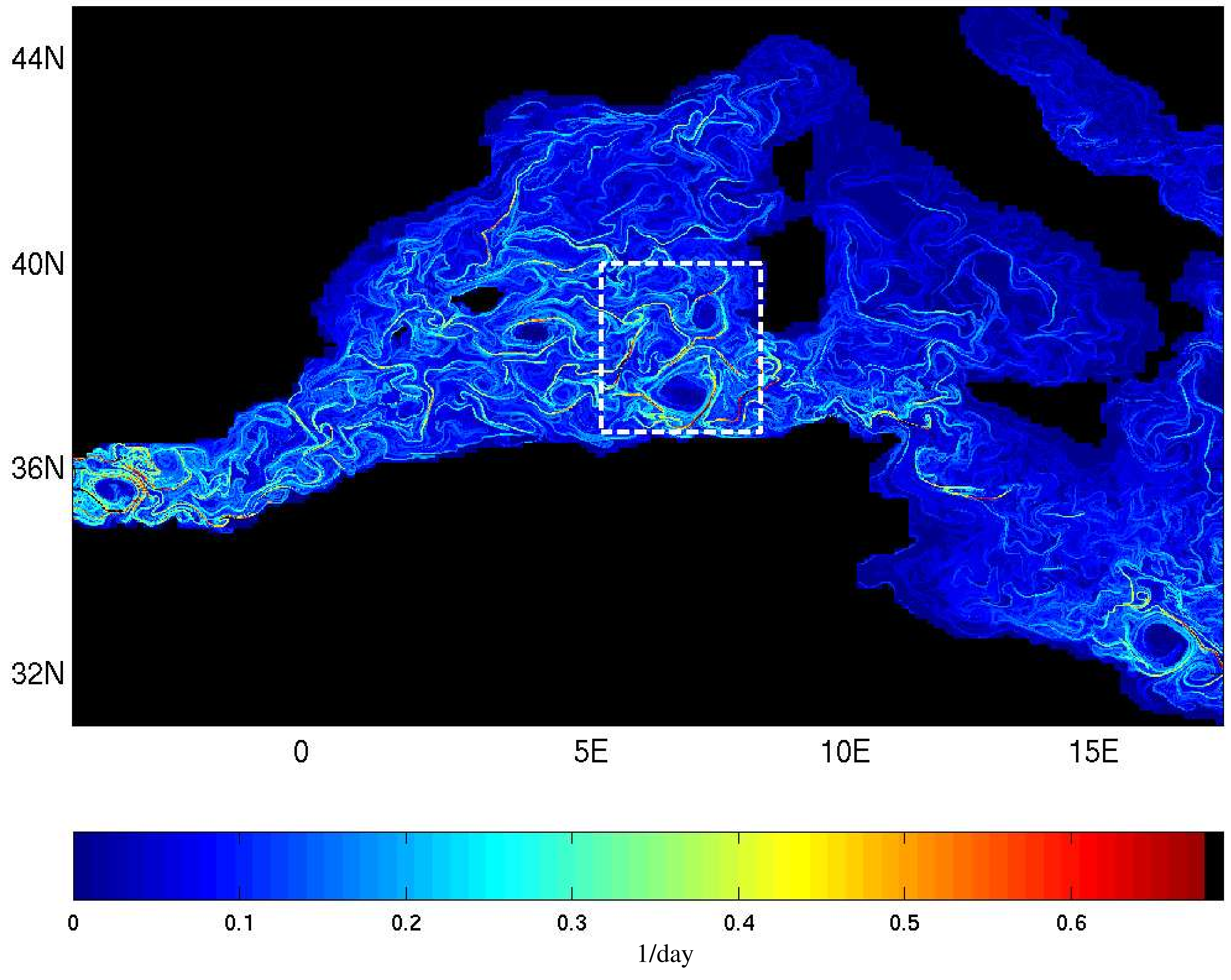


FSLE values from time-backwards trajectories



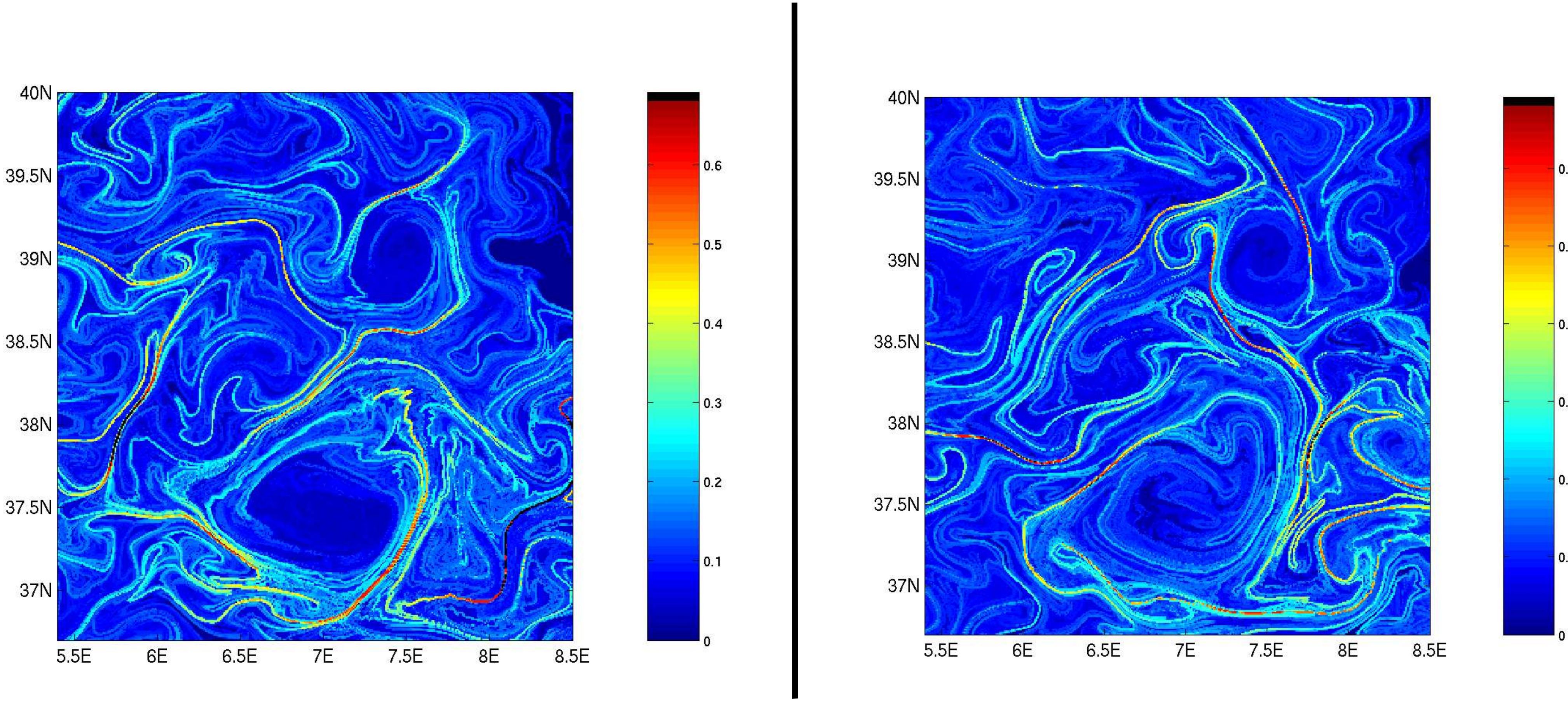
FSLE values from time-forward trajectories

Mesoscale Lagrangian structure of the Mediterranean surface



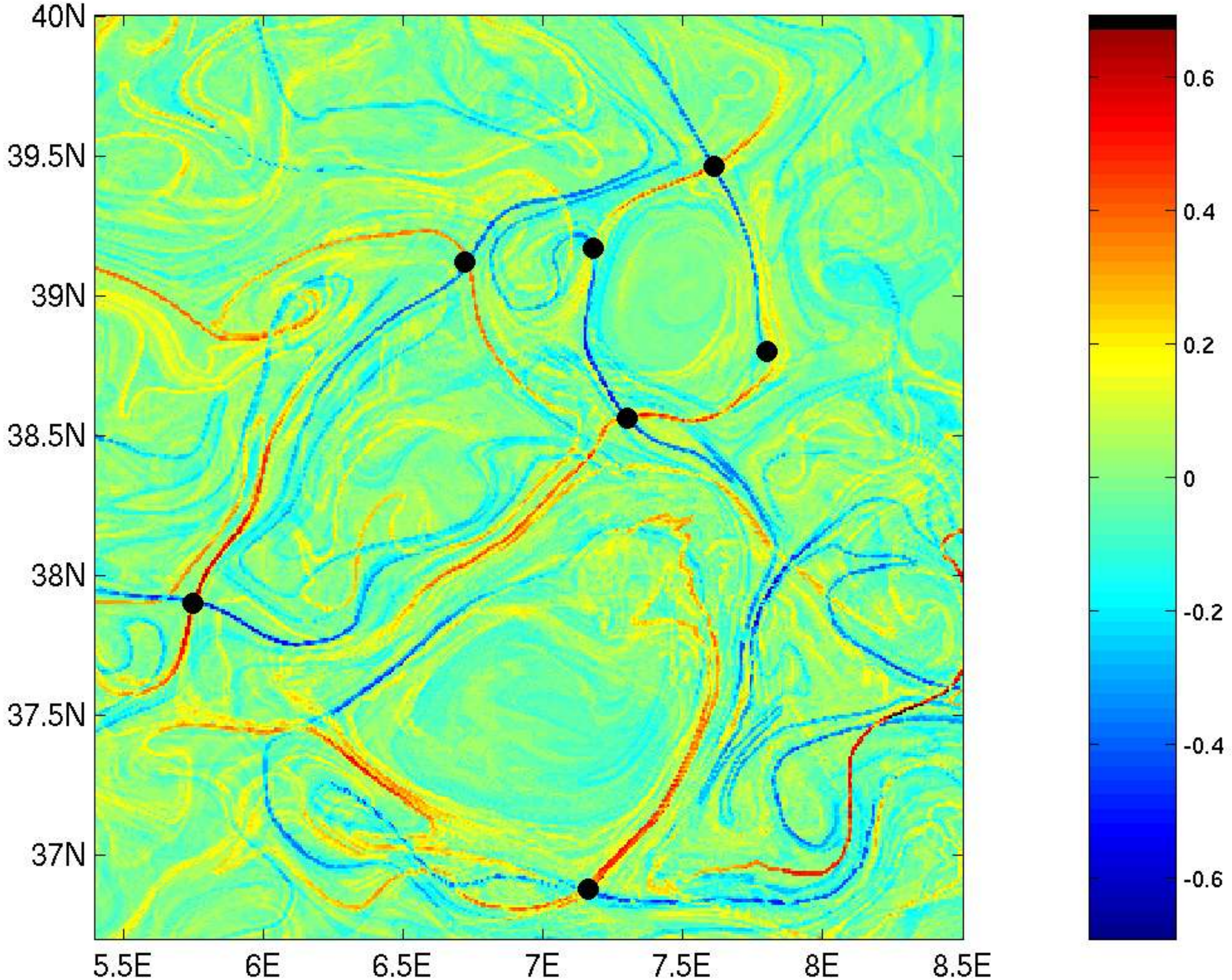
Initial separation of 0.02 deg. (approx. 2.2 km), final separation of 1 deg. (aprox 110 km)

Forward and backward FSLEs (enlargement)



- Incompressible field (statistically, backward and forward structures are similar)
- The filaments in the backward-FSLE picture appears as the forward-in-time ones but rotated 90 deg.
- **Structure below the data grid spacing!**

Intersection of stable and unstable manifolds: hyperbolic points



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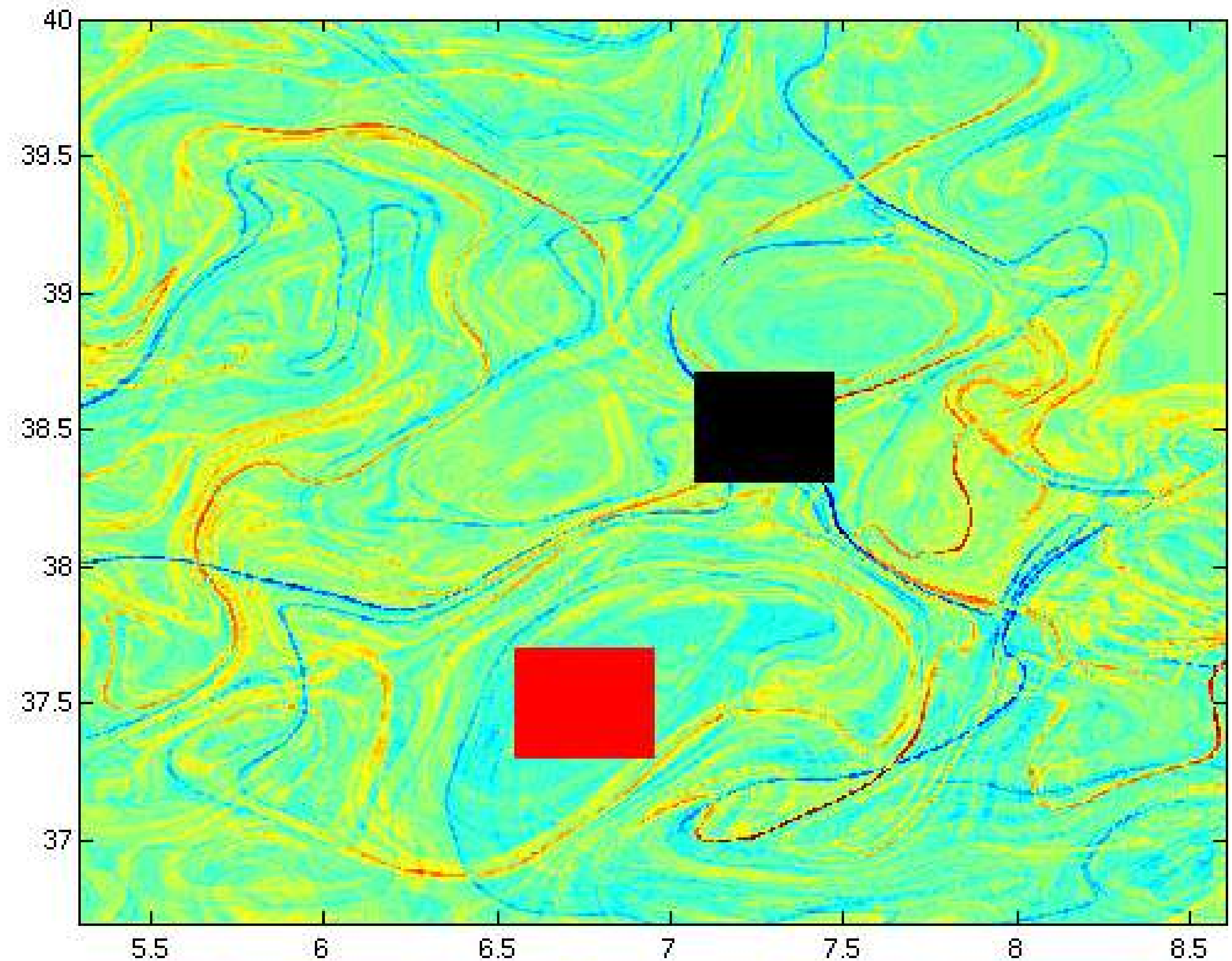

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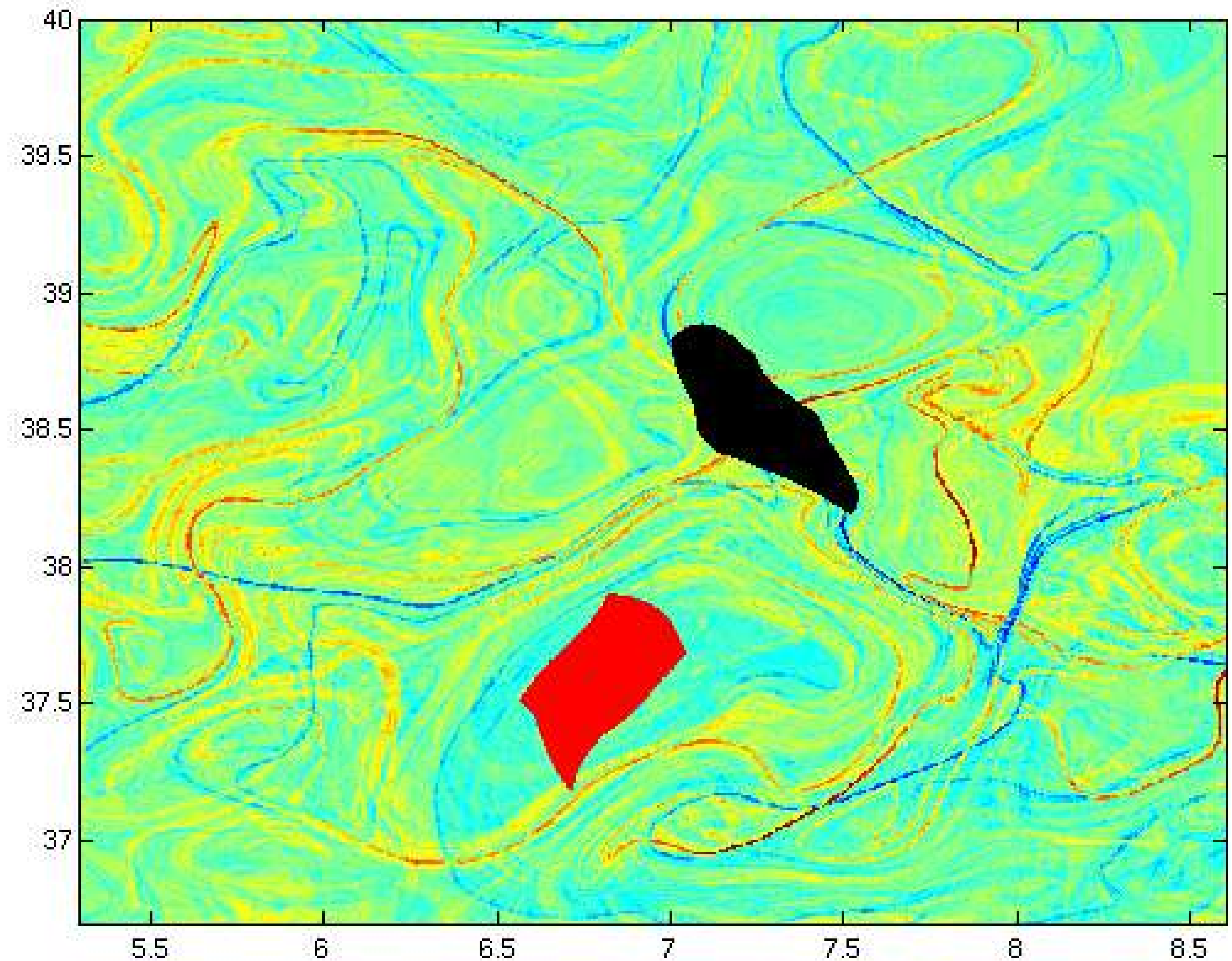

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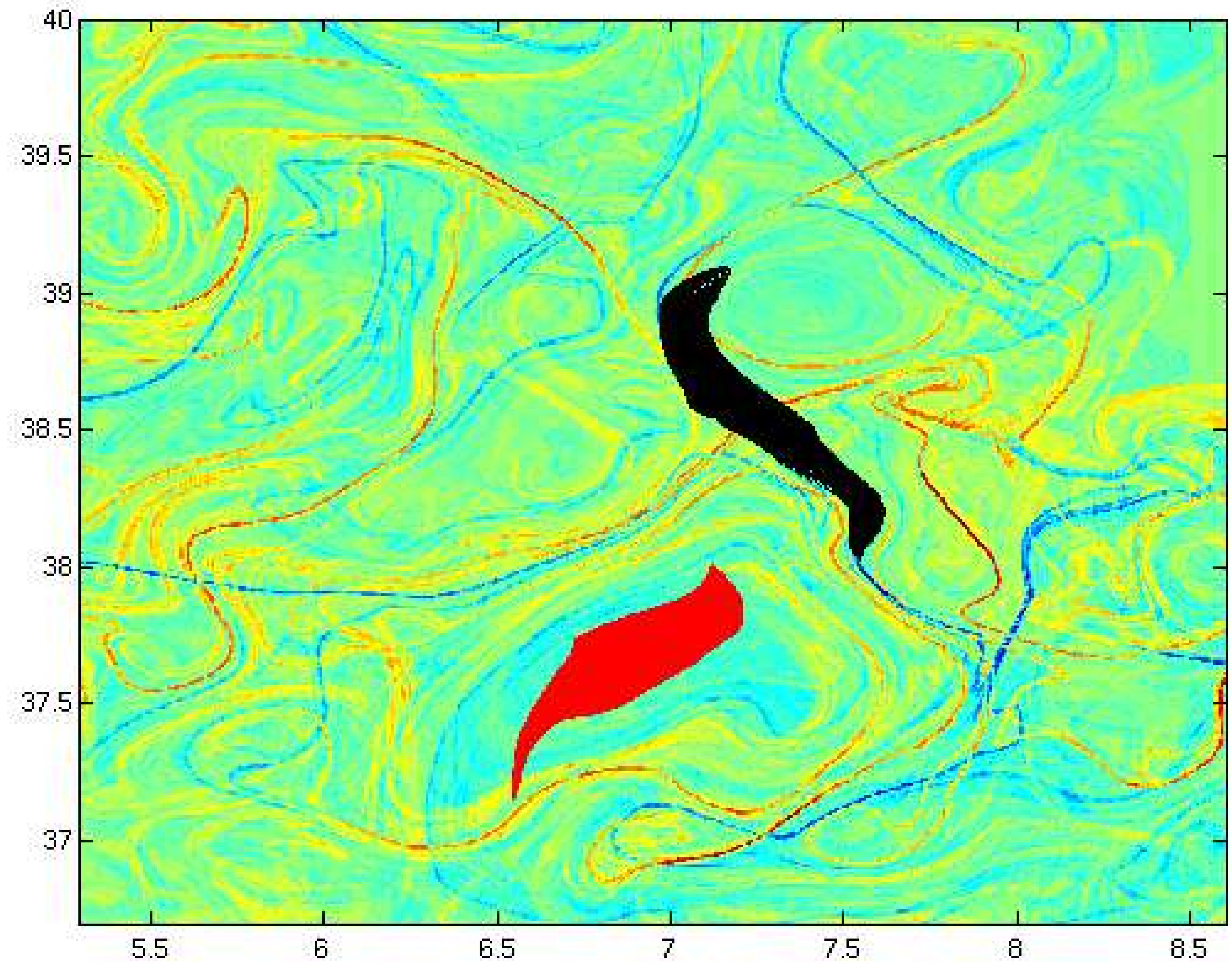

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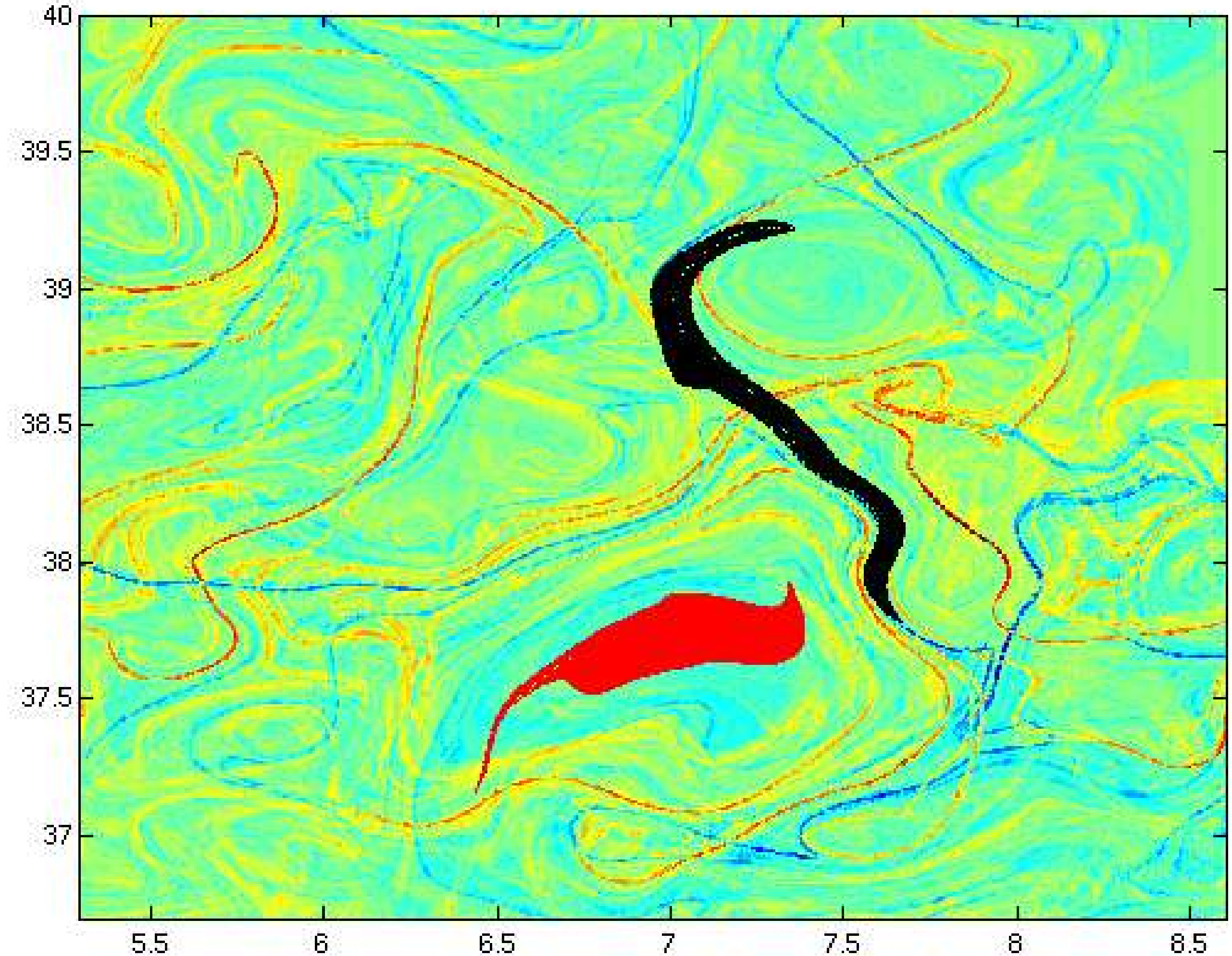

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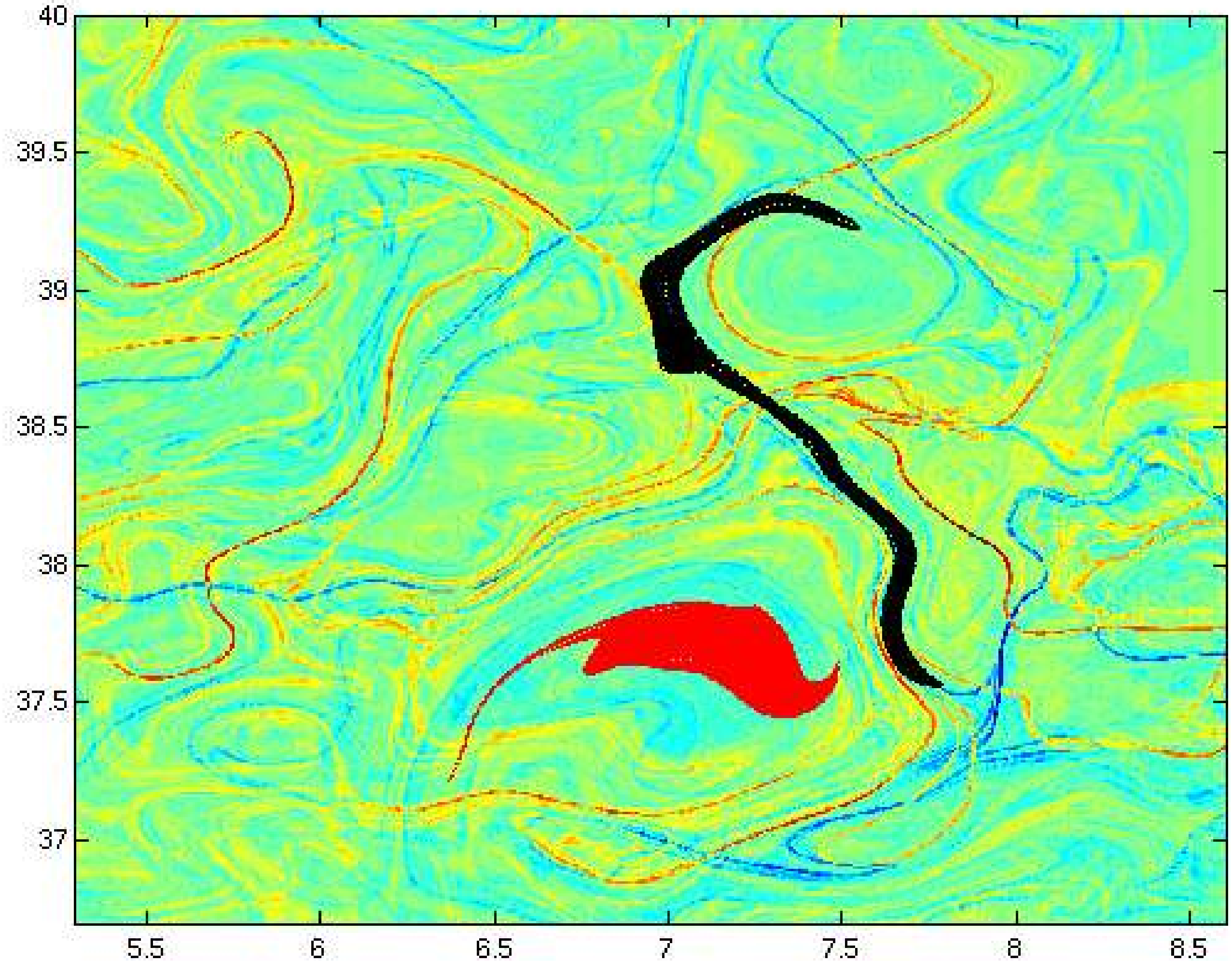

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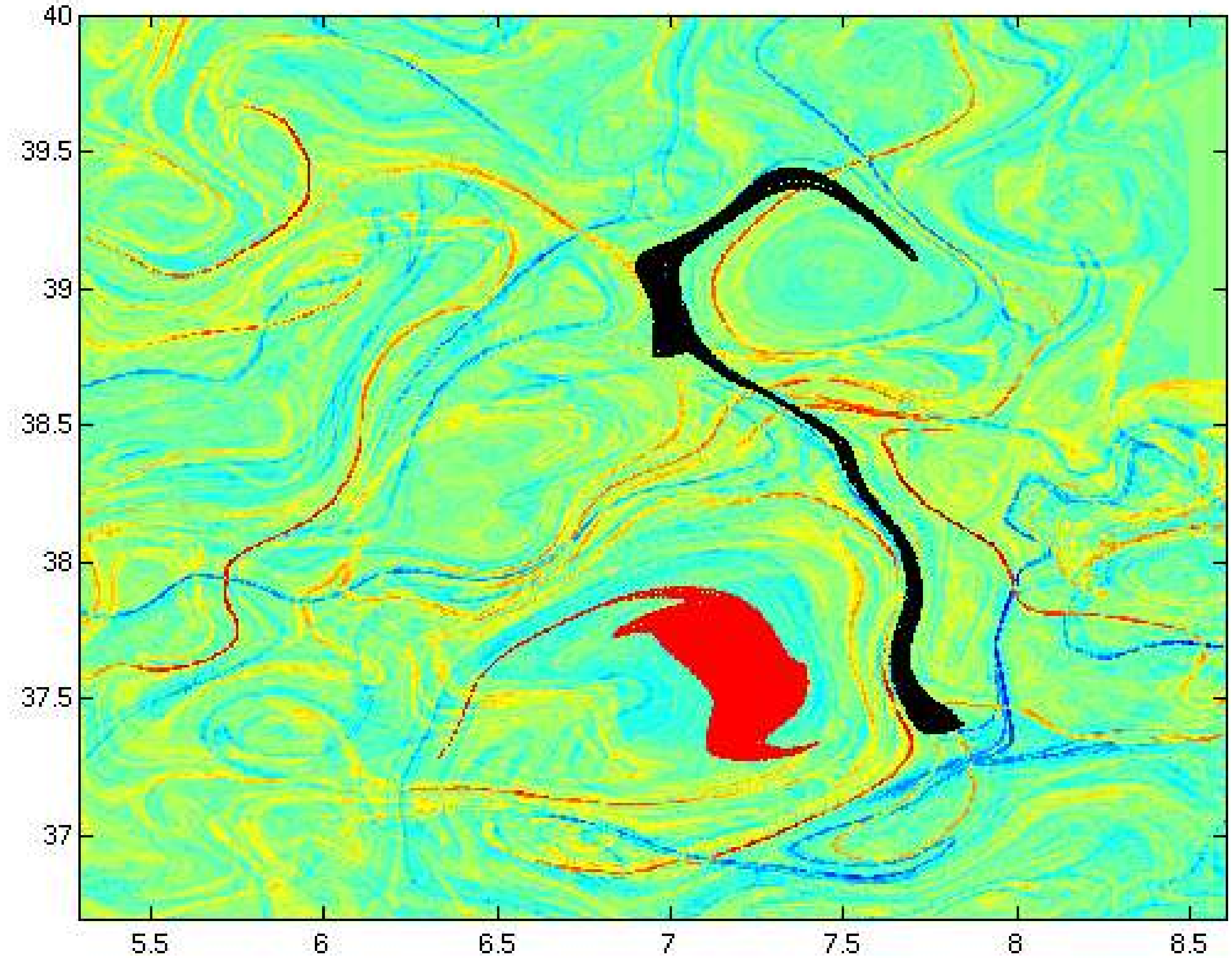

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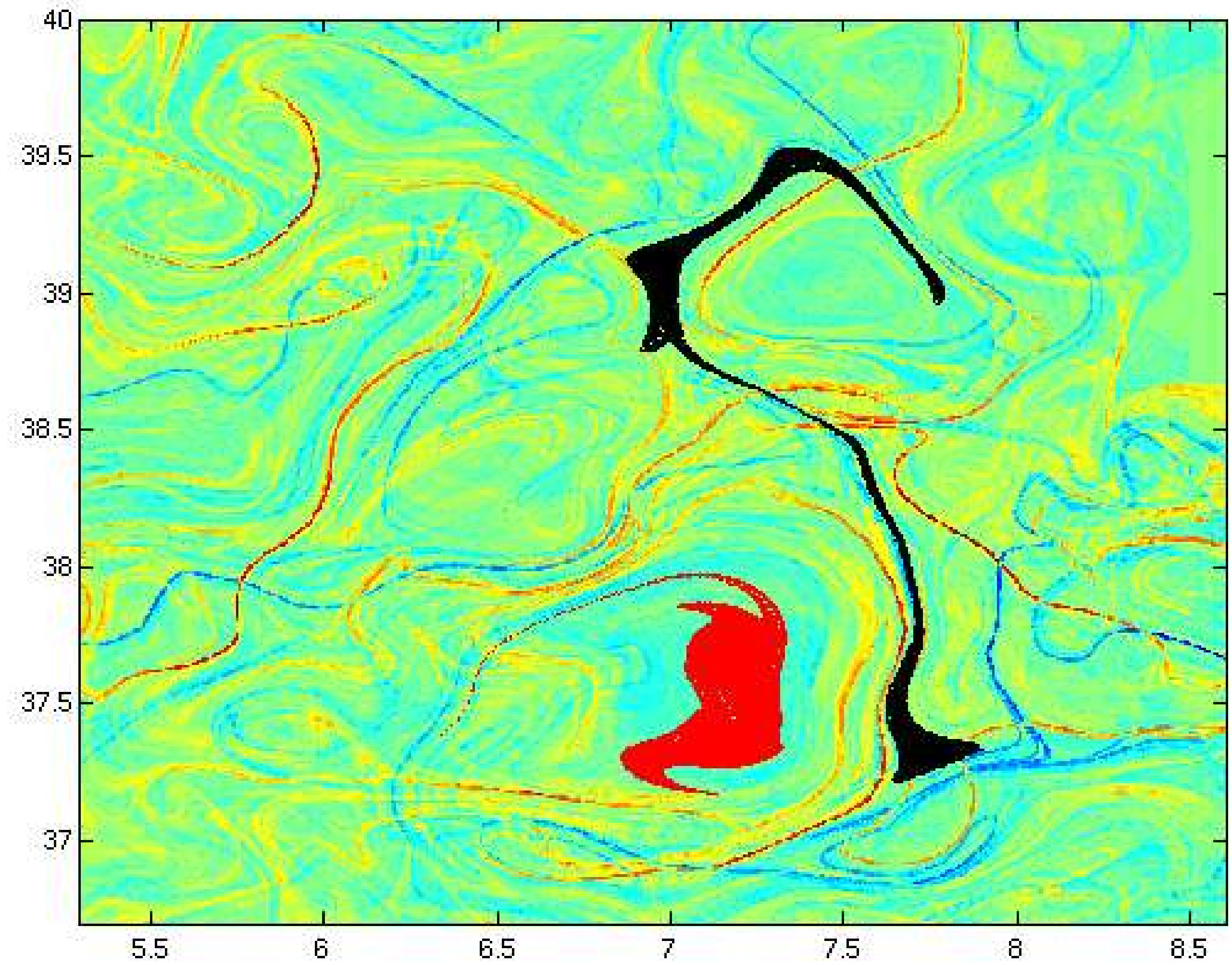

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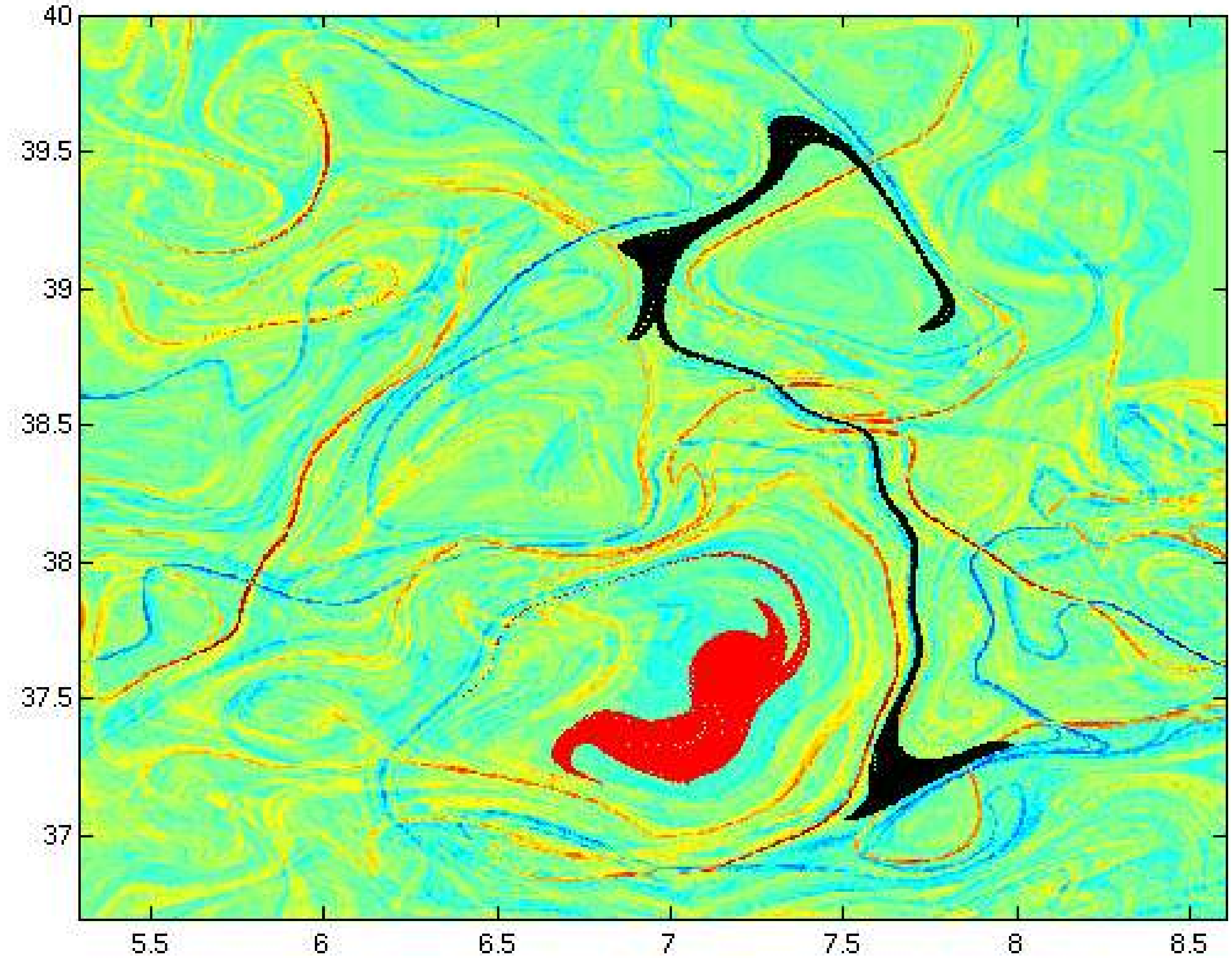

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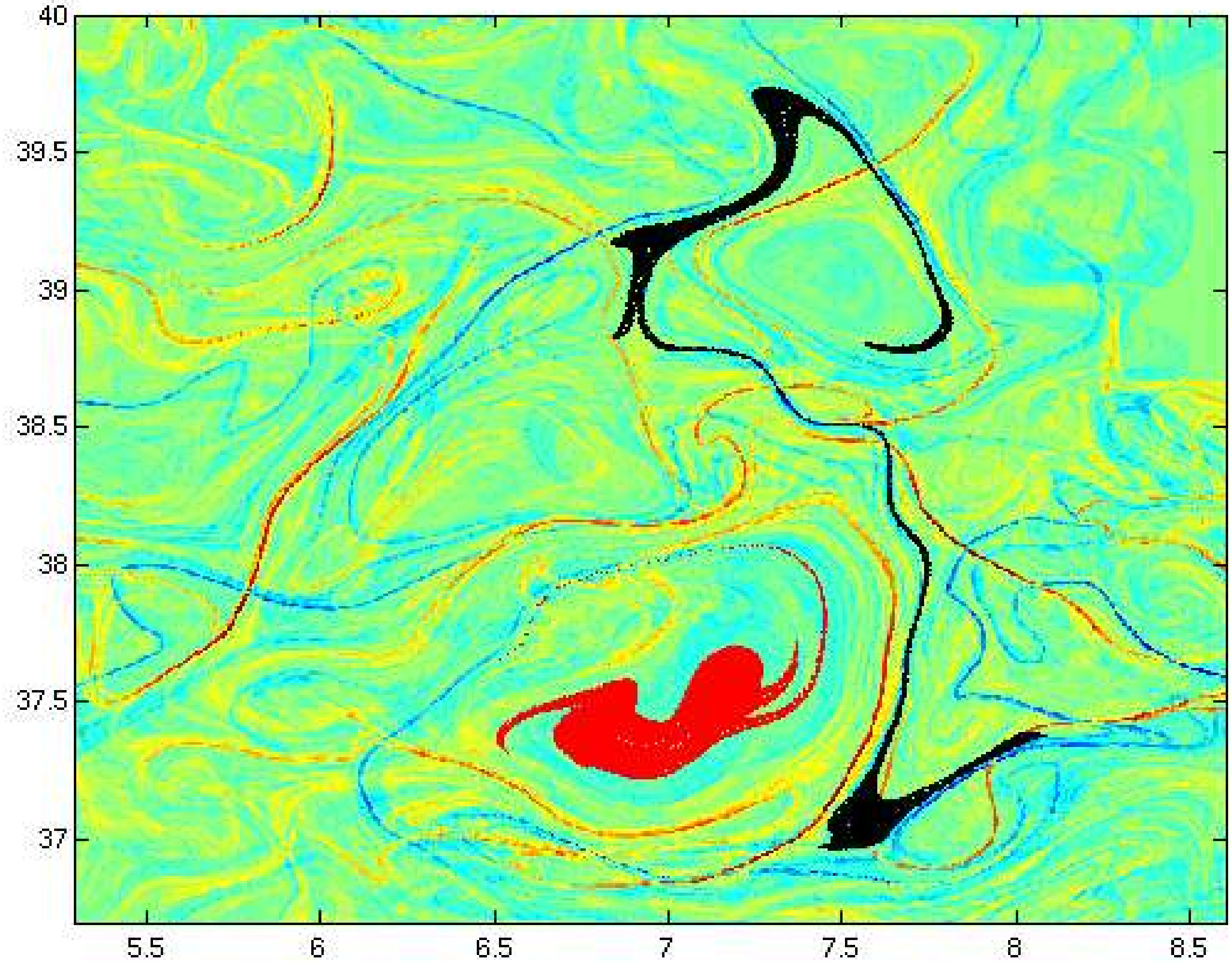

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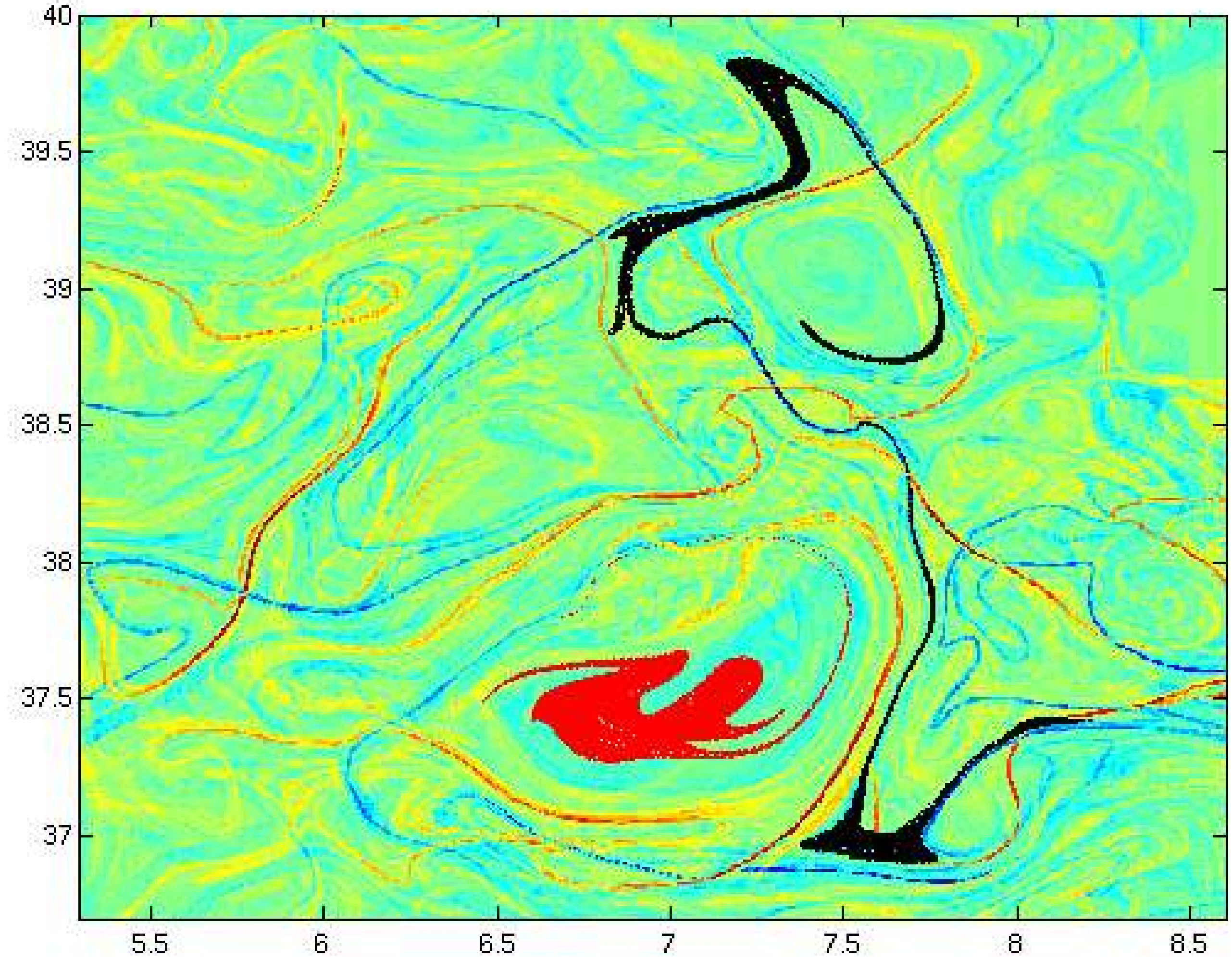

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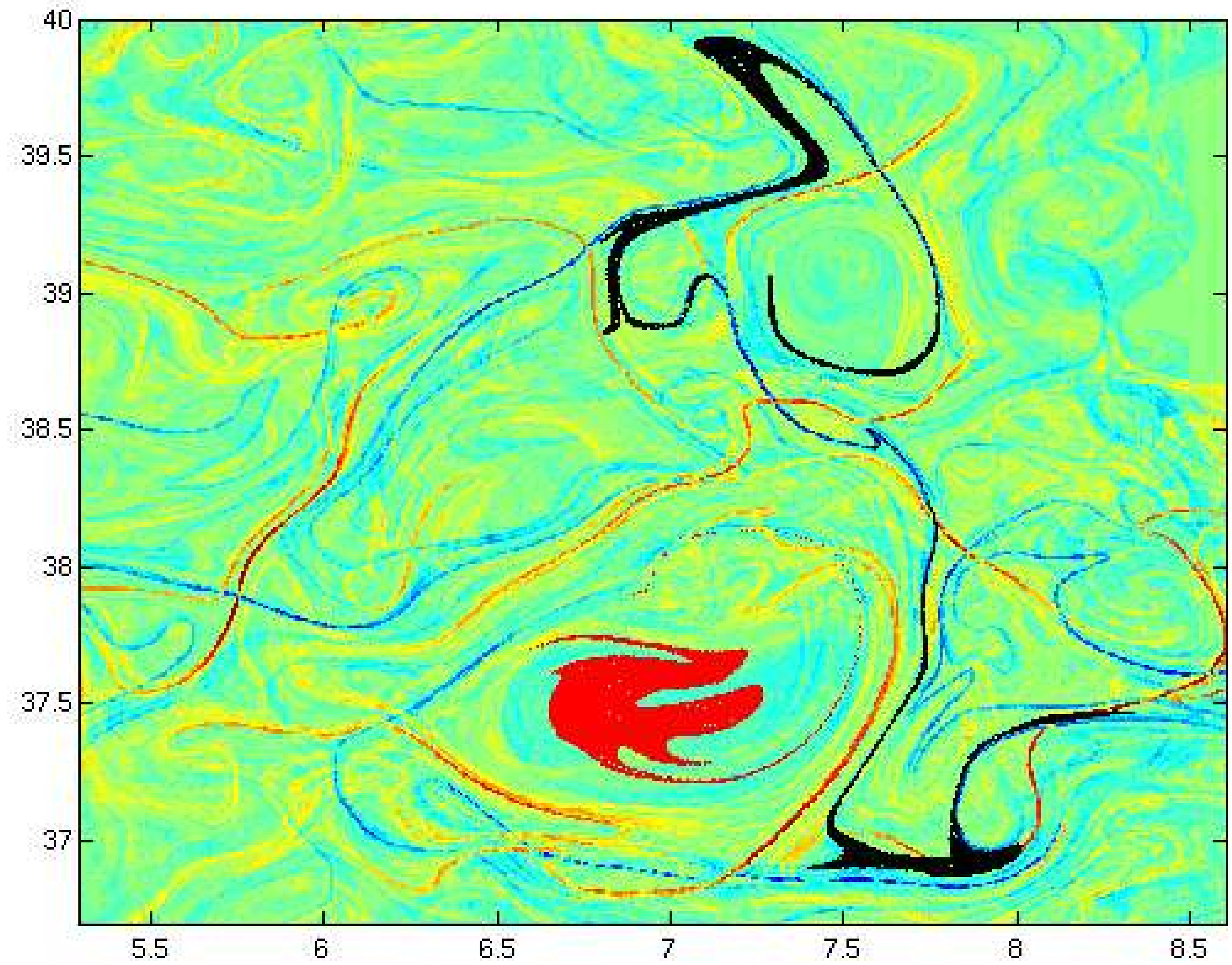
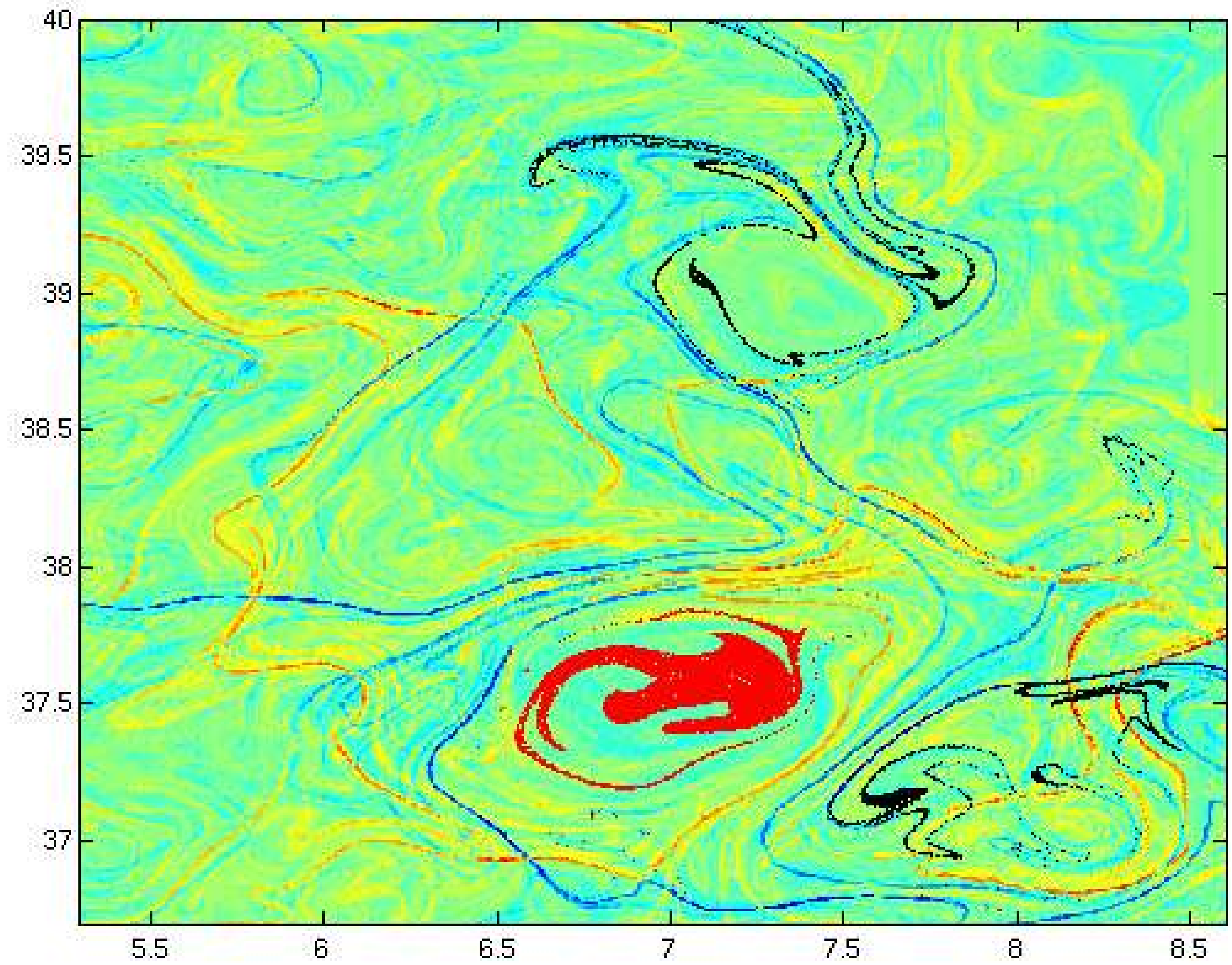
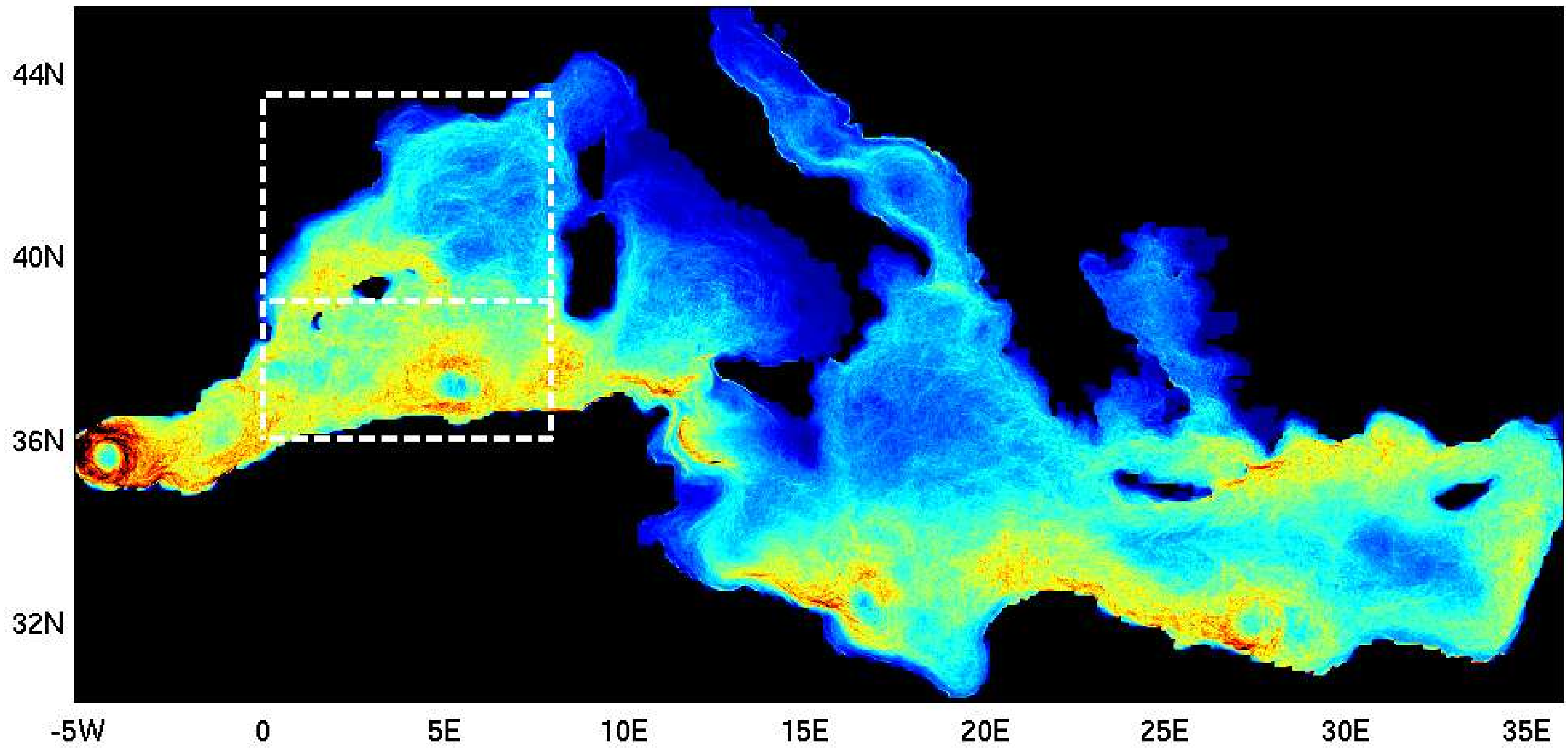


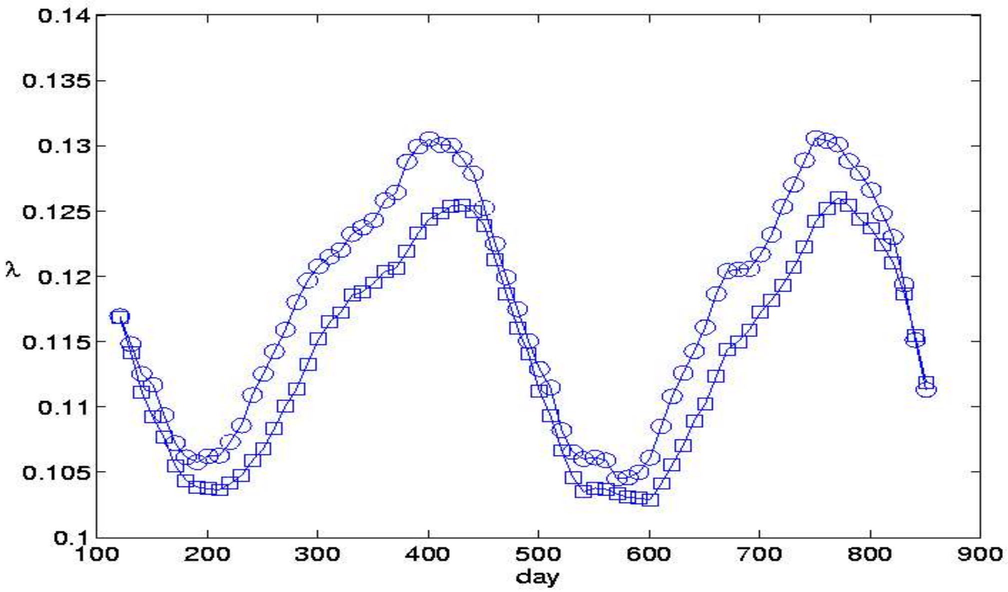
day 20

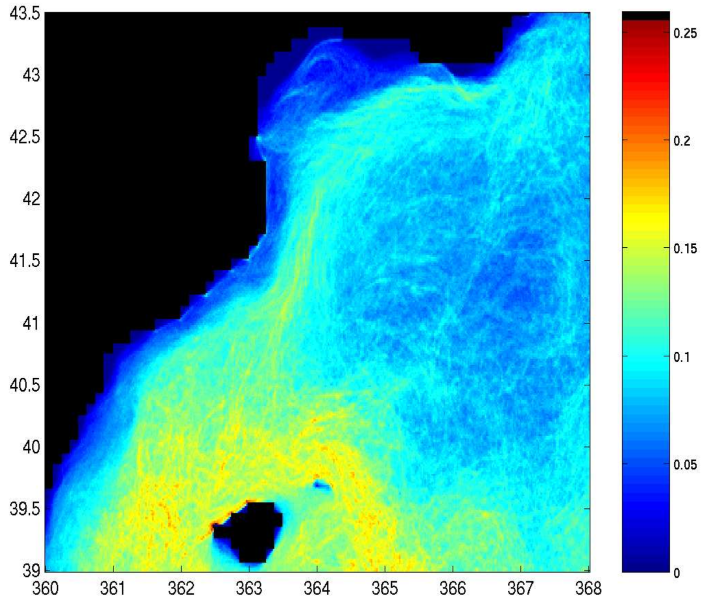


Miixing activity: time and space averages

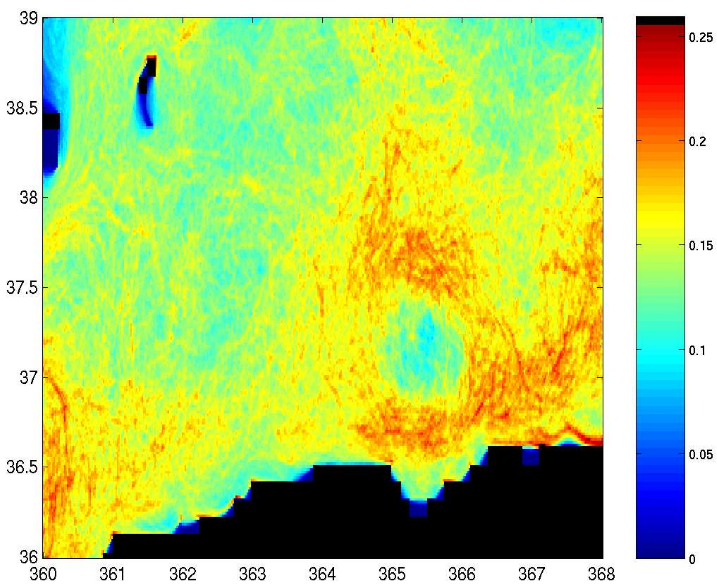


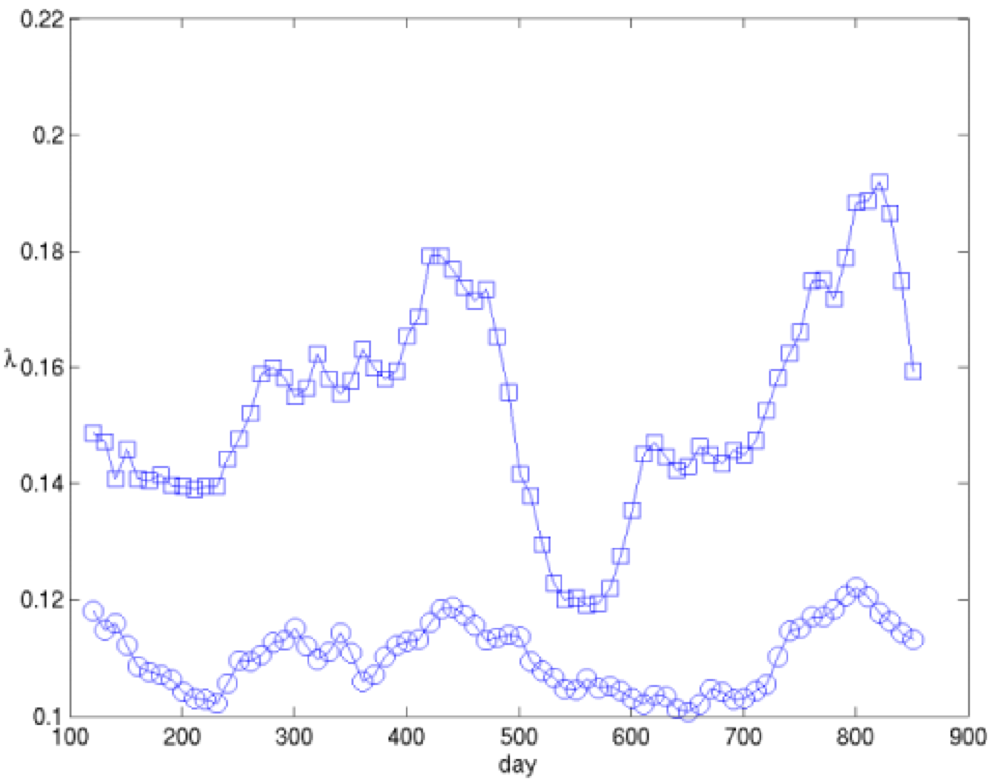
•The time average (1 year) divides the sea in regions of differentmixing activity.
•The space average (whole basin) shows seasonal variations.





An example of two regions with different mixing behaviour: north and south of the Balearic Islands





Second use of the FSLEs: lengthscale analysis

$$R(t) = x^2(t) - x^1(t)$$

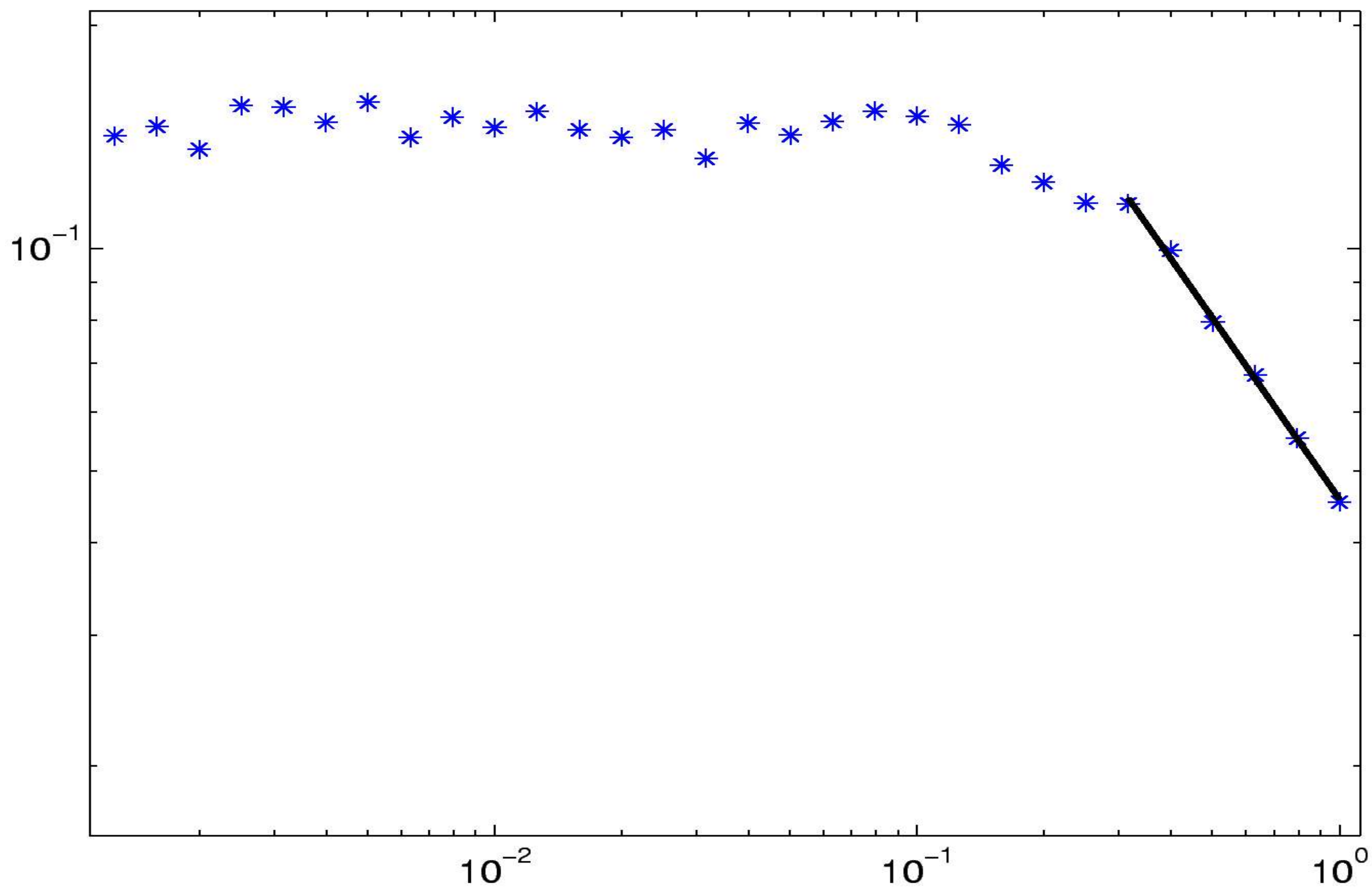
$$\frac{dR}{dt} = v^2(t) - v^1(t) = u(x^1(t) + R(t), t) - u(x^1(t), t)$$

The evolution of relative separations gives velocity correlations at scale *R*

The dependence of the FSLE from the initial displacement can thus be used to “probe” the diffusive behaviour of the velocity field at different lengthscale. Typically, given the Kolmogorov and the integral scale, the dependence is:

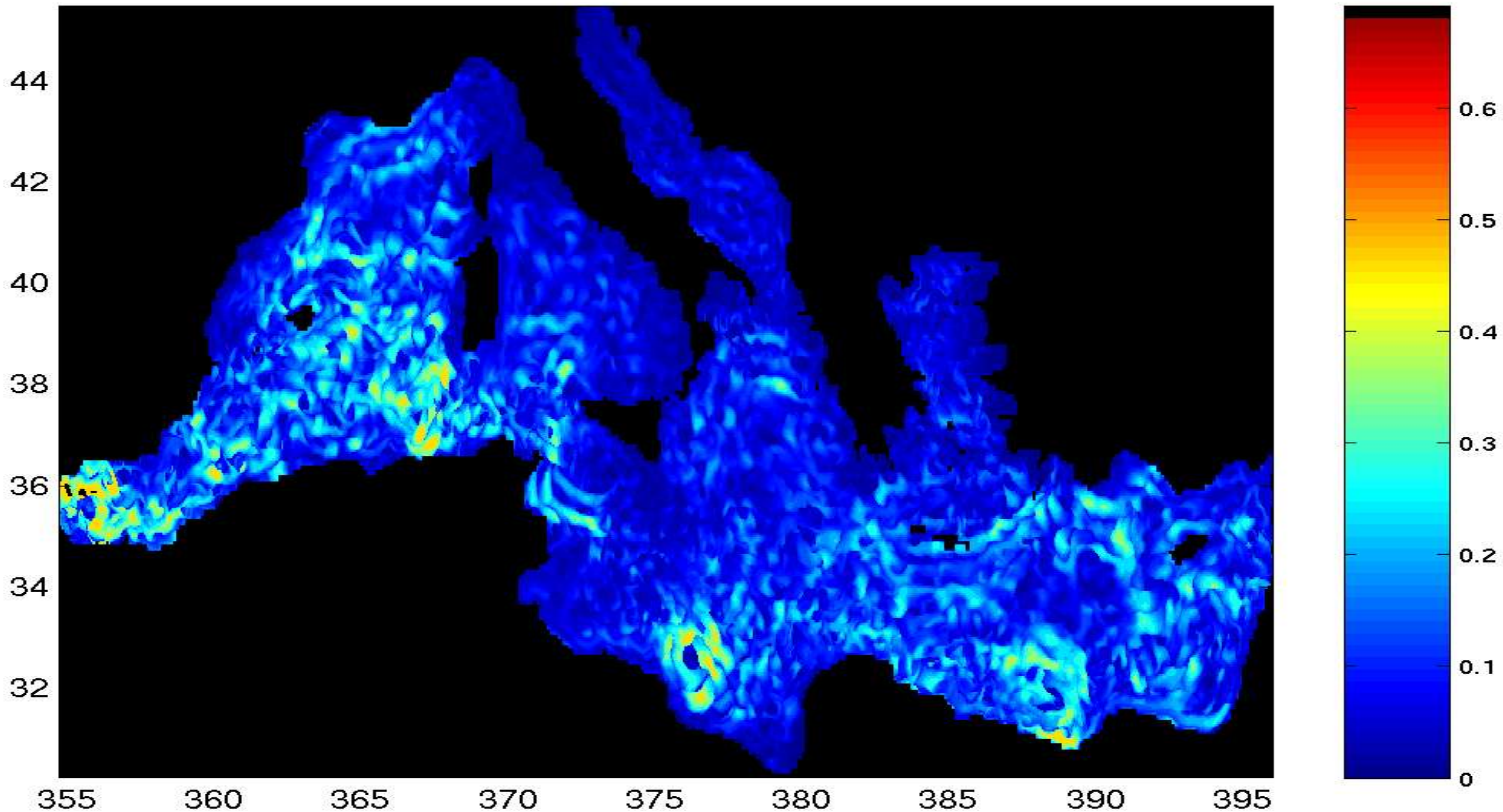
Linear behaviour	$\lambda(\delta) \approx \lambda$	$\delta << l_u$
Anomalous diffusion	$\lambda(\delta) \approx \mathrm{Cost}\ \ \delta^{-2/3}$	$l_u << \delta << L_0$
Diffusion	$\lambda(\delta) \approx \mathrm{Cost}\ \ \delta^{-2}$	$L_0 << \delta$

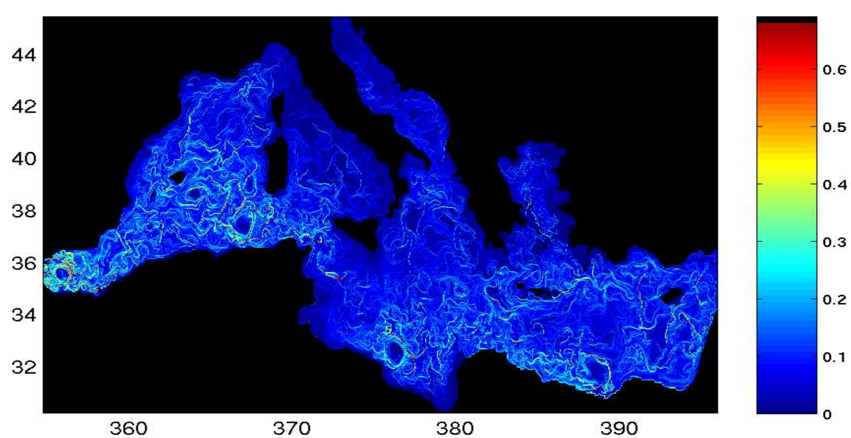
Scale dependence of the FSLE for the Mediterranean velocity data

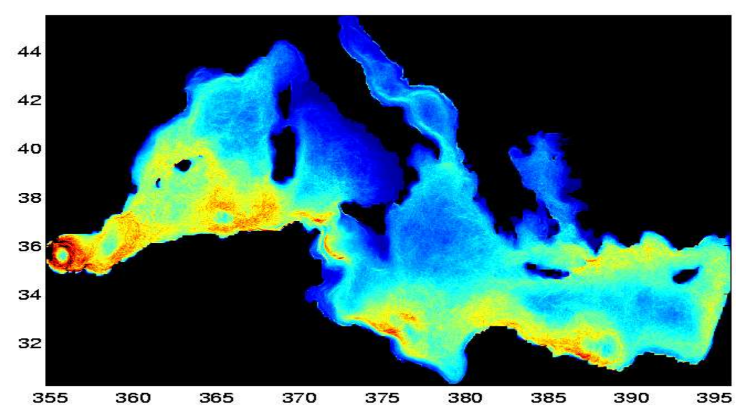


- The crossover is at the scale of the intergid sampling (1/8 deg.)
- The slope is approx. of 0.8, consistent with a Richardson law (slope of 2/3)
- The diffusive regime is not achieved

Structures above the Kolmogorov scale

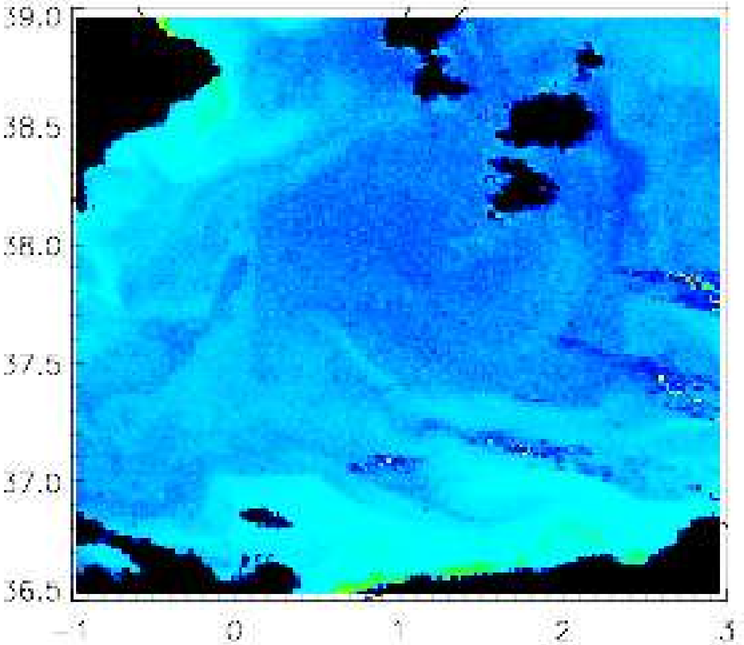






Real data

18 May 1998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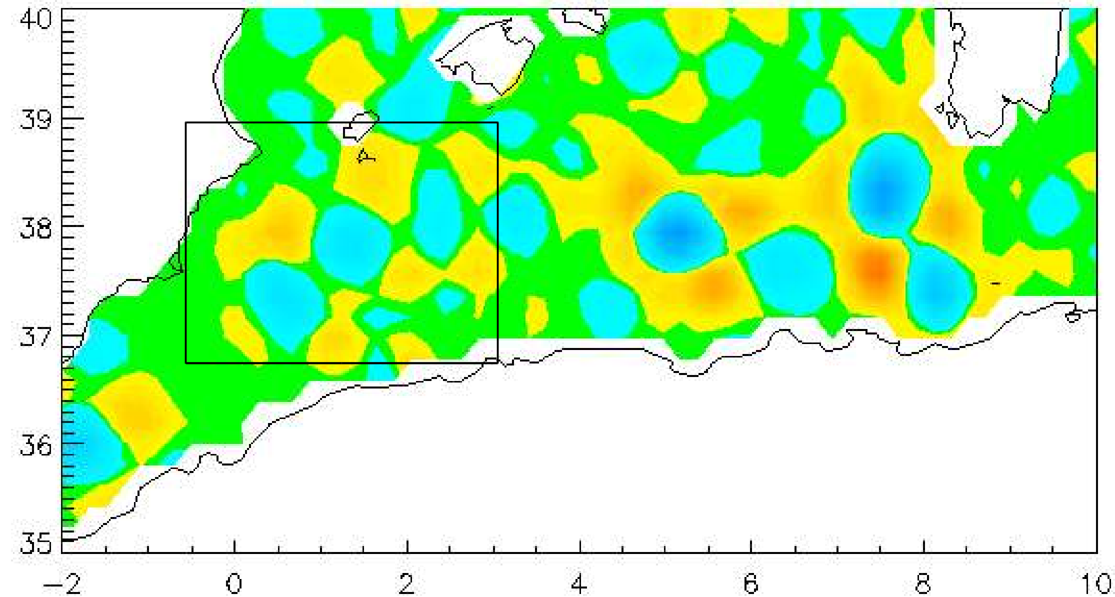
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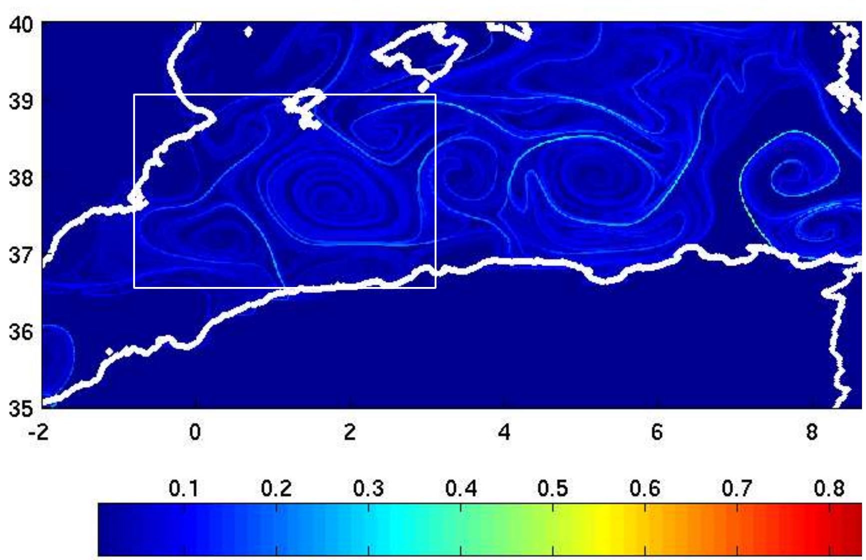
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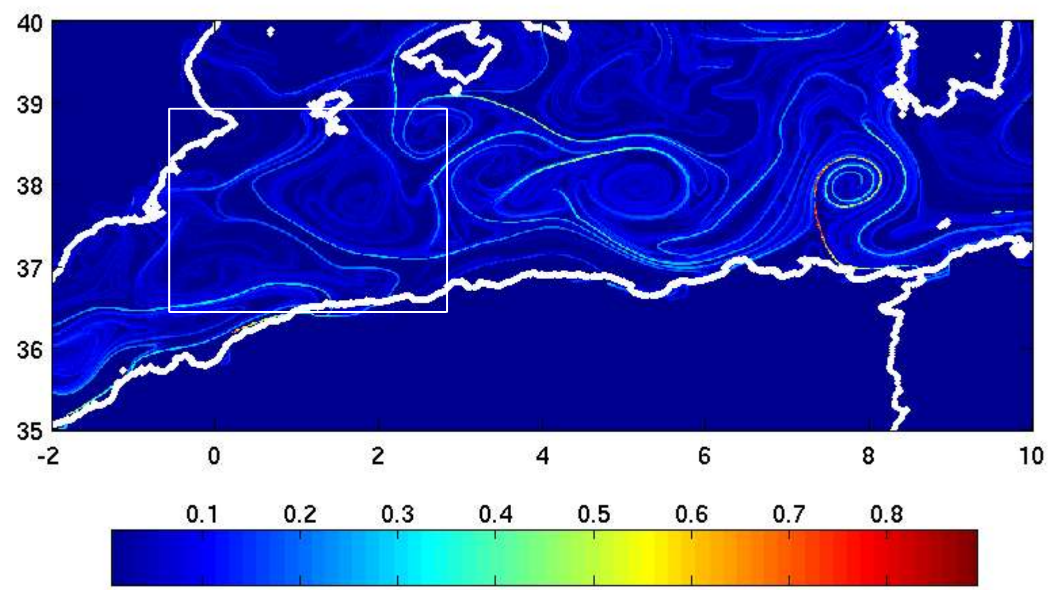
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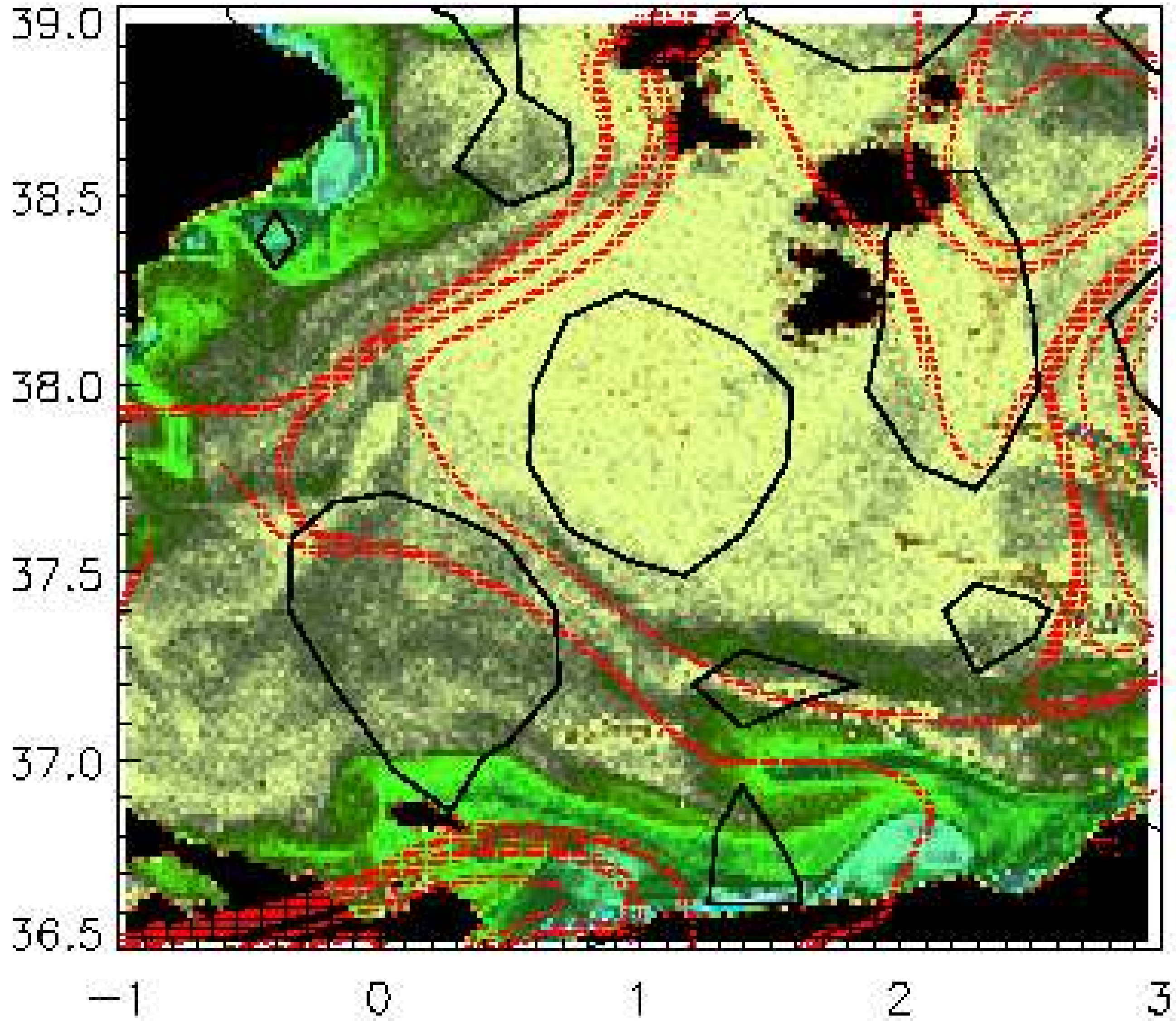

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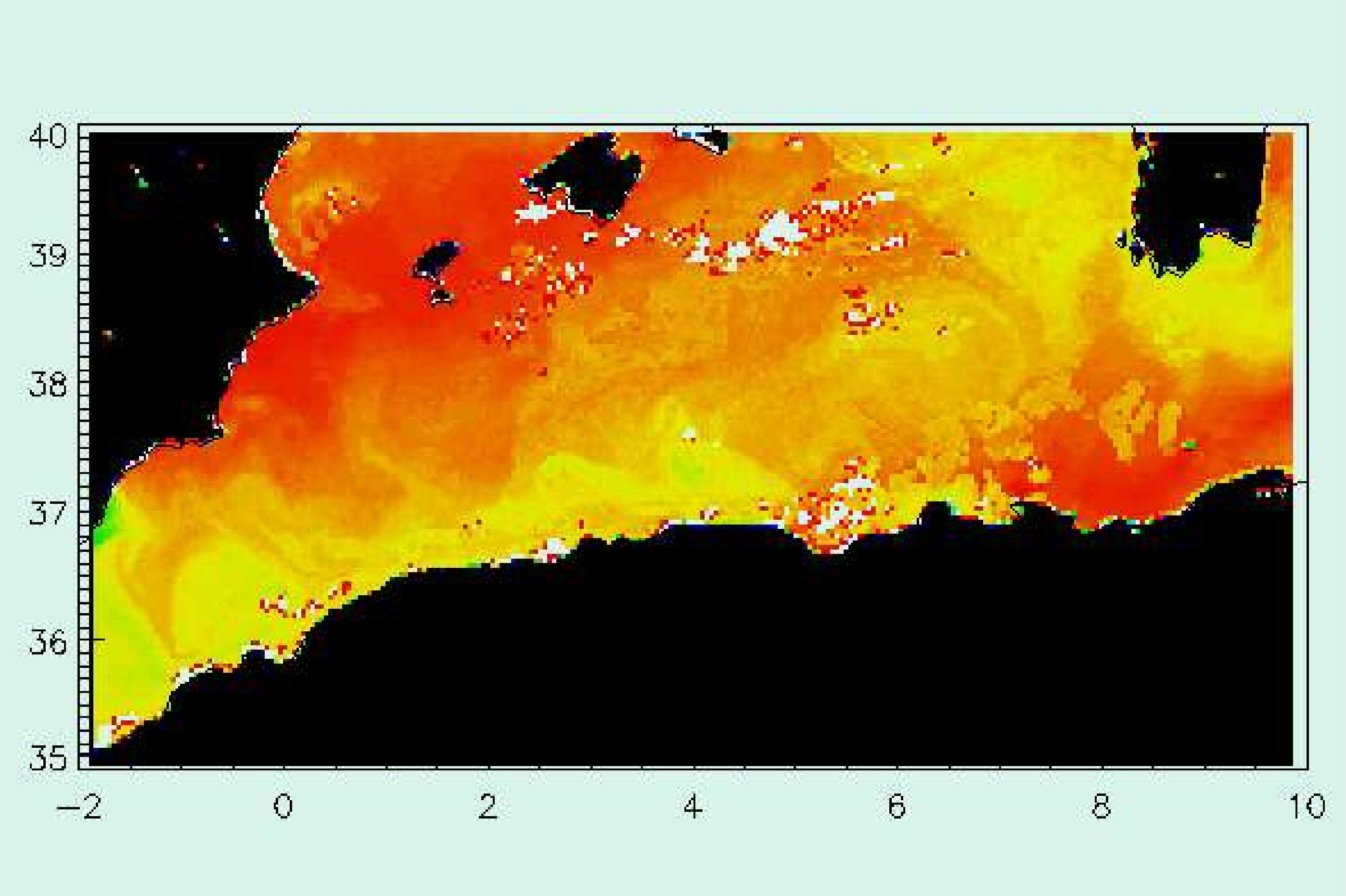
7 July 1996



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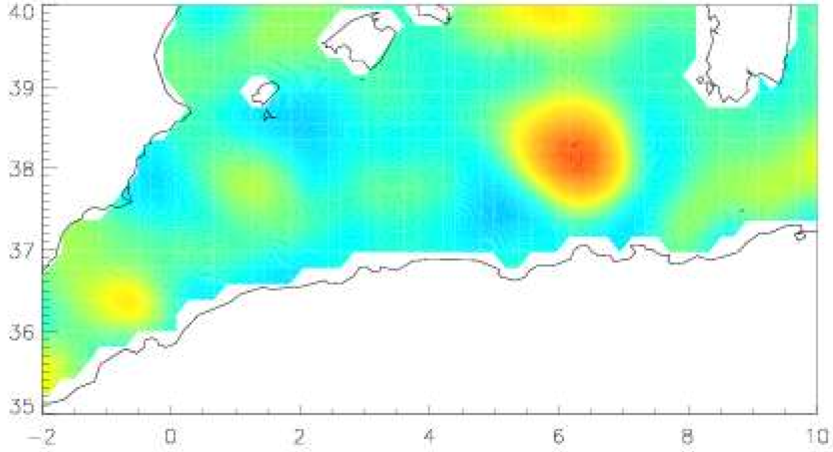

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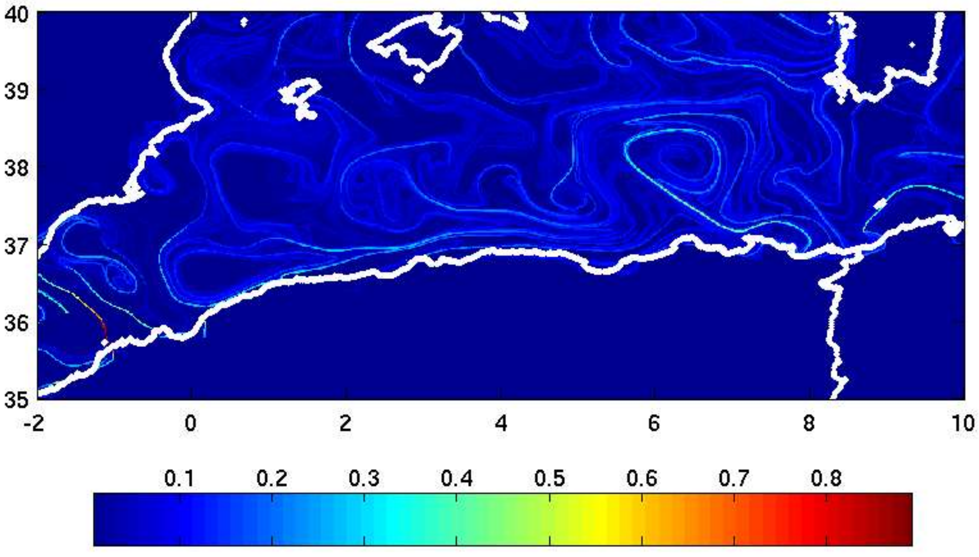

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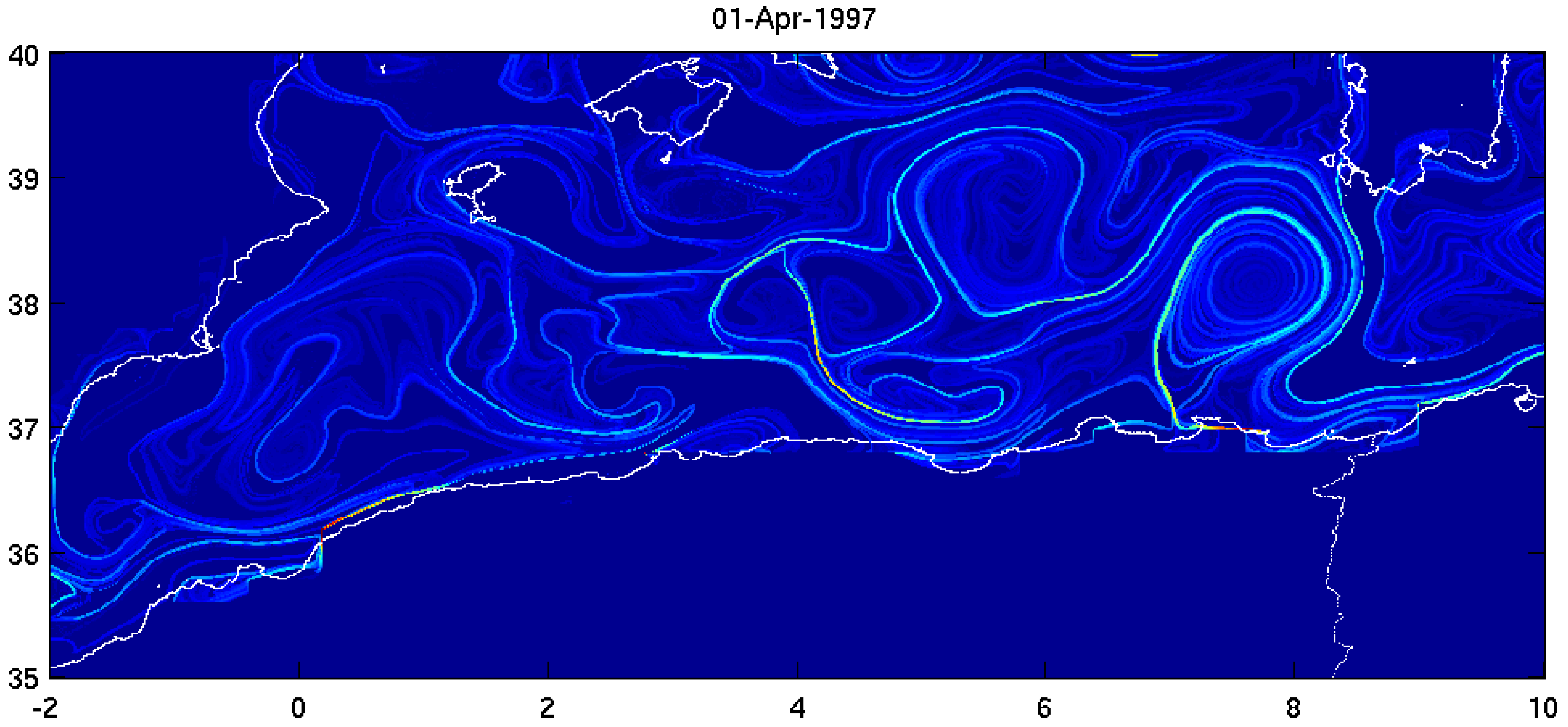
0.8

Dynamic properties


01-Apr-1997
40
39
38
37
36
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-2
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4
6
8
10

Conclusions

Ocean dynamics:

• The FSLEs can detect stable and unstable manifolds for Lagrangian hyperbolic points

• Chlorophyll and thermal patterns can be studied

In general:

• New (and better) data are available each year

• Nonlinear methods can be quantitatively applied

• Challenging test for the theory

• Important geophysical problems open

Work in collaboration with:

• Cristobal Lopez, Emilio Hernandez-Garcia (IMEDEA, Spain)

• Simulated data: Vicente Fernandez (IMEDEA, Spain)

• Altimetric, Chlorophyll, and temperature data: Jordi Isern-Fontanet, Emilio Garcia-Ladona (Marine Science Institute, Barcelona, Spain)

• Mean Dynamics Topography: Maria-Helene Rio (CLS, Toulouse, France)

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