

Voter model and interevent time distributions

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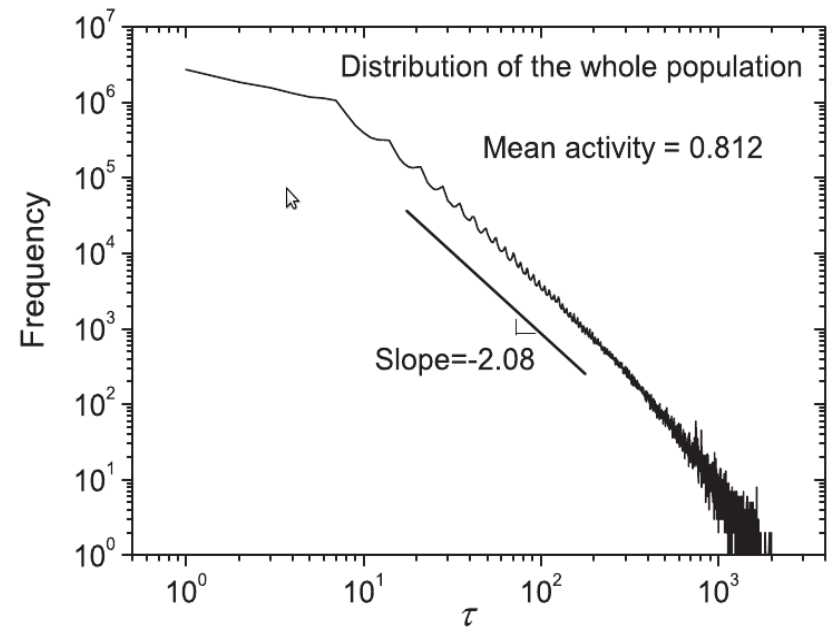
OUTLINE

- Real interevent time distributions.
- Typical update rules in agent based modeling.
- New rule introduction.
- Review on Voter model with different update rules.
- Application of the rule to the voter model.
- Conclusions.

Observed Interevent times distributions in human activities

Most of the interevent time distributions for different human activities display power law behaviour or at least heavy tails

Ex.: e-mail communications, surface mail communications, timing of financial trades, visits to public places, long-range travels, online games, response time of internauts, printing processes, phone calls...



Questions

- What's the origin of this particular timing?
 - Priority Queing Model (Barabási et al.)
 - Non-homogeneous Poisson process with cascades (Amaral et al.)
 - ...
- What's the effect of this timing on processes such as disease spreading, consensus formation...?
 - Effects on information diffusion (E.Moro, ...)
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Typical update rules in agent based modeling

- Asynchronous update: at each step the state of just one node is updated. Nodes are updated in a certain order that can be:
Random or Sequential.
- Synchronous update: All nodes are updated at the same time.

These update rules are implementing a homogeneous pattern of interaction, where the distribution of times between consecutive updates is either Poissonian or a delta function.

New update rule

Agents have an extra variable that is the time since their last change of state. τ

We will try to update the state of all nodes synchronously, but each node will update its state with certain probability $p(\tau)$.

How do we have to choose $p(\tau)$ so that we get a certain interevent time distribution?

Approximation for the interevent time distribution

We try to guess what kind of probability to choose by neglecting the specific dynamics.

$M(\tau)$: Desired interevent time distribution

$$M(\tau) = p(\tau) \left(1 - \sum_{j=1}^{\tau-1} M(j) \right)$$

Continuum limit:

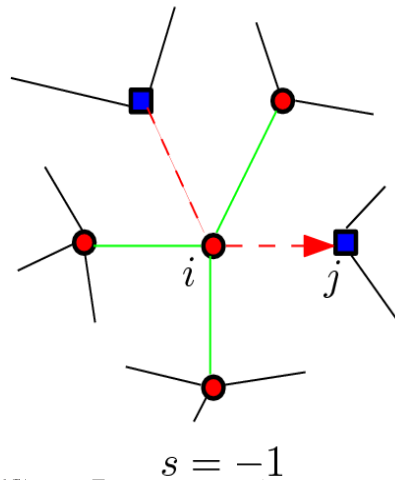
$$C(\tau) = 1 - \int_{\tau_{\min}}^{\tau} M(t) dt$$

$$\frac{d \ln(C(\tau))}{d\tau} = -p(\tau) \longrightarrow C(\tau) \propto \tau^{-\beta} \Rightarrow p(\tau) = \frac{\beta}{\tau}$$

Application: the voter model

--- n active links

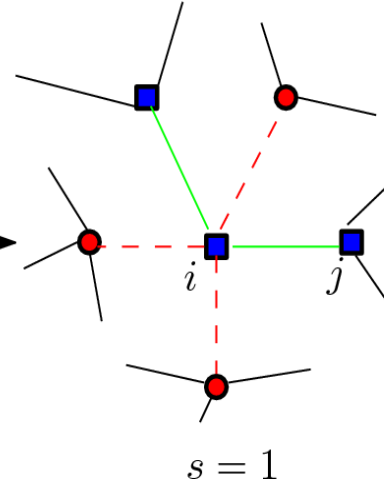
— $k - n$ inert links



$$prob = \frac{(1+v)}{2} \frac{n_+}{k}$$

$k - n$ active links

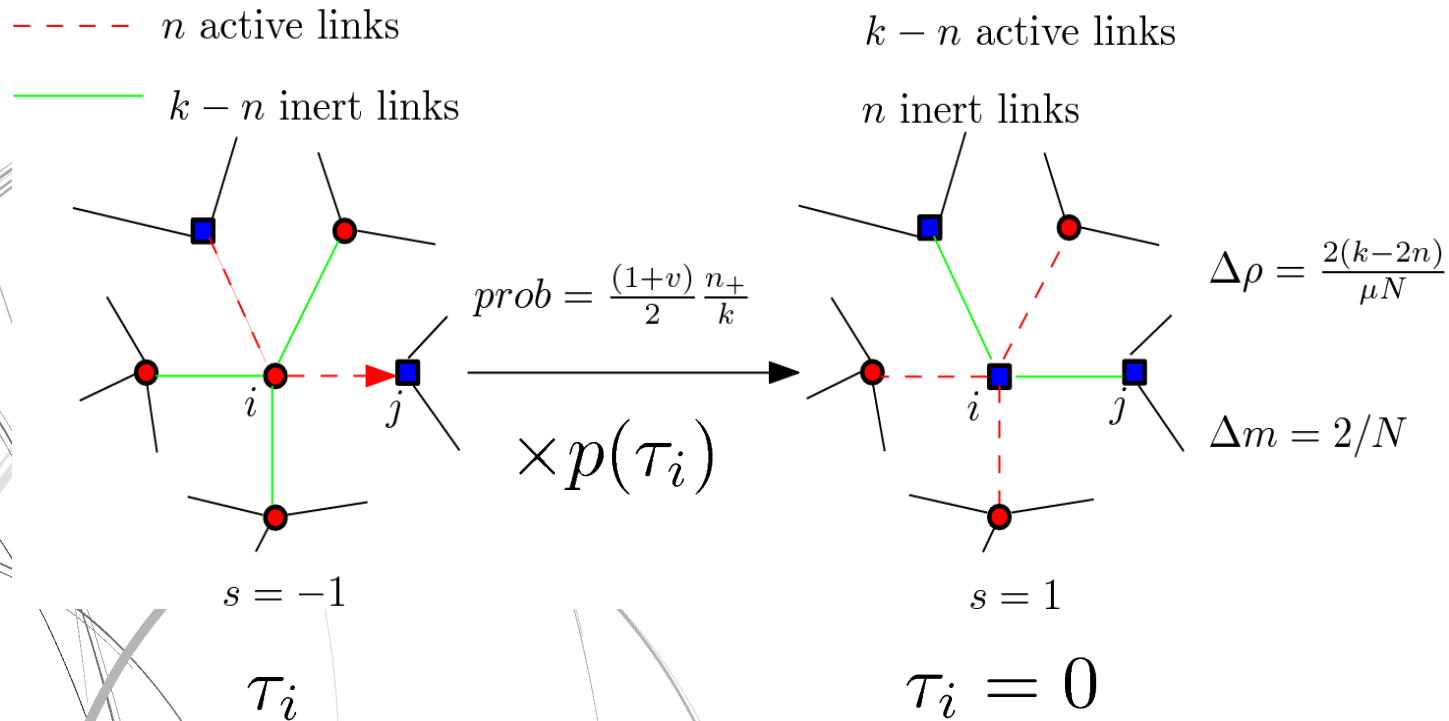
n inert links



$$\Delta\rho = \frac{2(k-2n)}{\mu N}$$

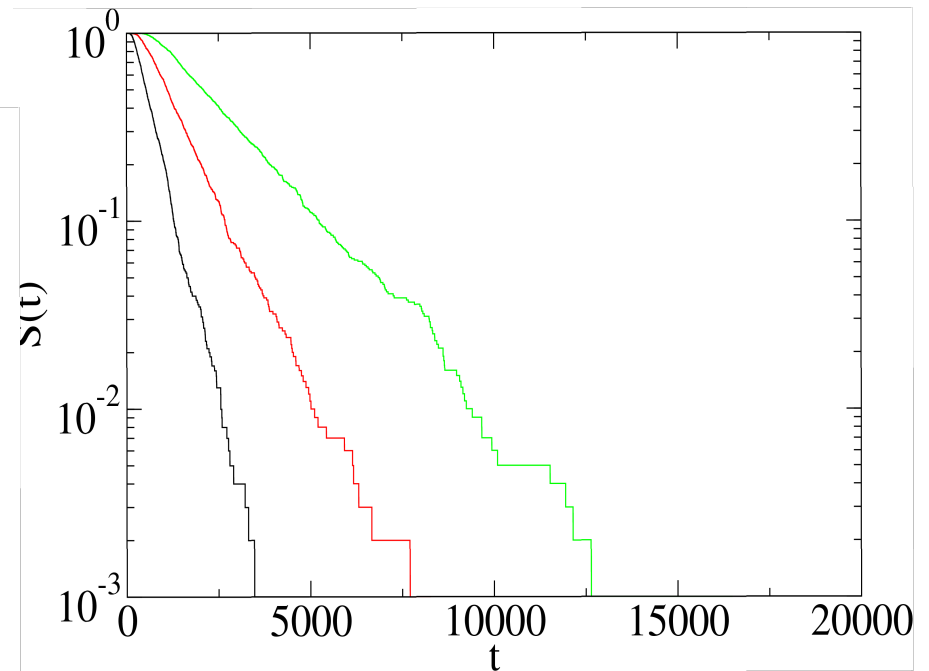
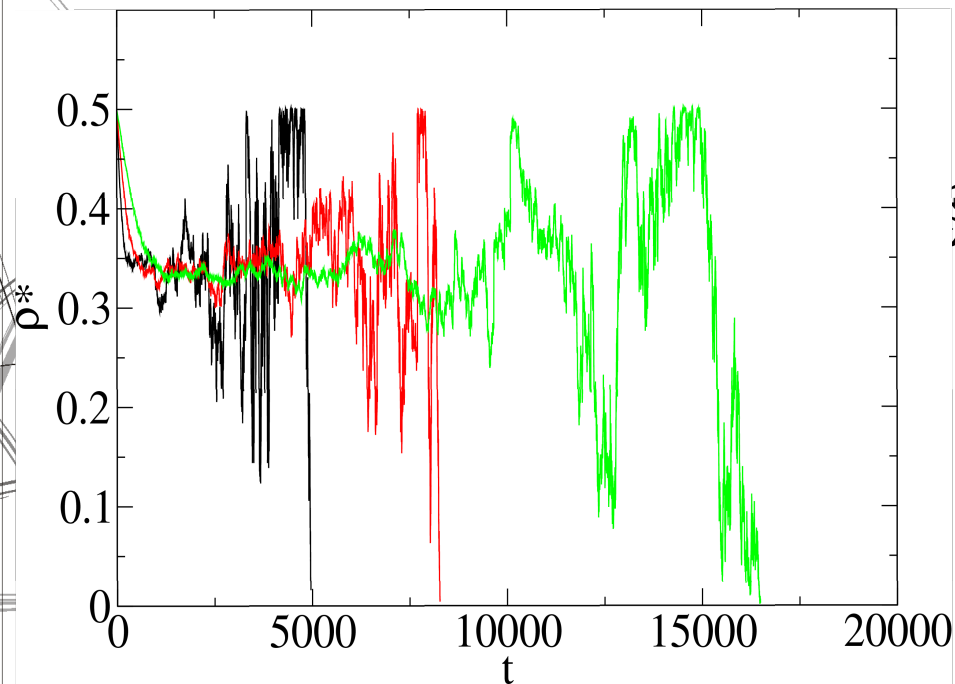
$$\Delta m = 2/N$$

Application: the voter model



Results on a complete graph

Usual update rules



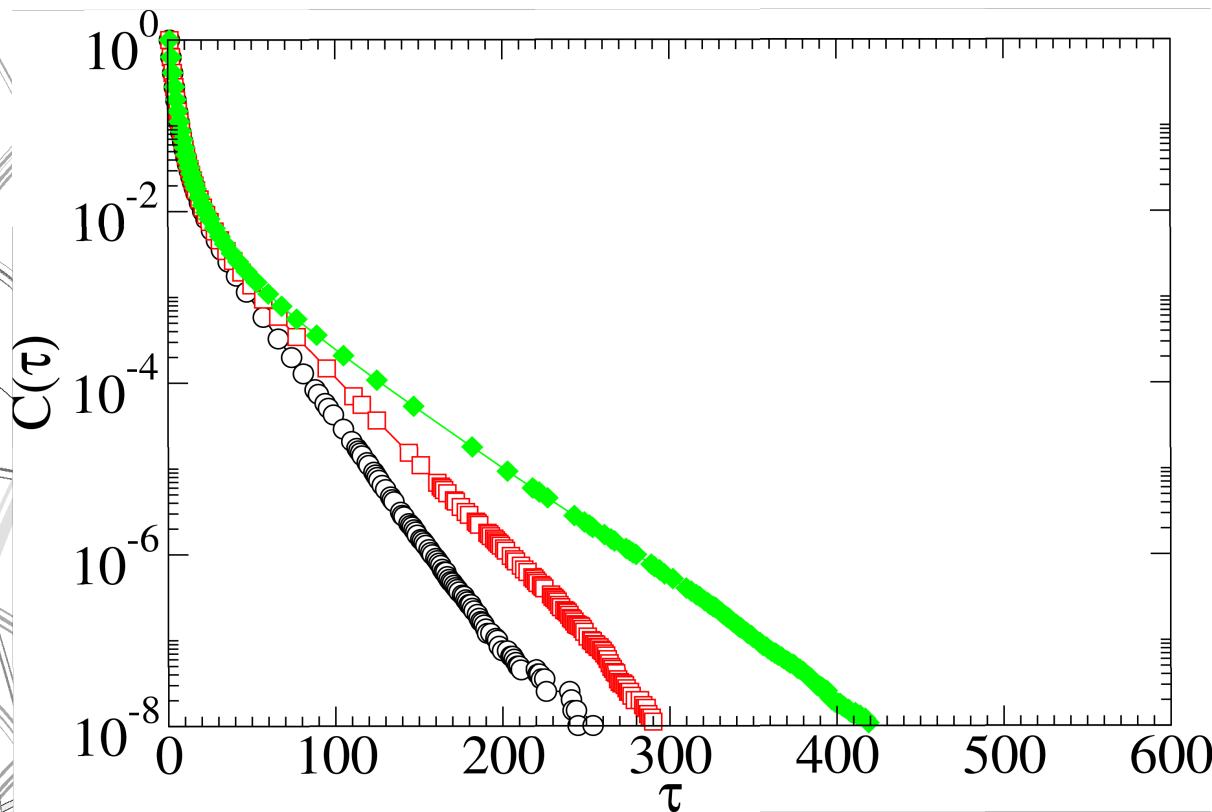
$$S(t) \propto e^{-t/\tau_{char}(N)}$$

$$\tau_{char} \propto N$$

Results on a complete graph

Usual update rules

$$C(\tau) \propto e^{-t/\tau_{char}(N)}$$

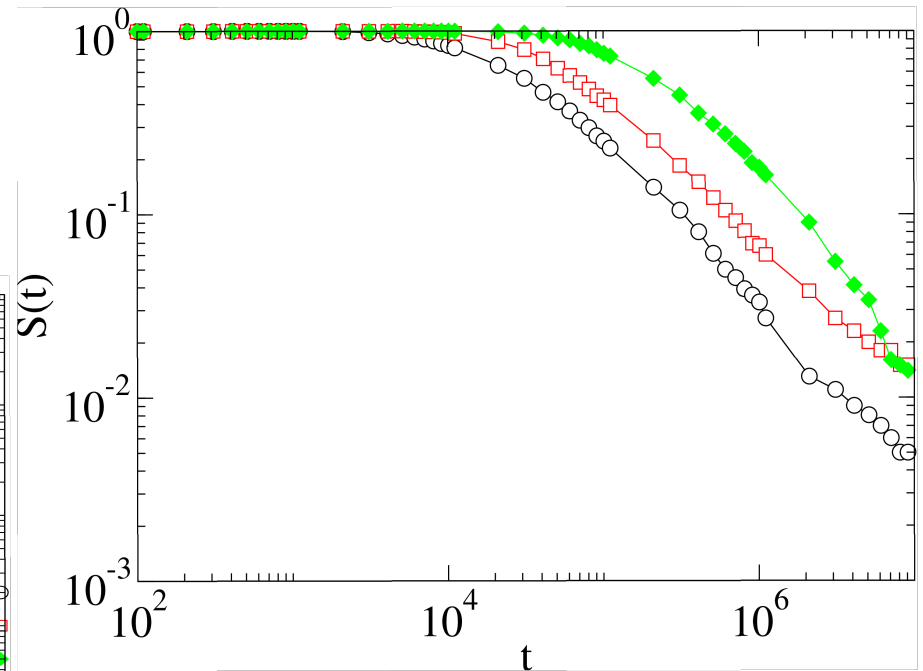
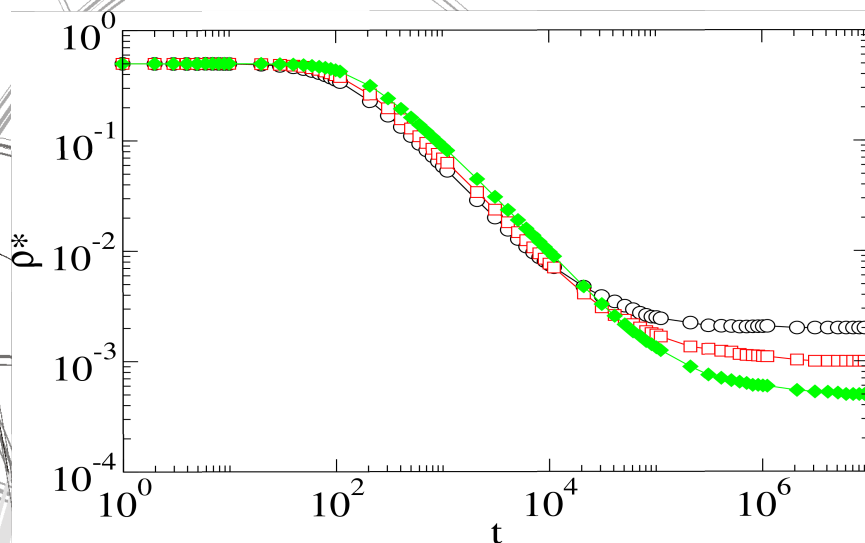


$$\tau_{char} \propto \sqrt{N}$$

Results on a complete graph

Memory update rule

$$p(\tau) = 1/\tau$$

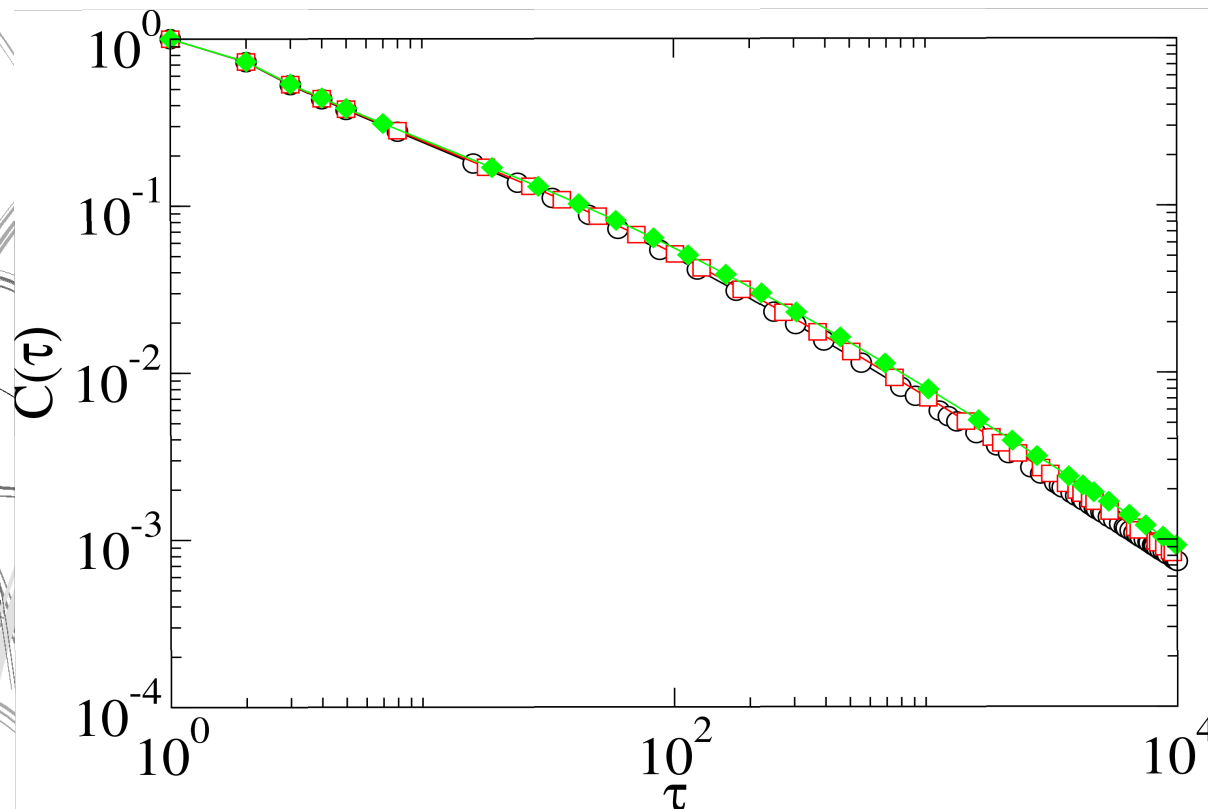


$$\alpha = 0.95$$

Results on a complete graph

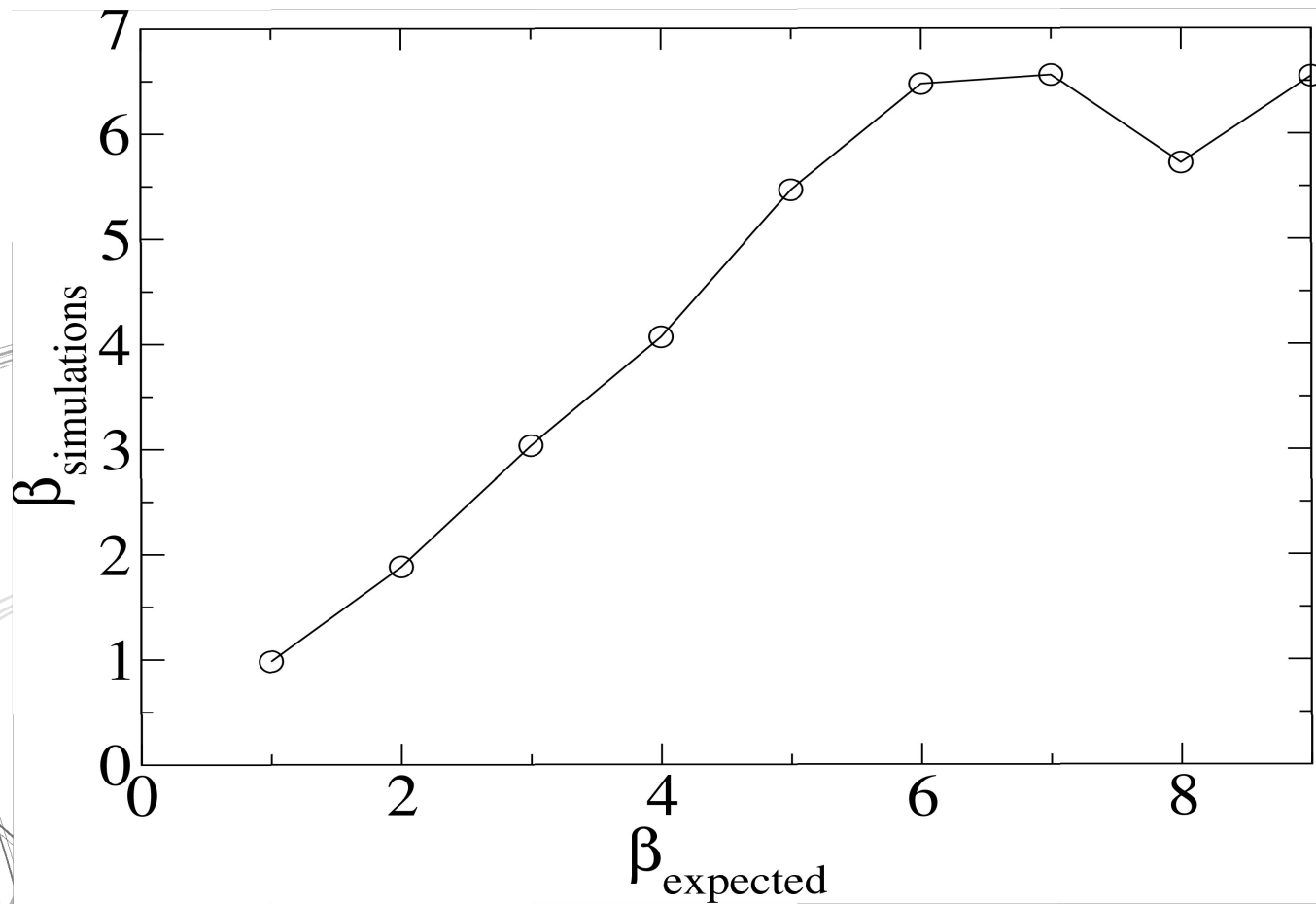
Memory update rule

$$p(\tau) = 1/\tau$$



$$C(\tau) \propto \tau^{-\alpha}$$

$$\alpha = 1$$



$$p(\tau) = \beta / \tau$$

Conclusions

- With the modified update rule we are able to account for broad interevent time distributions, though not able of controlling it totally.
- This special timing of interactions does make a difference in the macroscopic outcome of the model studied, so it should be taken into account before drawing any conclusions for real-life systems.

Thanks for the
attention!