Updating rules in social simulations

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Abstract

Microscopic models typically display a homogeneous pattern of activity based on a random (Poissonian) update. However, it has been recently found in empirical studies of human activities that the pattern of activity shows interevent time distributions with heavy tails. To capture this feature we propose an update rule, coupled to the dynamics, where the probability of an agent to interact depends on the time since the agent last changed its state. We study the voter model on different basic complex networks, such as complete, scale-free and random graph, with update probabilities leading to long tails. The macroscopic outcome of the model changes qualitatively. On the one hand, the characteristic time to reach consensus diverges. On the other, the system orders (reaches consensus) even in the infinite size limit, in contrast to the standard voter model that does not order in the infinite size limit or orders in finite networks due to finite size fluctuation.

Motivation

Microscopic models typically display a homogeneous pattern of interaction, leading to an exponential behavior of the interevent times distribution.

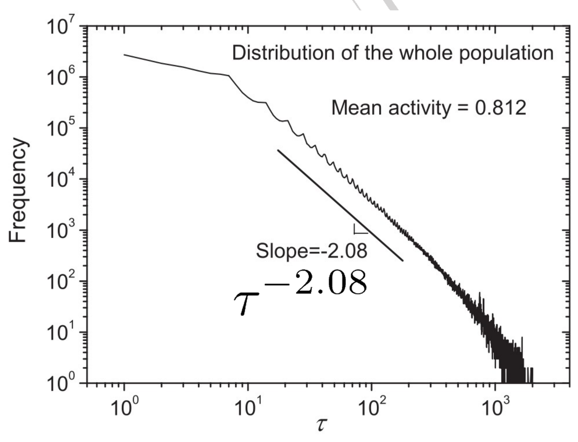


Fig 2. Interevent time distribution for ratings in an online DVD renting webpage **[1]**.

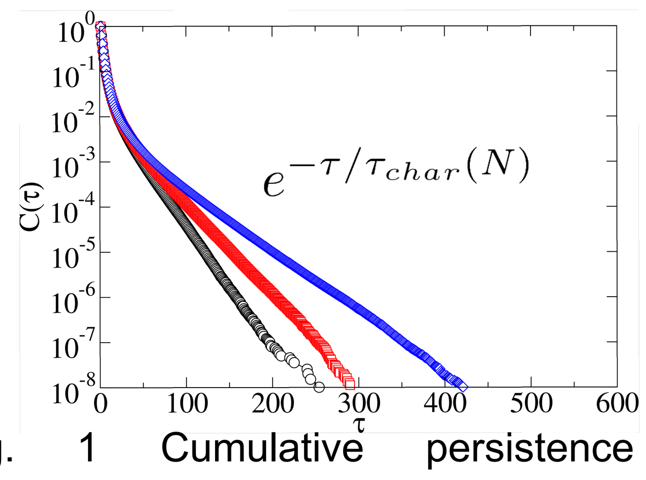


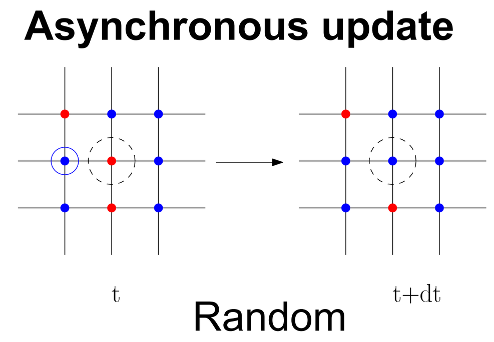
Fig. 1 Cumulative persistence time distribution for the voter model on complete graph under RAU.

Pattern of activity in human dynamics shows interevent time distributions with heavy tails. For example: timing of finantial trades, e-mail communications, phone calls...

Update rules

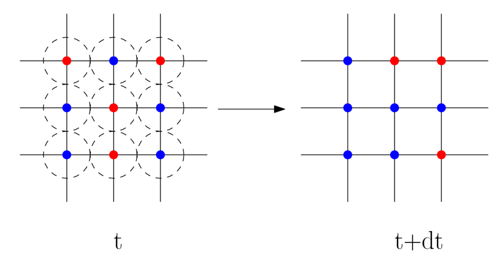
Usual update rules:
Implement a homogeneous pattern of interaction.

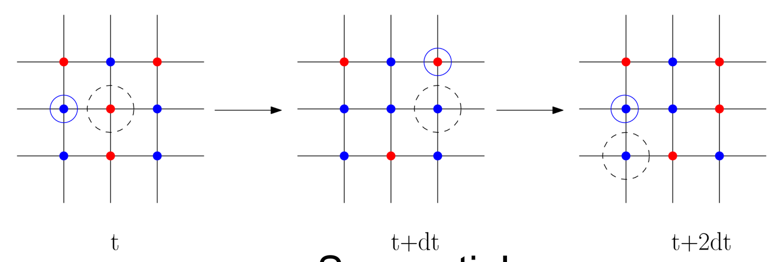
Asynchronous update



Random

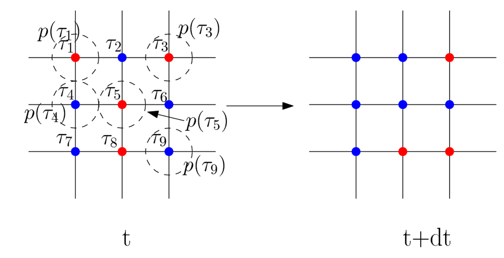
Synchronous update





Sequential

New update rule:
Every agent interacts with a probability that depends on its persistence time **[2]**.



$p(\tau) = \beta_p/\tau$

$\Rightarrow C(\tau) \propto \tau^{-\beta}$

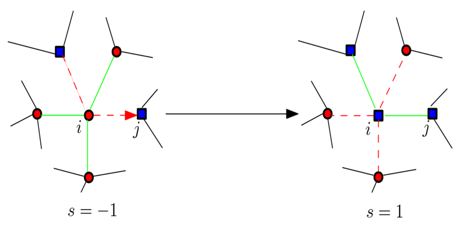
$\text{with} \beta_p = \beta$

Application: the voter model

Voter model dynamics

Agents : s binary variable (opinion)

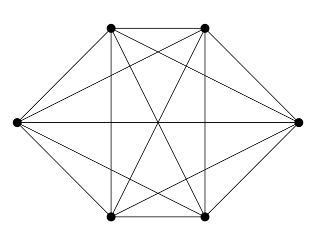
Interaction: imitation of random neighbour.



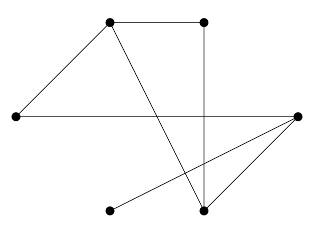
For all usual updates and for complete, random and scale-free graphs characteristic times in $e^{-\tau/\tau_{char}(N)}$

	$\rho(t)$	$S(t)$	$C(\tau)$
$\tau(N)$	$\propto N$	$\propto N$	$\propto \sqrt{N}$

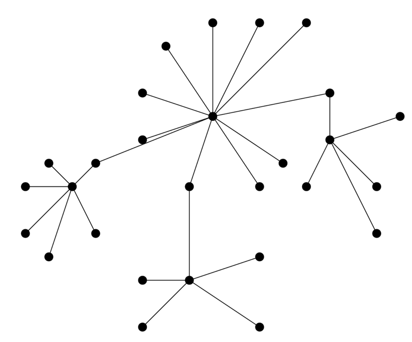
Interaction networks



Complete graph



Random graph



Scale-free graph

Quantities describing the model

Density of active links (averaged over all runs/over all active runs) $\rho(t)/\rho^*(t)$

Survival probability $S(t)$

Persistence/interevent time cumulative distribution $C(\tau)$

Results with usual update rules

$\rho(t)$ decays exponentially and $\rho^*(t)$ reaches a plateau independent of system size (no consensus in infinite size limit).

$S(t)$ decays exponentially. Characteristic time to consensus well defined.

$C(\tau)$ Exponential, due to underlying Poisson process.

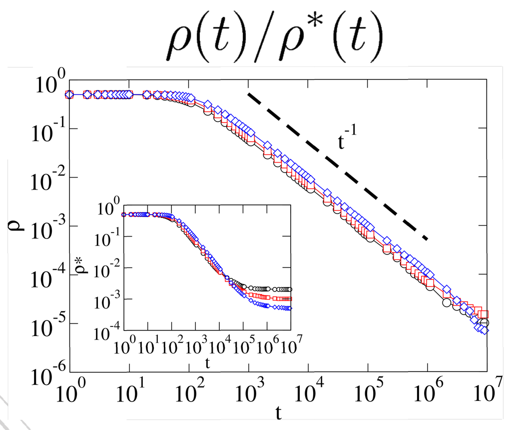
Results with new update rule with $p(\tau) = 1/\tau$

$\rho(t)$ decays as a power-law and $\rho^*(t)$ reaches a plateau that is smaller for bigger system size (consensus in infinite size limit).

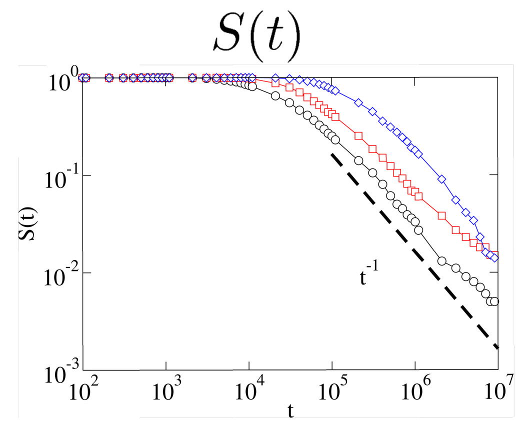
$S(t)$ decays as a power law with exponent less or equal 1. Characteristic time to consensus diverges.

$C(\tau)$ Heavy tail, power-law.

$\rho(t)/\rho^*(t)$



$S(t)$



$C(\tau)$

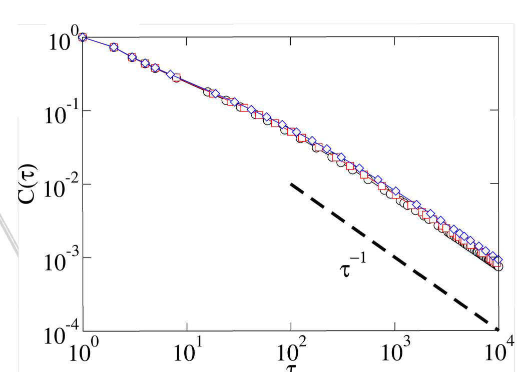


Fig 3. Behaviour in the case of a complete graph. Random and scale-free are similar.

Exponents in these power-laws

	$\rho(t)$	$S(t)$	$C(\tau)$
C.G.	0.98	0.98	0.99
R.G. <k>=8	0.46	0.43	0.51
S.-F. <k>=6	0.35	0.32	0.45

Different exponents in the tail of $C(\tau)$

By changing the parameter β_p in the function $p(\tau)$ we are able to get different exponents in the cumulative persistence time distribution $C(\tau)$.

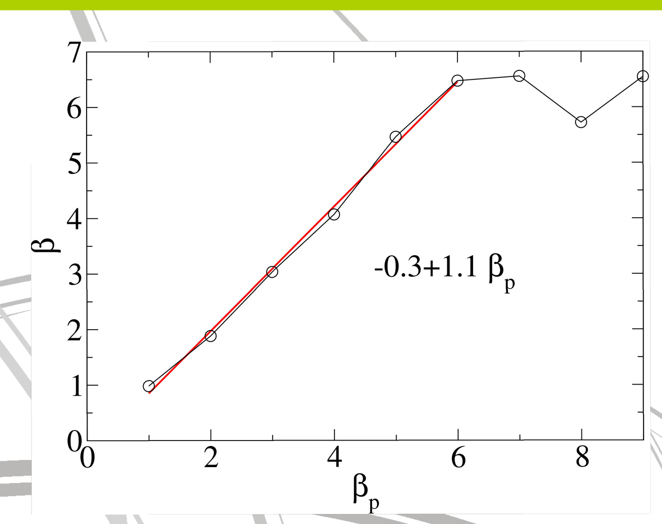


Fig 4. In complete graph good agreement $\beta \simeq \beta_p$.

References:
[1] T.Zhou *et al.* EPL 82 (2008), 28002
[2] H.U.Stark et al. PRL 101 (2008), 018701

Conclusions

•For the new update rule with memory function $p(\tau) = 1/\tau$:
· Different outcome of the model than for usual updatings: the system orders even in the infinite size limit.
· The characteristic time for consensus diverges.
• The persistence time distribution develops a heavy tail
•Interpretations of the outcome (order/disorder) of a model could be misleading, as the are not robust against this modification.