Mesoscale three-dimensional Lagrangian Coherent Structures

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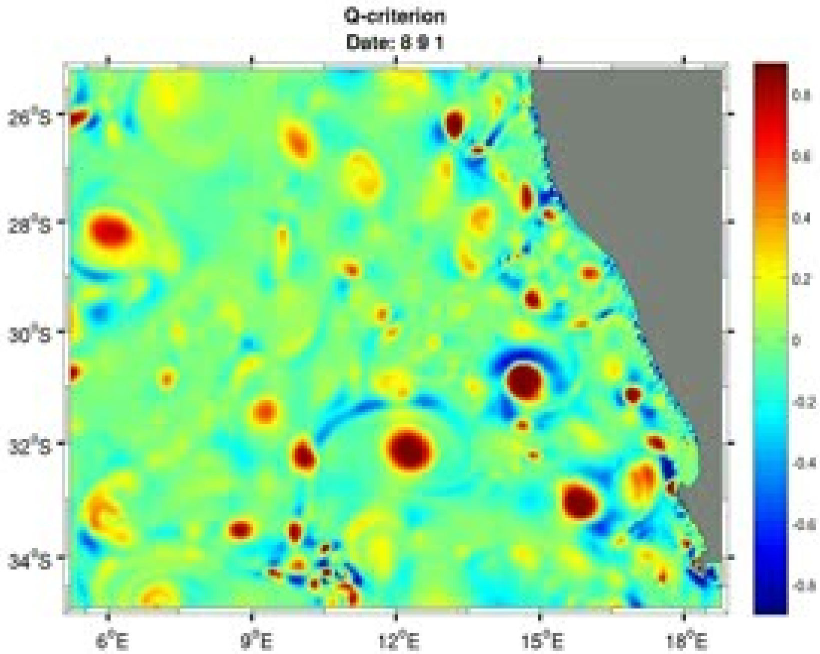
“Features of the flow field that last long enough to be observed...”

e.g. for an eddy with size D and velocity scale U we could say a lifetime t >> D/U

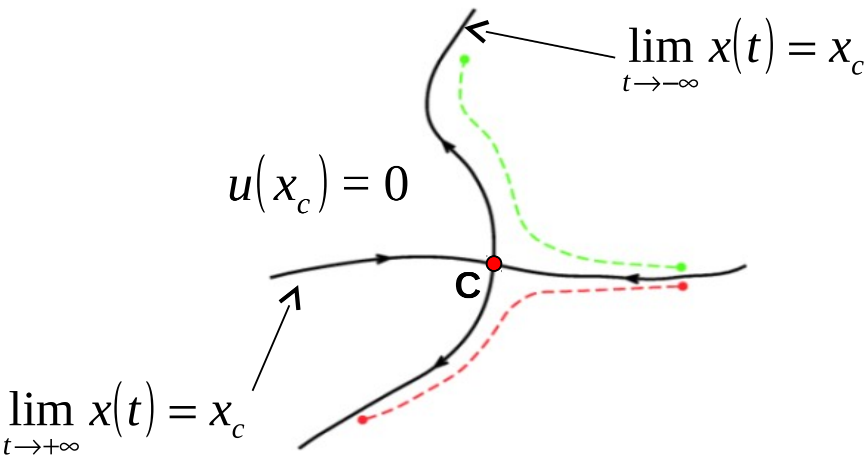
Identity could be defined by characteristic values of vorticity, ssh anomaly, etc.



Q-criterion (Hunt et al, 1988): $$Q=\frac{1}{2}\left(\|\boldsymbol{\Omega}\|^2-\|\boldsymbol{S}\|^2\right)$$

- In the Eulerian perspective we look at spatial distributions of flow quantities to look for coherence.
- But, is this relevant for fluid transport?
- The Lagrangian perspective looks at material properties of the fluid flow, such as transport or deformation of tracer patches.
    - More natural way of describing transport and mixing.
    - Can be studied using tools from the theory of dynamical systems.

Stable/unstable manifolds of hyperbolic trajectories govern the behaviour of solutions in their vicinity



Separatrices partition the phase space in dynamically distinct zones

The classical Lyapunov exponent gives the rate at which two initial conditions, infinitesimally separated, grow in the infinite time limit:

$$\lambda_C = \lim_{t\to\infty} \lim_{d_0\to 0} \frac{1}{t} \log \frac{|d(t)|}{|d_0|}$$

• If positive, the system is said to be chaotic.

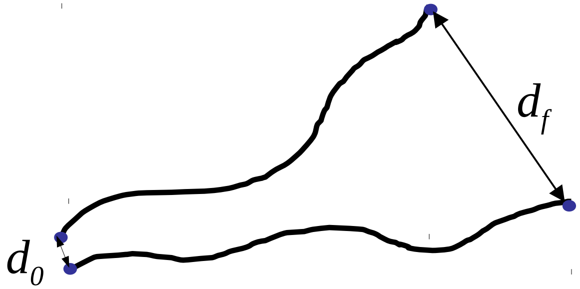
•In practice difficult to calculate for “real world” systems because of asymptotic character.

Non asymptotic measures derived from the classical Lyapunov exponent:

- Finite-time Lyapunov exponent (FTLE): measures the rate of growth of a infinitesimal separation in a finite time interval

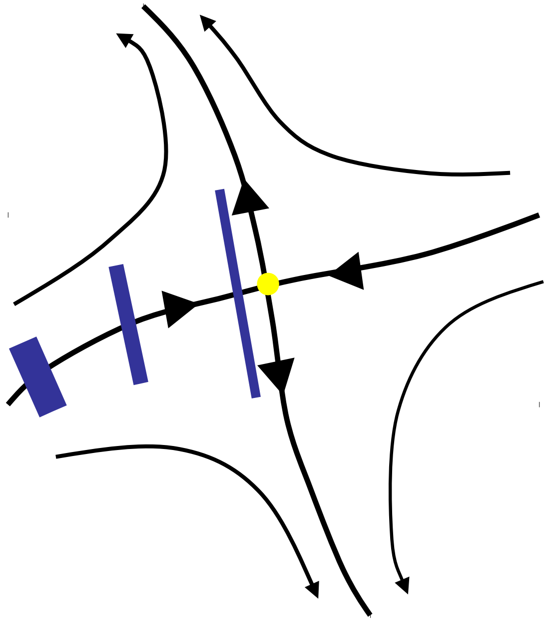
- Finite-size Lyapunov exponent (FSLE[1]): measures rate of growth of a finite separation between two limits:

$$\lambda = \frac{1}{\tau} \log\left(\frac{d_f}{d_0}\right)$$



[1]Aurell et al., Phys. Rev. Lett. **77, 1262 (1996)**
Boffetta et al., J. of Phys. A, **30, 1 (1997)**

The FSLE can be used to detect stable and unstable manifolds of “hyperbolic points” because higher rates of particle separation near these structures will result in larger values of the FSLE.



Algorithm (d'Ovidio et al, 2004):

- Define initial and final distances d0 and df
- Setup a regular grid of initial conditions with grid spacing d0
- Release particles from initial conditions and track trajectories
- Calculate time t until interparticle separation reaches df.
- Compute 1/t*log(df/d0) at each initial location

- Requires trajectory information only.

- Robust, does well with noisy data (Hernandez-Carrasco et al,2012).

Other tools include:

FTLE (Lapeyre,Huhn,Prants,Özgökmen,Kirwan,...)
Distinguished Hyperbolic Trajectories (Wiggins,Mancho,Branicki)
M-function(Mancho, Mendoza, de la Camara)
...

FSLE computation in 3D:

• The 2D algorithm can be extended to 3D directly, but...

- Vertical velocities in the ocean are generally much smaller than horizontal velocities, usually by two orders of magnitude.
- Thus, while in the horizontal particles separate hundreds of kilometers, in the same period, they only separate a hundred of meters in the vertical.

... so we use a *quasi-3d* scheme: we calculate FSLEs layer to layer, computing the time between two horizontal distances, but particle dynamics is 3d.

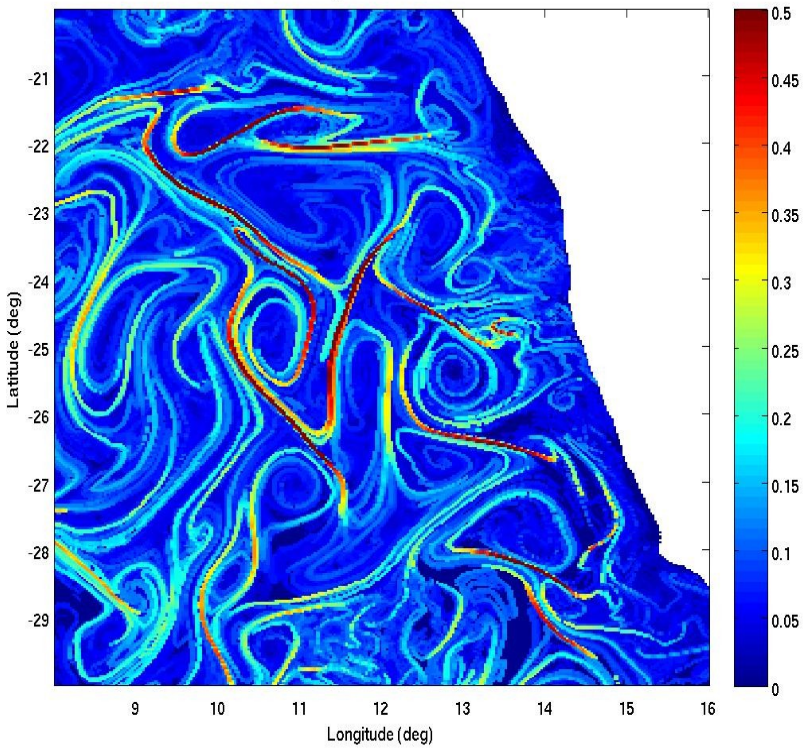
Lagrangian coherent structures can be identified with the maxima of the FSLE fields (Haller and Yuan, 2000; Joseph and Legras, 2002).

FSLE fields

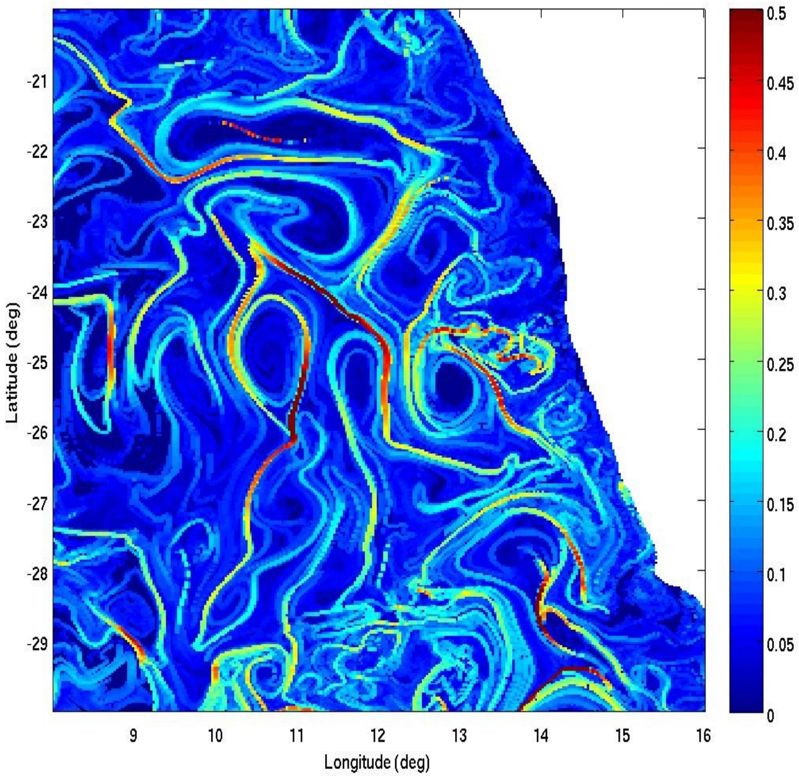
•Velocity field from ROMS simulation of flow off the coast of Benguela (LeVu et al,2011), resolution 1/12º

• δ_0=1/36º (~3km), δ_f=100 km, Particles tracked for six months max.

Unstable manifolds integrated backward



Stable Manifolds integrated forward



• High FSLE concentrate on thin structures

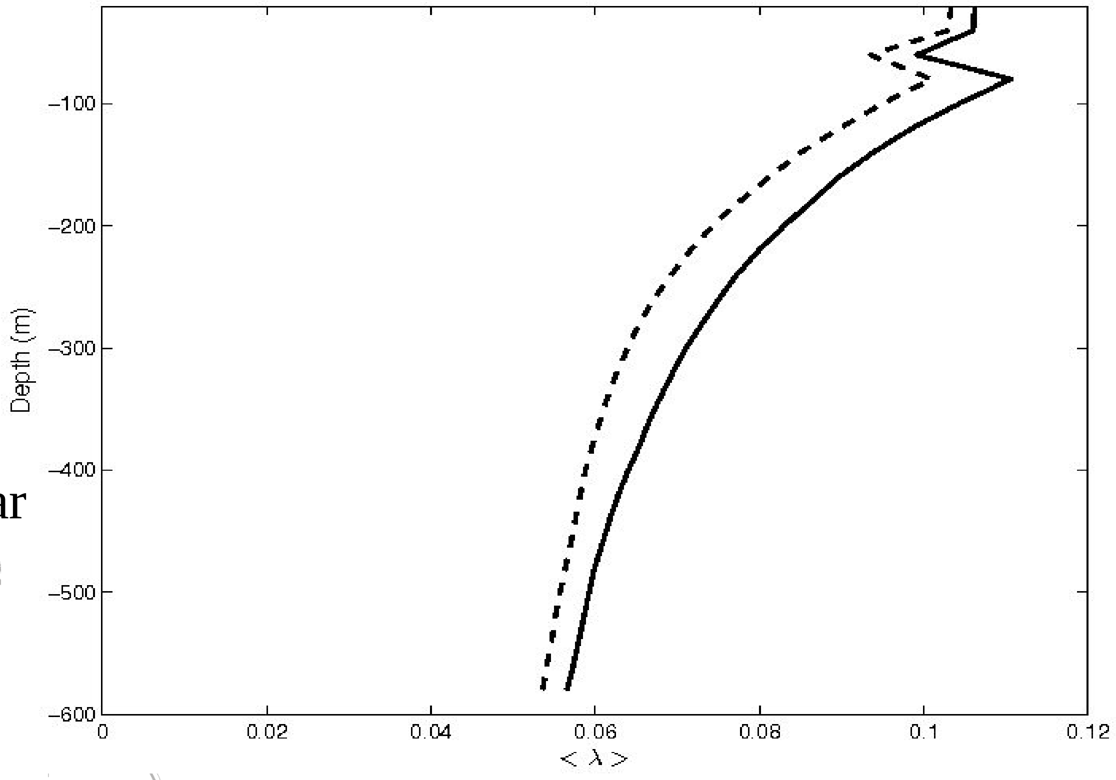
• Proliferation of high FSLE lines indicate chaotic behaviour of particle trajectories and intense non-local mixing

Averaged vertical profiles of backward/forward FSLE over 1 month (September)

- FSLE values decreased with depth

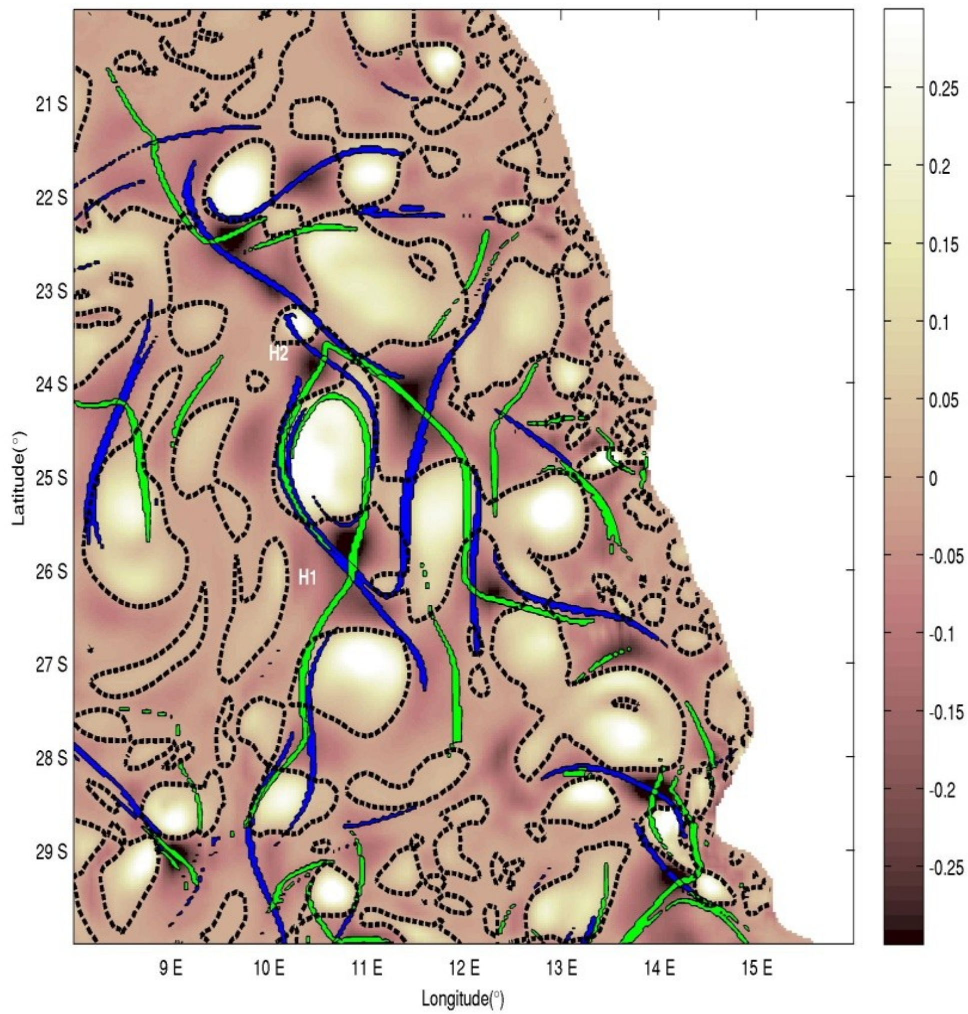
- A maximum is found around 100 m depth

- Forward FSLE average similar to backward FSLE average.



- In the Eulerian perspective, a mesoscale eddy is revealed by a patch of positive Q-criterion.

- Lagrangian methods reveal the eddy as the space enclosed by tangencies and intersections of stable/unstable manifolds of two hyperbolic points H1 and H2



Q-criterion map and high FSLE patches (>60% of max);
blue – unstable; green - stable

In three dimensions, Lagranian coherent structures are hidden in the volume data, so they must be extracted using the concept of ridge, a region that maximizes the scalar field in the normal direction to it.

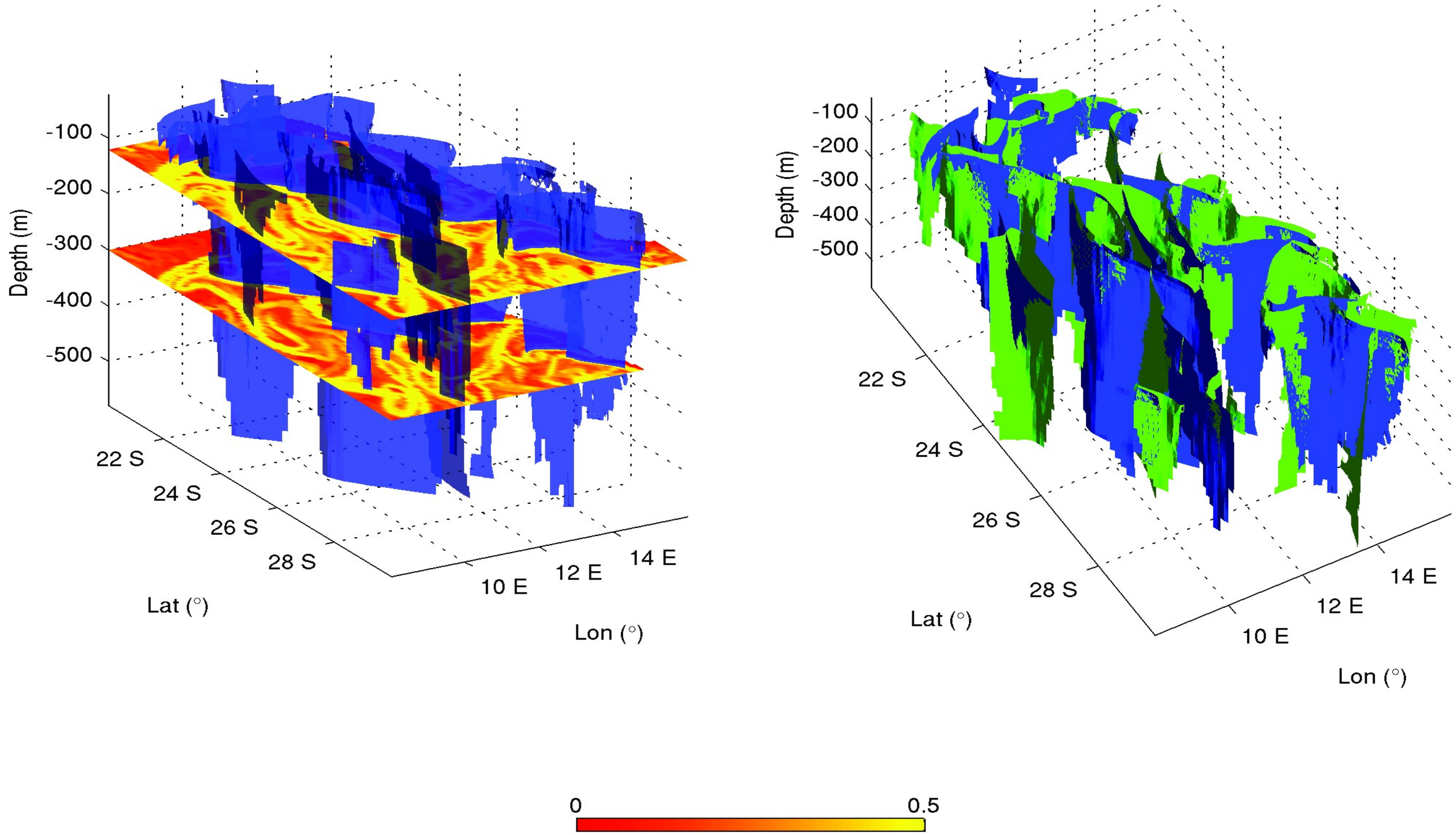
Conditions for a ridge point P of a scalar field *f* in *n*-dimensional space (Lekien, 2005):

- The field *f* attains a local extremum at P
- The direction normal to the ridge at P is the direction of fastest descent of the field *f*.

With $g_i=\partial_i f$ and $h_{ij}=\partial^2_{ij} f$ the hessian matrix of *f* with eigenvalues α^k ($\alpha_1>...>\alpha_n$) and eigenvectors e^k, a *d*-dimensional ridge point is given by (Eberly et al, 1994):

$$\forall_{d<k\leqslant n} \; g_i e_i^k=0 \wedge \alpha^k<0$$

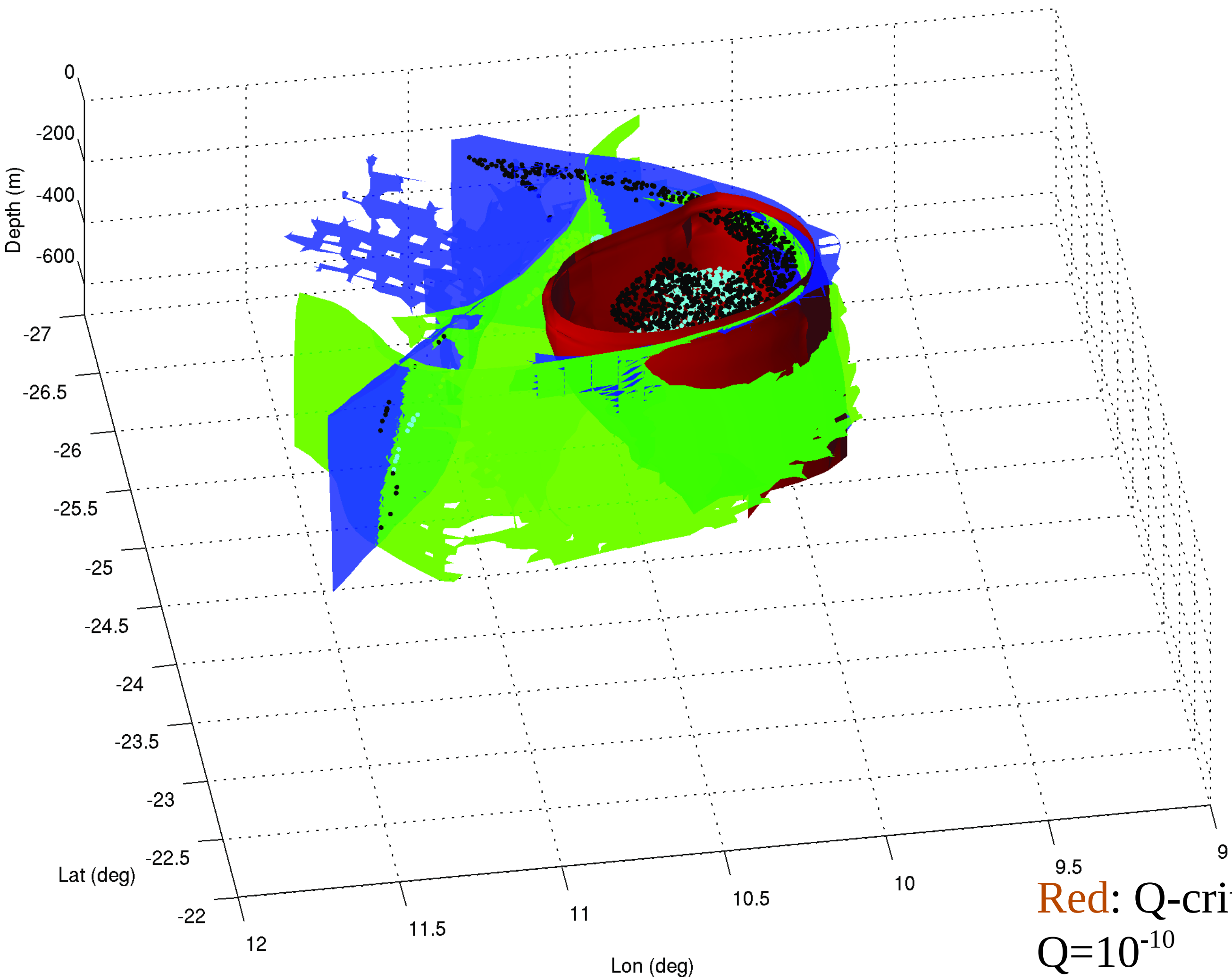
The extraction method (Schultz et al,2008) marches through the grid checking the ridge condition. Ridge points with α^3 lower than a predefined threshold are excluded. There is no systematic way of finding an “optimum” threshold.



•Quasi-vertical shape results from predominance of horizontal dispersion (Prof. Kirwan's talk this morning)

•The ridges fall on the FSLE maxima in 2D plots (Branicki et al, 2010)

•The ridge strength varies with depth.



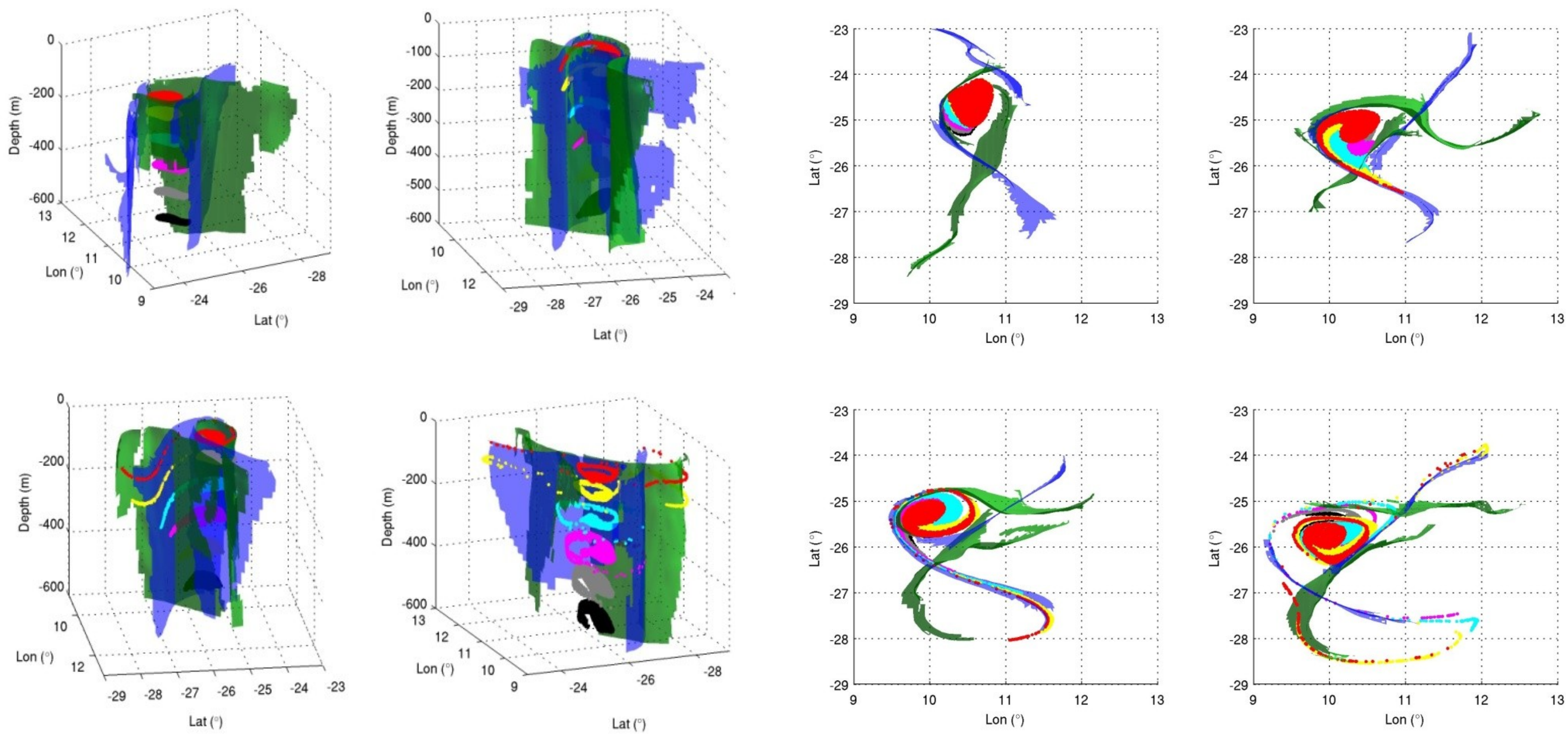
Red: Q-criterion isosurface at $Q=10^{-10}$

Green: Repelling LCS

Blue: Attracting LCS

Mesoscale eddy transport

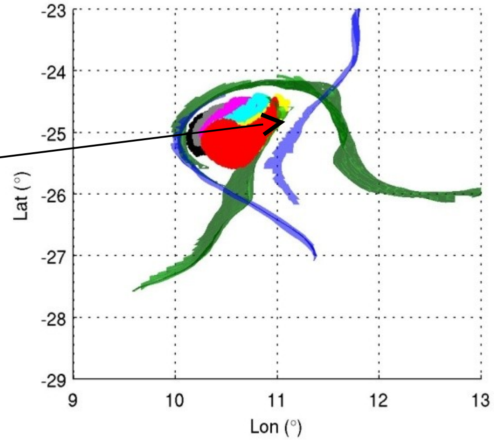
Mesoscale eddie are known to be able to carry differentiated waters for long periods of time (e.g.,LeHan et al 2011).
Exchange with the exterior can occur with eddy interactions, mergers or filamentation.

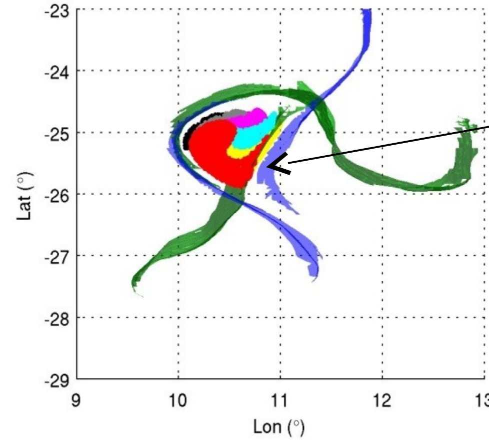


days 3, 13, 19 and 29.

Lagrangian structures surrounding the eddy prevent exchange of material with exterior waters, but filamentation occurs along unstable manifolds.

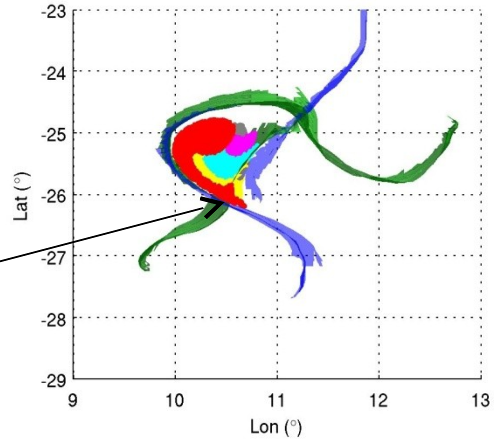
Day 7: a “gate” is open between the stable and unstable manifolds of 2 hyperbolic points.

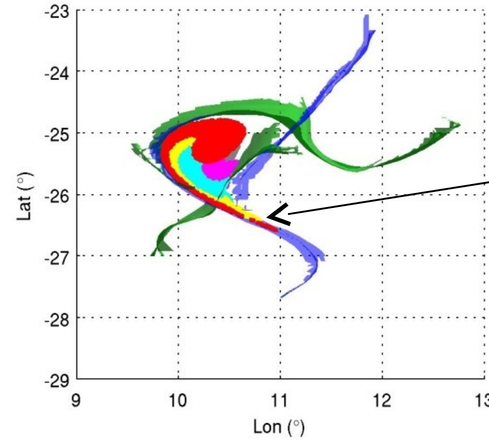




Day 9: some material has crossed the gate and flows along the stable manifold.

Day 11: Material that flowed along the stable manif. Inside and out of the eddy reaches the hyperbolic point….



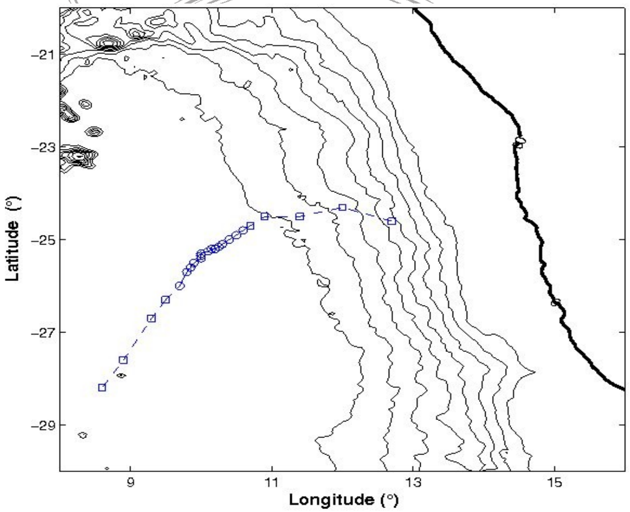


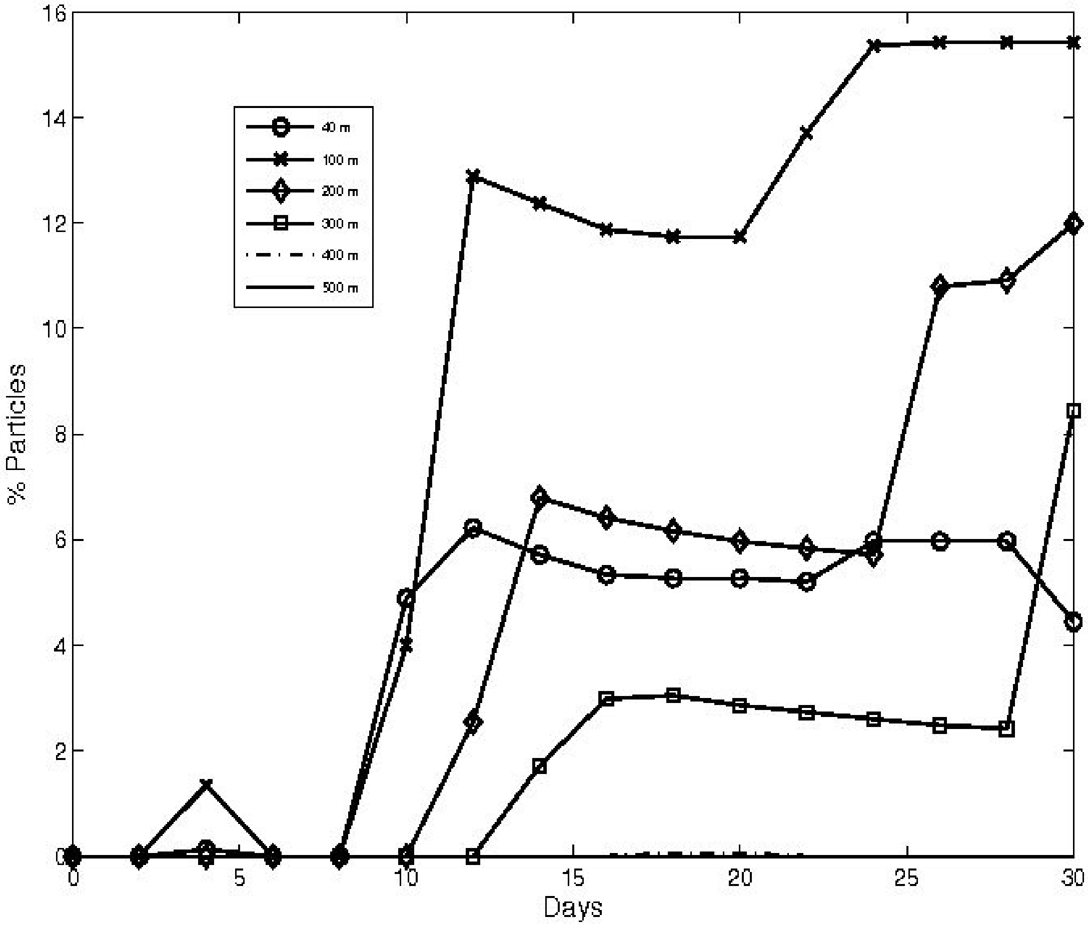
Day 13: … and is stretched along the unstable manifold forming the filament.

See also Velasco Fuentes (2005)

Percentage of particles leaving a circle of radius 100km centered on the eddy center:

Filamentation is more intense at 100 m depth





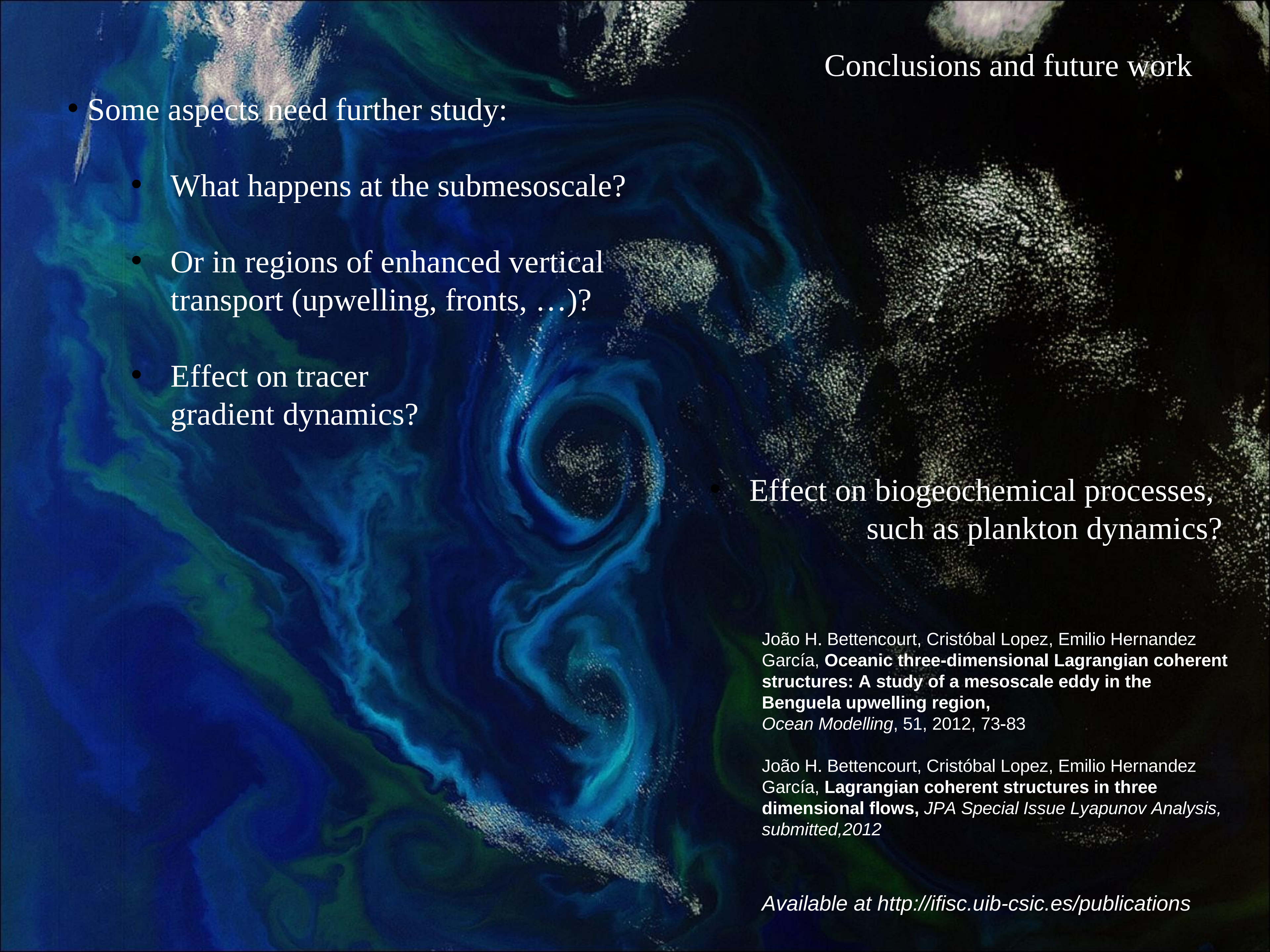
- The Lagrangian view of coherent structures can highlight aspects of material transport that are missed by traditional eulerian tools.

- Lagrangian coherent structures can be educed with tools from dynamical systems theory.

- At the mesoscale, they populate the ocean giving rise to vigorous lagrangian mixing.

- In three dimensions they show up as quasi-vertical structures because of the predominance of horizontal dispersion.

- They can control material transport into and out of an mesoscale eddy.



Conclusions and future work

- Some aspects need further study:
  - What happens at the submesoscale?
  - Or in regions of enhanced vertical transport (upwelling, fronts, …)?
  - Effect on tracer gradient dynamics?
  - Effect on biogeochemical processes, such as plankton dynamics?

João H. Bettencourt, Cristóbal Lopez, Emilio Hernandez García, **Oceanic three-dimensional Lagrangian coherent structures: A study of a mesoscale eddy in the Benguela upwelling region,** *Ocean Modelling*, 51, 2012, 73-83

João H. Bettencourt, Cristóbal Lopez, Emilio Hernandez García, **Lagrangian coherent structures in three dimensional flows,** *JPA Special Issue Lyapunov Analysis, submitted,2012*

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