Threshold model with external influence

J. C. González-Avella, V. M. Eguíluz, M. San Miguel

Fernando Vega-Redondo

Matteo Marsili

IFISC

A basic puzzle posed by innovation diffusion is why there is often a long lag between an innovation's first appearance and the time when a substantial number of people have adopted it.

Consider three basic types of innovation diffusion models, each arising from a different account of how innovations spread.

1. Contagion. People adopt an innovation when they come in contact with someone who has already adopted.

2. Social threshold. People adopt when enough other people in the group have adopted.

3. Social learning. People adopt once they see enough evidence among prior adopters to convince them that the innovation is worth adopting.

The Model

There is large population of agents, $N = \{1; 2; :::; n\}$, placed on a given Indirected network.

Every player $i \in N$ chooses one of two alternative actions, action S=-1 or action S=+1.

Dynamics

At the beginning of every *t*, they receive a signal on the relative payoff of the two actions (E). ***with p→E=+1 and (1-p) →E=-1***

if $E = \mathbf{S}_i$ *nothing happens*.

if $E \neq \mathbf{S}_i$ *then*

If and only if the fraction of neighbors with opposite action is greater than a threshold T then Si→-Si

Question:

What is the relationship between *p* (the quality of the signal) and τ (the threshold for action change) that underlies the spread and consolidation of action 1?

Mean field analysis.

X(t)= *fraction of agents choosing action 1 at some t*

$$\dot{x} = -(1-p)x\ \theta(1-x-\tau) + p(1-x)\ \theta(x-\tau)$$

where $\theta(z) = 1$ if $z \geq 0$ while $\theta(z) = 0$ if $z < 0$. It is useful to divide the analysis into two cases:

Case I: $\tau > 1/2$

In this case, it is straightforward to check that

$$\begin{array}{rcc} x < 1-\tau & \Longrightarrow & \dot{x} = -(1-p)x < 0 \\ 1-\tau < x < \tau & \Longrightarrow & \dot{x} = 0 \\ x > \tau & \Longrightarrow & \dot{x} = p(1-x) > 0 \end{array}$$

So, it follows that correct social learning occurs iff $p > \tau$.

Case II: $\tau < 1/2$

In this case, we find:

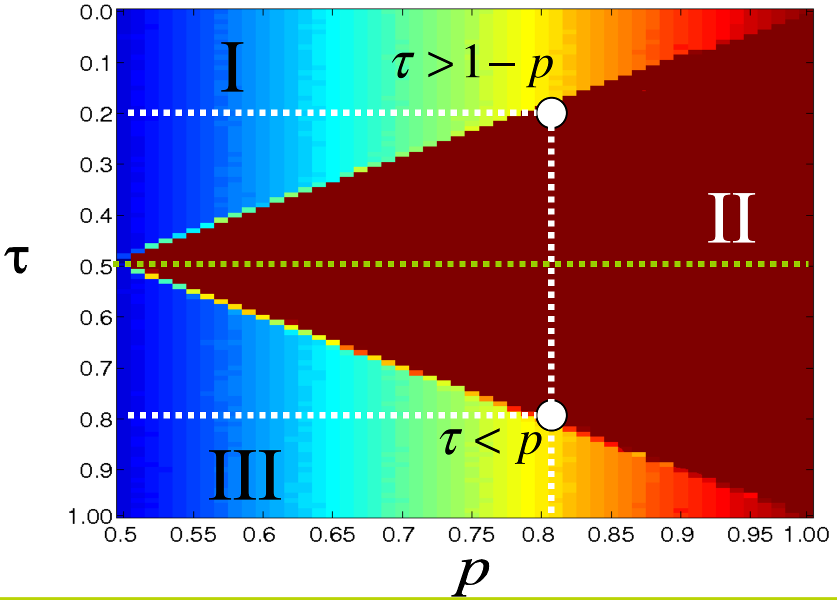
$$\begin{array}{rcc} x < \tau & \Longrightarrow & \dot{x} = -(1-p)x < 0 \\ \tau < x < 1-\tau & \Longrightarrow & \dot{x} = p - x \\ x > 1-\tau & \Longrightarrow & \dot{x} = p(1-x) > 0 \end{array}$$

And, therefore, correct social learning occurs iff $p > 1-\tau$.

To sum up, we can combine both cases simply stating that mean-field analysis predicts that correct social learning occurs if, and only if,

$$p > \max\{\tau, 1-\tau\}. \qquad (1)$$

Phase diagram (p,τ)



Phase I: Disorder (active)

Phase II: Order

Phase III: Disorder (Frozen)

Phase I para $\tau < 0.5$

Phase III para $\tau > 0.5$

case $\tau > 0.5$

$t < p$

$p = 0.8$

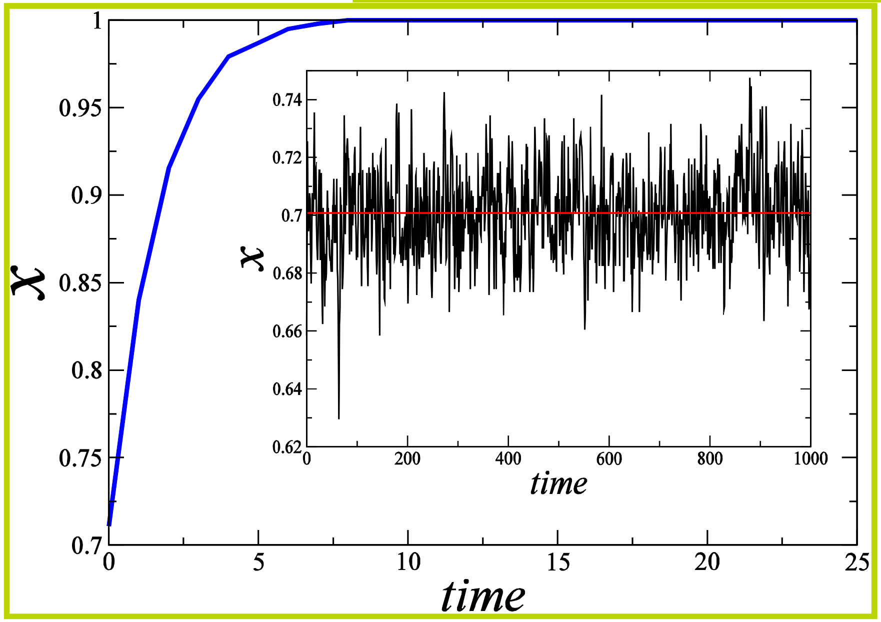
$\tau < 0.8$

case $\tau < 0.5$

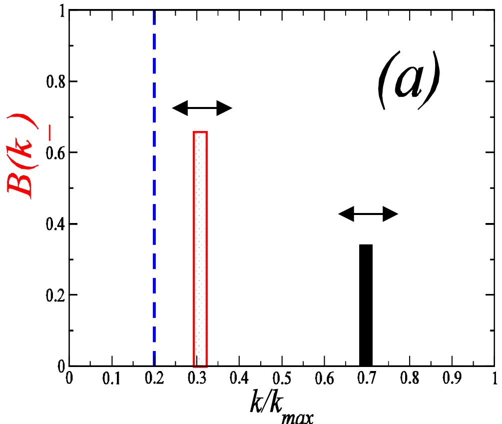
$\tau > 1 - p$

$p = 0.8$

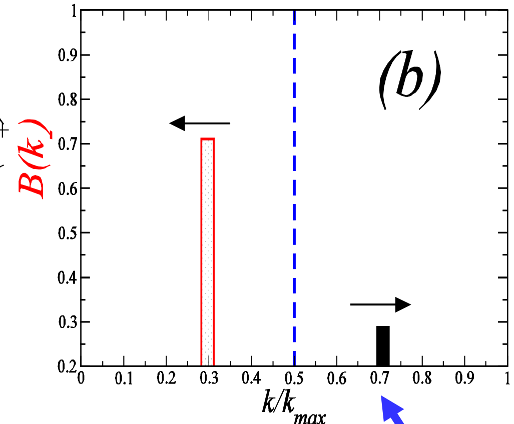
$\tau > 0.2$



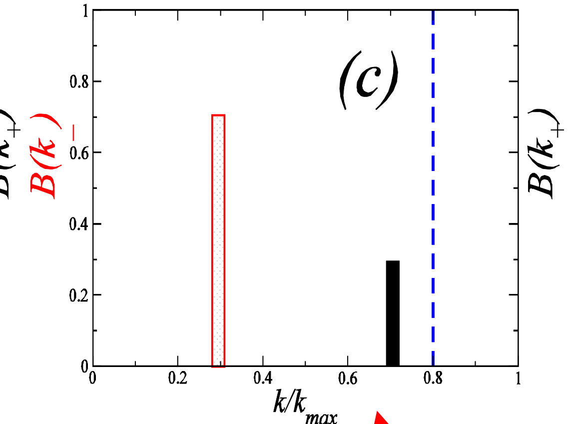
Fase I

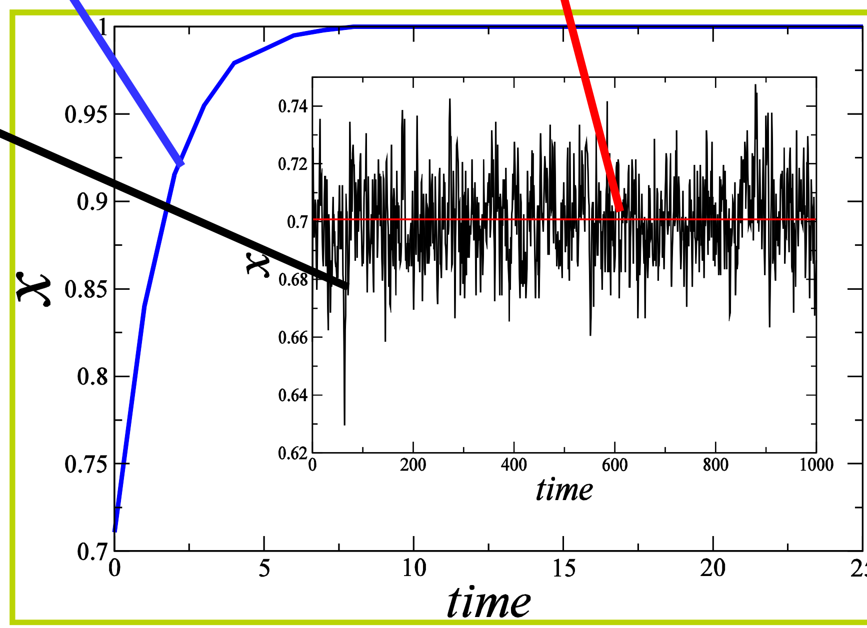


Fase II



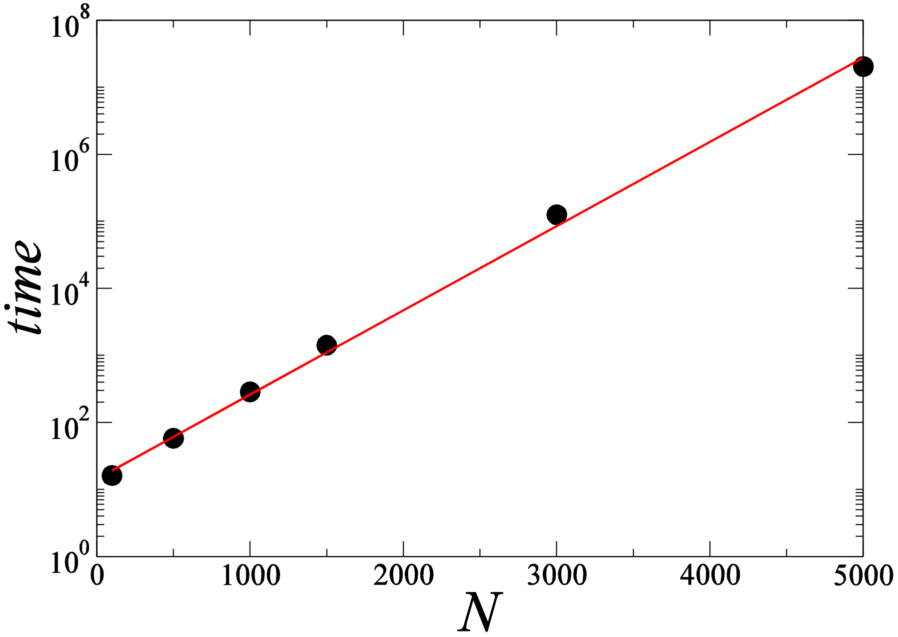
Fase III





$B(k_\pm) \rightarrow$ Distribución de probabilidad de que un nodo con estado ∓ 1 tenga una fracción de nodos vecinos con estado ∓ 1

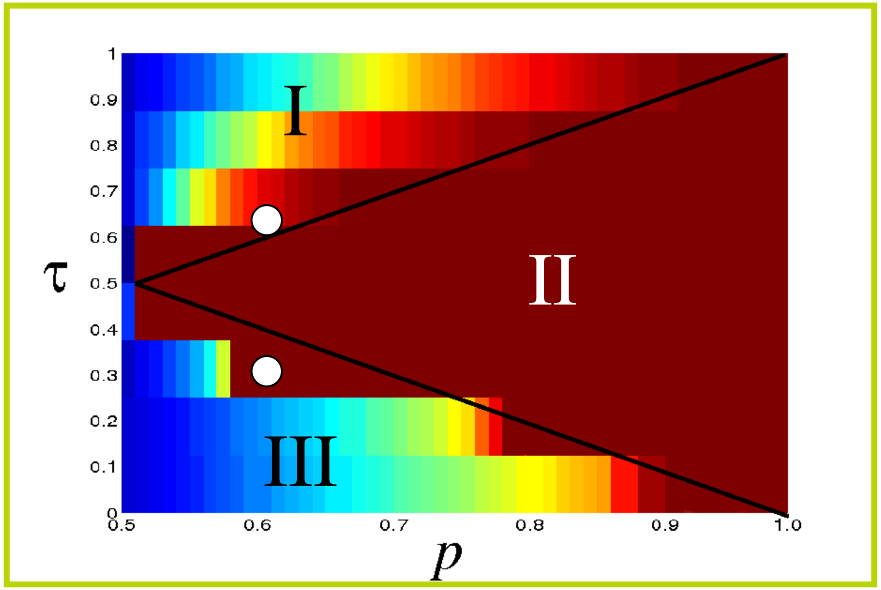
Time scale



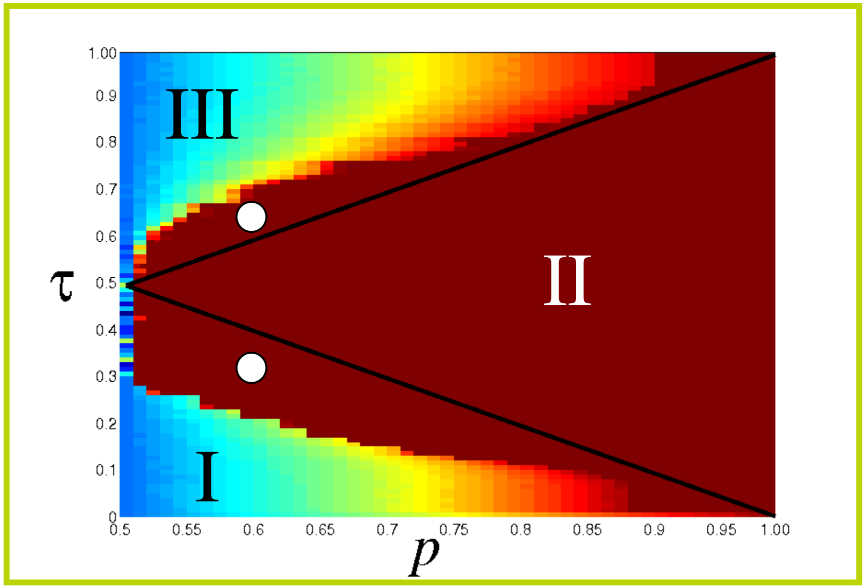
$$t \sim e^{N}$$

Phase I: N→œ; $x \approx p$

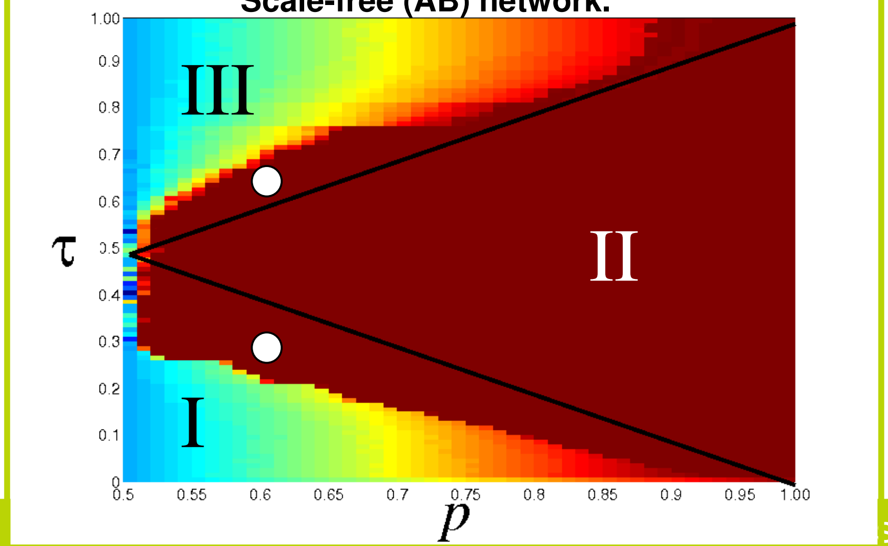
Regular lattice with k=8. System size 10^4



Random network <k> = 8. (Poisson)

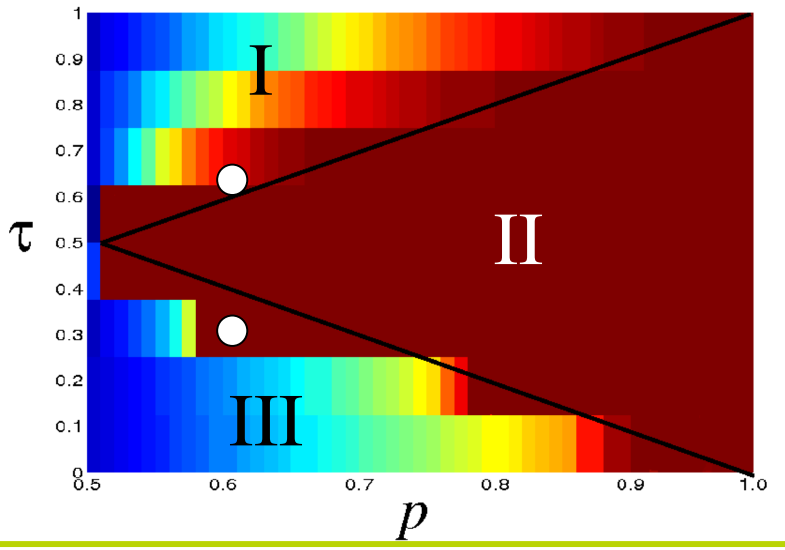


Scale-free (AB) network.



$$p = 0.60$$
$$\tau = 0.30$$
$$\tau = 0.60$$

Phase diagram (p,τ)

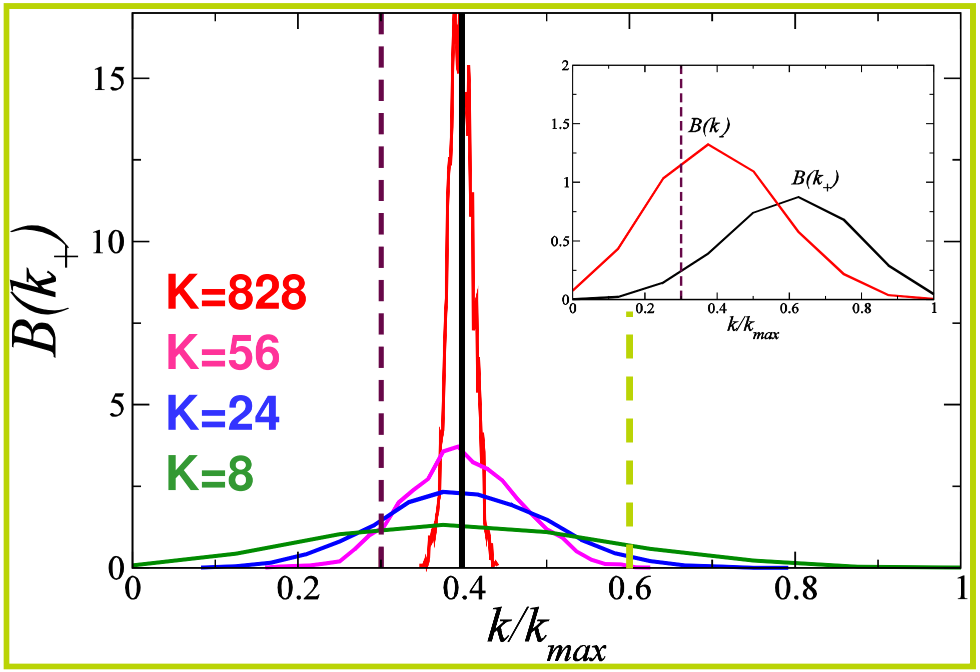


Regular lattice with k=8. N= 10^4

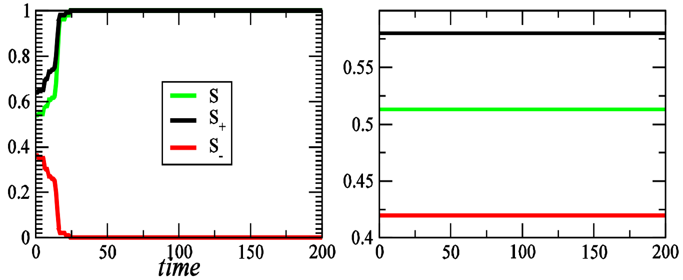
$p = 0.60$

$\tau = 0.30$

$\tau = 0.60$



N = 100; T = 0.75; p = 0.60; 2d lattice with k=56



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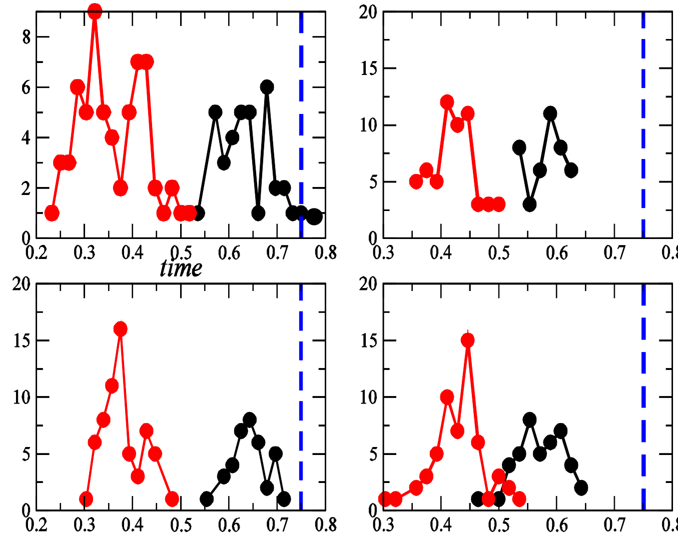

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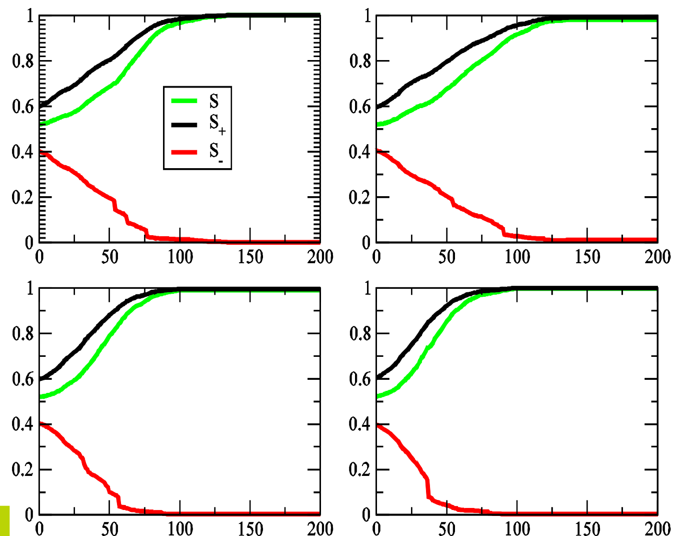

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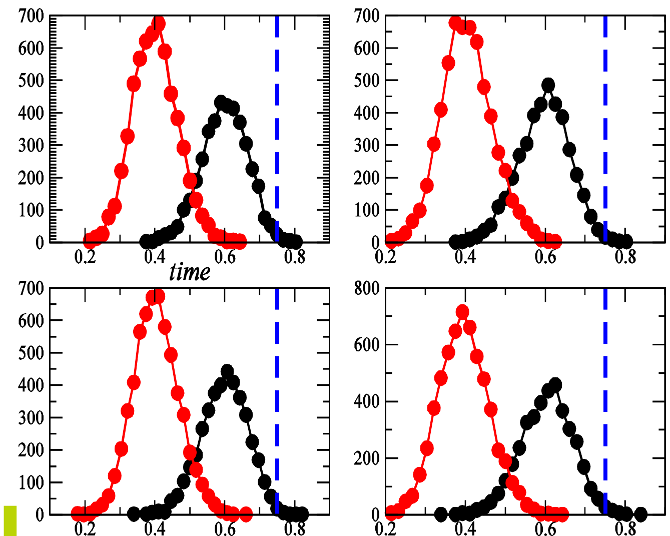
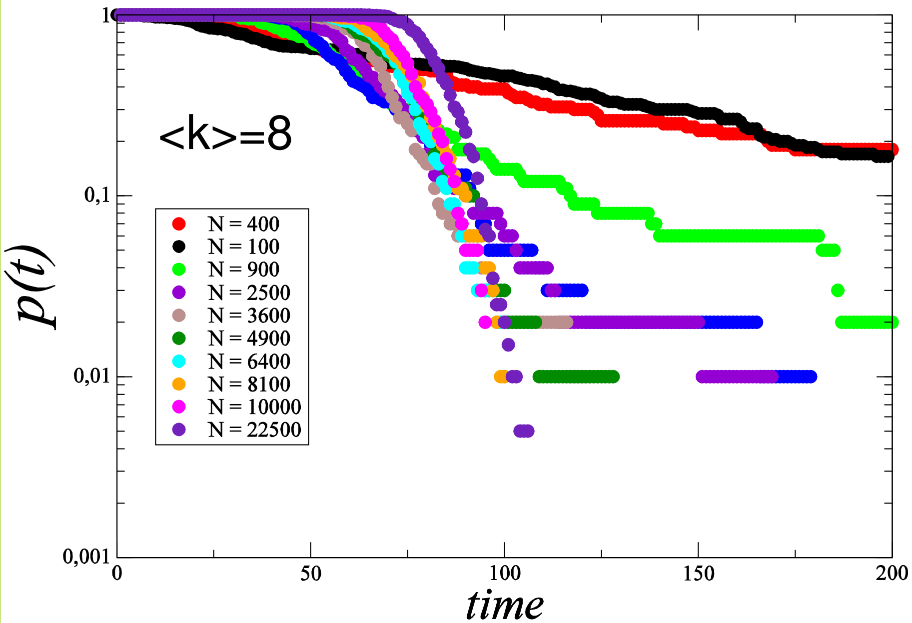


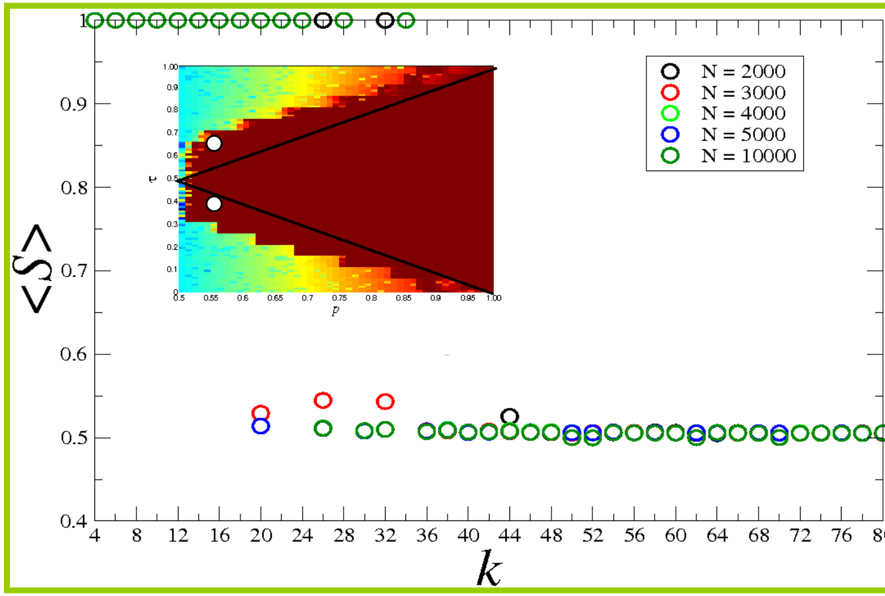


Random network.

$p = 0.55$

$\tau = 0.35$

$\tau = 0.75$



Summary

The mean-field analysis and the simulations deliver the same message: depending on the quality of the signal, neither too strong nor too weak peer effects in action adjustment (as measured by the magnitude of τ) is required for correct social learning at the overall population level.

Local interactions are more efficient to promote social learning that the case of global interaction.

Introduce heterogeneity among the agents.