Coherent Structures in Three Dimensional Flows

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Abstract

Coherent Structures are known to drive biological dynamics, from plankton to top predators, thus it is very important to be able to characterize them in realistic three dimensional flows. The Finite-Size Lyapunov Exponent (FSLE) is a measure of particle dispersion in fluid flows and the ridges of this scalar field locate regions of the velocity field where strong exponential separation between particles occur. These regions are referred to as Lagrangian Coherent Structures (LCS). We have identified LCS in two different 3D flows: a canonical turbulent velocity field, that is the turbulent flow between two parallel stationary plates, driven by a pressure gradient in the mean flow direction and a primitive equation model (ROMS) simulation of the oceanic flow in the Benguela region.

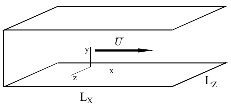
Turbulent Channel Flow

We used the DNS algorithm of **Channelflow.org** to simulate turbulent channel flow at low Reynolds number. The simulation domain is a periodic (in x and z) box.

The simulation parameters follow Moser et al (1999):

$$(L_X, L_Y, L_Z) = (4\pi, 2L, 4/3\pi)$$

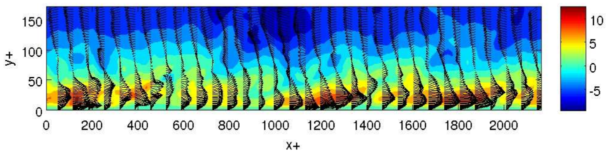
$$N_X \times N_Y \times N_Z = 128 \times 129 \times 128$$



In terms of wall variables the Reynolds number of the flow is:

$$R_{e\tau} = \frac{L u_\tau}{\nu} = 172$$

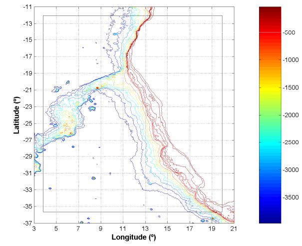
with the friction velocity: $u_\tau = \sqrt{\frac{\mu}{\rho} \frac{d\overline{U}}{dy}\Big|_{y=0}}$



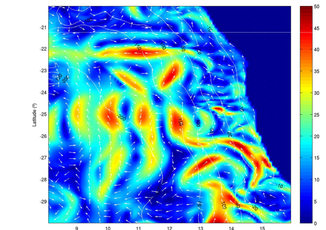
Snapshot of the turbulent velocity field in the bottom half of the channel at z=Lz/2.

Benguela Ocean Flow

The three dimensional velocity field used in this study was obtained from a ROMS (Regional Ocean Modeling System) of the Benguela ocean region (LEGOS, Toulouse). The horizontal resolution was 1/12º and in the vertical direction 32 terrain following layers were used. The model was run with climatological forcing.



Limits of the domain of the velocity field set used in the FSLE field calculation (black line) in the Benguela ocean region. The set contained two years of daily averaged horizontal (u,v) and vertical (w) velocity components. Color bar shows depth values in meters.



Snapshot of the horizontal velocity field (cm/s) at layer 16 (200 m depth at the offshore area). There is a strong mesoscale activity for which the Benguela ocean region is known.

FSLE Algorithm

The Finite Size Lyapunov Exponent measures the time τ it takes for two fluid particles initially at a distance δ_0 to separate by a distance δ_f

The FSLE fields were calculated using a 3D version of the algorithm used by d'Ovidio et al.(2004).

The velocity field domain is covered by a regular grid with mesh spacing δ_0. At time t_0, a fluid particle **P** is released from every grid node, together with particles that are at a distance δ_0 from **P** in the three coordinate directions.

The algorithm then tracks the particles during a specified time interval *T*, checking, at each integration time step t if the distance between **P** and any other particle is greater than δ_f. If so, the FSLE for the initial location of P, $\mathbf{x}(\mathbf{P},t_0)$ is

$$\Lambda(\mathbf{x}, t_0, \delta_0, \delta_f) = \frac{1}{\tau} \ln\left(\frac{\delta_f}{\delta_0}\right)$$

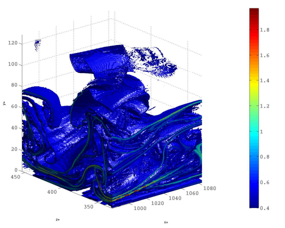
If at the end of the tracking interval the distance is lower than δ_f, the FSLE for the spatial location of **P** at t_0 is set to zero.

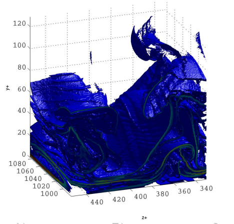
The particles positions do not coincide in general with the spatial locations where the velocity field is specified, so we use linear interpolation in all spatial directions and also time to obtain velocity values at the particle's position at the required integration time step.

3D LCS Isosurface Visualization

Lagrangian Coherent Structures are defined as ridges in the FSLE field. Intuitively, a ridge can be defined as if a person was standing on a ridge moving perpendicularly to it would lead this person down.

Turbulent channel flow was previously studied with lagrangian techniques (Direct Lyapunov Exponent) by Green et al (2007) who showed that lagrangian measures were able to define more precisely the boundaries of coherent structures when compared to eulerian measures.





The two figures above show a 3D depiction of the LCS. The very thin volumes enclosed by the blue 0.4 isosurfaces contain the points where the FSLE value is larger than 0.4, whereas the "empty" volumes contain the FSLE from 0 up to 0.4. The maximum FSLE for this sub domain of the turbulent velocity field at the instant considered is approximately 2, so it can be said that the higher FSLE values are contained in very thin volumes and it is expected that as we increase the FSLE grid resolution - decreasing δ_0 - these volumes will get thinner, approaching 2D surfaces. Moving away from this surfaces, perpendicularly to them one would see lower values of the FSLE.

3D LCS Ridge Visualization

The isosurface based visualization only gives an approximation of the 3D LCS. By using a rigorous mathematical definition of ridge (Eberly et al, 1994), the ridge surfaces can be extracted directly from the FSLE fields.

Ridge Definition

For a smooth scalar field f in n - dimensional space a d - dimensional height ridge is given by:

$$\forall_{d<i\leq n}\ \mathbf{g}^T \mathbf{e}_i = 0 \wedge \alpha_i < 0$$

with $\mathbf{g} = \nabla f$, α_i the eigenvalues of $\mathbf{H}$, the hessian of f, and $\mathbf{e}_i$ the eigenvector of $\mathbf{H}$ associated with α_i.

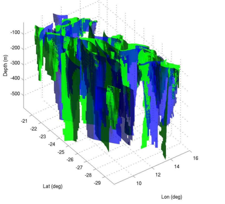
For $n = 3$ and $d = 2$ we have $\mathbf{g}^T \mathbf{e}_3 = 0 \wedge \alpha_3 < 0$.

For the extraction of the ridges we use the algorithm of Schultz et al (2010).

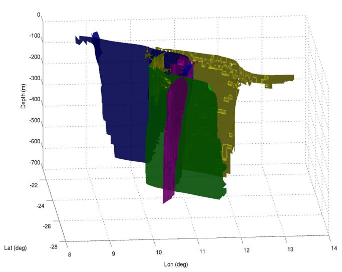
LCS in the Benguela region

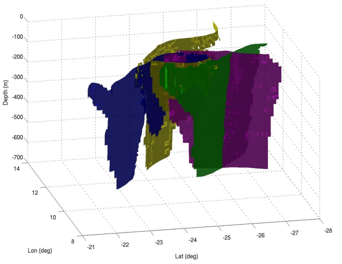
The ridge extraction algorithm will search for the points where the ridge criteria are met. The surfaces are defined based on the triangulation of the ridge points.

The figure to the left shows the attracting (blue) and repelling (green) LCS extracted from a snapshot of the 3D FSLE field. There is a vigorous mesoscale activity seen by the high number of LCS.



A Mesoscale Eddy from 3D LCS





A mesoscale eddy is identified in the figures above from the intersection of repelling - yellow (Y); magenta (M) - and attracting – blue (B); green (G) - LCS. The LCS stem from two hyperbolic intersections (M-G and Y-B) that on 2D plots would be seen as hyperbolic points.

The tangencies between LCS define boundaries to transport between the inside and outside of the mesoscale eddy.

References

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