Efficiency anlysis of forced

RATCHETS

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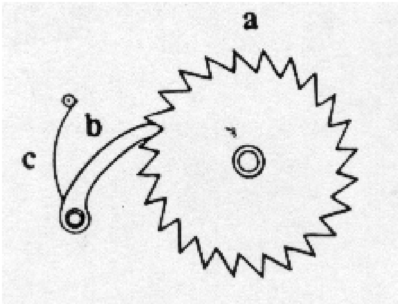
What do we understand by Ratchet?



Mechanical device that transmit intermittent rotary motion or permits a shaft to rotate in one direction but not in the opposite one.

- **Feynman**: usable work can be extracted due to the presence of net force or a macroscopic gradient.

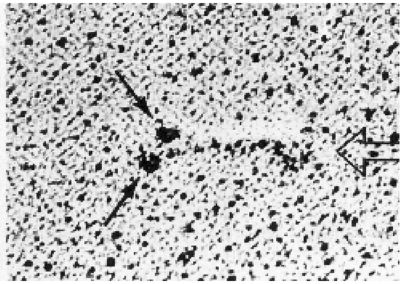
Brownian Domain ?

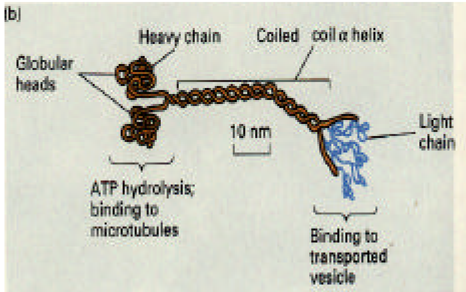
Molecular Motors

MOTOR PROTEINS

Kinesin: Transport of vesicles

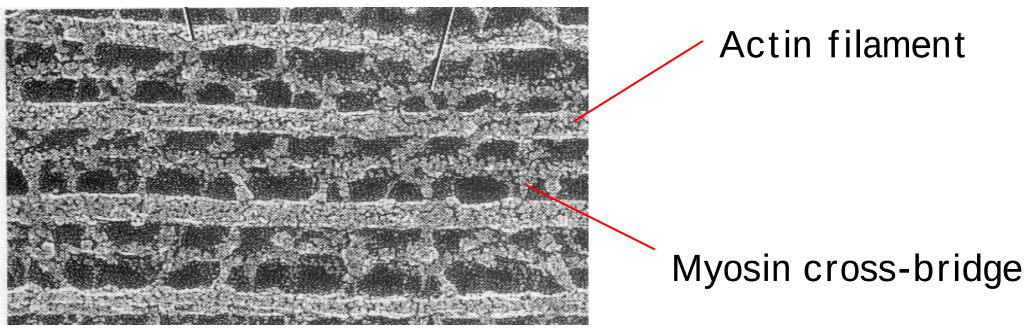
(F. Marchesoni, Phys. Lett. A, **237**, (1998) 126)





Myosin powers motility

- Cell motility and Cell shape (Sambeth and Baumgärtner, Phys. A **271** (1999) 48)
- *Muscle contraction* (Astumian and Bier, PRL, **72** (1994), 1766)



New addressed topics

- Model of kinesin (T. Vicsek, PRL '97)
- Asymmetric polymerization of actin filaments. (Sambet, PhysA '99)
- Electrons in Josephson juctions (Zapata, PRL '96)
- Chaotic transport (Mateos, PRL '99)
- Paradoxical games (Parrondo, Toral,unpubl.)

Novel technological applications

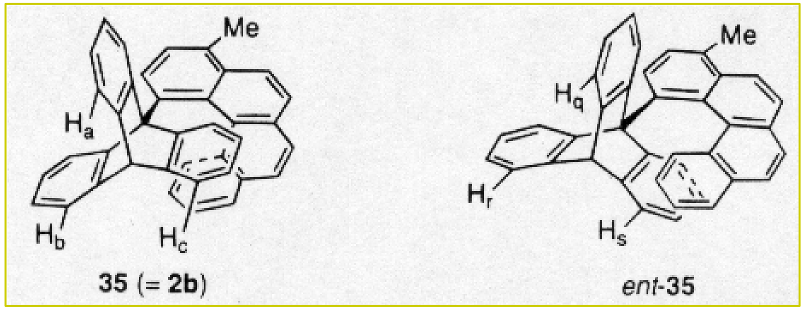
- Mass separation (Rousselet, Nature '94)
- Reduce vortex density in superconductors (Lee, Nature '99)

Experimental evidences?

Experimental findings

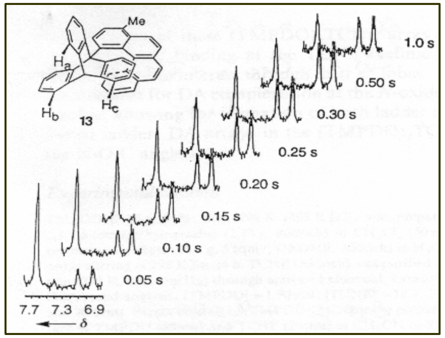
• Directed motion of rubidium atoms in optical bipotential
(Mennerat-Robilliard, PRL '99)

• Unidirectional rotary motion in molecular systems (Kelly, Nature '99)

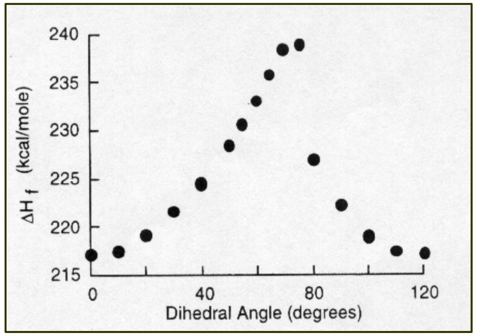


Triptycene (4) Helicene

Rotation evidence



Energy diagram



Rotation barrier
ΔH=22 Kcal/mol

Thermal Ratchet model

- Basic ingredients:
  - energy transducer (x)
  - external system (y)
  - the heat bath
  - the load against which the transducer works

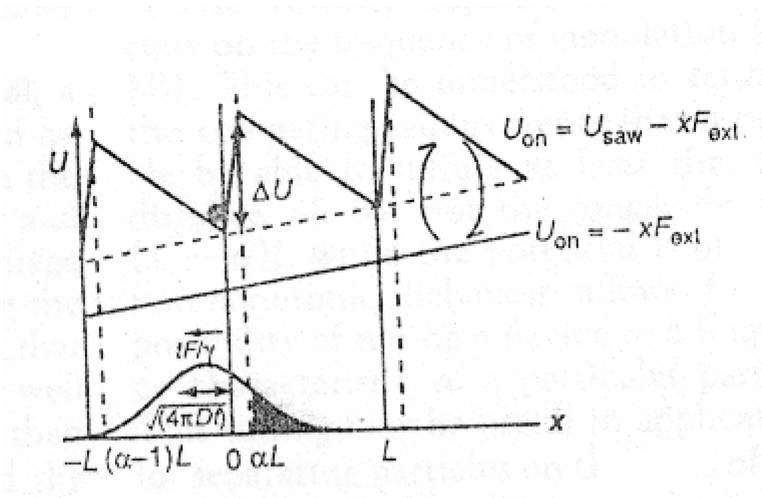
  - interaction potential: ***$U(x,y)=U_0(x,y)+Lx;\ U_0(x+l\ ,y)=U_0(x,y)$***

- Dynamics of the transducer:

  (overdamped)

$$-\frac{\partial U(x,y)}{\partial x}+\left[-g\,\frac{dx}{dt}+x(t)\right]=0$$

$$\langle x(t)x(t')\rangle = 2gk_B T d(t-t')$$



Is it possible to optimize the efficiency of the process at non-zero T?

Energy analysis

(Sekimoto, J. Phys. Soc. Japan, 66 (1997), 1234)

- Change in the total potential energy

$$\Delta U = U(x(t_f),y(t_f)) - U(x(t_i),y(t_i))$$

- Dissipation to the heat bath: D
- Consumption of energy: R=D+ΔU

Steady state of the transducer: $\left\langle \frac{dU}{dt} \right\rangle = L \left\langle \frac{dx}{dt} \right\rangle$

$$D = -\int \left[-g\frac{dx}{dt} + x(t) \right] dx(t) = \int \left[-\frac{\partial U(x(t),y(t))}{\partial x} \right] dx(t)$$

$$R = \int \left[dU(x(t),y(t)) - \frac{\partial U(x(t),y(t))}{\partial x} dx(t) \right] \qquad = \int \frac{\partial U(x,t)}{\partial t} dt$$

y(t) ?

Y(t+T)=y(t)

U(x,y(t)) —— U(x,t)

Efficiency: η= L<dx/dt>/ R

$$\frac{dx}{dt} = -\frac{\partial}{\partial x}\left[V_0(x) + V_L(x)\right] + F(t) + \sqrt{2T}\,x(t)$$

Time averaged current

$$J = \left\langle \frac{dx}{dt} \right\rangle_{st} = -L + \frac{1}{t} \lim_{t \to \infty} \int_{t}^{t+t} \left\langle -\frac{\partial}{\partial x}\left(V_0 + V_L\right) \right\rangle dt$$

Input of energy

$$R = \frac{1}{t} \int_{x(nt)}^{x(n+1)t} F(t)\,dx(t)$$

Analytical Solutions?

Fokker-Planck eq.

$$\partial_t P(x,t) + \partial_x j(x,t)$$

$$j(x,t) = \left(-\frac{\partial}{\partial x}\left(V_o + V_L\right) + F(t) - T\frac{\partial}{\partial x} \right) P(x,t)$$

F(t)=A

P(x,t)=P(x+l,t)=P(x,t+t) $\Longrightarrow$ $J = \frac{1}{t}\int_{0}^{t} j(x,t)\,dt$

Quasi-static limit

• F(t) has a square wave form and changes slowly enough compared to any other frequency in the system

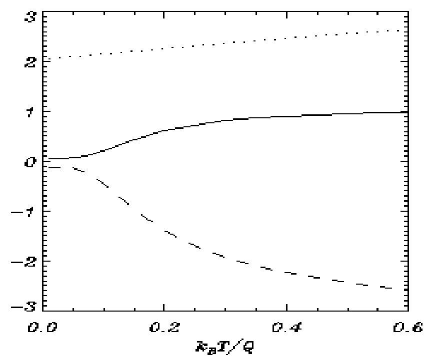
$$J = \frac{1}{2}\left[J(A) + J(-A)\right]$$

$$h = \frac{L\left[J(A) + J(-A)\right]}{A\left[J(A) - J(-A)\right]}$$

A=3

• *J(-A) < 0*

$$h = \frac{L}{A}\left(\frac{1 - \left|\frac{J(-A)}{J(A)}\right|}{1 + \left|\frac{J(-A)}{J(A)}\right|}\right)$$

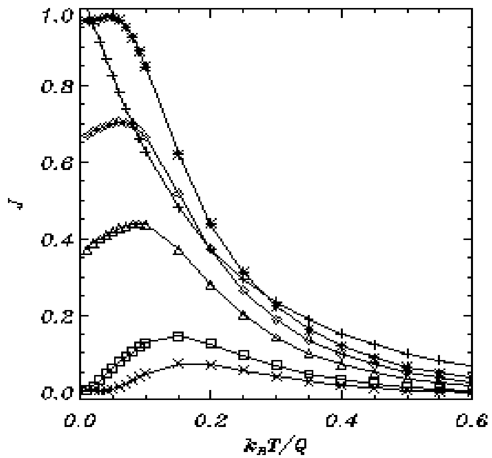


Kamegawa (PRL '99): η cannot be optimized at finite T.

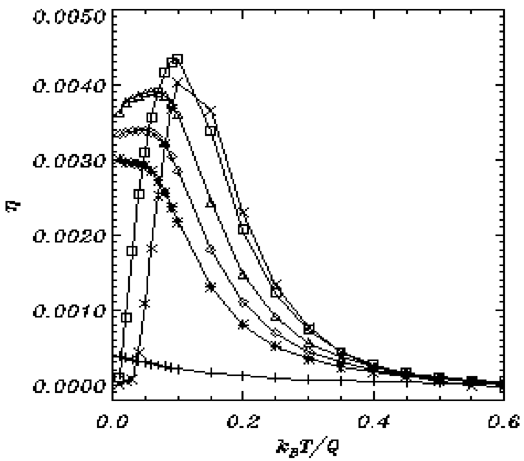
Takagi (PRE '99)
Bao (Phys.A '99)
Dan et al (PRE '99)

?

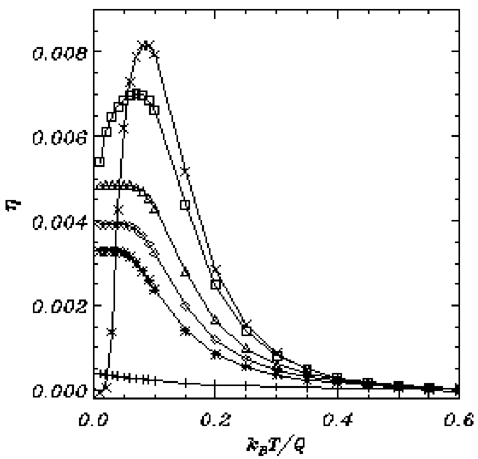
Numerical Results (OVERDAMPED)



Current(J) and efficiency (η) measured at different values of the amplitude of the driving force: A=1.0 (X); A=1.3 (á); A=2.0(△); A=2.5 (◇); A=3.0(ø); A=6.0 (+).

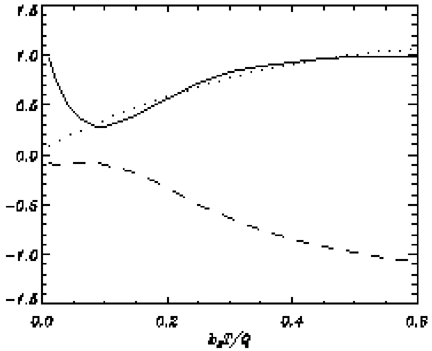


t =6

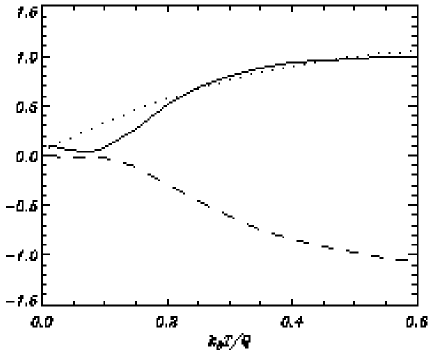


t =50

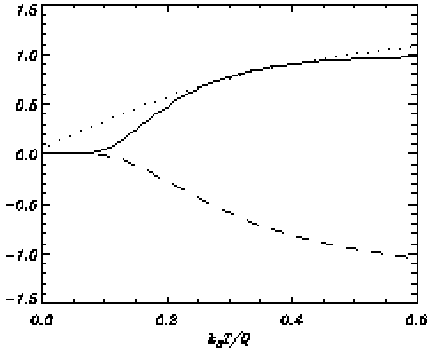
A=1.3



t =6



t =50



t = ∞

J(A) (· · ·); J(-A) (- - -); |J(-A)/J(A)| (___)

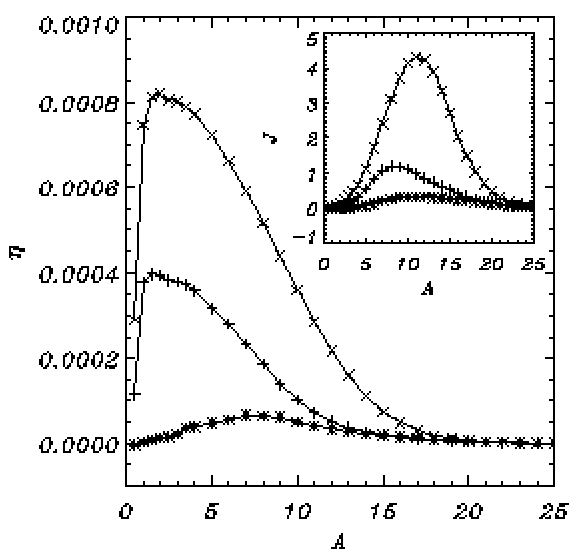
INERTIA RATCHETS (underdamped)

$$\dot{x} = v$$
$$m\dot{v} = -bv - \frac{\partial}{\partial x}\left[V_0(x) + V_L(x)\right] + F(t) + \sqrt{2T}\,x(t)$$

Role of
Mass
&
Friction

Amplitude of the forcing

T=1, b=0.5

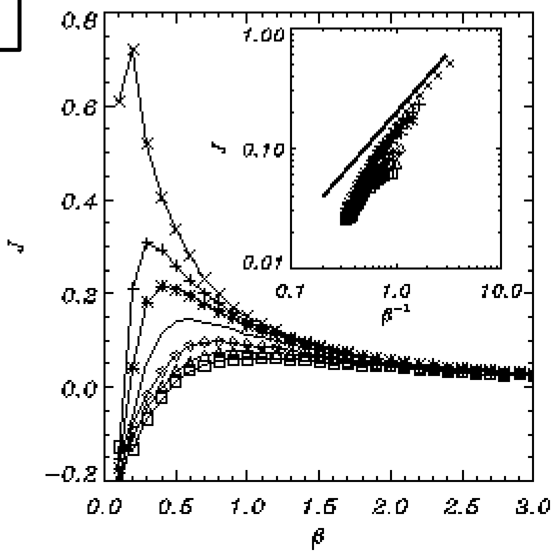


- **Amplitudes are selected to optimize the current and the efficiency**

Current(J) and efficiency (η) at different values of the mass: μ=0.01 (X); μ =0.05 (+); μ =0.1(ø).

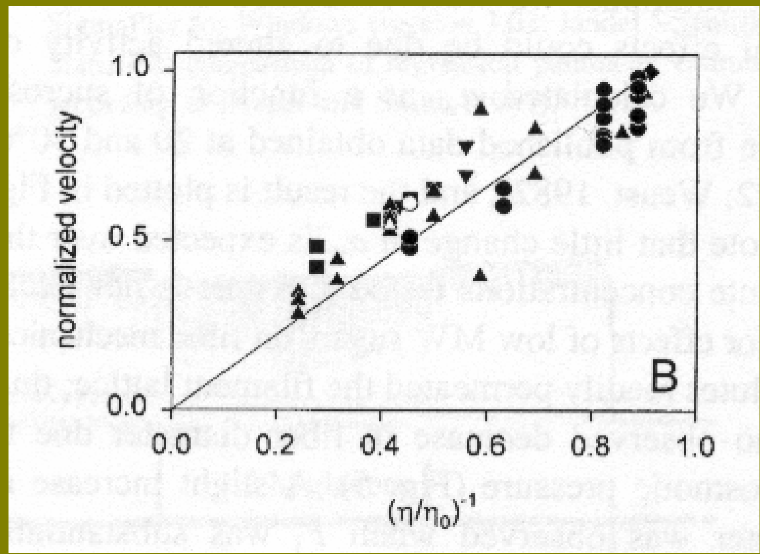
Friction

T=1, A=3



Current behavior with the viscosity at different values of the mass, from µ=0.01 (X) to µ =0.1 (á).

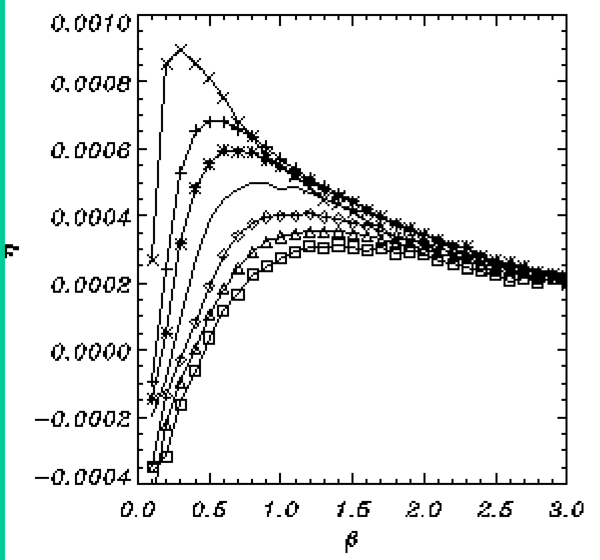
Inset: Molecular motors operate in the overdamped regime. Shortening of the velocity is found to be proportional to the solution viscosity



Effect of the viscosity on skinned rabbit fibers. Viscosity within the myosin filaments is controlled by adding low MW sugars that decreases the chemical reaction kinetics.

(Taken from P. Bryant Chase et al. Biophys. J. **74** (1998) 1428.)

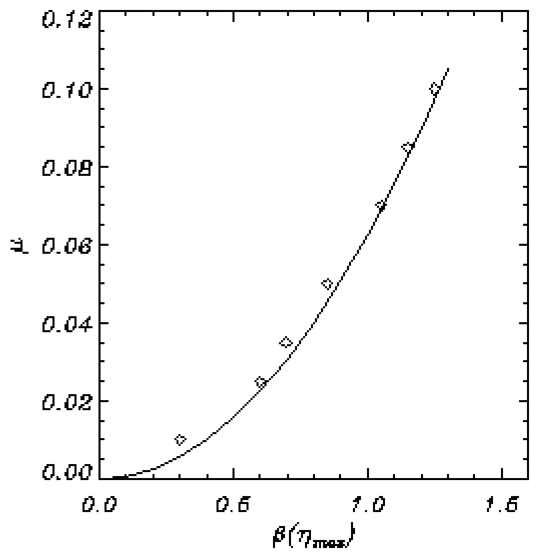
Friction: effect on the efficiency



Behavior of the efficiency with the viscosity at different values of the mass, from μ=0.01 (X) to μ =0.1 (á).

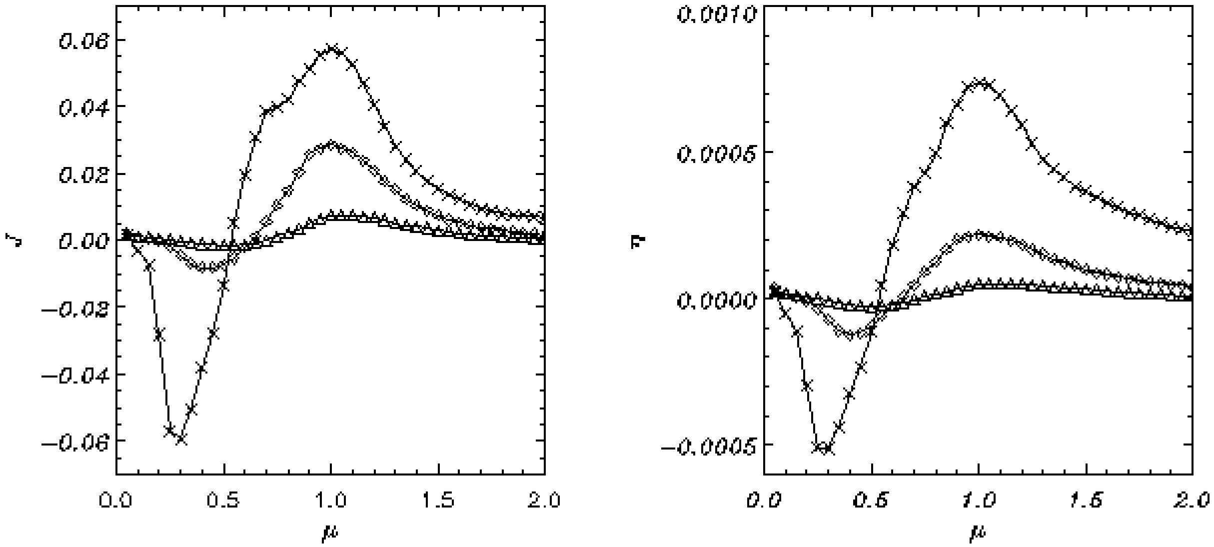
Condition of optimal efficiency

$$\sqrt{\frac{Tm}{b^2}} = L - a$$



Inertia: Mass separation

T=0.25, A=3

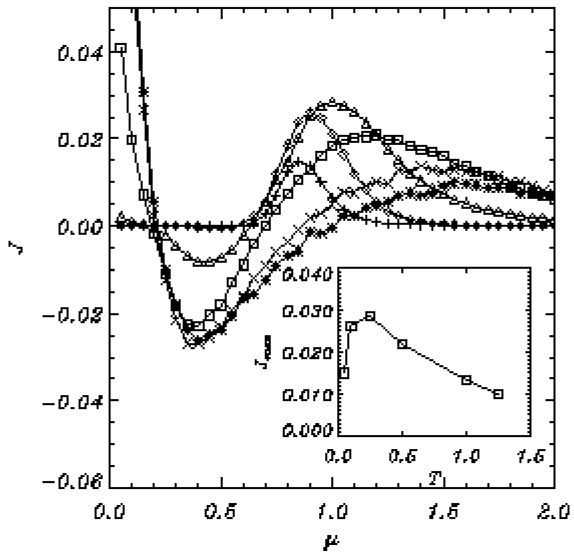


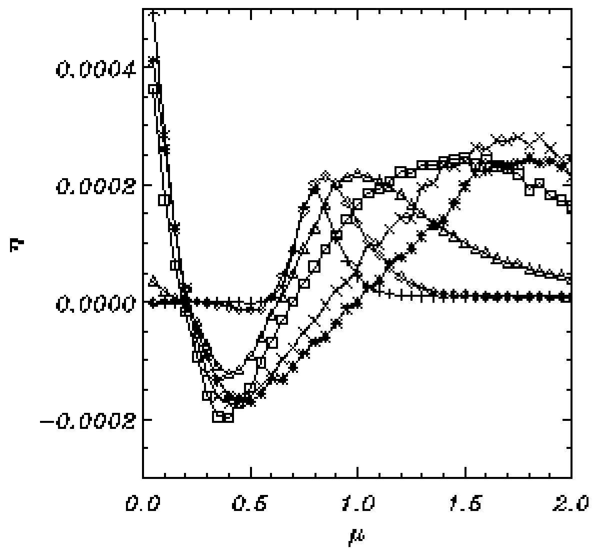
Current(J) and efficiency (η) at different values of the friction:
β=0.5 (X); β=1.0 (◇); β=1.5 (△).

Inertia: Noise intensity

b=1, A=3

Current(J) and efficiency (η) behavior at different temperatures:
T=0.05 (+); T=0.1 (◇); T=0.25 (△); T=0.5 (á);T=1.0 (X); T=1.25 (ø).





- Maximum of the current shifts to higher values of μ as T increases
- Inset plot: Current is maximized at a particular T.

stochastic resonance like effect

CONCLUSIONS

Thermal fluctuations facilitate the efficiency of the energy transformation.

OVERDAMPED REGIME:
Current decreases linerly with the viscosity. (Experimental evidence in rabbit muscle fibers)

UNDERDAMPED REGIME:
- Condition of optimal efficiency is proposed
- Current reversals at different friction strength and mass
- Evidence of stochastic like resonance effect (applicabiliy to ion selectivity in voltage sensitive channels).