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Effects of the topology and delayed connections in neuronal networks

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Abstract

In this study we investigate the local and global synchronization of an ensemble of delayed coupled neurons. We consider a circuit composed of neurons described by the Hodgkin and Huxley model with reciprocal delayed chemical connections, modeled as alpha function. We study the influence of different topologies on the synchronization properties of the network. Five different types of topologies are analysed: regular, small-world, random, scale-free and all-to-all. We also consider two kind of delay configurations: one in which the delays are homogeneous and another in which heterogeneous delays are considered.

We find that synchronized firing activity between one neuron and its neighbours (local synchronization) is achieved in all the networks while a high randomness in the connections is required to reach a global synchronized activity. While varying the coupling delay we find a resonant-like effect in the synchronization even in the all- to-all topology.

We also consider a Gaussian distribution of currents, allowing some neurons to operate in the sub-threshold state for low coupling strength. For higher couplings, synchronized states re-emerge. We observe that neurons with currents close to the threshold can develop oscillations death.

Introduction

> Our system

> 10^3 neurons (Hodgkin-Huxley model)

$$C_m \dot{v}_i = I_i - I_i^{ion} - I_i^{syn}$$

$$I_i^{ion} = -g_{Na} m^3 h (v_i - E_{Na}) - g_K n^4 (v_i - E_K) - g_L (v_i - E_L)$$

$$\dot{m} = \alpha_m(v)(1-m) - \beta_m(v) m$$
$$\dot{n} = \alpha_n(v)(1-n) - \beta_n(v) n$$
$$\dot{h} = \alpha_h(v)(1-h) - \beta_h(v) h$$

>Reciprocal delayed chemical connections (alpha functions)

$$I_i^{syn} = -\frac{g_{max}}{N_i} \sum_{spikesnn} \alpha(t - t_{spikes} - \tau)(v_i(t) - E_{syn})$$

$$\alpha(t) = \frac{1}{\tau_d - \tau_r}\left(\exp(-t/\tau_d) - \exp(-t/\tau_r)\right)$$

> Network Topology

Regular, Small World, Random, Scale-Free and All to All

>Delay

Homogeneous distribution

Heterogeneous distribution (Gamma function)

> Current

Single and distributed current

> Synchronization measure

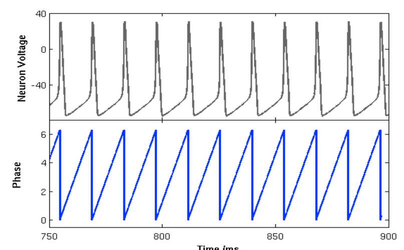
Phase of the neuron

$$\varphi(t) = \begin{cases} 2\pi \frac{(t-\tau_k)}{(\tau_{k+1}-\tau_k)} & \textit{firing neurons} \\ 0 & \textit{non firing neurons} \end{cases}$$

Local and Global synchrony

$$s_i(t) = \frac{1}{n} \sum_{j \in neigh(i)} \sin^2\left(\frac{\varphi_i(t) - \varphi_j(t)}{2}\right) \qquad s_i'(t) = \frac{1}{N} \sum_{j=1}^{N} \sin^2\left(\frac{\varphi_i(t) - \varphi_j(t)}{2}\right)$$

$$S^{loc} = \lim_{T\to\infty} \frac{1}{T} \int \left(\frac{1}{N}\sum_{i=1}^{N} s_i\right) dt \qquad S^{glob} = \lim_{T\to\infty} \frac{1}{T} \int \left(\frac{1}{N}\sum_{i=1}^{N} s_i'\right) dt$$



Results

Emergent States

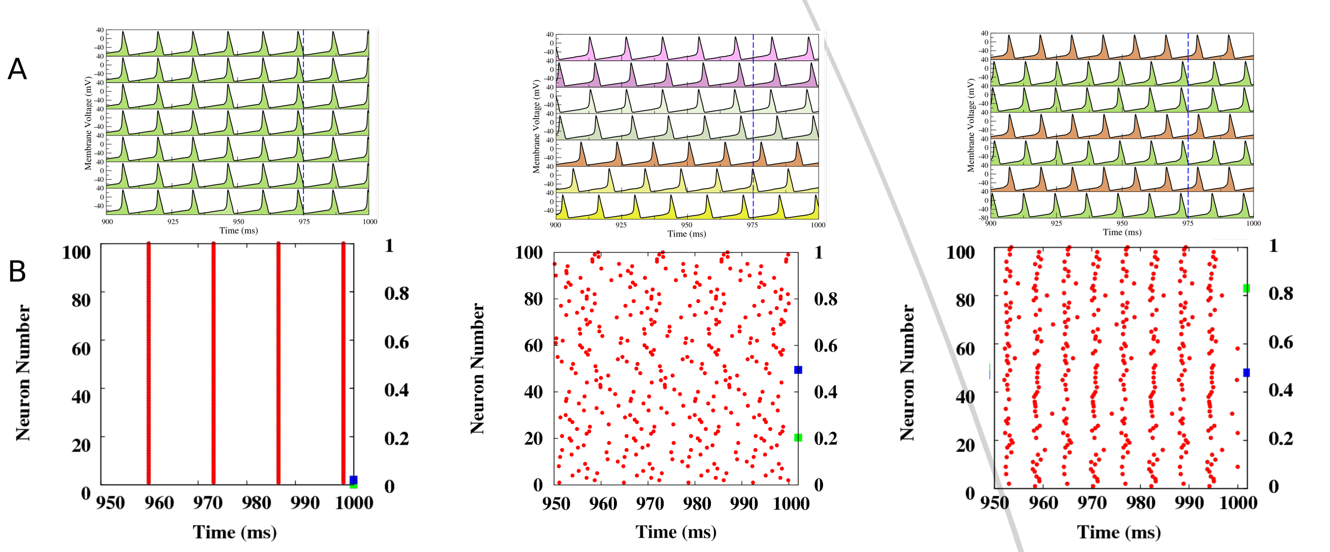


Figure 1. Panel A. Time Traces of the membrane potential of one neuron and its neighbors for different values of the delay in a random network. In Phase state: τ_0/τ=0.82, Out of phase state: τ_0/τ=0.96, Anti Phase state: τ_0/τ=1.1. For a coupling strength of 0.8 mS/cm². **Panel B:** Raster plots of spikes for the first 100 neurons for a random network. On the right y-axis are plotted the values of the local (green) and global (blue) synchronization indexes.

Effect of the topology (homogeneous delays)

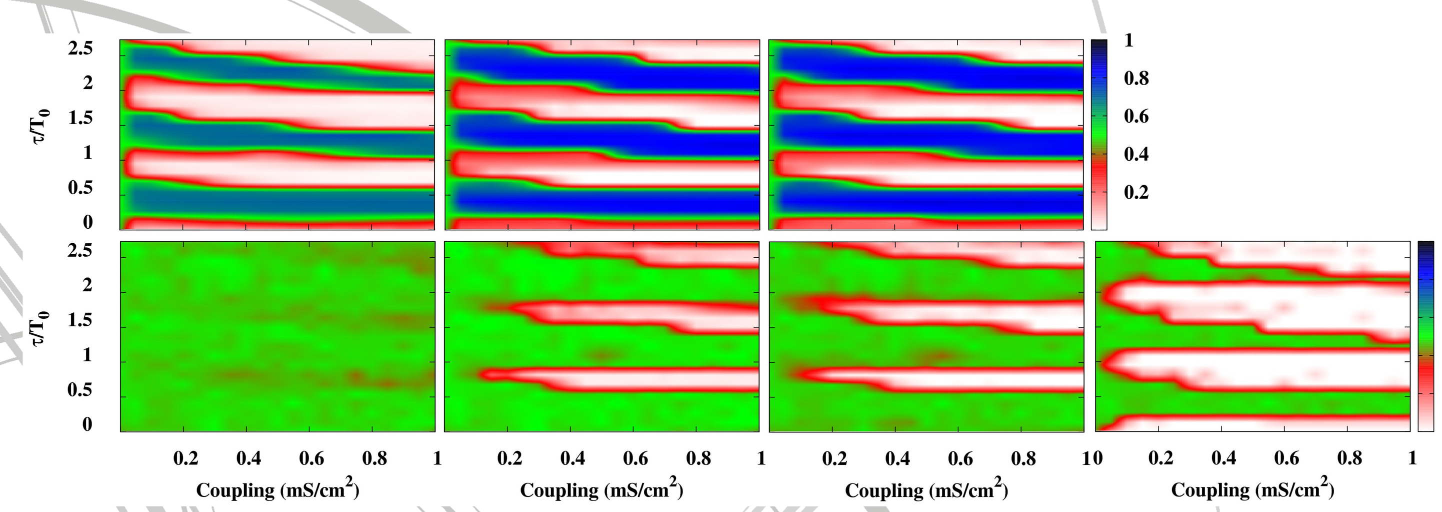


Figure 2. Local and global synchronization indexes for the different network topologies as function of the coupling strength and the delay of the connections. An index value of zero means synchronization, in-phase state, whereas a value of one represent an anti-phase state.

Effects of the diversity (heterogeneous currents)

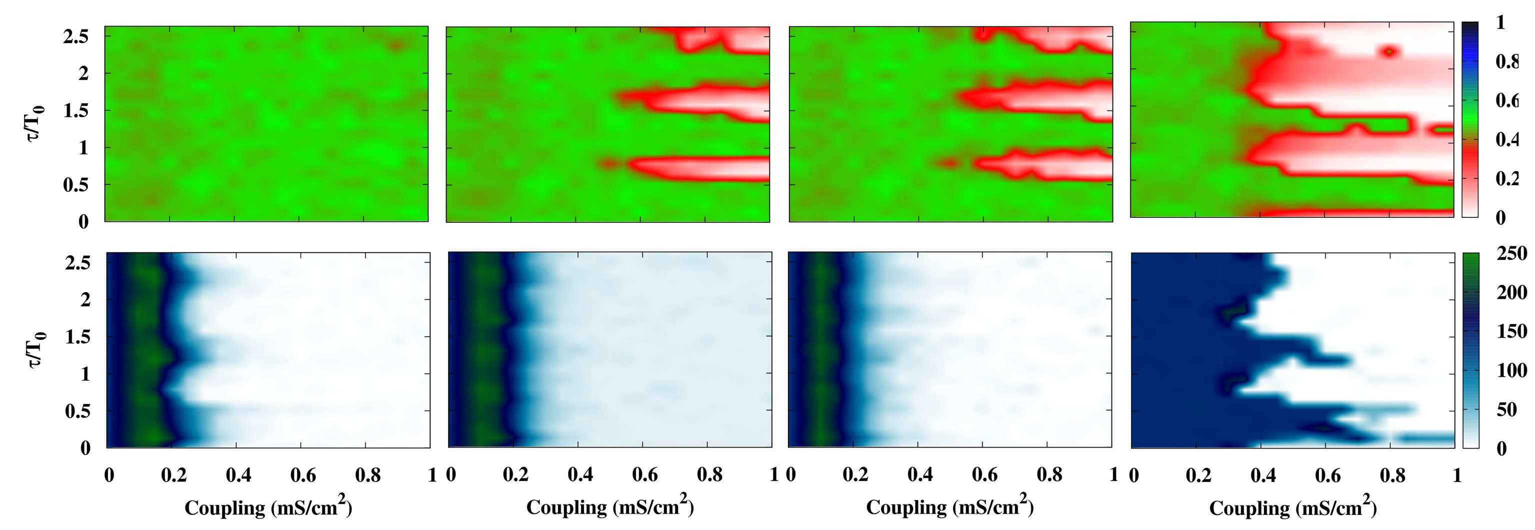
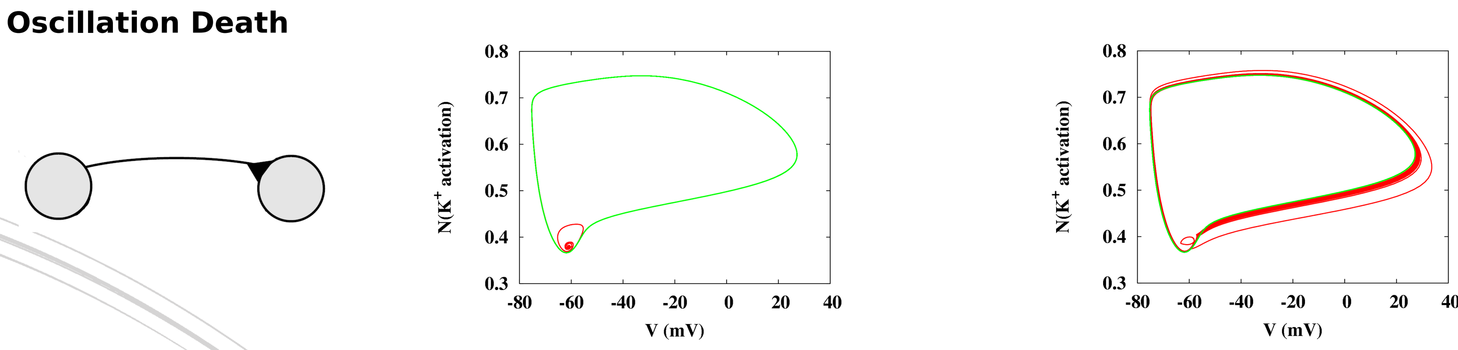


Figure 4. Global synchronization index (upper panel) and number of non firing neurons (bottom panel) for different topologies as function of the coupling strength and the delay in the connections. We consider a Gaussian distribution of currents centered at 9 μA/cm² with dispersion 2.5 μA/cm².

Oscillation Death



Understanding oscillations death: a simple motif of two coupled neurons. Phase space representation (N vs V) of stable-unstable limit cycle solutions of the HH equations for two different coupling values and the same bias current (6.3 μA/cm²). Left panel shows annihilation of repetitive firing cause by a excitatory synaptic input, showing the collapse of the oscillations to a singular point. Right panel shows repetitive firing for a higher value of the coupling strength.

Effect of the delays (heterogeneous delays)

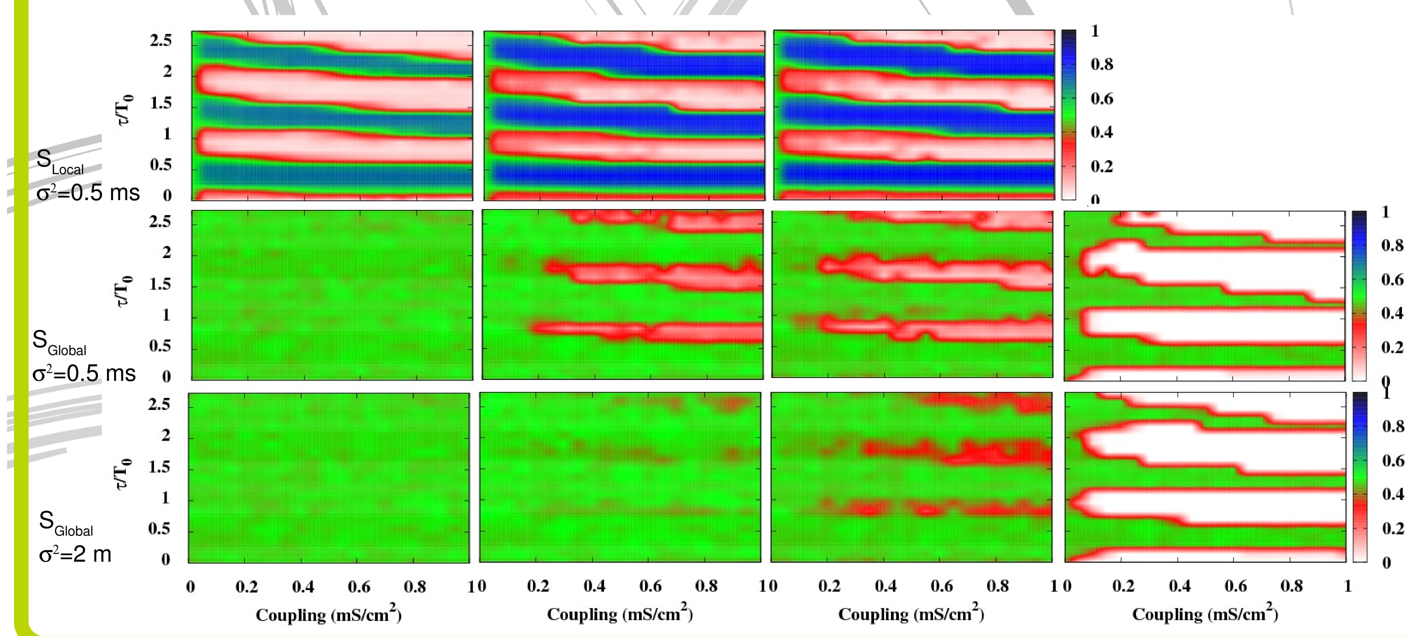
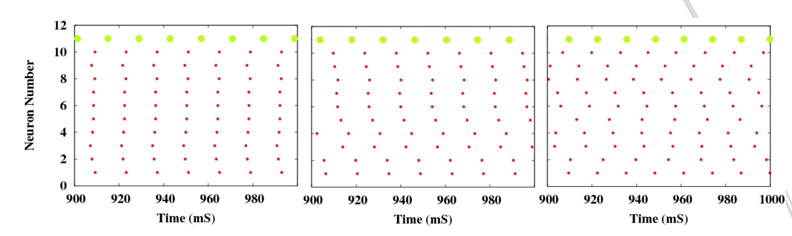
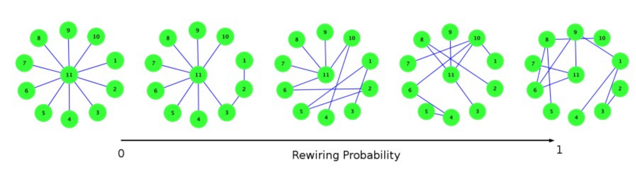


Figure 3. Local and global synchronization index for different network topologies as function of the coupling strength and the mean delay in the connections. Delays were generated according to a gamma distribution. The mean value was varied between 0.2 to 40 ms, and the variance was maintained constant at 0.5 ms in the middle panel and 2 ms in the lower one.

Why is the scale free topology more robust than the random one?





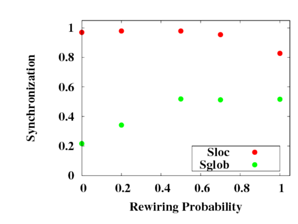


Figure 4. Raster plot for eleven neurons in a star-like network, for a coupling delay of 0.7mS/cm², mean delay 6 ms and variance 0.5 ms. The rewiring probability was varied between zero and one. In this way we change the topology from a star-like network (pr=0) to a random one (pr=1). This simple example allowed us to understand the robustness of the scale-free topology against the delay dispersion.

Conclusions

- The variation of the conduction delays reveals a resonant-like effect in the synchronization indexes in all the networks.
- Synchronized firing activity between one neuron and its neighbours is achieved in all the networks. A high randomness in the connections is required to reach a global synchronized state.
- For heterogeneous delays we observed that global synchronization is lost when the dispersion of the delay distribution increases.
- The effect of the distribution of currents gives rise to a new state, the oscillations death.