Thermally driven out-of-equilibrium Kondo system

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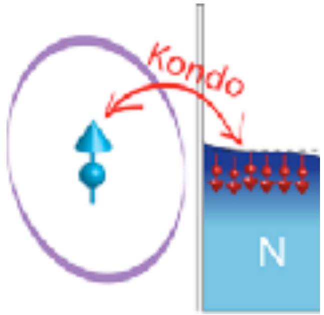
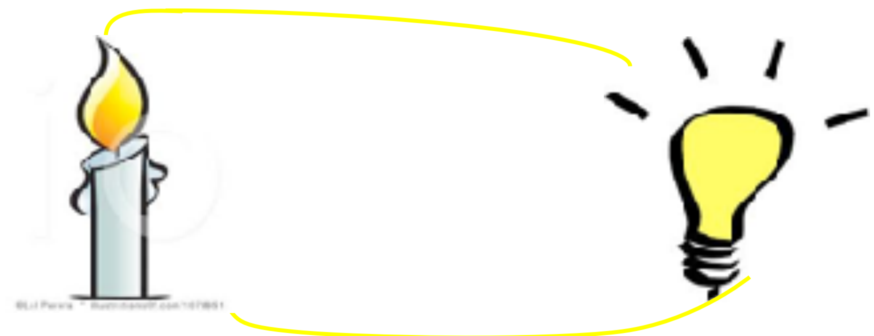


Phys. Rev. Lett. 121, 096801



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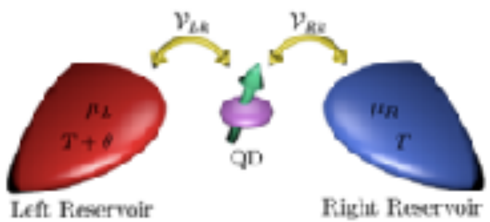
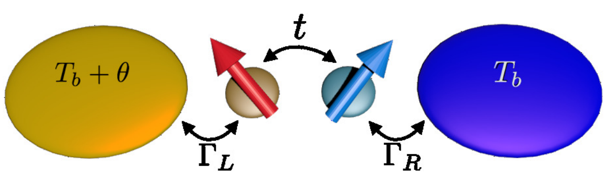
Outline

- Thermoelectric transport
- Kondo effect in quantum dots
- Thermally driven single dot
- Theoretical approaches
- Results(I)
- Thermally driven two impurity Kondo system
- Theoretical model
- Results (II)
- Conclusions

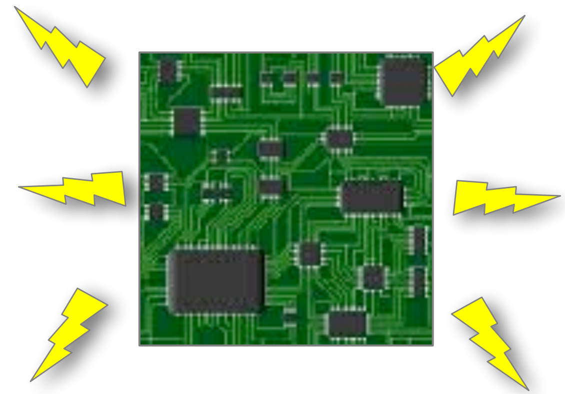




First experimental evidences of Kondo effect

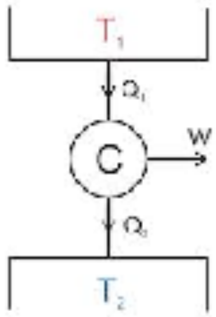
D. Goldhaber-Gordon *et al.*, *Nature* **391**, 156 (1998).
S. M. Cronenwett, T. H. Oosterkamp, L. P. Kouwenhoven, *Science* **281**, 540 (1998).
J. Schmid, J. Weis, K. Eberl, K. v. Klitzing, *Physica B* **256**, 182 (1998).

Thermolectricity*



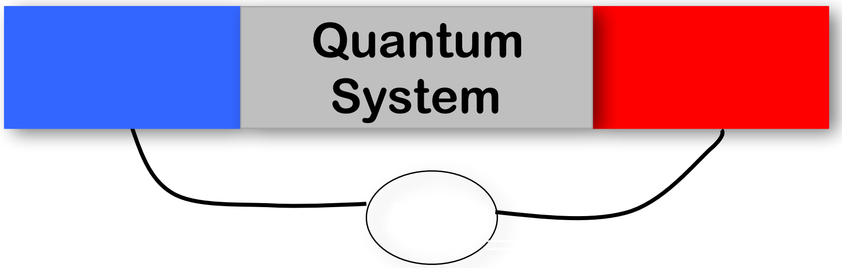
Circuit heat generation is one key limiting factor for scaling device speed

Waste heat recovery: typically 30-40 % efficiency for heatengines, waste heat 60%



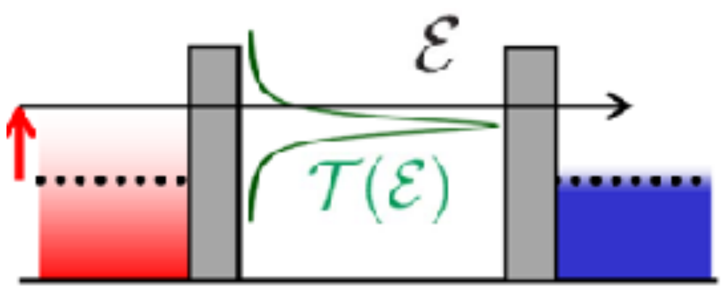
THERMOELECTRICS is recognized as a potentially transformative energy conversion technology: heat is directly converted into electricity and vice versa

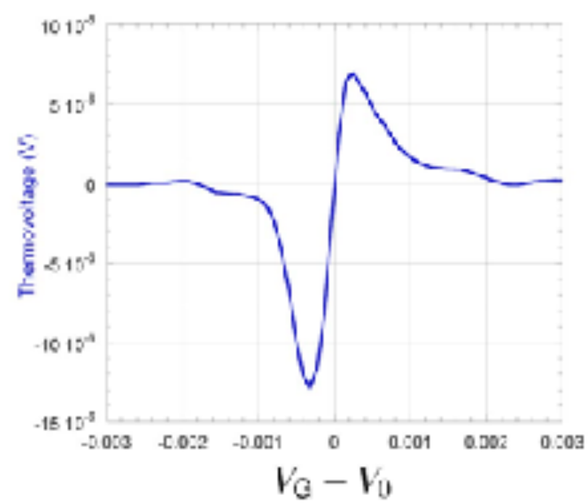
Thermolectric transport*



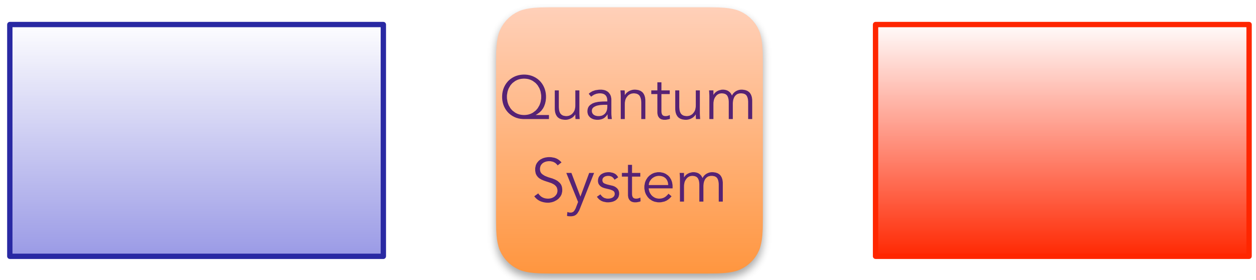
Large thermal gradients large as 13K/mV
R. Venkatasubramanian et al., Nature 413, 597 (2001)

Rectification effects due to interactions
Thermopower changes sign, A.A.M. Staring et al.,
EPL 22, 57 (1993)



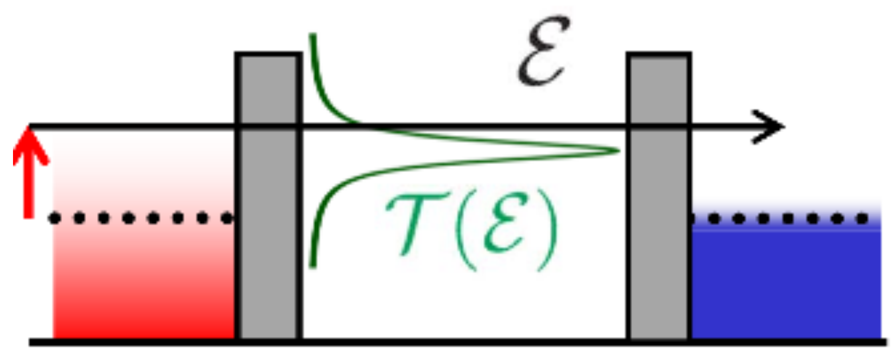


Thermolectric transport*



Nanosystems exhibit REMARKABLE THERMOELECTRIC properties

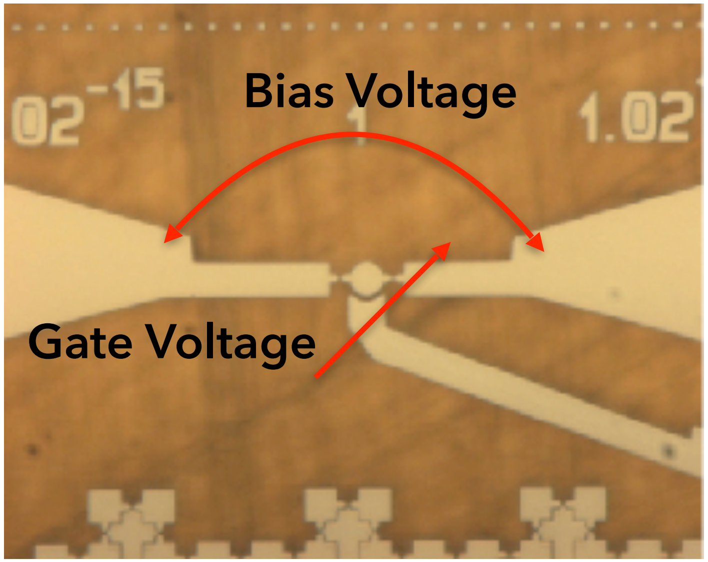
Specially when transport properties depend very much on **energy** as in the case of **quantum dots**

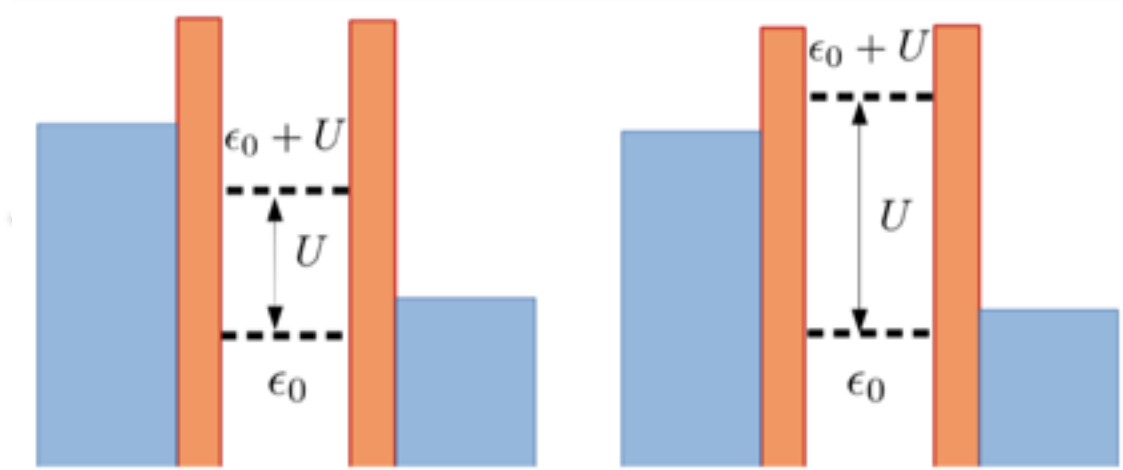


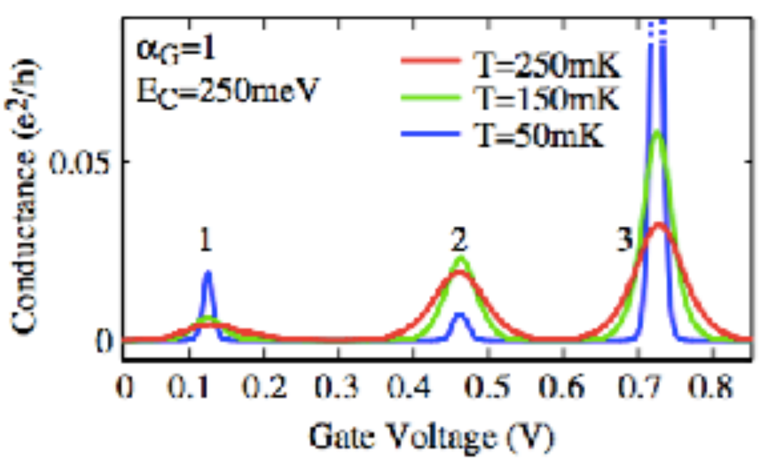
Quantum Dots

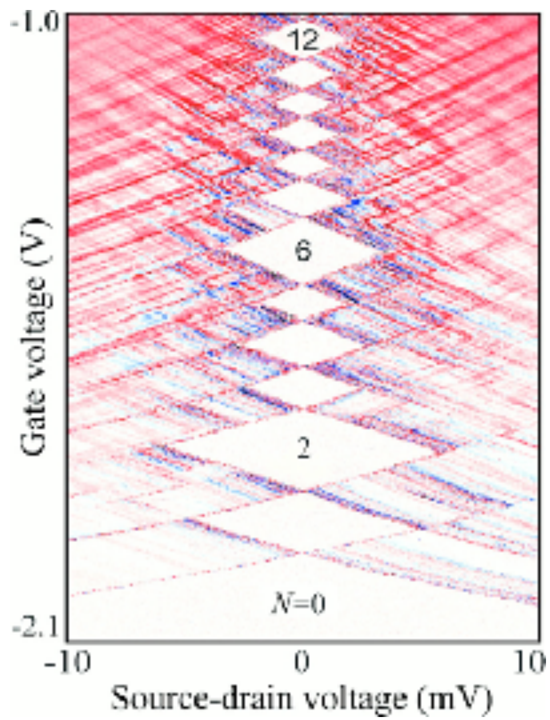
Coulomb blockade phenomena

Quantum dot: 0D system









L. P. Kouwenhoven, Science 1997

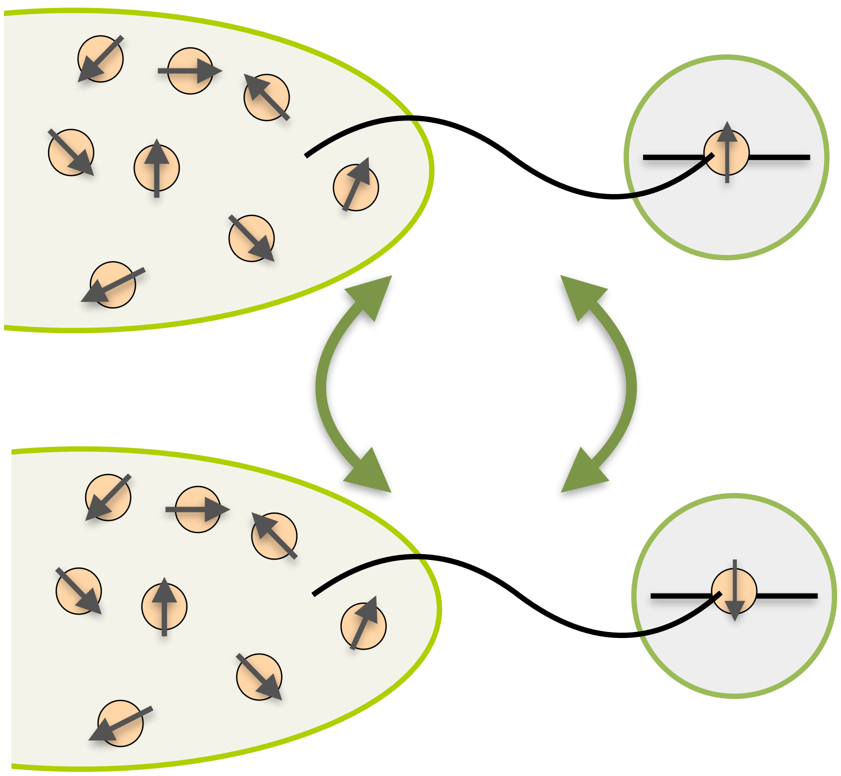
Energy levels
Quantized

Coulomb energy
$E_c = q^2/2C$

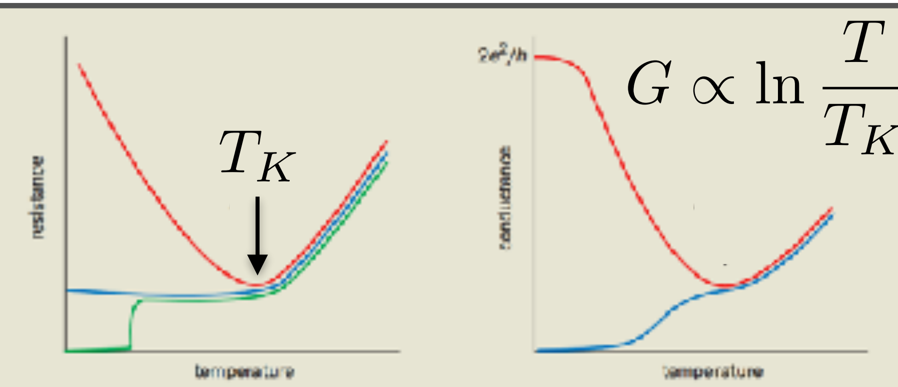
The Kondo Effect in metals

Resistance Minimum in Dilute Magnetic Alloys" Progress of Theoretical Physics 32 (1964) 37
J. Kondo

Creation of a spin singlet

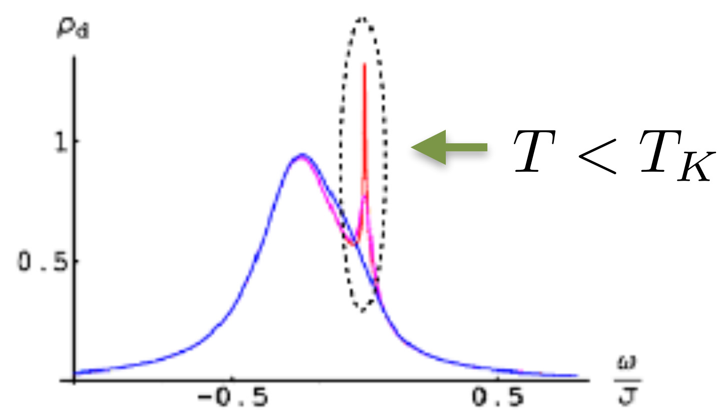


Interaction between the metal and impurity spins

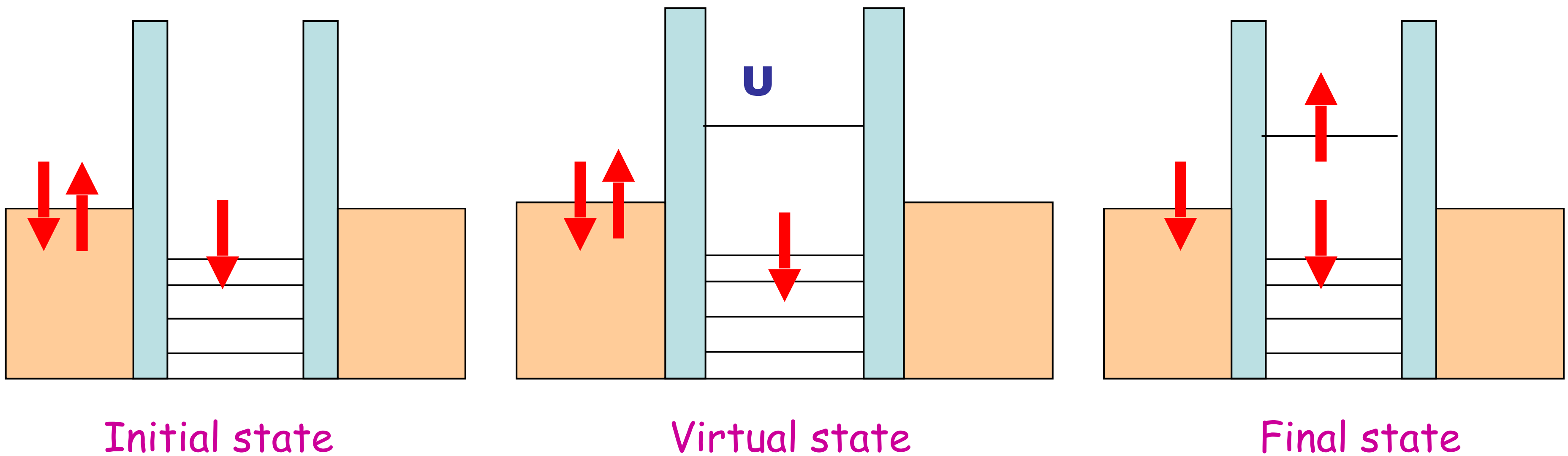


Resistance minimum in a given temperature

Temperature Kondo: characteristic energy scale

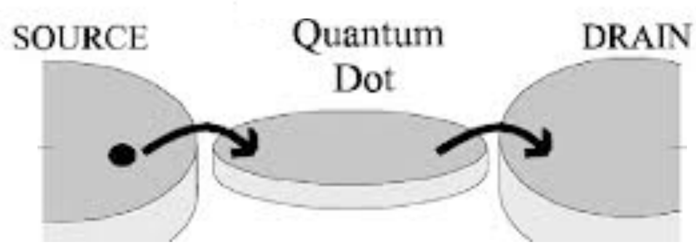


L.I. Glazman and M.E. Raikh JETP Lett. 47, 452(1988),
T.K..Ng and P.A. Lee, Phys. Rev. Lett. 61, 1768 (1988)



The resulting correlated motion gives rise to a Kondo resonance in the quasiparticle density of states at the Fermi energy

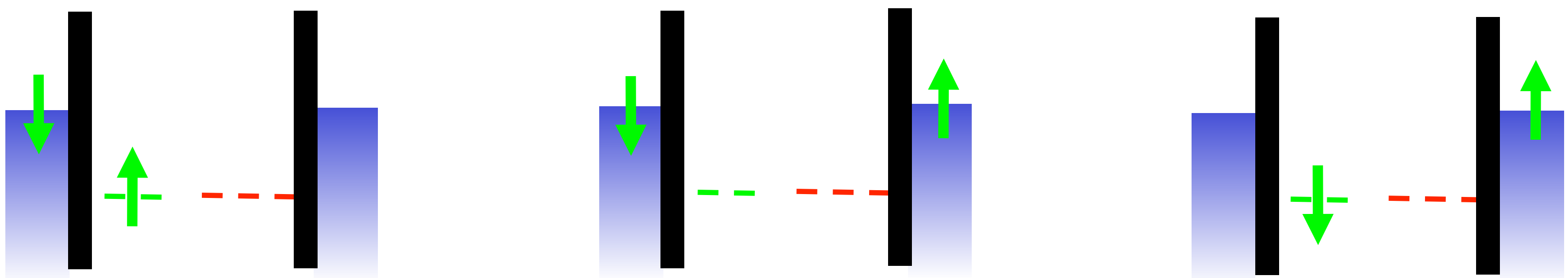
Due to this resonance the transmission through the quantum dot is perfect, the so-called “unitary limit”

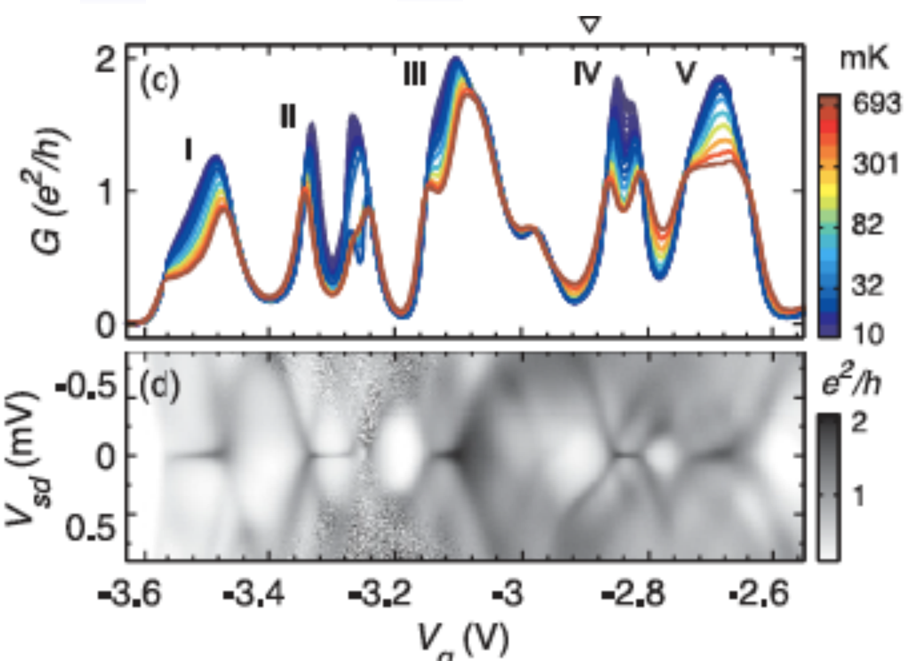


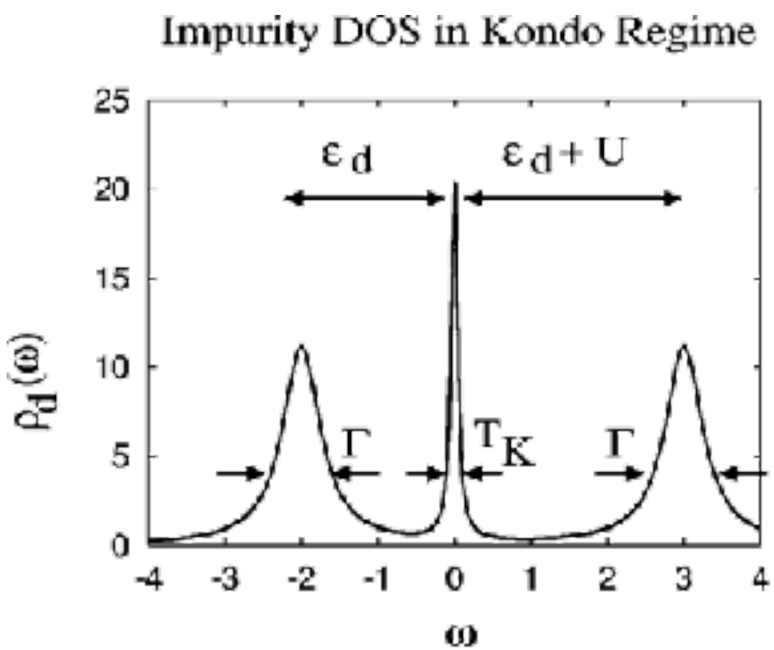
Unitary limit in the even valleys when the temperature is lowered

Appearance of the Zero Bias Anomaly in the Coulomb diamond

SPIN KONDO

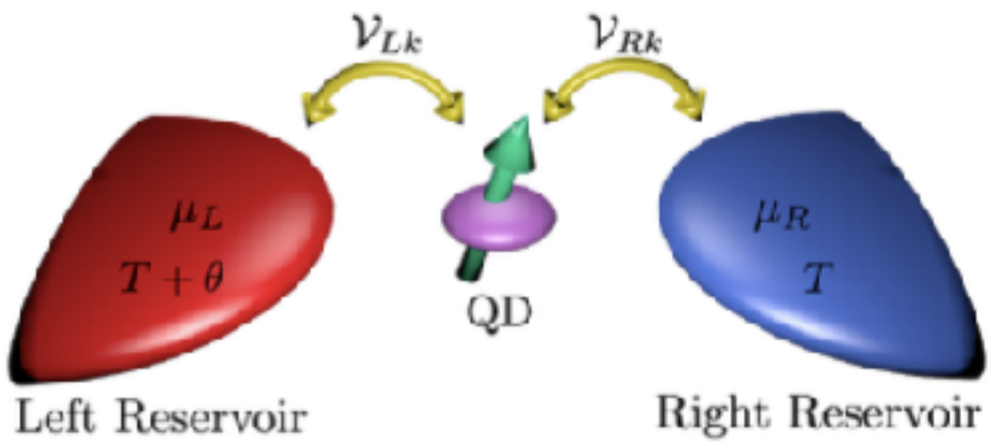


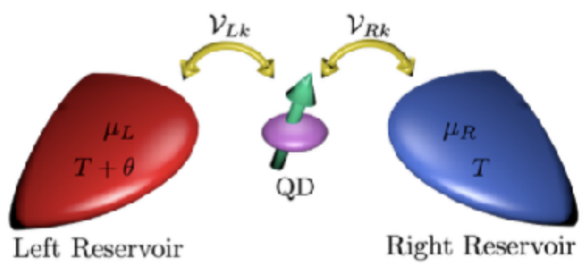


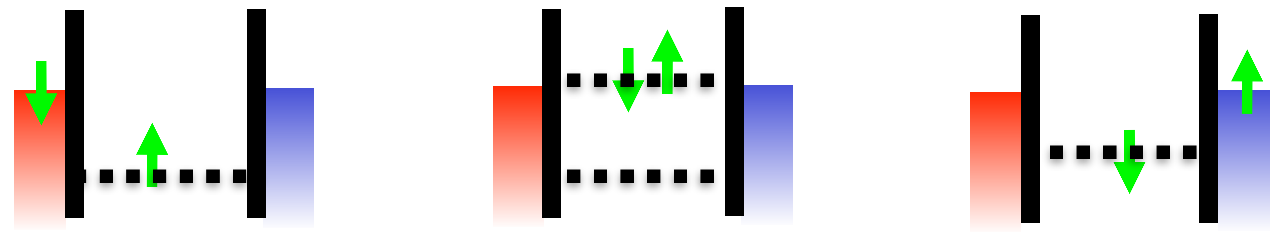


PHYSICAL REVIEW B 84 , 245316 (2011)

Single Quantum Dot case







Kondo Hamiltonian $\mathcal{H}_{\mathrm{K}} = \mathcal{H}_0 + \mathcal{H}_1$

$$\mathcal{H}_0 = \sum_{\alpha k \sigma} \varepsilon_{\alpha k} C^{\dagger}_{\alpha k \sigma} C_{\alpha k \sigma} \qquad \mathcal{H}_1 = \sum_{\alpha k \sigma \beta q s} \mathcal{J}_{\alpha\beta}(t) x_{\sigma s} C^{\dagger}_{\alpha k \sigma} C_{\beta q s}$$

$$x_{\sigma s} = \delta_{\sigma s}/4 + \hat{S}_l s^l_{\sigma s}$$

Localized and delocalized electrons: spin operators

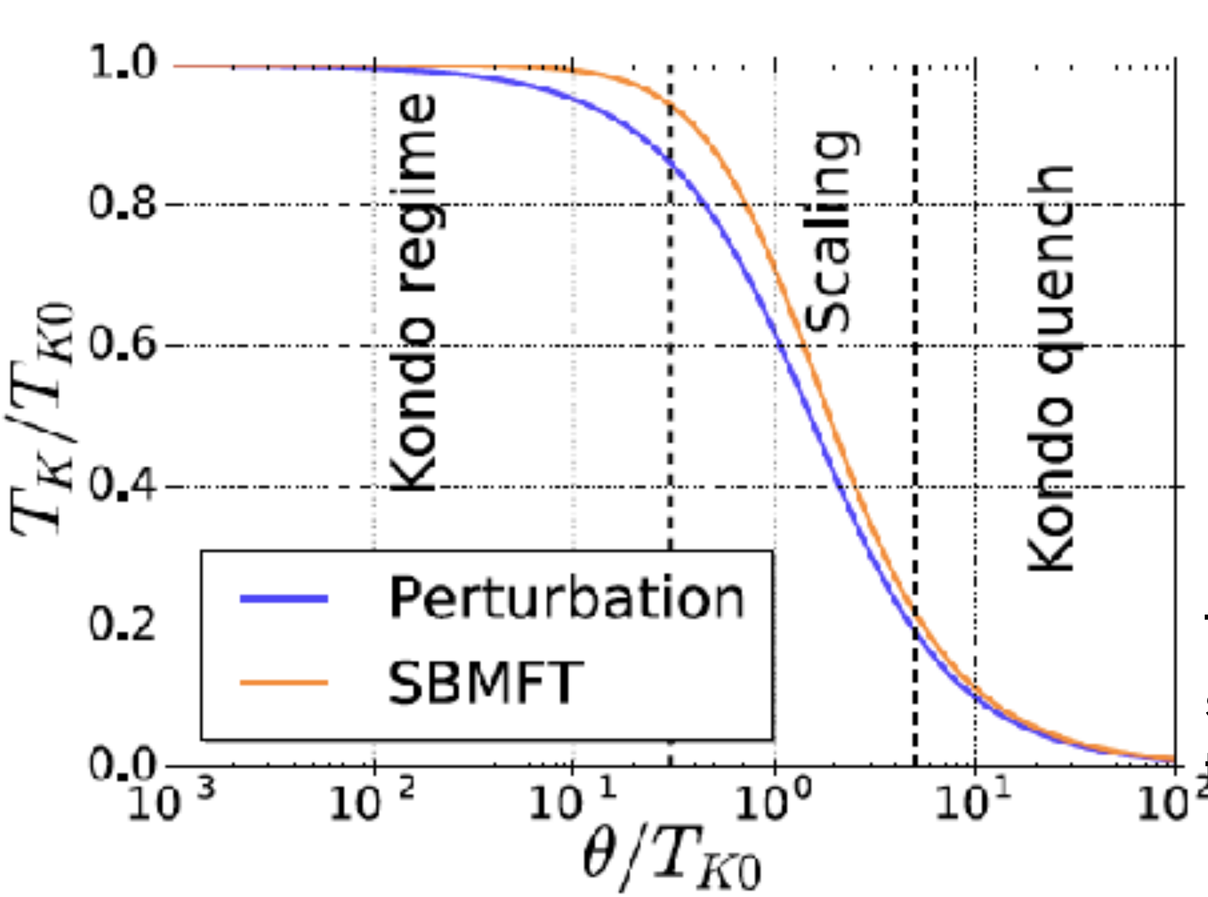
$S_l,\ l = x, y, z$

$$\mathcal{J}_{\alpha\beta}(t) = \mathcal{J}^{(0)}_{\alpha\beta} \exp\left(-\frac{ie}{\hbar}[V_\alpha - V_\beta]t\right),$$

$$\mathcal{J}^{(0)}_{\alpha\beta} = -\mathcal{V}_\alpha \mathcal{V}_\beta U/[\varepsilon_d(U+\varepsilon_d)]$$

Poor man scaling procedure

$$G = -\frac{3e^2\pi}{4\hbar}\nu^2[\mathcal{J}_{LR}^{(0)}]^2\left(1-\frac{\nu}{2}(\mathcal{J}_{LL}^{(0)}+\mathcal{J}_{RR}^{(0)})\ln\left|\frac{k_B^2 T_L T_R}{D_0^2}\right|\right) - \frac{e^2\pi}{4\hbar}\nu^2[\mathcal{J}_{LR}]^2. \qquad (11)$$



$$k_B T_{K0} = D_0 \exp\left[\frac{\pi\varepsilon_d(U+\varepsilon_d)}{U\Gamma}\right]$$

$$\Gamma_\alpha = \pi\nu|\mathcal{V}_\alpha|^2$$

$$\Gamma = \Gamma_L + \Gamma_R$$

$$T_K(\theta) = \sqrt{\left(\frac{\theta}{2}\right)^2 + T_{K0}^2} - \frac{\theta}{2}$$

The Kondo scale is determined when the second order contribution in perturbation theory prevails on the first order

Thermally driven Kondo effect:Slave boson representation

Anderson Hamiltonian

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{dot}} + \mathcal{H}_{\text{tun}}$$

$$\mathcal{H}_{\text{leads}} = \sum_{\alpha k\sigma} \varepsilon_{\alpha k} C^{\dagger}_{\alpha k\sigma} C_{\alpha k\sigma}$$

$$\mathcal{H}_{\text{dot}} = \sum \varepsilon_d d^{\dagger}_{\sigma} d_{\sigma} + U d^{\dagger}_{\uparrow} d_{\uparrow} d^{\dagger}_{\downarrow} d_{\downarrow},$$

$$\mathcal{H}_{\text{tun}} = \sum_{\alpha k\sigma} \mathcal{V}_{\alpha k} C^{\dagger}_{\alpha k\sigma} d_{\sigma} + \text{H.c}$$

$$U \to \infty$$

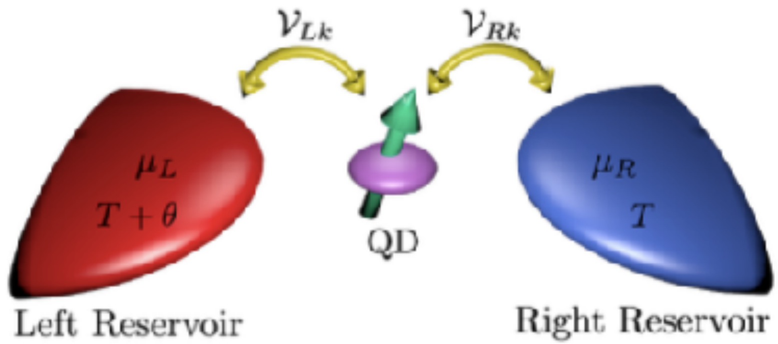
$$d_{\sigma} = b^{\dagger} f_{\sigma}$$

pseudofermion operator f_{σ}

boson field operator $b^{\dagger}$

$$\mathcal{H}_{\text{Lag}} = \lambda \left(b^{\dagger} b + \sum_{\sigma} f^{\dagger}_{\sigma} f_{\sigma} - 1 \right)$$

$$\mathcal{H}_{\text{tun}} = \sum_{\alpha k\sigma} \mathcal{V}_{\alpha k} C^{\dagger}_{\alpha k\sigma} b^{\dagger} f_{\sigma} + \text{H.c.}$$



Mean-field approach

$$\langle b \rangle = \tilde{b},$$

$$\sum_{\alpha k\sigma} \tilde{\mathcal{V}}_{\alpha k} G^{<}_{f\sigma,\alpha k\sigma}(t,t) = -iN\lambda|\tilde{b}|^2/\hbar$$

$$\sum G^{<}_{f\sigma,f\sigma}(t,t) = i(1 - N|\tilde{b}|^2)/\hbar$$

$$G^{<}_{f\sigma,f\sigma}(t,t') = -(i/\hbar)\langle f^{\dagger}_{\sigma}(t') f_{\sigma}(t)\rangle$$

$$G^{<}_{f\sigma,\alpha k\sigma}(t,t') = -(i/\hbar)\langle C^{\dagger}_{\alpha k\sigma}(t') f_{\sigma}(t)\rangle$$

Mean field parameters

$$\tilde{\Gamma} = |\tilde{b}|^2 \Gamma$$ Kondo width, Kondo temperature

$$\tilde{\varepsilon}_d = \varepsilon_d + \lambda$$ Kondo peak position

Mean field equations

$$\sum_\alpha \frac{\Gamma_\alpha}{\Gamma}\left[\log\left|\frac{2\pi k_B T_\alpha}{D}\right| + \psi\left(\frac{1}{2} + \frac{i\tilde{\varepsilon}_d + \tilde{\Gamma}}{2\pi k_B T_\alpha}\right)\right] = \frac{\pi N \varepsilon_d}{2\Gamma}$$

Electrically driven

$$\tilde{\Gamma}(V)\tilde{\varepsilon}_d = 0$$

$$\tilde{\varepsilon}_d^2 - \left(\frac{eV}{2}\right)^2 - [\tilde{\Gamma}(V)]^2 = -(k_B T_{K0})^2$$

Thermally driven

$$\tilde{\varepsilon}_d = 0$$

$$\tilde{\Gamma} = k_B T_{K0} e^{-\frac{\pi^2}{12}\frac{T_L^2+T_R^2}{(T_{K0})^2}}$$

Mean field parameters

Electrically driven

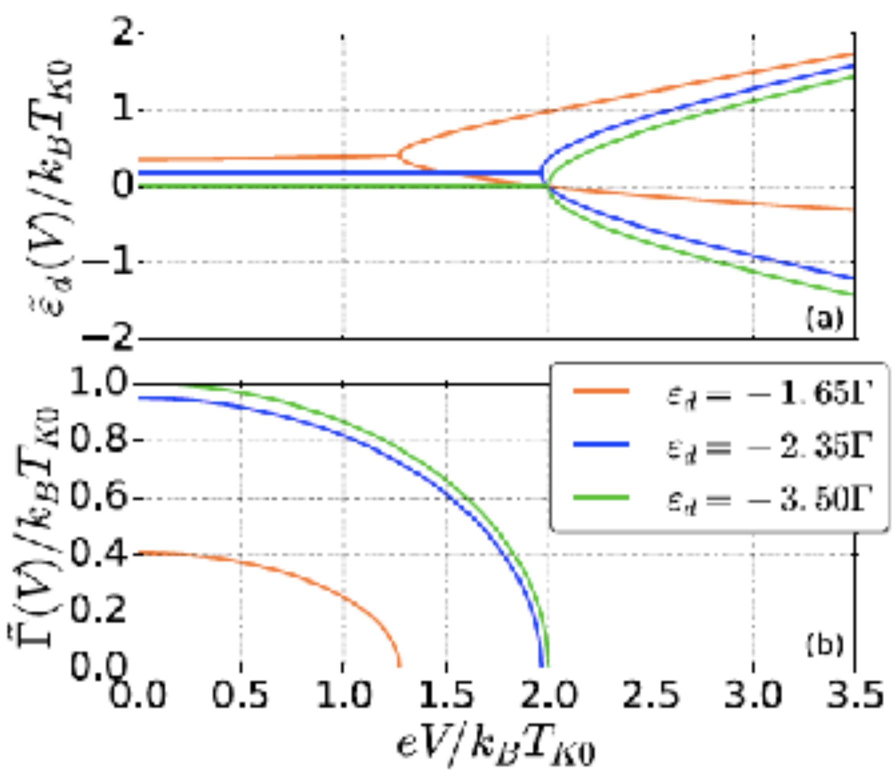


FIG. 3. Position of the SBMFT (a) renormalized energy level $\tilde{\varepsilon}_d$ and (b) width $\tilde{\Gamma}$ as a function of the applied voltage for different dot level positions. The case $\varepsilon_d = -3.5\Gamma$ agrees with the analytical result given by Eq. (24). Parameters: $D = 100\Gamma$, $k_BT = 0$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Bias voltage induced Kondo splitting

Thermally driven

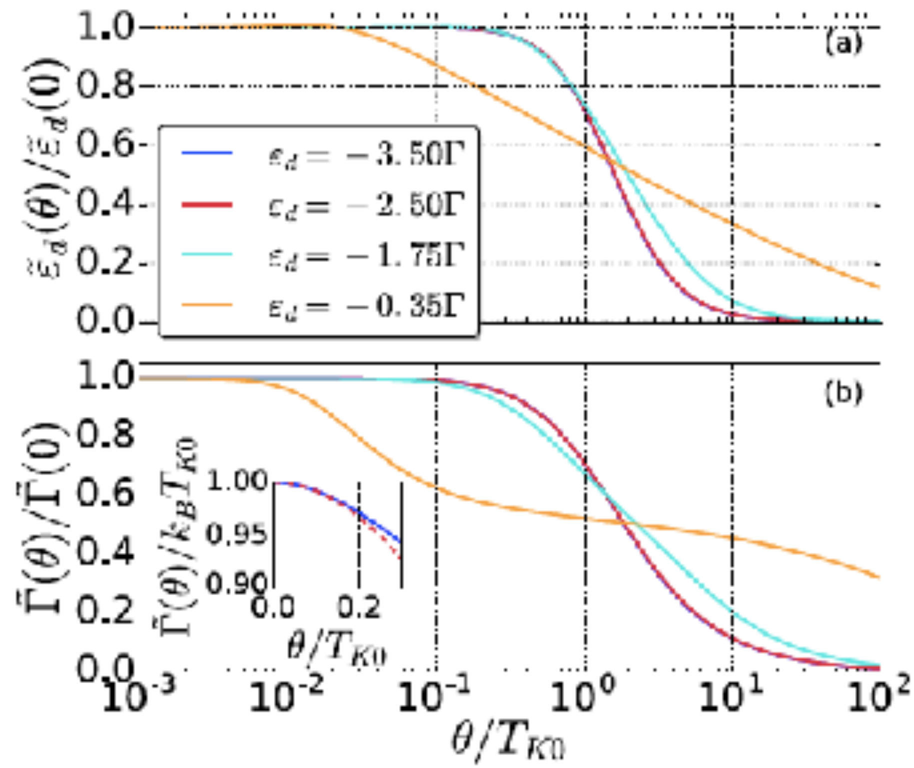


FIG. 4. (a) Renormalized dot gate position $\tilde{\varepsilon}_d$ and (b) resonance width $\tilde{\Gamma}$ as a function of the thermal bias θ for different ε_d values within SBMFT. (Inset) Resonance width versus the thermal bias from a numerical calculation (solid line) and from the analytical expression given by Eq. (27) (dashed line) for $\varepsilon_d = -3.5\Gamma$. Parameters: $D = 100\Gamma$, $k_BT = 0$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Thermal bias Kondo width quenching

Electric, and Thermoelectrical transport

$$I = -\frac{e}{h}\int_{-\infty}^{\infty} d\omega \sum_{\sigma} \frac{4\Gamma_L\Gamma_R}{\Gamma}\mathrm{Im}[G^r_{\sigma,\sigma}(\omega)][f_L(\omega) - f_R(\omega)],$$

$$L_2 = \frac{4\pi^2 e k_B^2}{3h}\tilde{\Gamma}_L\tilde{\Gamma}_R\frac{\tilde{\varepsilon}_d}{\tilde{\varepsilon}_d^2 + \tilde{\Gamma}^2}$$

Electrically driven

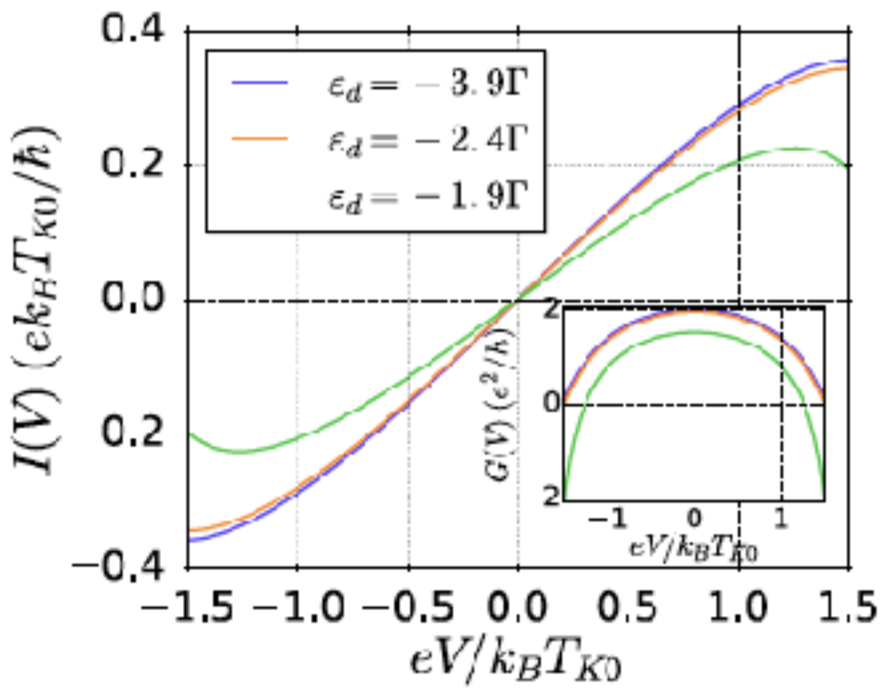


FIG. 5. Current-voltage characteristics of a single level quantum dot in the Kondo regime using slave-boson mean-field theory for different values of the gate voltage (level position). (Inset) Differential conductance of the quantum dot as a function of the applied voltage. Parameters: $D = 100\Gamma$, $k_BT = 0$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Unitary limit in the deep Kondo regime

Thermally driven

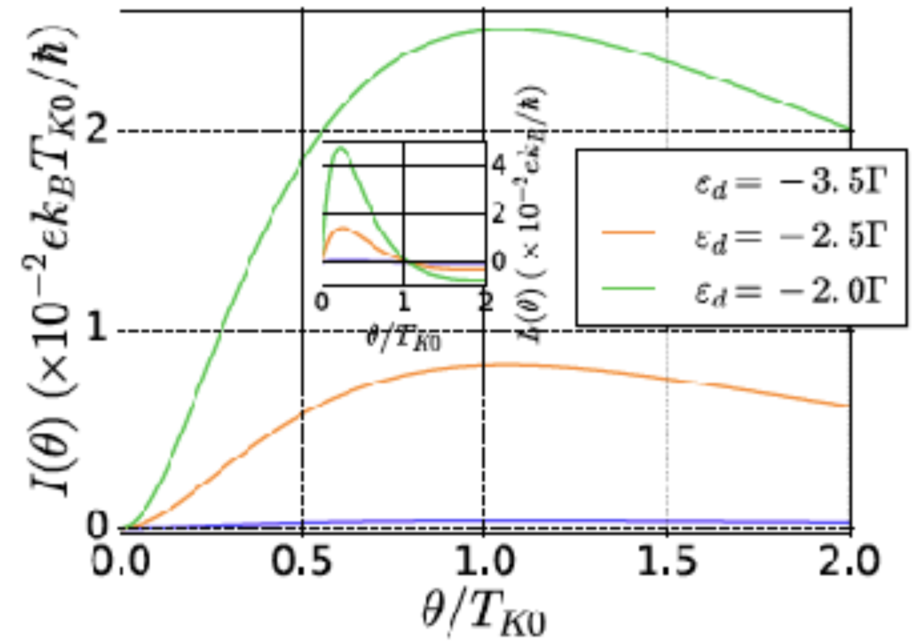


FIG. 6. Thermocurrent as a function the thermal gradient θ of a single level quantum dot in the Kondo regime using slave-boson mean-field theory for different values of the dot gate position. Inset: thermoelectric conductance as a function of the thermal bias for the same dot gate positions. Parameters: $D = 100\Gamma$, $k_BT = 0$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Larger thermocurrent when charge fluctuations are present

By using EOM tenchique the dot Green function reads

$$G^r_{\sigma,\sigma}(\omega) = \frac{1-\langle \tilde{n}_{\bar{\sigma}}\rangle}{\omega - \varepsilon_d - \Sigma_0 + U\Sigma_1/[\omega - \varepsilon_d - U - \Sigma_0 - \Sigma_3]} + \frac{\langle \tilde{n}_{\bar{\sigma}}\rangle}{\omega - \varepsilon_d - \Sigma_0 - U - U\Sigma_2/[\omega - \varepsilon_d - \Sigma_0 - \Sigma_3]}$$

$$T_{K0} \approx \sqrt{2\Gamma U}\exp\left[-\frac{\pi|\varepsilon_d|(U+\varepsilon_d)}{2\Gamma U}\right]$$

Self-energies are determined and the dot occupation is computed self-consistently

$$\Sigma_0 \quad \Sigma_1 \quad \Sigma_2 \quad \Sigma_3 \quad \langle \tilde{n}_{\bar{\sigma}}\rangle$$

O. Entin-Wohlman, et at., Phys. Rev. B 71 035333 (2005)

Equilibrium spectral density

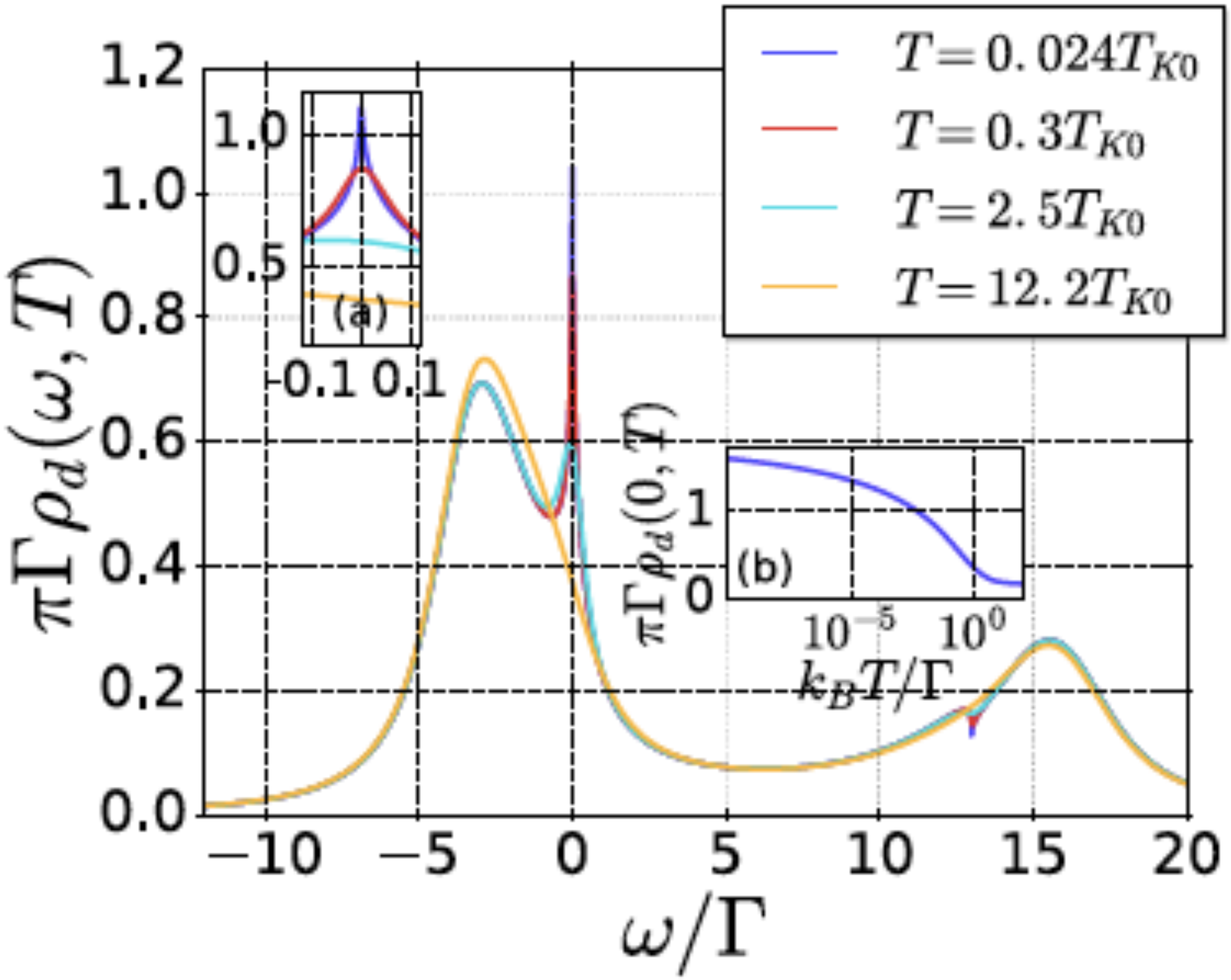


FIG. 8. Finite-U quantum dot spectral density at equilibrium for different background temperatures. Parameters: $\varepsilon_d = -3.5\Gamma$, $D = 100\Gamma$, and $U = 20\Gamma$. (Insets) (a) Detail of the dot spectral density of states around the Fermi energy. (b) Height of the Kondo peak as a function of the background temperature.

Finite U calculation

The Kondo peak height decreases as the background temperature enhances

Here, no dephasing is included

Infinite-U Equation of motion technique

Electrical and thermal driven cases: spectral density

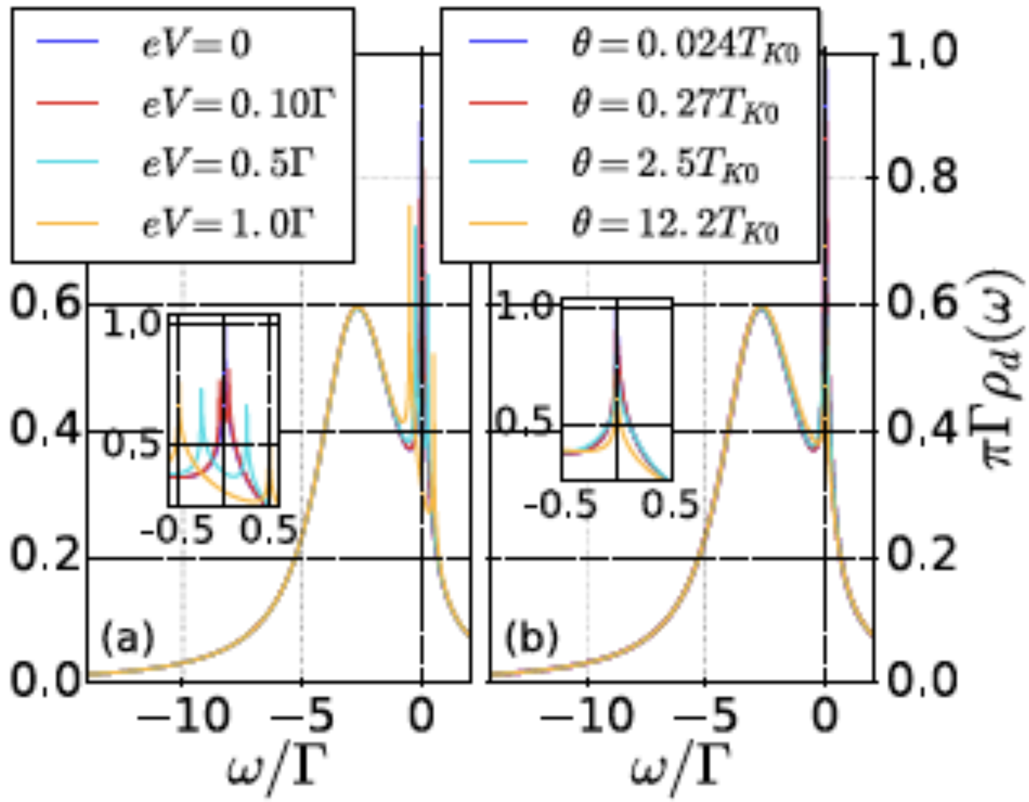


FIG. 7. (a) Nonequilibrium infinite-U quantum dot spectral density of states for different eV values. (Inset) Detail of the density of states around the Fermi energy ($\varepsilon_F = 0$). (b) Nonequilibrium infinite-U quantum dot spectral density of states for different thermal gradients. The background temperature is set at $T = 0.024T_{K0}$. (Inset) Detail of the density of states around the Fermi energy ($\varepsilon_F = 0$). Parameters: $\varepsilon_d = -3.5\Gamma$, $D = 100\Gamma$, and $T = 0.024T_{K0}$.

The Kondo peak splits with the bias voltage

Kondo peak is quenched with the thermal bias

Artifact of the model: the cold reservoirs always yields a Kondo singularity even for large thermal gradients

Finite U electric, and thermoelectrical transport*

Electrical current

Electrically driven

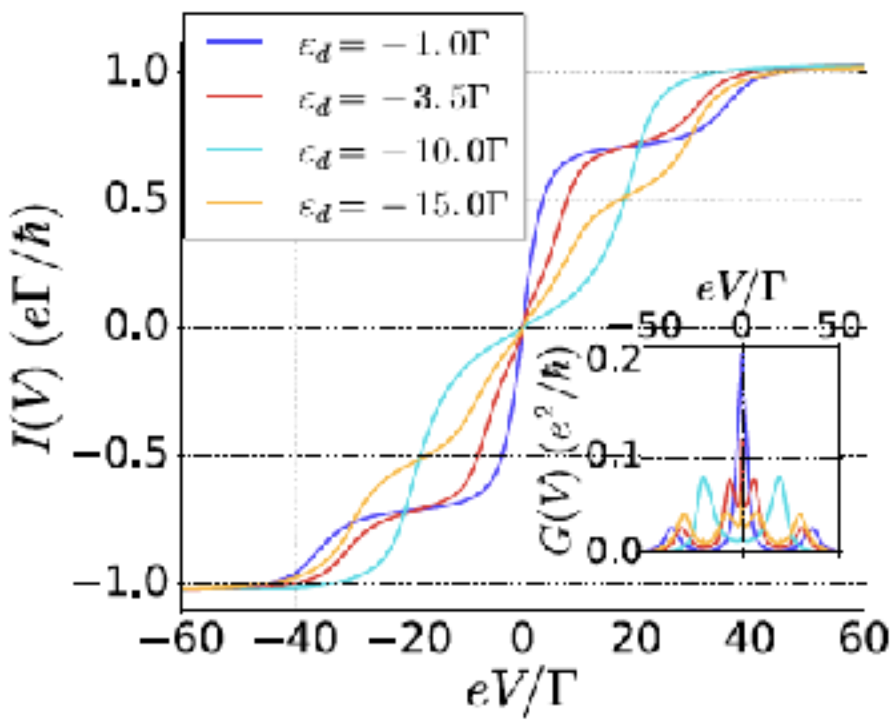


FIG. 9. I-V characteristics at different dot level positions. (Inset) Differential electric condutance vs voltage bias for the given values of the energy level. Parameters: $D = 100\Gamma, k_BT = 0.0001\Gamma, U = 20\Gamma$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Thermally driven

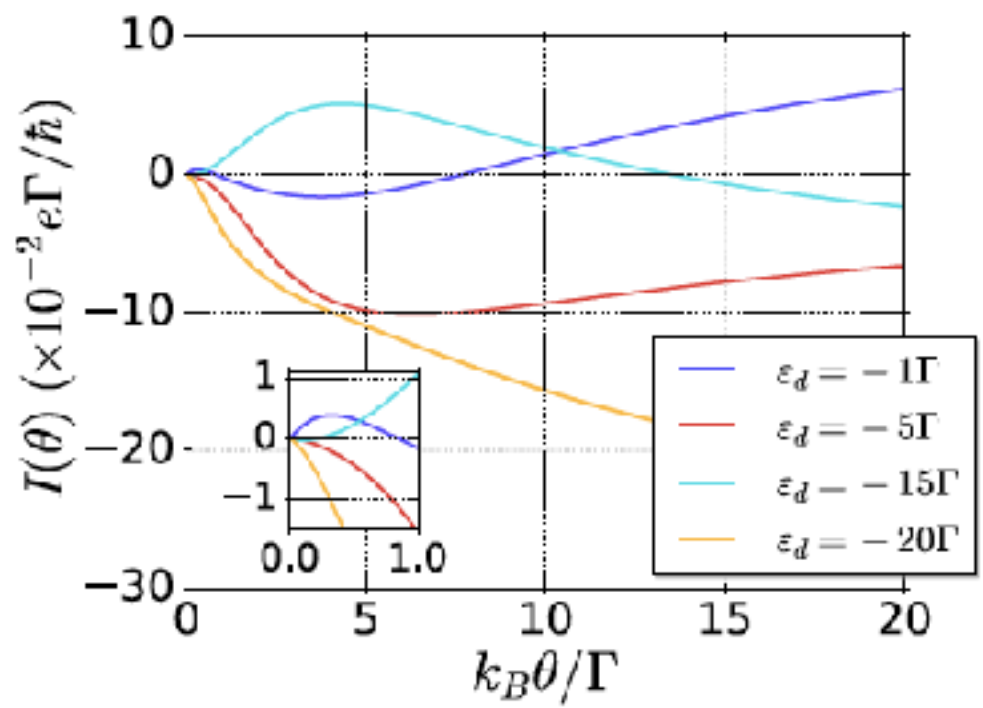
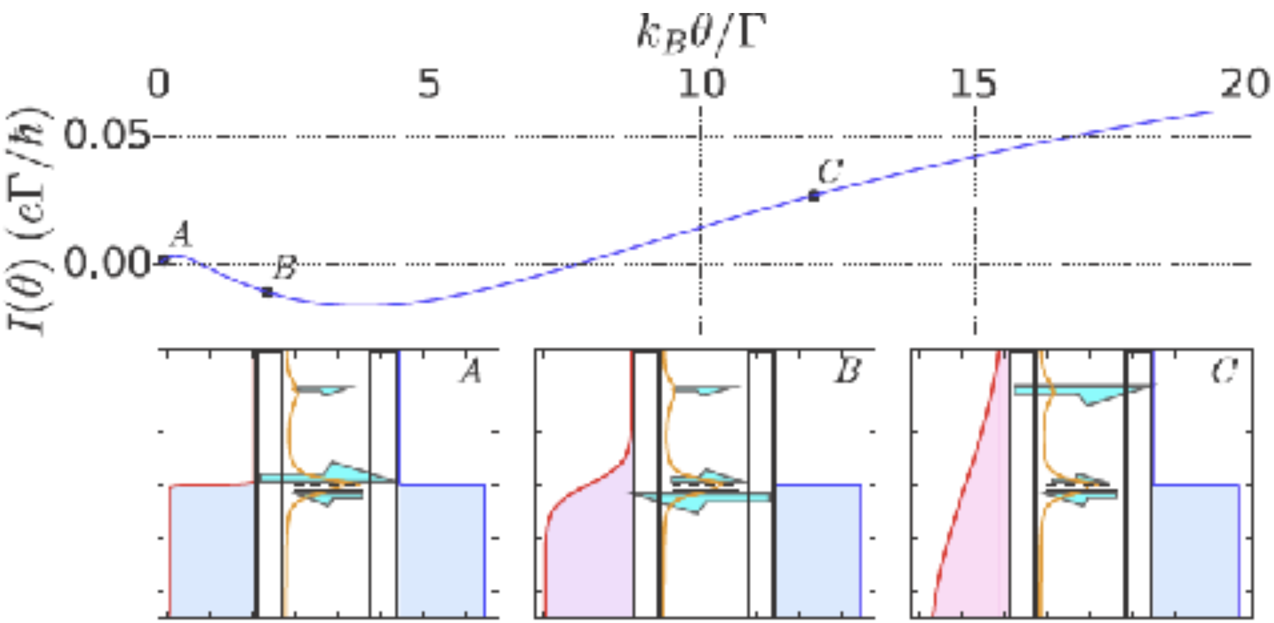


FIG. 10. Thermocurrent vs the thermal gradient for different dot gate positions. (Inset) Detail of the thermocurrent at low thermal gradients. Parameters: $U = 20\Gamma$, $D = 100\Gamma$, $k_BT = 0.01\Gamma$, and $\Gamma_L = \Gamma_R = \Gamma/2$.

Steps at the dot resonances: mean-field peaks and Kondo singularity

Non trivial zero at the spin and charge fluctuation energy scales

Nontrivial zeros at the thermoelectrical current in the finite U case

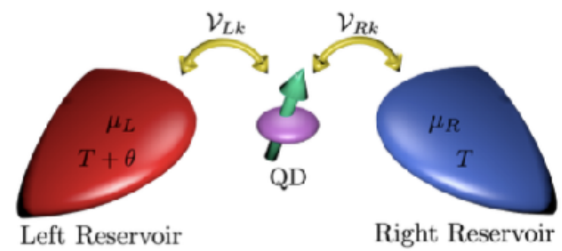


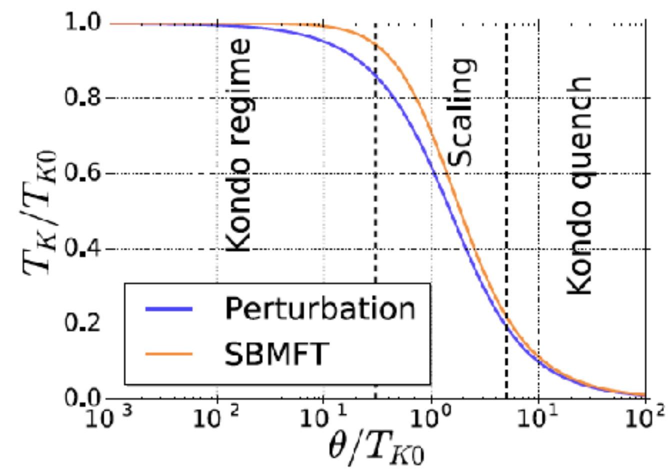
A e-like dominant Kondo peak
B h-like dominant Kondo peak and e-like contribution from the mean field peak
C e-like dominant from the second mean field peak, the others are h-like

The different electron-like and hole-like contributions of the Kondo and mean-field peaks leads to the appearance of the nontrivial zeros in the thermoelectrical current when the thermal bias increases

System

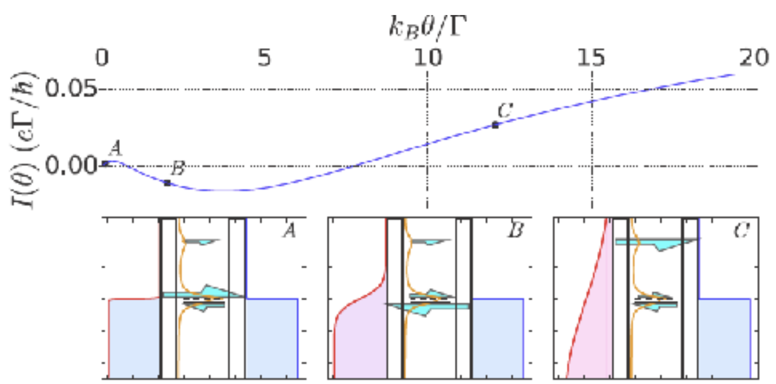
- A single quantum dot is considered in the Kondo regime.
- The Kondo effect is quenched with the thermal bias: results from perturbation theory, SBMFT, and EOM



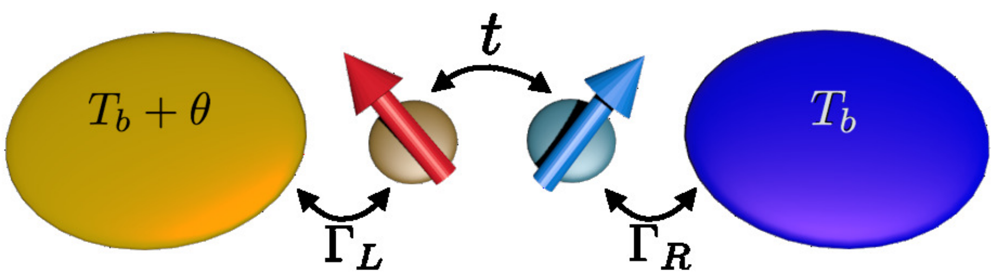


Transport

- Thermocurrent
  - Nonlinear behavior
  - Appearance of nontrivial zeros



Double Quantum Dot case

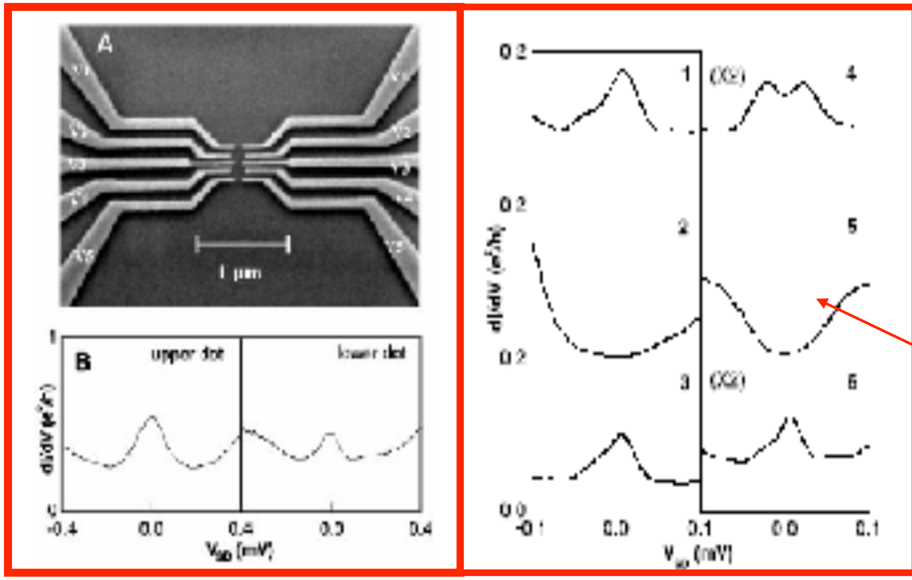


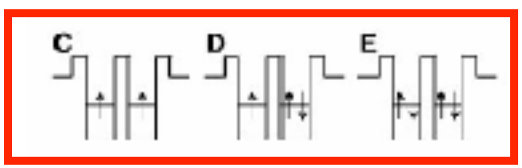
The artificial two-impurity Kondo problem *

The Kondo Effect in an Artificial Quantum Dot Molecule

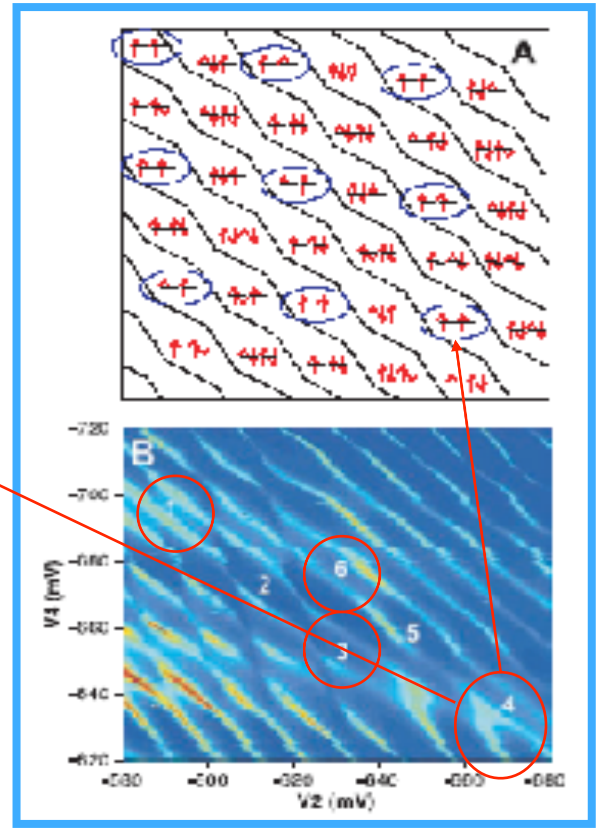
H. Jeong, A. M. Chang and M. R. Melloch

Science **293** (5538), 2221-2223.
DOI: 10.1126/science.1063182





Δ=45μeV



Kondo

1
3
4
6

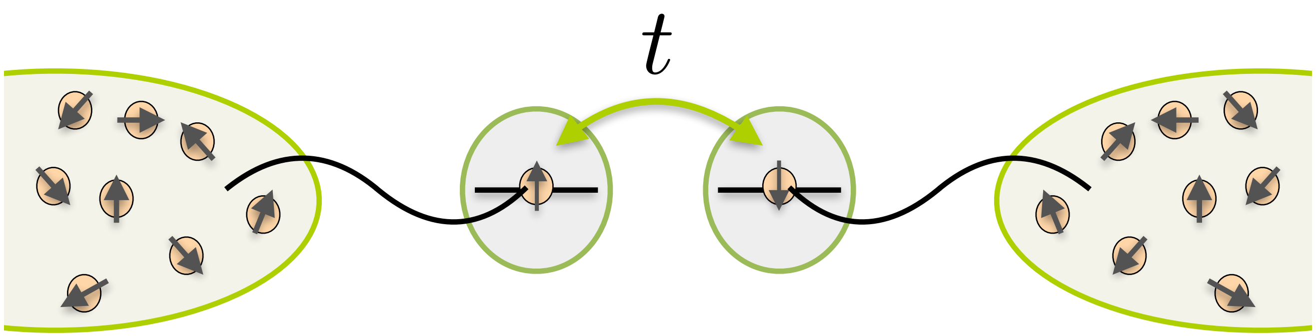
Hybridization of Kondo singlets into a coherent superposition of Kondo states

Bonding-Antibonding Kondo states!

R. Aguado, D. C. Langreth, *Phys. Rev. Lett.* **85**, 1946 (2000).

R. Aguado, D. C. Langreth, Phys. Rev. B 67, 245307 (2003)

Two-Impurity Kondo System



Two magnetic impurities in the Kondo regime

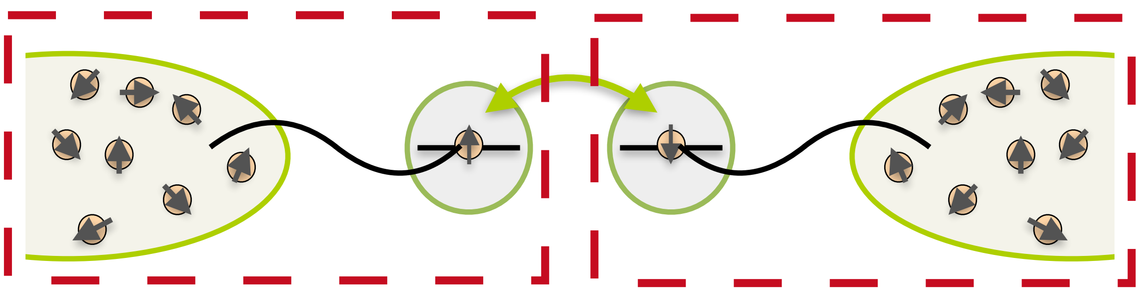
Crossover at $J = J_c$

$$J = \frac{4t^2}{U}$$

Superexchange

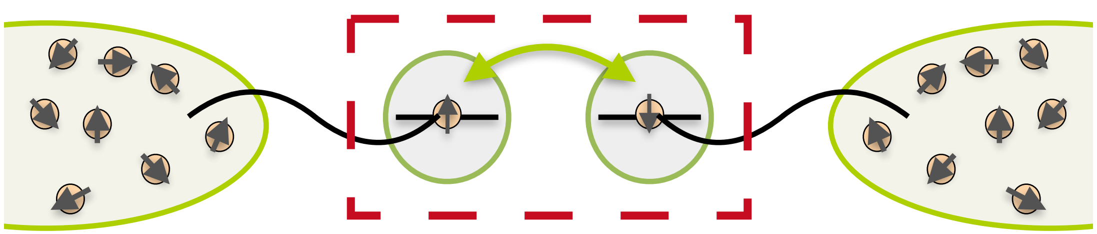
t → *Interdot tunneling amplitude*

U → *Intradot Coulomb interaction*



Two Kondo Impurities

$$J < J_c$$

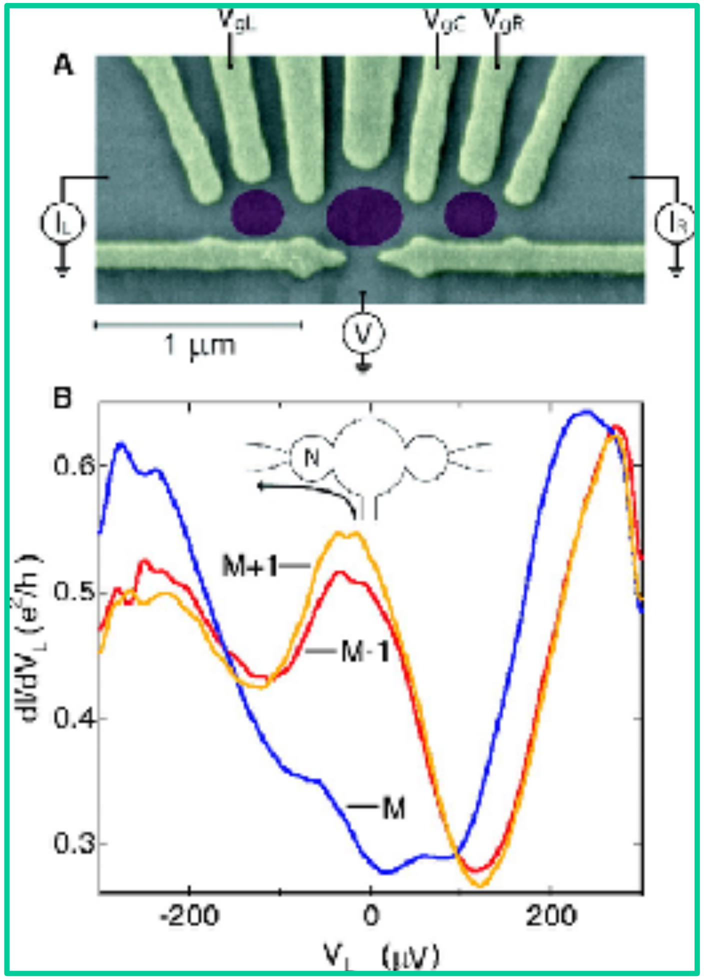


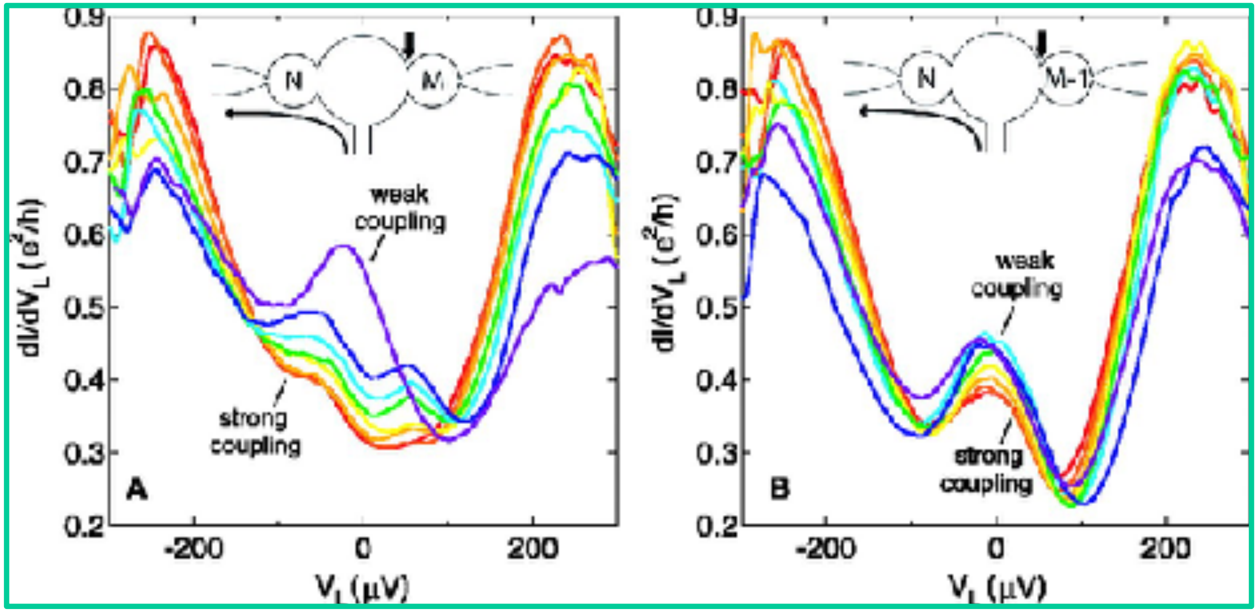
Spin-singlet between dots

$$J > J_c$$

Tunable Nonlocal Spin Control in a Coupled-Quantum Dot System

N. J. Craig, et al., Science 304, 565 (2004)





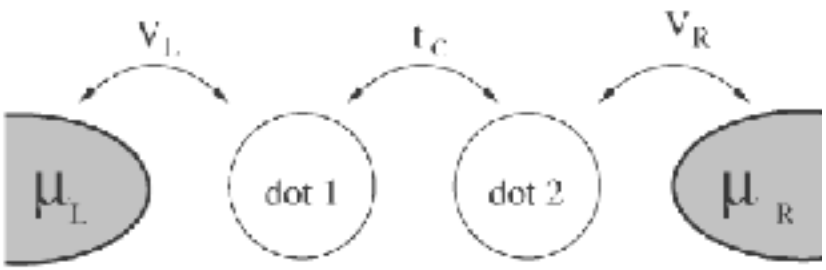
Coherent control of quantum dot spins by a nonlocal RKKY-like interaction

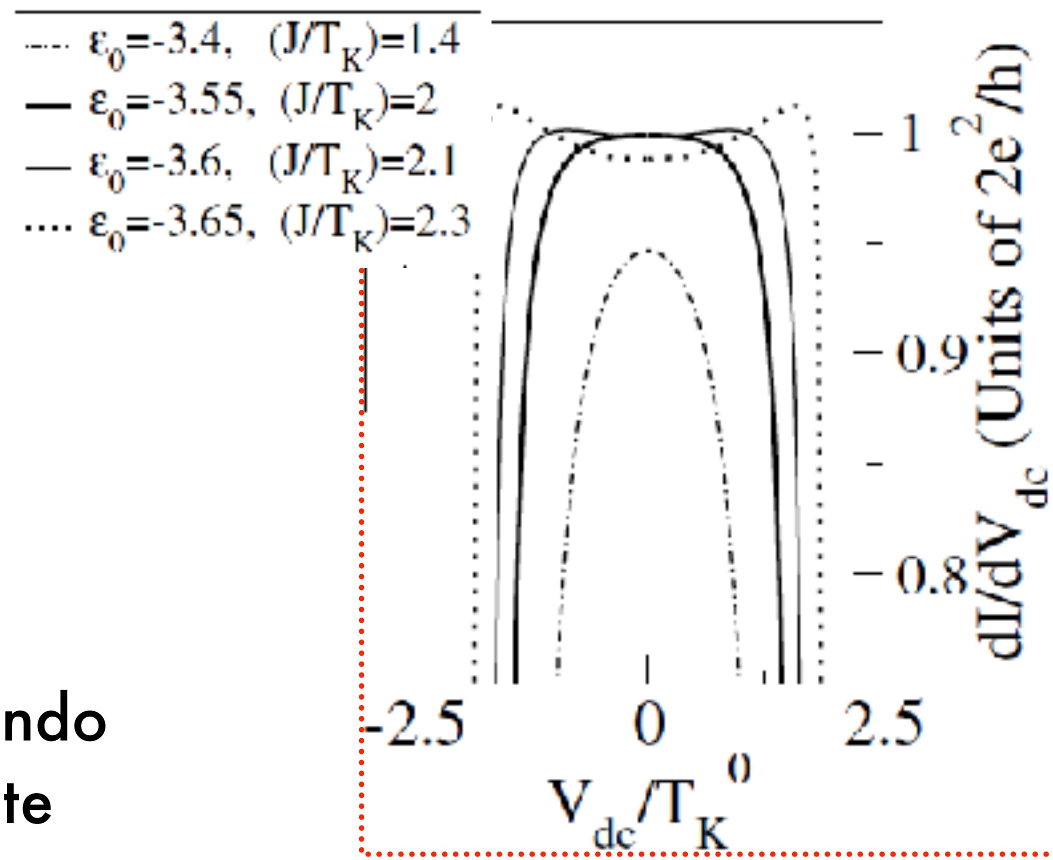
$$\mathcal{H}_J = J\hat{S}_L \cdot \hat{S}_R$$

R. López et al., Physical review letters 89, 136802 (2003)

Electrically driven

$$\mathcal{H}_J = J\hat{S}_L \cdot \hat{S}_R$$

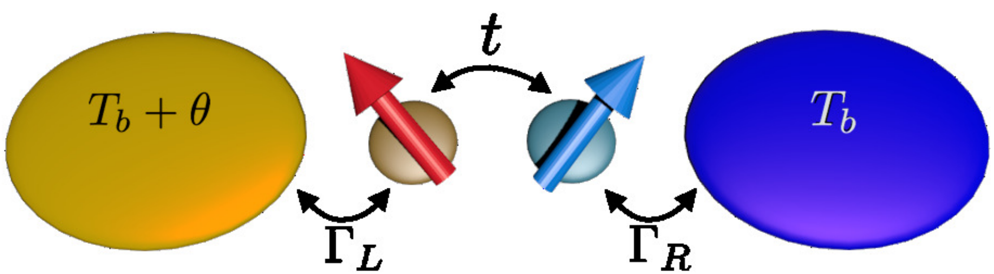




Smooth transition from the Kondo state to the AF spin singlet state around $(J/T_K) = 2.5$

Serial QDs: Crossover from the Kondo phase to the antiferromagnetic phase

Hamiltonian for the serial double quantum dot



We consider the slave-boson $U \to \infty$ Anderson Hamiltonian

Lead Hamiltonian

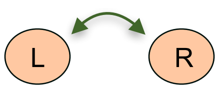
$$\mathcal{H}_C = \sum_{\alpha k \sigma} \varepsilon_{\alpha k} C^{\dagger}_{\alpha k \sigma} C_{\alpha k \sigma}$$

$\alpha = \{L, R\}$ Dots and leads

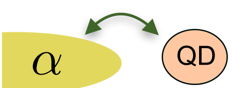
$\}$ ε_α Dot Energy level

Quantum dots Hamiltonian

$$\mathcal{H}_d = \sum_{\alpha\sigma} \varepsilon_\alpha f^{\dagger}_{\alpha\sigma} f_{\alpha\sigma} + \sum_{\sigma} t(f^{\dagger}_{L\sigma} f_{R\sigma} + \mathrm{H.c.})$$



t Dot-dot Tunnel amplitude



$\mathcal{V}_{\alpha k}$ Lead-dot Tunnel amplitude

Tunneling Hamiltonian

$$\mathcal{H}_t = \sum_{\alpha k \sigma} \mathcal{V}_{\alpha k \sigma} C^{\dagger}_{\alpha k \sigma} b^{\dagger}_{\alpha} f_{\alpha\sigma} + \mathrm{H.c.}$$

λ_α Lagrange Multiplier

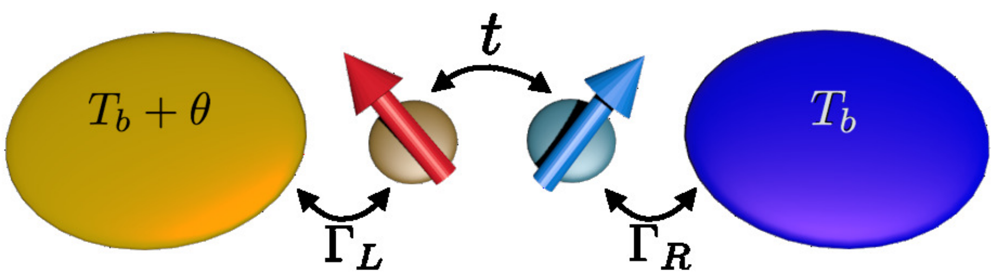
$b^{\dagger}_{\alpha}$ Creates a vacuum state

Lagrange Hamiltonian

$$\mathcal{H}_l = \sum_{\alpha} \lambda_\alpha \left(b^{\dagger}_{\alpha} b_\alpha + \sum_{\sigma} f^{\dagger}_{\alpha\sigma} f_{\alpha\sigma} - 1 \right)$$

Valid in the **Fermi Liquid Regime**

Mean-field Equations



We assume the *slave-boson mean-field theory*

$$b_\alpha \to \tilde{b}_\alpha \equiv \langle b_\alpha \rangle$$

Two conditions need to be satisfied

Boson equation of motion

$$\partial_t b_\alpha = 0$$

$$\sum_\sigma G^<_{f\alpha\sigma,f\alpha\sigma}(t,t) = \frac{i}{\hbar}\left(1 - N|\tilde{b}_\alpha|^2\right)$$

Lagrange condition

$$b^\dagger_\alpha b_\alpha + \sum_\sigma f^\dagger_{\alpha\sigma} f_{\alpha\sigma} = 1$$

$$\sum_{k\sigma}[g^r_{\alpha k}]^{-1} G^<_{f\alpha\sigma,f\alpha\sigma}(t,t) = -\frac{i}{\hbar}\lambda_\alpha N|\tilde{b}_\alpha|^2$$

$G^<_{f\alpha\sigma,f\alpha\sigma}(t,t)$
Lesser Green's function

N
Spin degeneracy

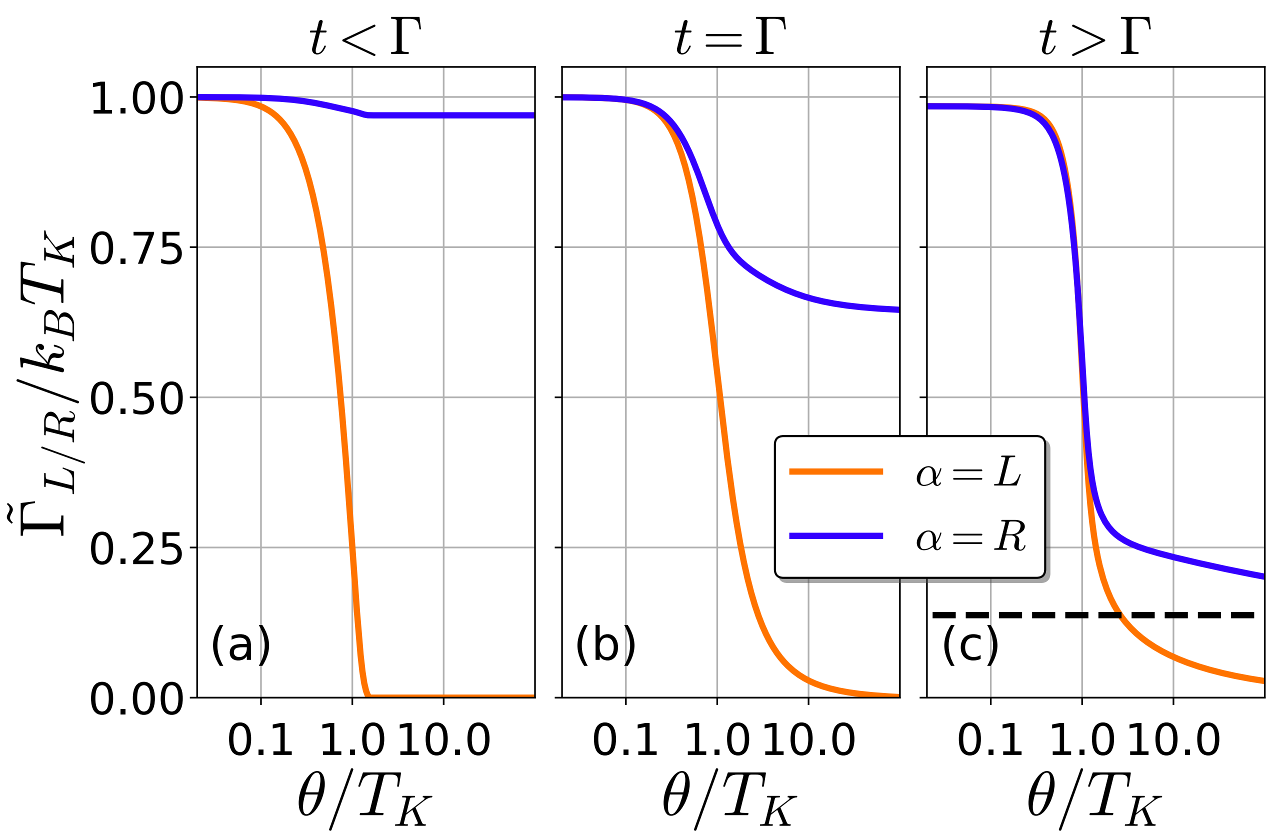
Two renormalized parameters

$\tilde{\Gamma}_\alpha = \Gamma_\alpha|\tilde{b}_\alpha|^2$ Kondo resonance width

$\Gamma_\alpha = \pi\rho_\alpha|\mathcal{V}_\alpha|^2$

$\tilde{\varepsilon}_\alpha = \varepsilon_\alpha + \lambda_\alpha$ Kondo resonance position

Temperature driven case for the Kondo temperature



$$T_{K\alpha} \equiv \tilde{\Gamma}_\alpha(\theta)$$

Renormalized width determines the role of the Kondo effect

Three different regimes depending on

t and $\Gamma = \Gamma_L + \Gamma_R$

Weakly coupling

$t < \Gamma$

$\tilde{\Gamma}_L$ Rapid decrease

$\tilde{\Gamma}_R$ Almost unaffected

Intermediate

$t \approx \Gamma$

Both $\tilde{\Gamma}_\alpha$ affected

Strong coupling

$t > \Gamma$

$\tilde{\Gamma}_\alpha$ behave similarly

Bonding and anti-bonding states

Heat Current

$$Q = \frac{1}{h}\int d\omega[f_L(\omega) - f_R(\omega)](\omega - \mu_\alpha)\mathcal{T}(\omega)$$

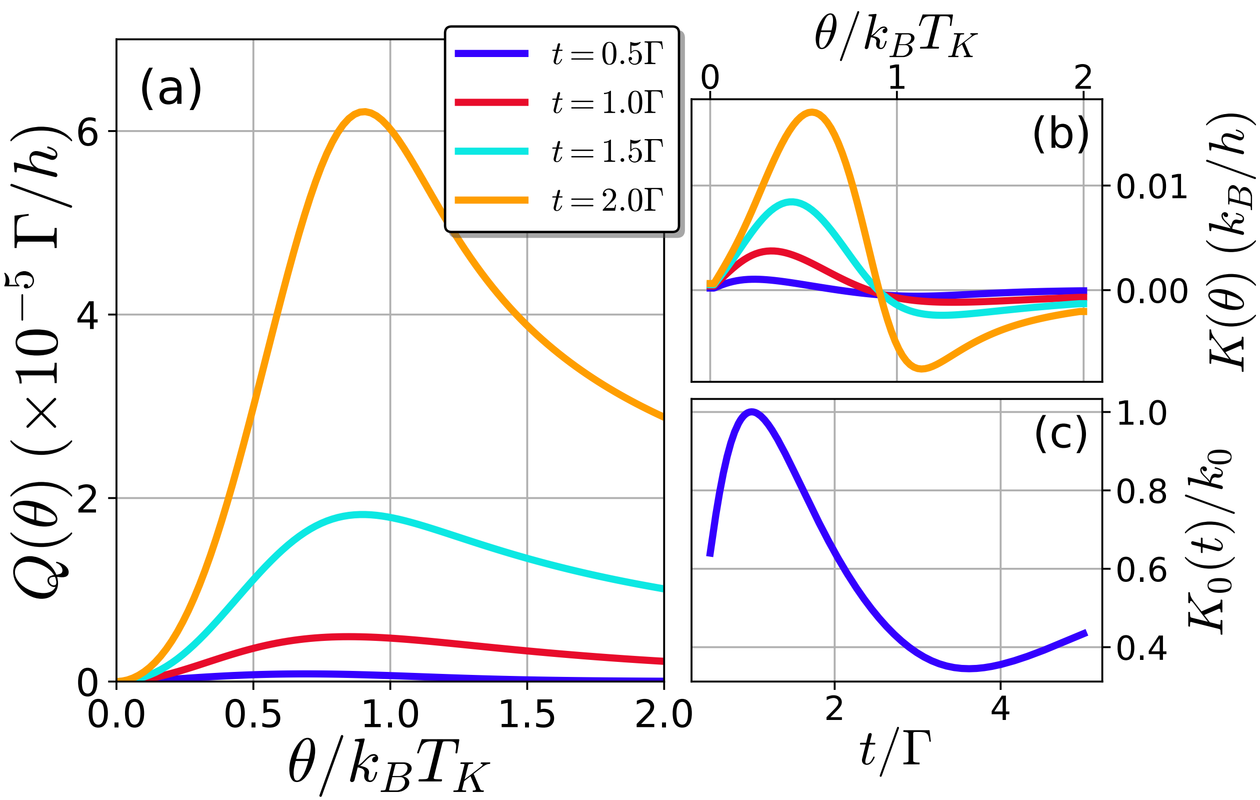
Transmission function

$$\mathcal{T}(\omega) = 4\sum_\sigma \tilde{\Gamma}_L\tilde{\Gamma}_R\left|G^r_{fL\sigma,fR\sigma}\right|^2$$

Fermi function

$$f_\alpha(\omega) = \frac{1}{1+\exp\frac{\omega-\mu_\alpha}{k_BT_\alpha}}$$

$$\mu_\alpha = \varepsilon_F$$
$$\theta = T_L - T_R$$



- Heat current more intense in the **strong coupling regime**
- It presents a maximum and vanishes at large θ.
- **Thermal diode** behavior.
- It obeys Fourier's law.

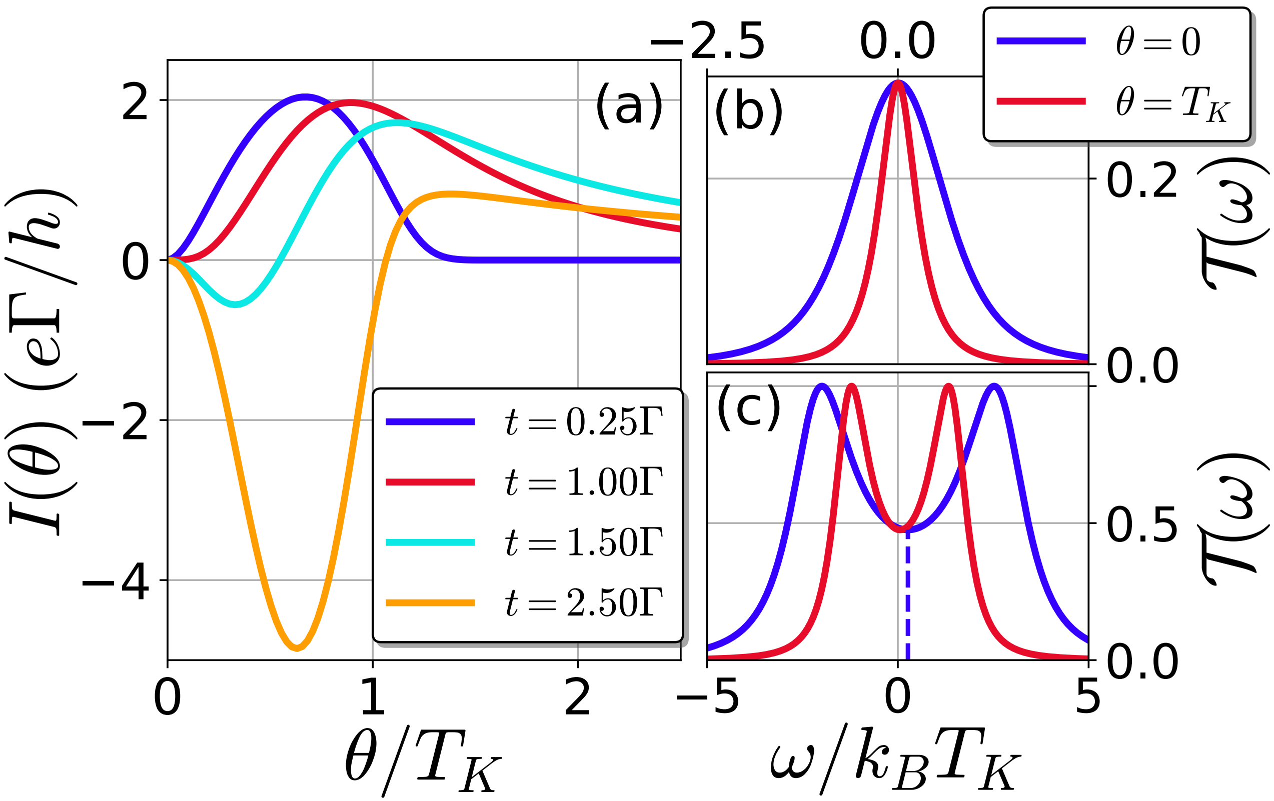
$$K_0 \approx \mathcal{T}(\varepsilon_F)$$
$$Q \approx K_0\theta$$

$$I = \frac{e}{h}\int d\omega [f_L(\omega) - f_R(\omega)]\mathcal{T}(\omega)$$

Transmission function

$$\mathcal{T}(\omega) = 4\sum_{\sigma}\tilde{\Gamma}_L\tilde{\Gamma}_R \left|G^r_{fL\sigma,fR\sigma}\right|^2$$

$$\mu_\alpha = \varepsilon_F$$
$$\theta = T_L - T_R$$



- Nonlinear behavior of the current
- Thermoelectric response depends highly on the ratio

$$t/\Gamma$$

- Nontrivial zeros due to asymmetric transmissions
- Vanishing current at large thermal biases

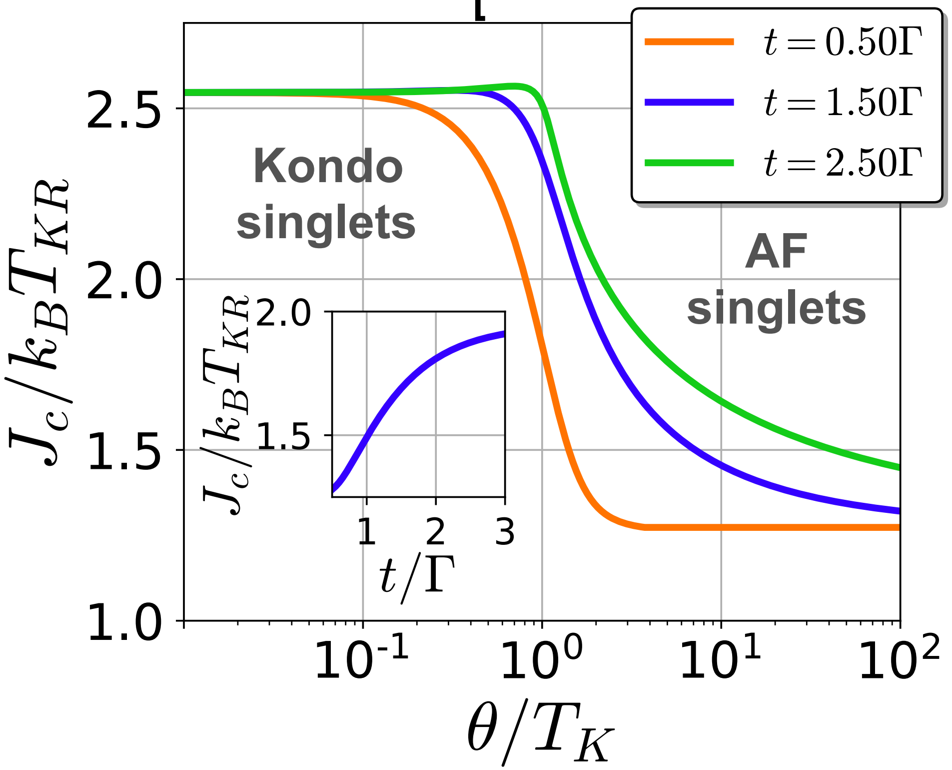
Antiferromagnetic Kondo crossover

Case of relevant antiferromagnetic interactions. $\mathcal{H}_J = J\hat{S}_L \cdot \hat{S}_R$

A crossover can occur between the Kondo singlet and the dot-dot AF singlet.

Critical dependent on Kondo Temperatures

$$\frac{J_c}{k_B T_{KR}} = \frac{4}{\pi}\left(1 + \frac{T_{KL}}{T_{KR}}\right)$$



- A transition of the critical value occurs:

$$J_c = \frac{8}{\pi} k_B T_{KR} \rightarrow J_c = \frac{4}{\pi} k_B T_{KR}$$

- Kondo correlations are reinforced at large tunneling amplitudes.

P. Simon, R. López, and Y. Oreg, Phys. Rev. Lett. 94, 086602 (2005)

Conclusions

System

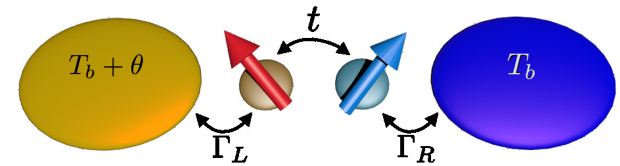
- A serially-coupled double quantum dot is considered in the Kondo regime.
- Depending on t/Γ, one can identify different regimes:
  - Weakly coupled regime
  - Intermediate coupled regime

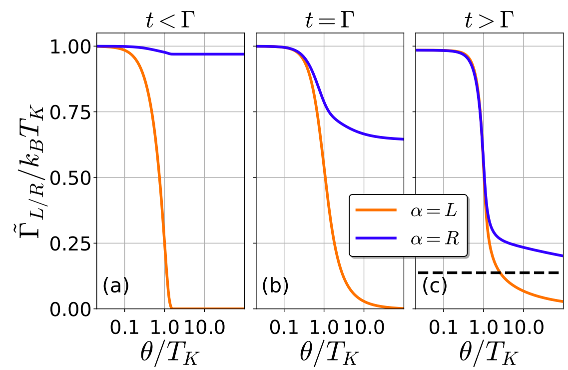
Transport

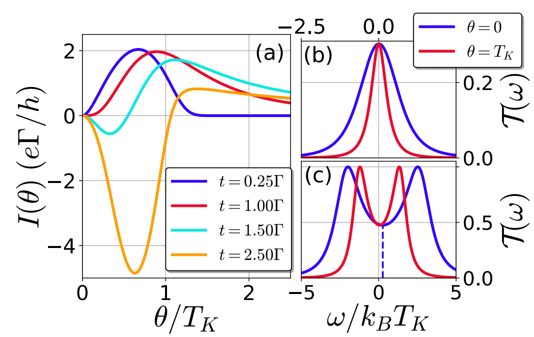
- Heat current
  - Vanishing at large θ .
  - Thermal diode behavior.
- Thermocurrent
  - Nonlinear behavior.

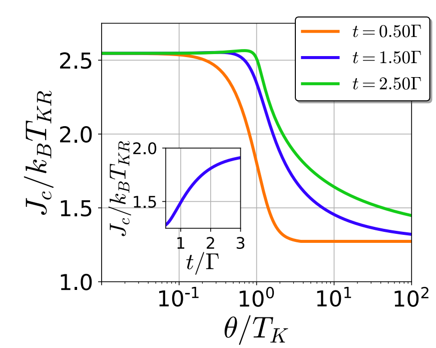
Critical value

- Tunneling modifies the critical value of the crossover.









Phys. Rev. Lett. 121, 096801

THANK YOU

for your attention