Disappearance of the classical limit: Entangled oscillators at room temperature

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Introduction

■ Continuous variable entanglement between two linearly coupled harmonic oscillators under the presence of a common bath has been studied (PRL. 100, 220401 (2008)) :

Entanglement is present even at finite (but low) temperature.

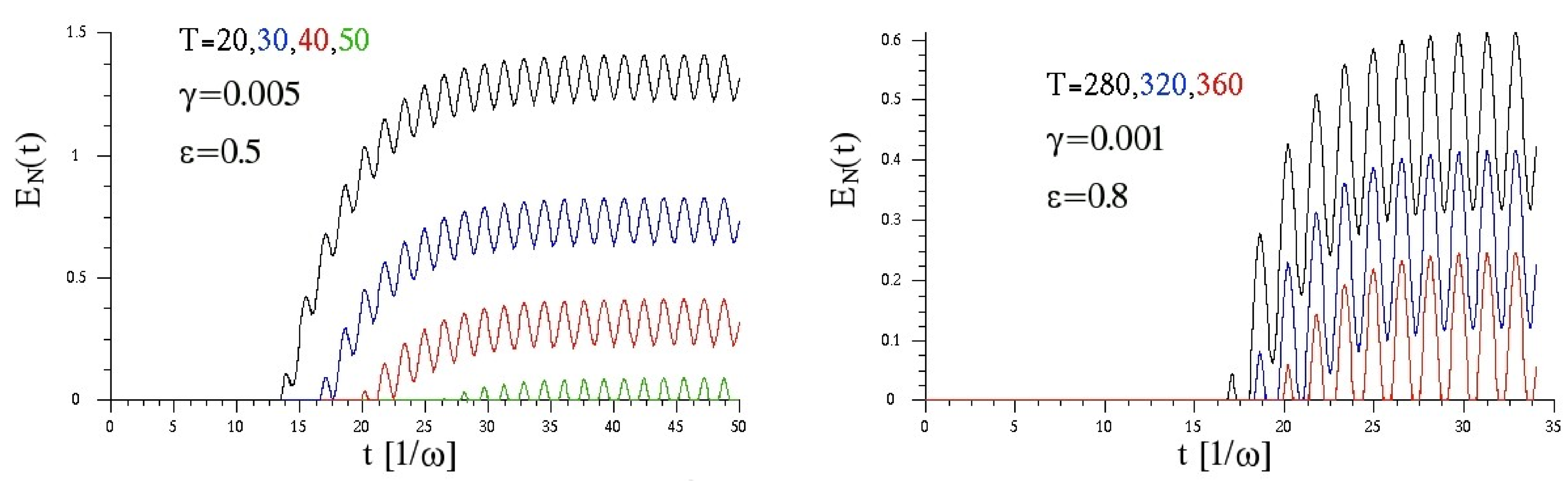
■ The same situation with two independent baths, or different oscillator frequencies yields a fast elimination of any entanglement (PRA 76, 022312 (2007)). The latter is clearly the most realistic situation.

■ Our purpose: Study the realistic situation and compensate the loss of entanglement by parametrically driving the oscillators. Result:

Entanglement at high temperatures can be achieved

Entanglement production

■ With a sinusoidal (resonant) time-dependence in the eigenfrequencies, the eigenmodes get highly squeezed, despite the bath's influence. An asymptotic nonzero amount of entanglement is thus produced:



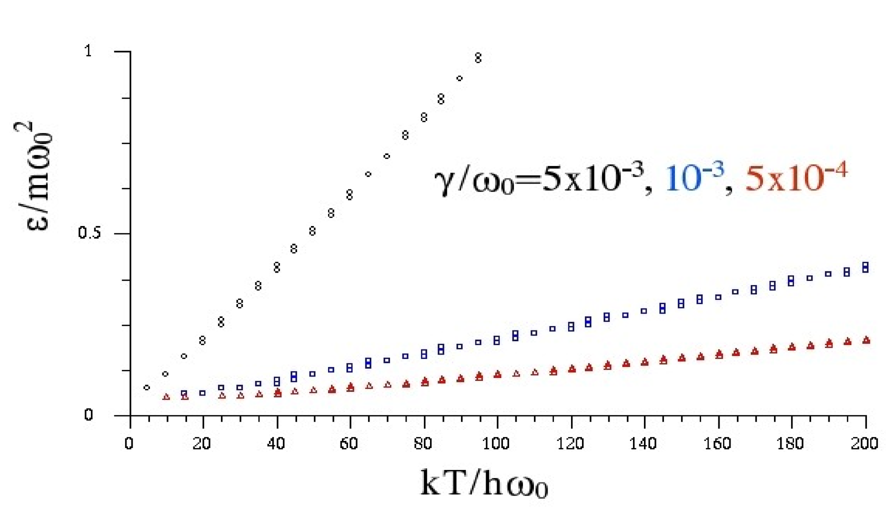
■ Drawback: the energy keeps growing all the time! This protocol can be used to produce entanglement in unfavourable conditions.The latter is to be used afterwards, but fast enough so that the bath does not spoil the achieved entanglement.

■ The competition between driving (squeezing) and the bath can be understood in terms of the two-oscillator purity and the purity of each. The lower/upper bounds on entanglement for a two-mode gaussian state given in PRL 92, 087901 (2004) coincide in our case, and yield:

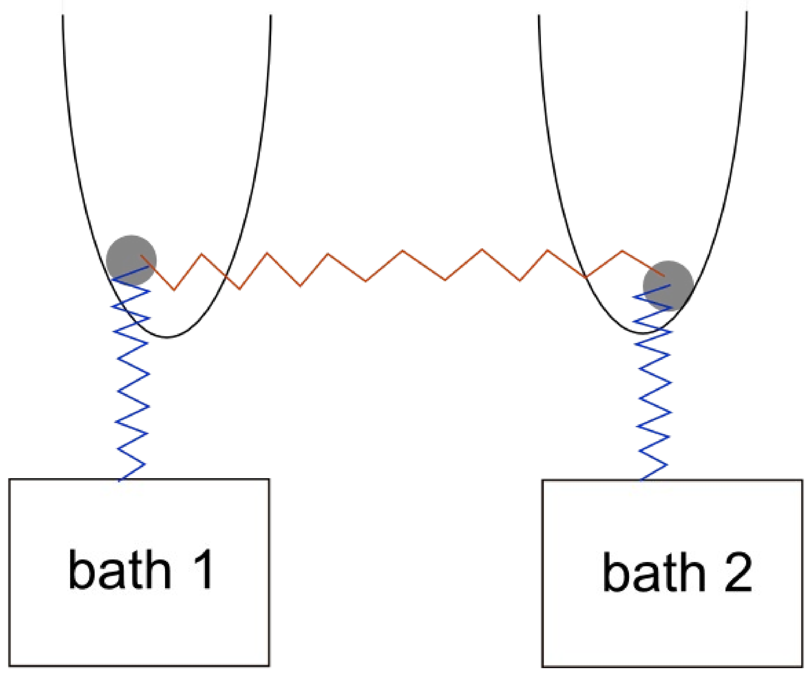
$$E_N = 1 + \log_2 \mu - \log_2 \mu_R$$

when both purities (global and reduced) go to zero exponentially fast. It can be analytically shown that they do, and that both rates coincide. This explains the fact that entanglement reaches an asymptote.

■ Numerical evidence shows that a diagram of presence(absence) of entanglement can be plot for a given coupling to the bath:



The model



■ We study two harmonic oscillators with time dependent frequency and/or coupling:

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2(t)x_1^2 + \frac{1}{2}m\omega^2(t)x_2^2 + c(t)x_1x_2$$

■ They are coupled to two independent Ohmic baths, with a Hamiltonian of the form:

$$H_{\text{bath+int.}} = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}m_i\omega_i^2(q_i - \frac{c_i x_{1(2)}}{m_i\omega_i})^2$$

■ The normal modes

$$x_\pm = \frac{1}{\sqrt{2}}(x_1 \pm x_2)$$

with time-dependent eigenfrequencies

$$\omega_\pm^2(t) = \omega^2(t) \pm c(t)$$

are described by a dissipative quantum master equation, whose coefficients depend on the initial temperature (we consider the two oscillators and bath initially at the same temperature T).

Summary

■ We have shown that the regime at very high temperatures in a realistic situation can be still regarded as *inside* the quantum regime. Thus the classical limit clearly depends on how far from thermodynamic equilibrium is the system.

■ The evolution of entanglement in terms of the two-mode and one-mode purities allows for an interpretation of its asymptotic behaviour: entanglement is present when the purity of the global state (2 oscs.) is higher than the reduced purities (1 osc.), as expected.

■ A “phase” diagramme for the presence(absence) of entanglement can be approximately drawn, which depends on the initial temperature of system+bath and the driving amplitude.

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