EXCITABLE MEDIA IN CHAOTIC FLOWS

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-Excitable Systems (ES) and applications

-Open and closed chaotic flows

-Numerical results for two dimensional ES in flows

-Reduced models:

a)Baker map models

b)One dimensional filament model

Excitable media are reacting systems that, despite of having a stable homogeneous and stationary state, display a nontrivial response to perturbations above a threshold.

FitzHugh-Nagumo + Difussion

$$\frac{du}{dt} = ku(u-a)(1-u) - v + D\nabla^2 u$$

$$\frac{dv}{dt} = \epsilon(u - fv) + D\nabla^2 v$$

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$u = F(u, v)$

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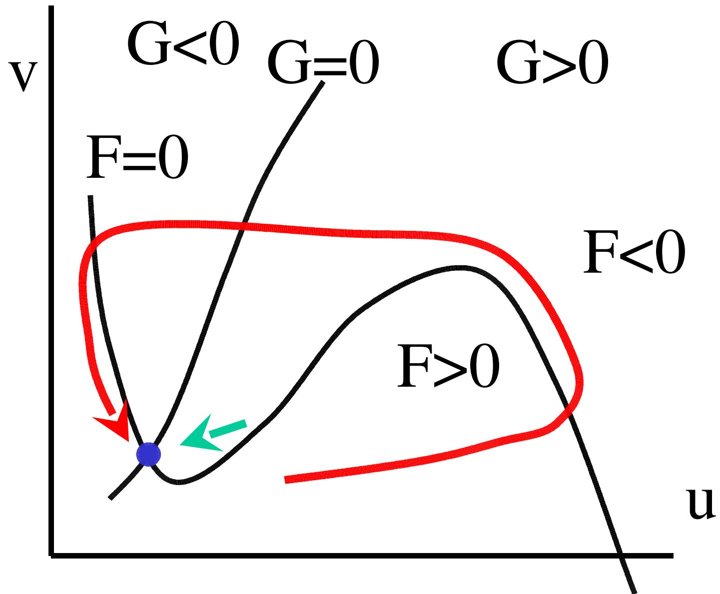
$v = G(u, v)$

u fast, v slow



Excitable behaviour is typical of biological or chemical systems: Beloutsov-Zabhotinsky chemical reaction, neuron systems, plankton dynamics

Plankton Blooms: Explosive and localised growth of phyto or zooplankton populations

It is important the advection

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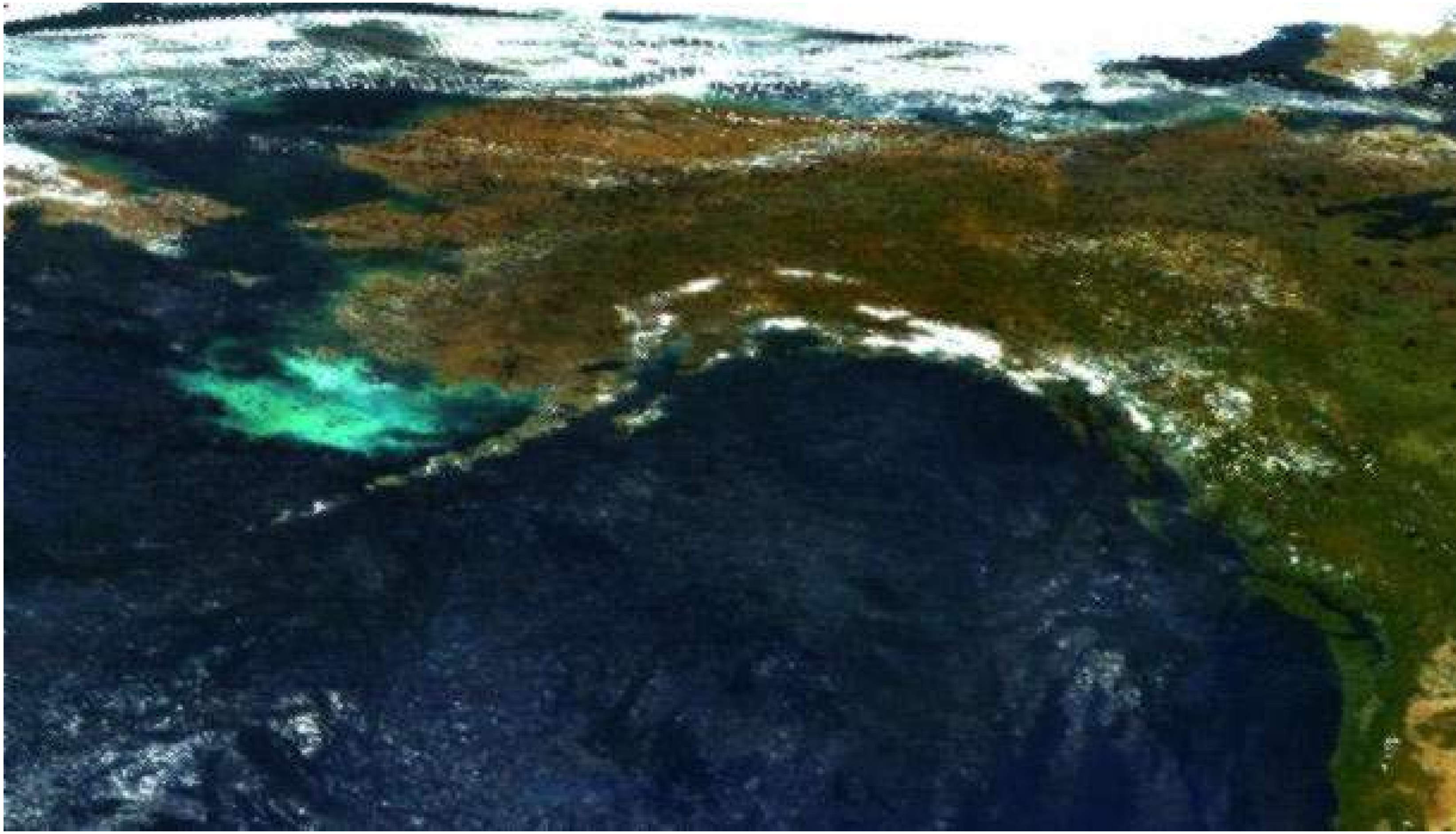


EXCITABLE DYNAMICS + DIFFUSION + ADVECTION

$$\frac{dC_1}{dt} + V\nabla C_1 = kC_1(C_1 - a)(1 - C_1) - C_2 + D\nabla^2 C_1$$

$$\frac{dC_2}{dt} + V\nabla C_2 = \epsilon\left(C_1 - fC_1\right) + D\nabla^2 C_2$$

We study the response of the system to a perturbation

V chaotic flow that can be open or closed

Characterised by a positive Lyapunov exponent

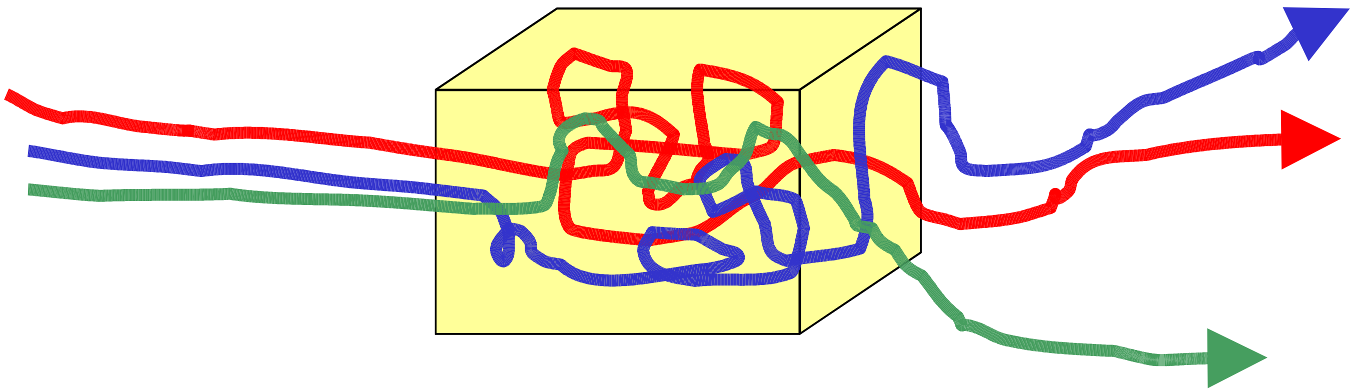
λ

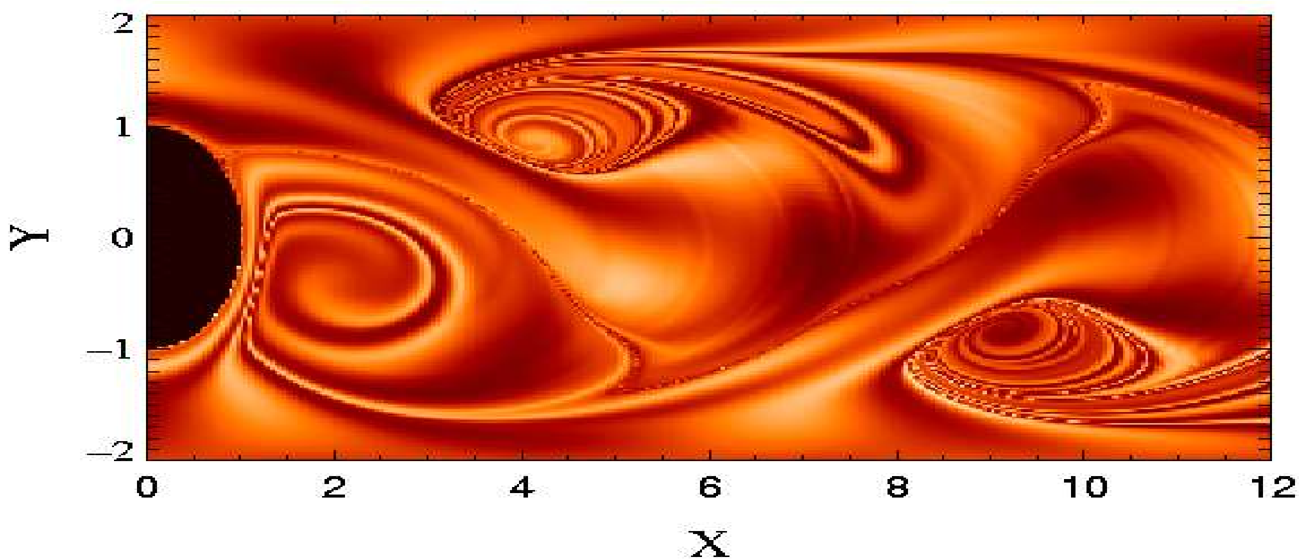
Closed flows: All fluid trajectories are bounded (all particles remain in a bounded region) and there is mixing in the whole fluid.

E.g: flow in a lake, in a closed vessel (numerically periodic boundary conditions)

Open flows: Fluid particles enter the system and, typically, after some time leave the system. There is just mixing (Lyapunov positive) in a fractal set of measure zero, the chaotic saddle, corresponding to a set of trajectories never leaving the system

E.g.: flow after cylindrical object, flow circunventing an island





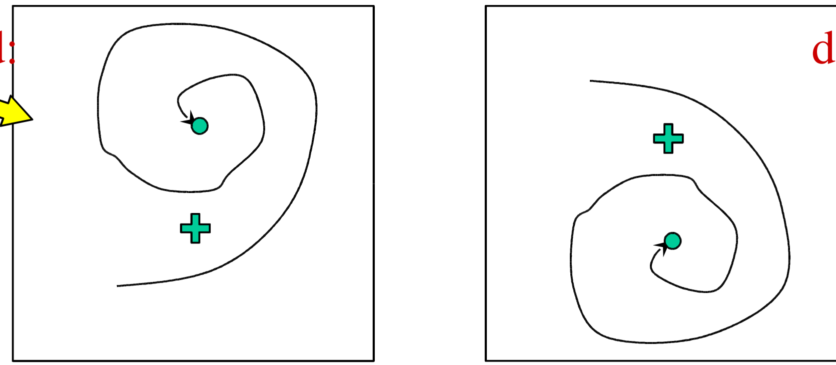
2
1
0
−1
−2
Y
0
2
4
6
8
10
12
X

OPEN FLOW:

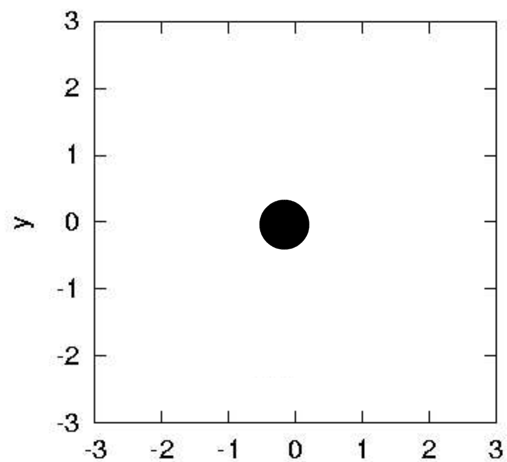
A simple incompressible two-dimensional flow leading to chaotic advection with escape:

THE BLINKING VORTEX-SINK FLOW:

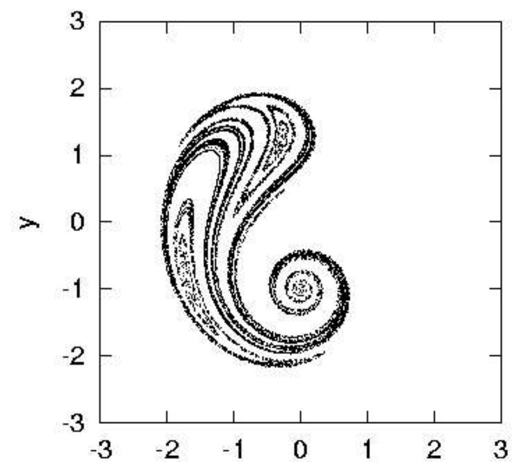
during half a period:



during the other half



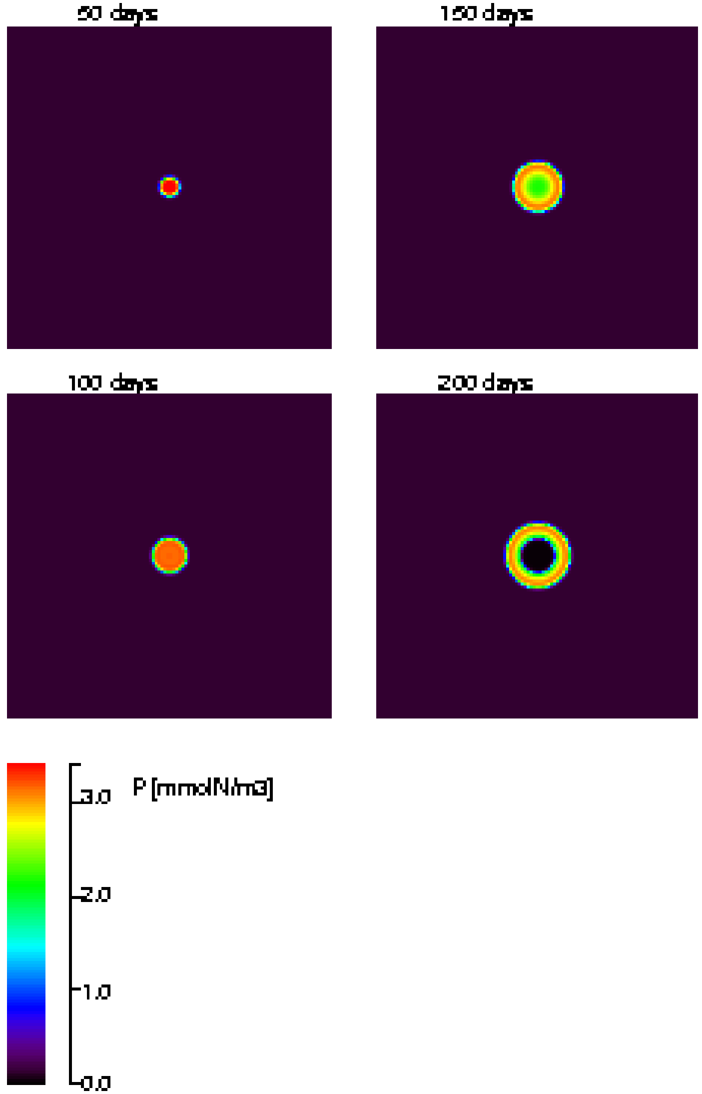
After some time ...



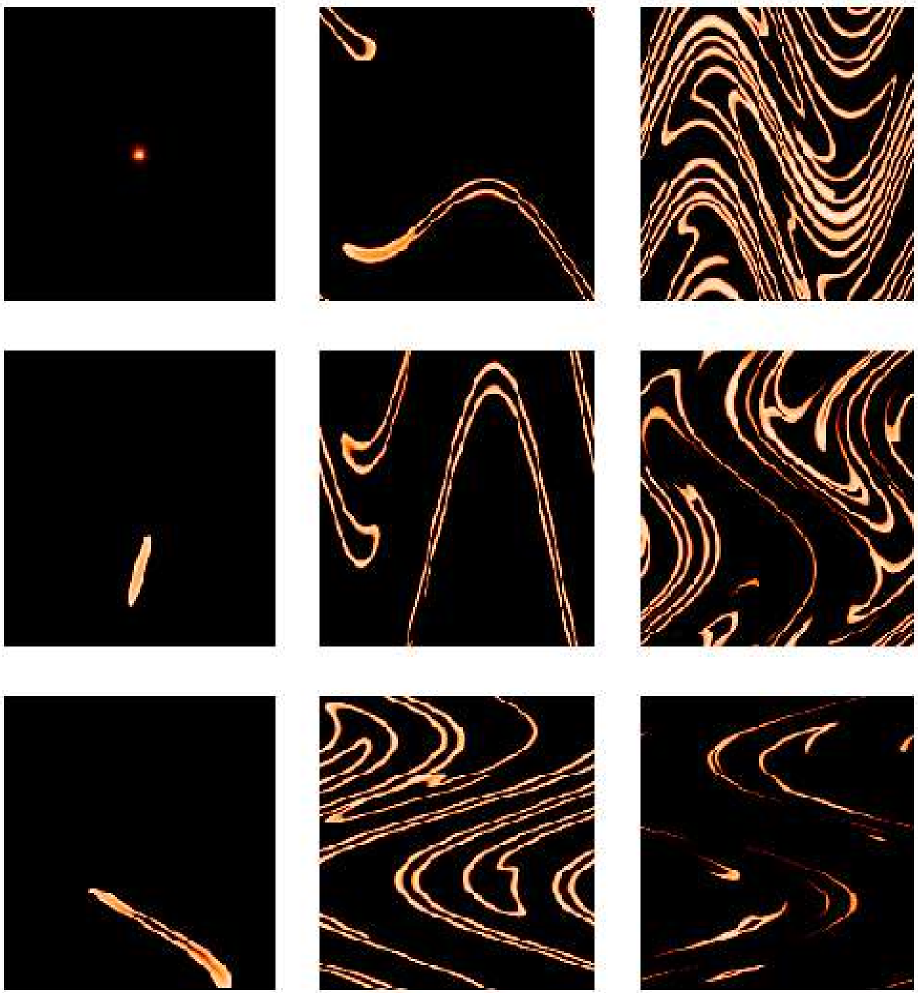
Almost-all particles escape from the system, except the ones lying on the fractal chaotic saddle.

Numerical results:

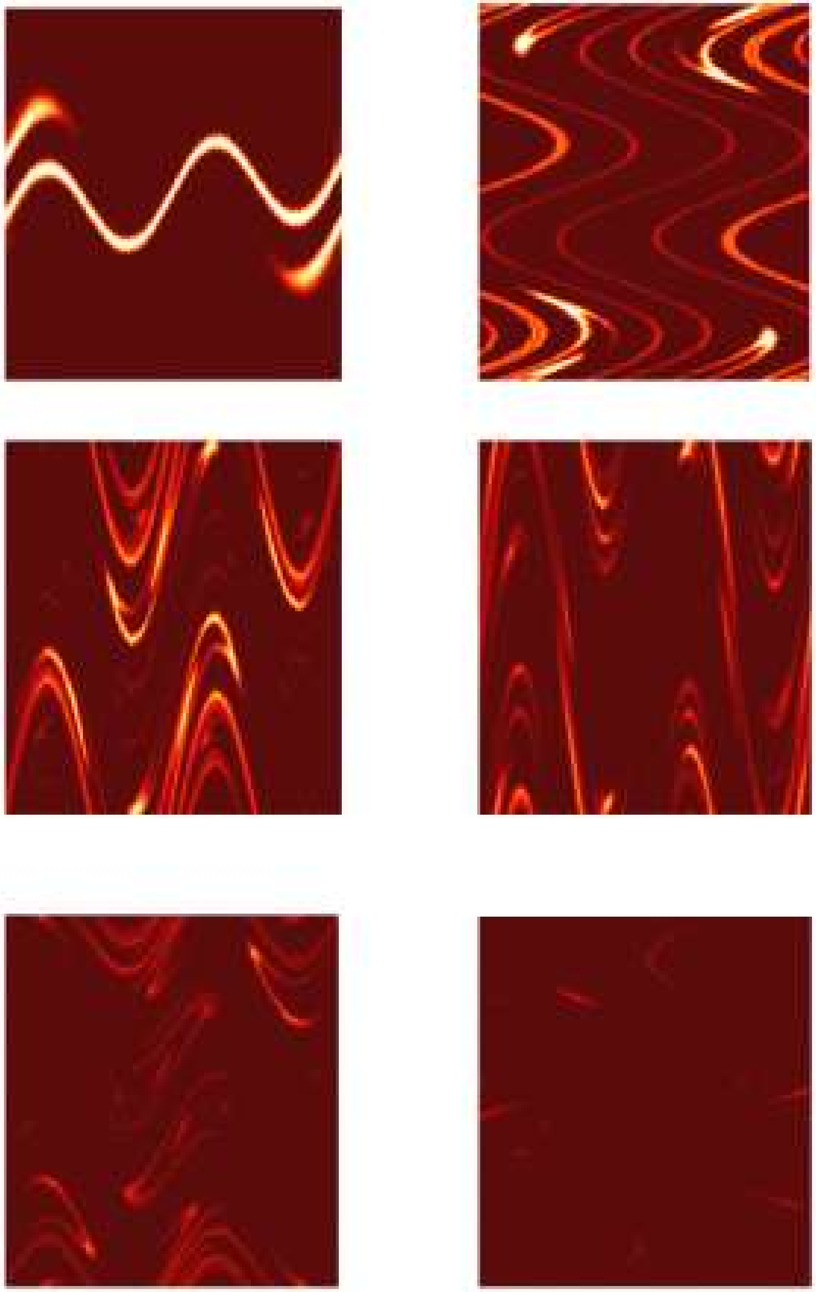
NO FLOW



Closed flow



Very slow stirring: excitation waves travel into the system: incoherent excitation

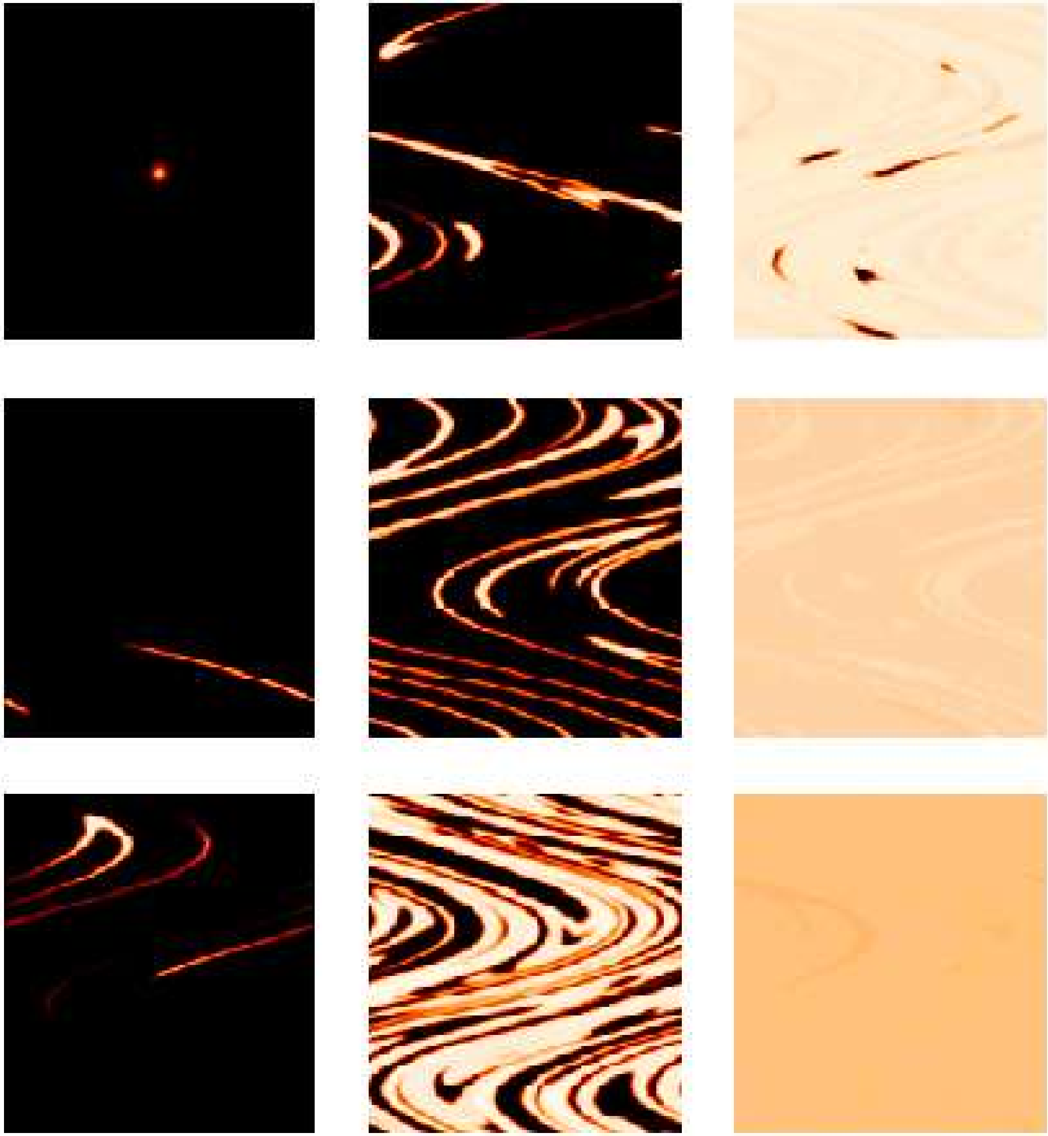


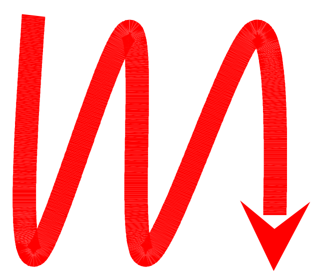
Fast stirring: filament dilution

stops excitation propagation

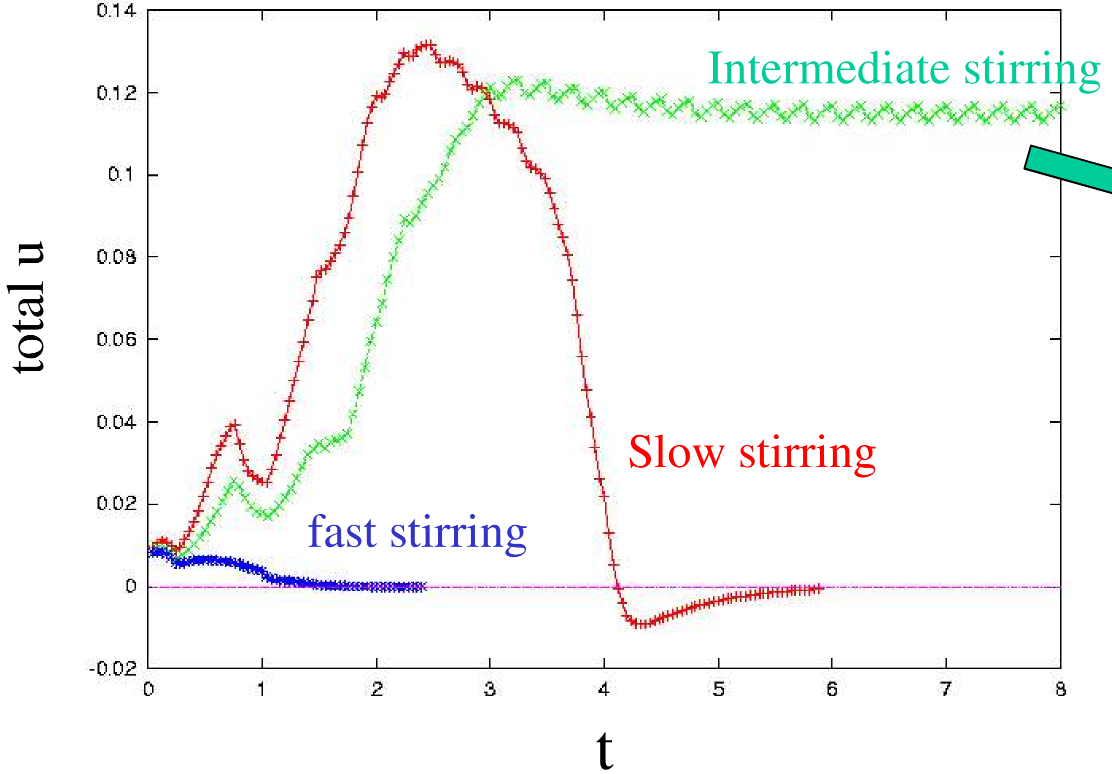
Intermediate stirring:

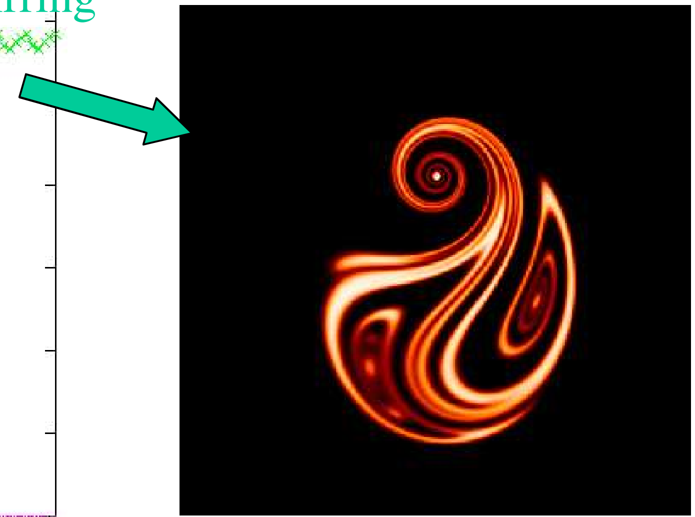
coherent excitation





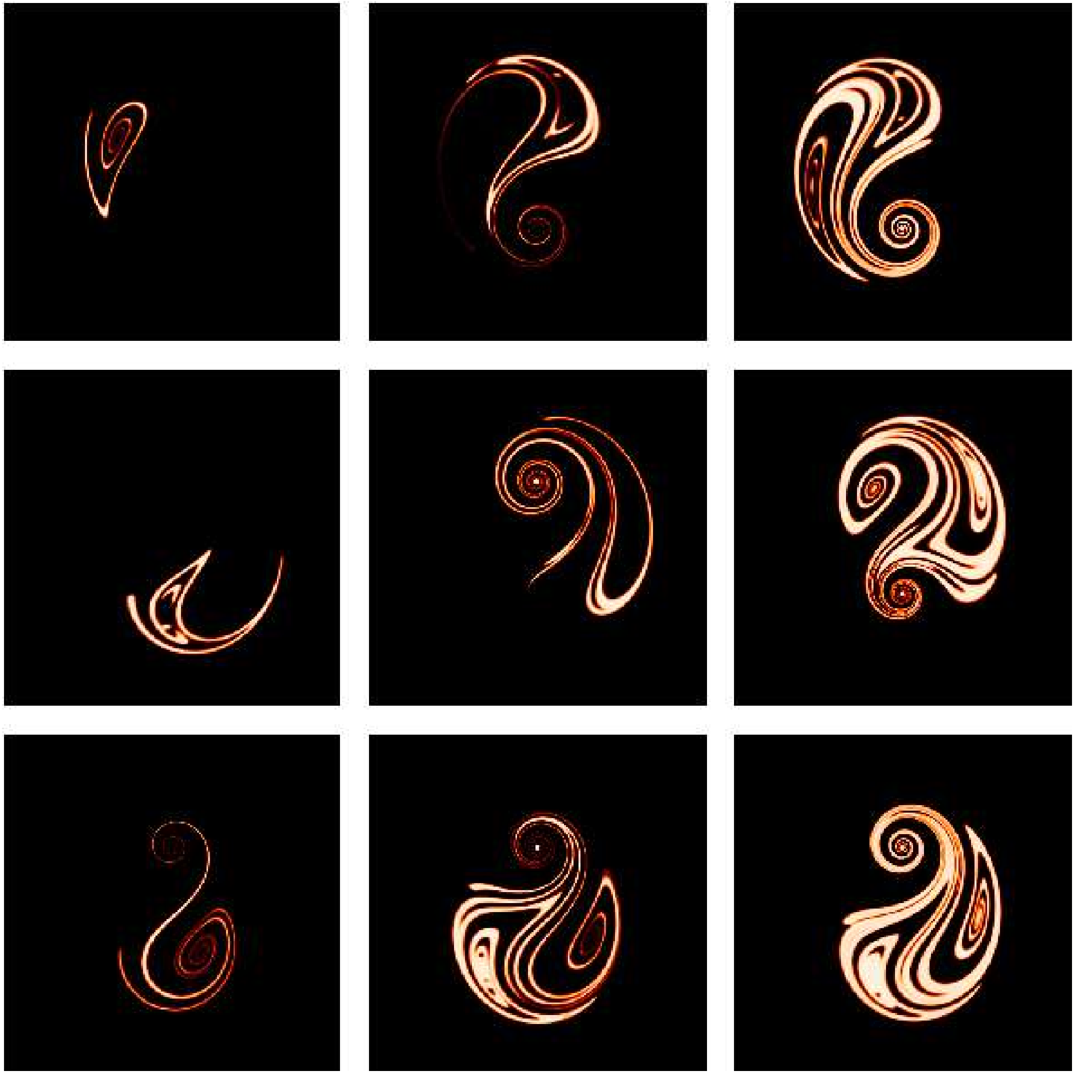
temporal sequence





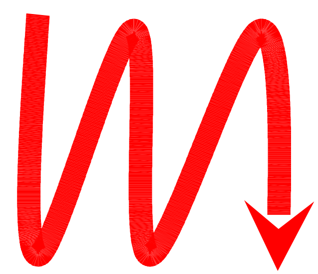
OPEN FLOW

At fast stirring, the excitation is destroyed, as in the closed-flow case. At slow stirring, the excitation pattern is transient, as in the absence of flow.
A novel phenomenon occurs at intermediate stirring speeds.



In a range of intermediate stirring speeds, a permanent excitation pattern, linked to the chaotic saddle, remains in the system:

The flow has rendered permanent an otherwise transient excitation phenomenon

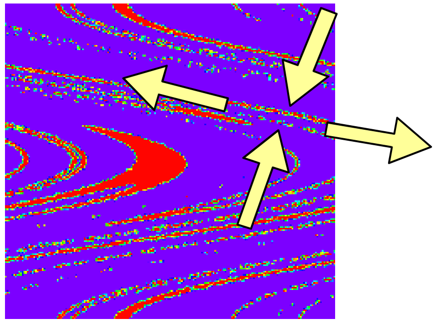


temporal sequence

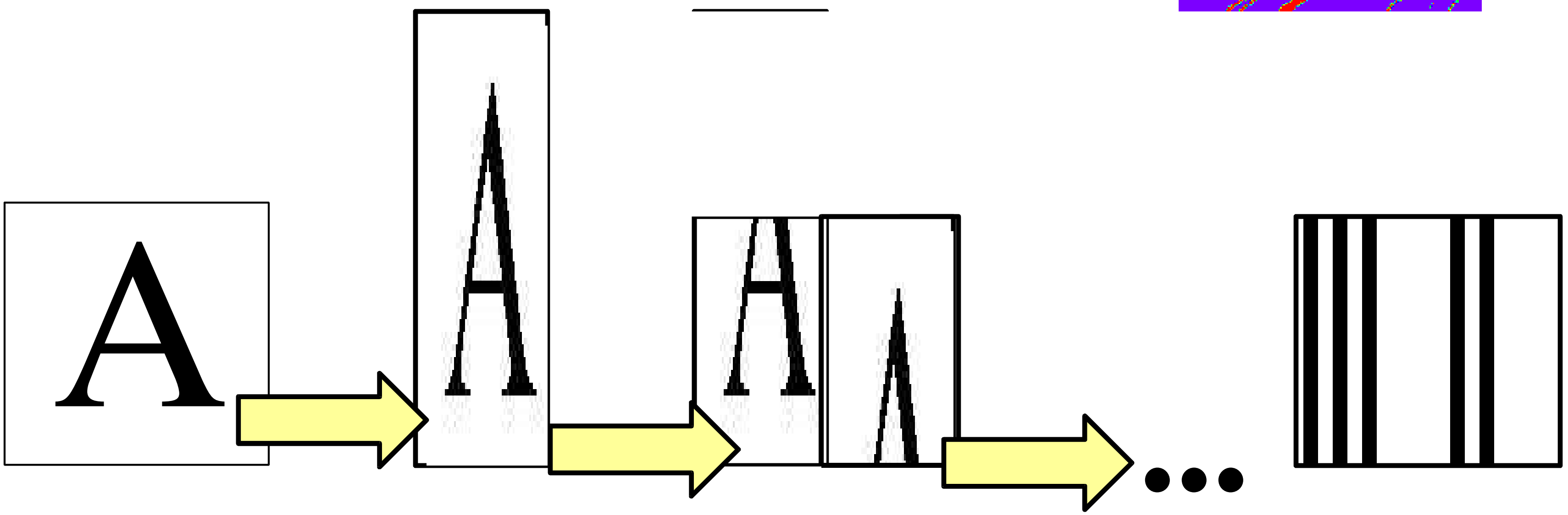
REDUCED MODELS

In order to understand the above process we have introduced two types of one-dimensional reduced models:

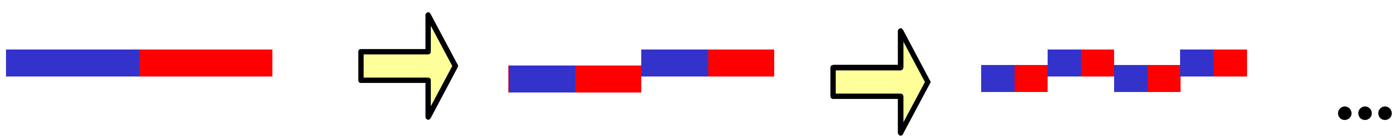
- A BAKER-LIKE model
- A FILAMENT model



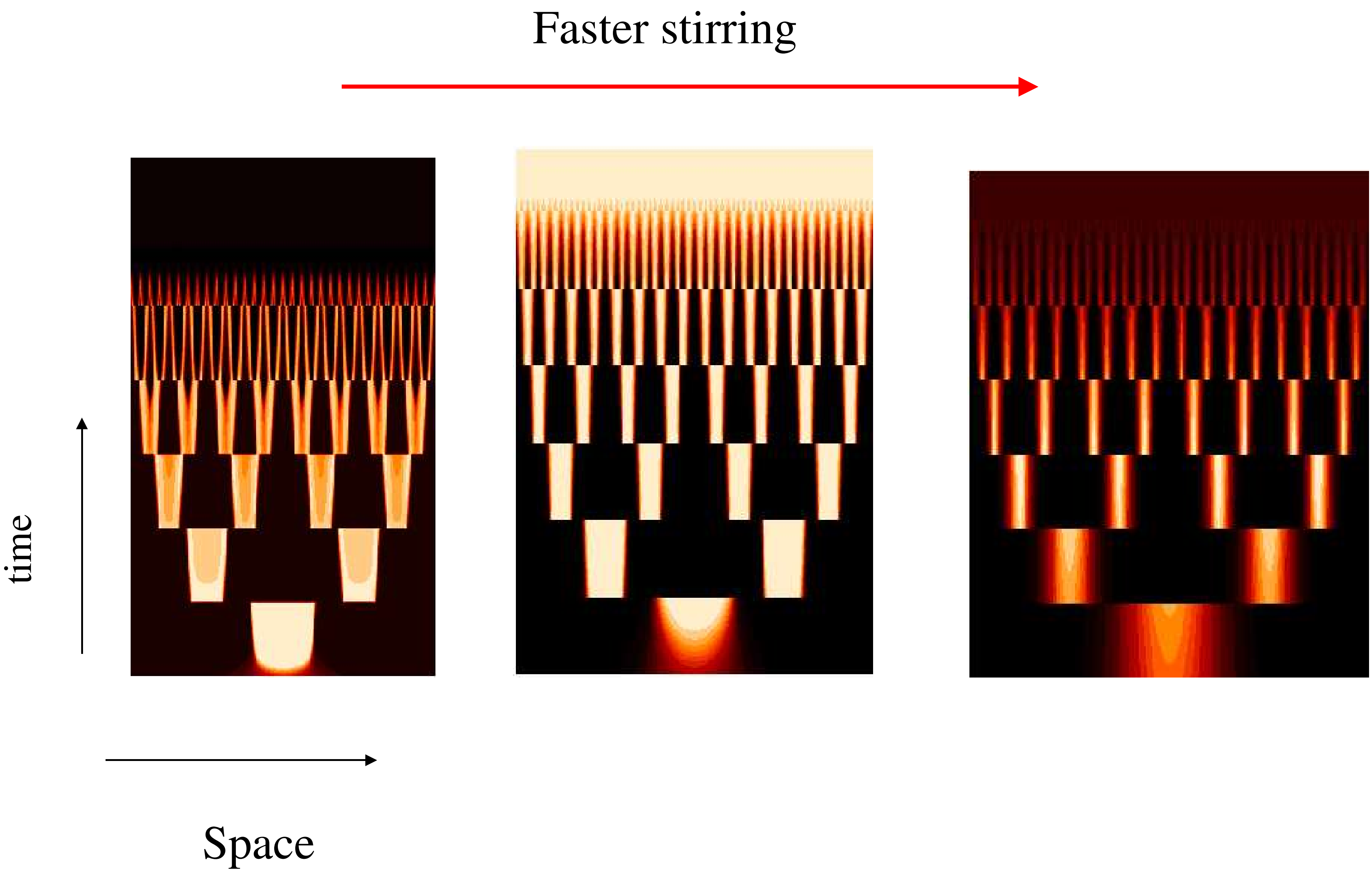
In the BAKER model, stretching and folding by the chaotic flow is represented by a simple geometrical procedure:



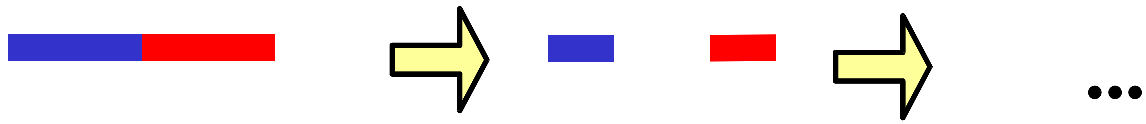
After some iterations, the process is essentially onedimensional, transverse to the filaments in the chaotic flow:



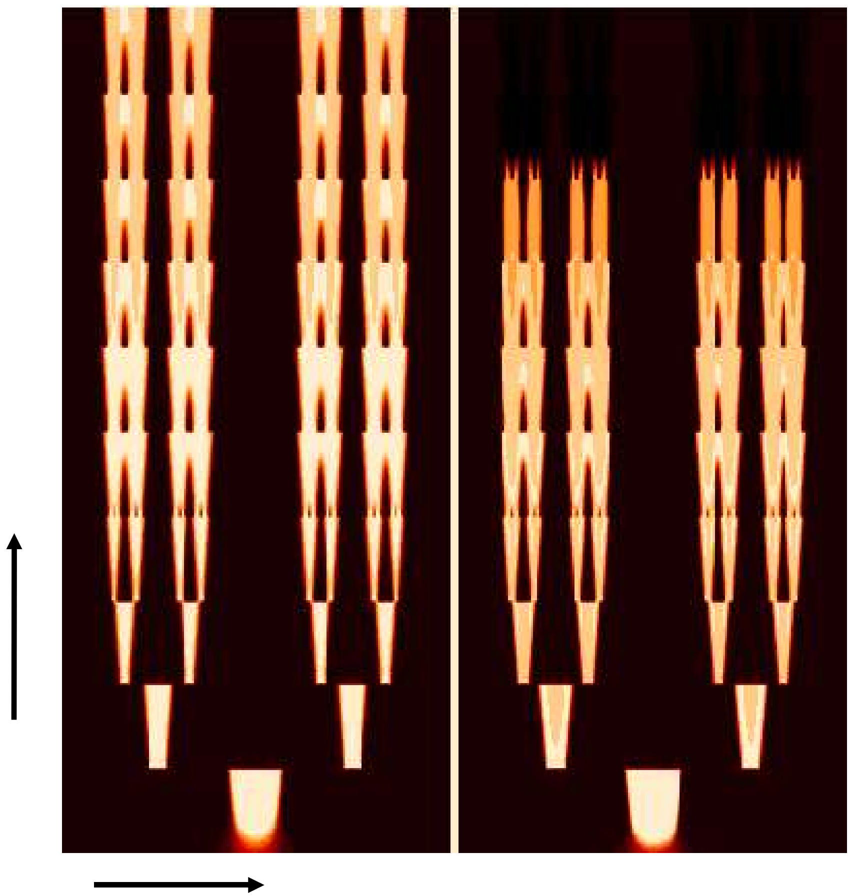
Baker map model for the closed flow



A simple modification gives to the baker model characteristics of an OPEN flow:

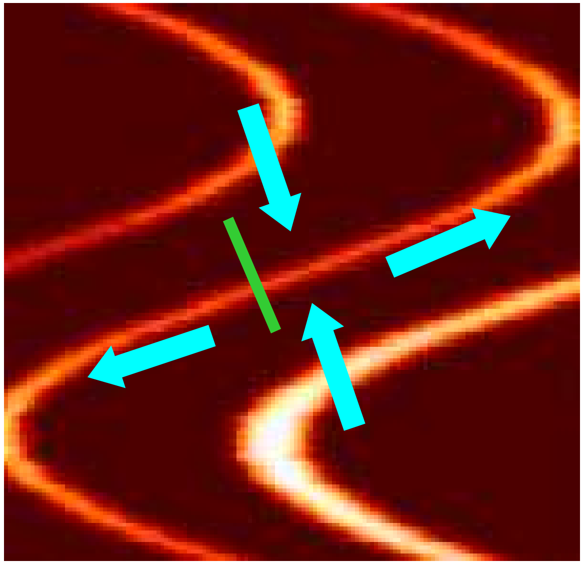


Intermediate stirring: selfsustained pattern



Again, the qualitative behavior of the two-dimensional flows is recovered, including the permanent pattern found at intermediate stirring speed

Filament model



Assume locally the same strain flow :

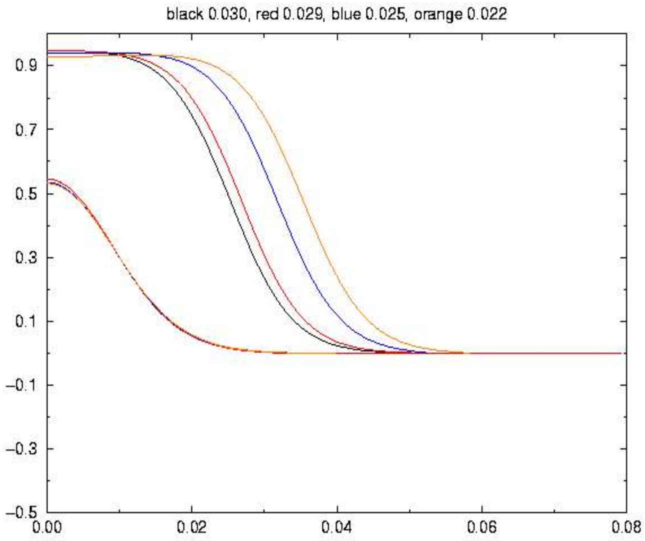
$$v_x = -\lambda x$$
$$v_y = \lambda y$$

Neglect the stretching direction y

$$\frac{\partial C_1}{\partial t} - \lambda x = f(C_1, C_2) + D\frac{\partial^2 C_1}{\partial x^2}$$

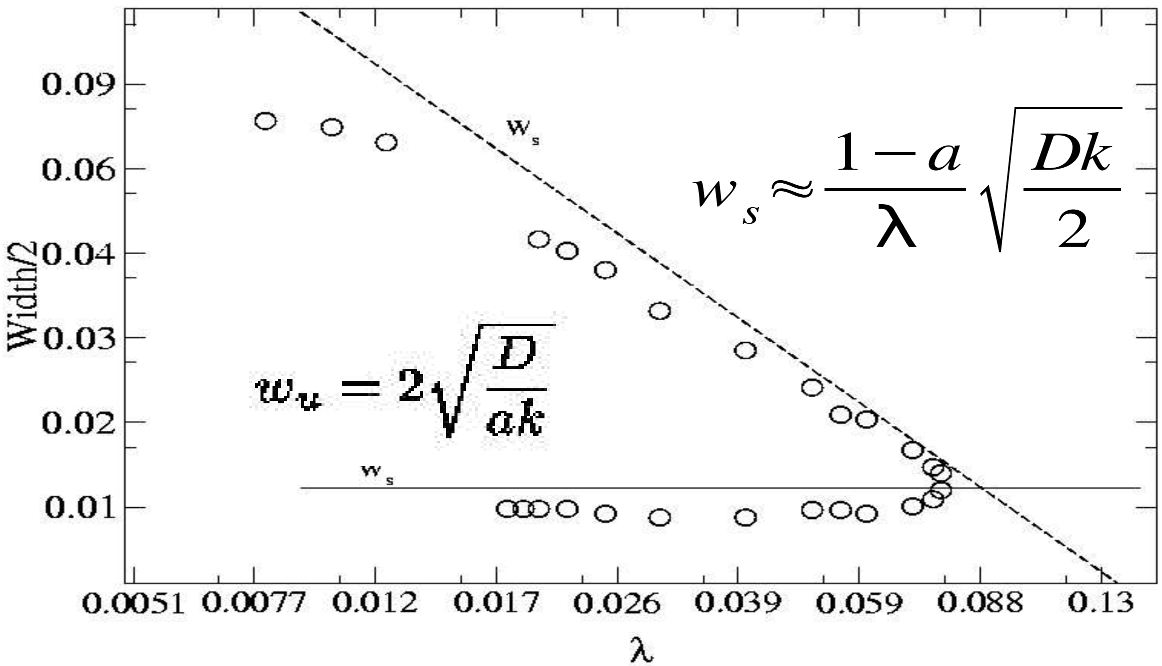
$$\frac{\partial C_2}{\partial t} - \lambda x = g(C_1, C_2) + D\frac{\partial C_2}{\partial x^2}$$

There is a range of values of lambda for which we can obtain a stationary (stable) one hump solution. This is the coherent excitation

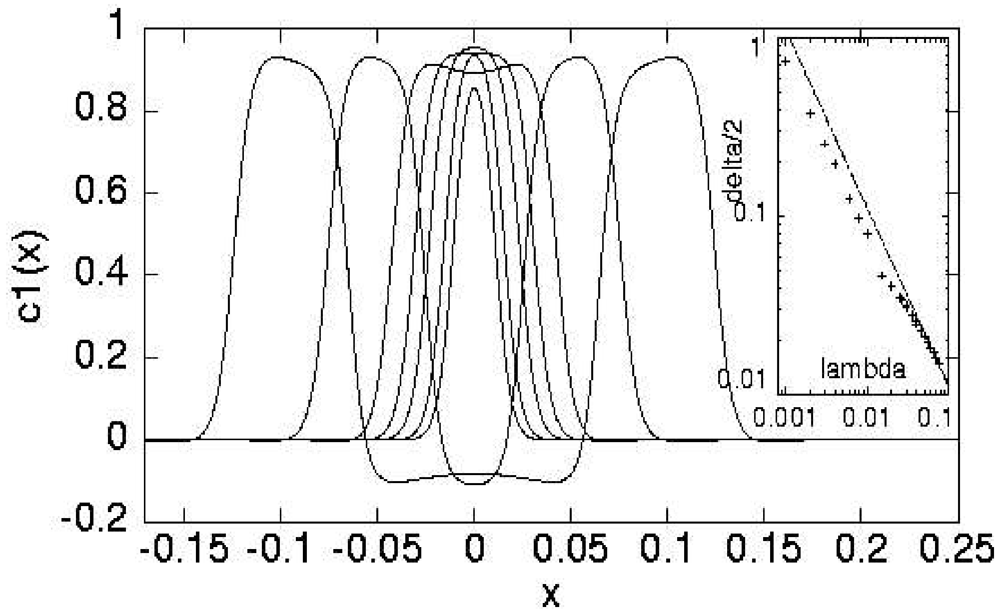


With the shooting method we can also obtain an unstable solution for the same value of lambda.

Increasing the stirring the unstable and stable solutions approach and this is why dissapears the coherent excitation for fast stirring.



Decreasing stirring there appears double humped solutions that also can explain the lost of the coherent excitation in the two dimensional simulations



SUMMARY AND OUTLOOK

- **The interplay between excitable dynamics and chaotic flow leads to interesting novel phenomena. In particular, there is a kind of broad resonance between chemical and hydrodynamic time scales such that:**
  - **In closed flows, it leads to a global coherent excitation [1].**
  - **In open flows, it leads to the stabilization of excitation patterns that are transient in other circumstances, or in the absence of flow [2].**

- **The introduction of reduced models allows to identify Lagrangian stretching and folding as the essentials ingredients in the flow to produce the above phenomena. A simple filament model allows also some quantitative predictions.**