Constructive effects of diversity in the synchronization of a model for the circadian clock in mammals

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Abstract

In this work we analyze the role of the light-dark cycle and constructive diversity in the dynamics of a system of circadian neurons [1]. We introduce neuronal heterogeneity in the form of quenched noise, by rescaling the individual neuronal periods by a scaling factor drawn from a normal distribution. The system response to the light-dark cycle periodicity is studied as a function of the interneuronal coupling strength, external forcing amplitude and neuronal heterogeneity. We show that the right amount of diversity helps the system to respond globally in a more coherent way to the external forcing. Our proposed mechanism for neuronal synchronization under external periodic forcing is based on heterogeneity-induced oscillators death, damped oscillators being more entrainable by the external forcing than the self-oscillating neurons.

Model equations and order parameters

$$\tau_i \frac{dX_i}{dt} = \nu_1 \frac{K_1^4}{K_1^4 + Z_i^4} - \nu_2 \frac{X_i}{K_2 + X_i} + \nu_c \frac{KF}{K_c + KF} + \frac{L_0}{2}(1 + \sin \omega t)$$

$$\tau_i \frac{dY_i}{dt} = k_3 X_i - \nu_4 \frac{Y_i}{K_4 + Y_i}$$

$$\tau_i \frac{dZ_i}{dt} = k_5 Y_i - \nu_6 \frac{Z_i}{K_6 + Z_i}$$

$$\tau_i \frac{dV_i}{dt} = k_7 X_i - \nu_8 \frac{V_i}{K_8 + V_i}$$

$$F = \frac{1}{N} \sum_{i=1}^{N} V_i$$

dynamic variables:
X – concentration mRNA (*per/cry*),
Y – protein (PER/CRY),
Z – active protein (inhibitor) and
V – neurotransmitter

τ_i rescales individual periods (Gaussian distributed)

Synchrony order parameter [2]

Ranges between 1 (perfect synchronization) and 0 (no synchronization):

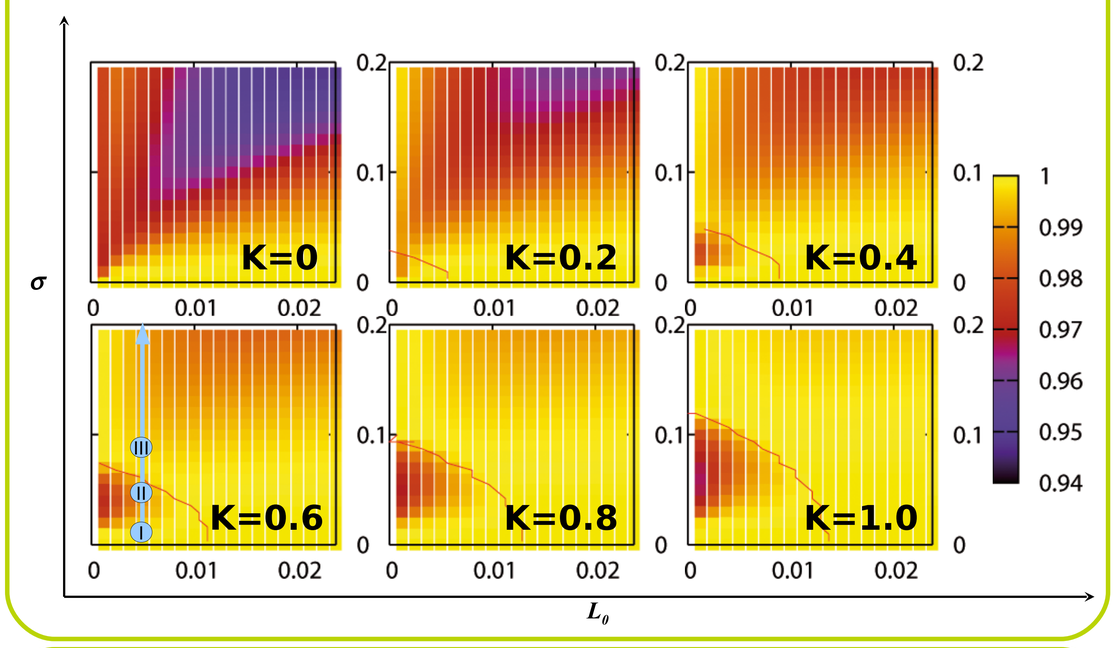
$$\rho = \sqrt{1 - \left\langle \frac{\frac{1}{N}\sum_{i=1}^{N} |V_i(t) - F(t)|^2}{\frac{1}{N}\sum_{i=1}^{N} |V_i(t)|^2} \right\rangle_t}$$

Response to external frequency [3]

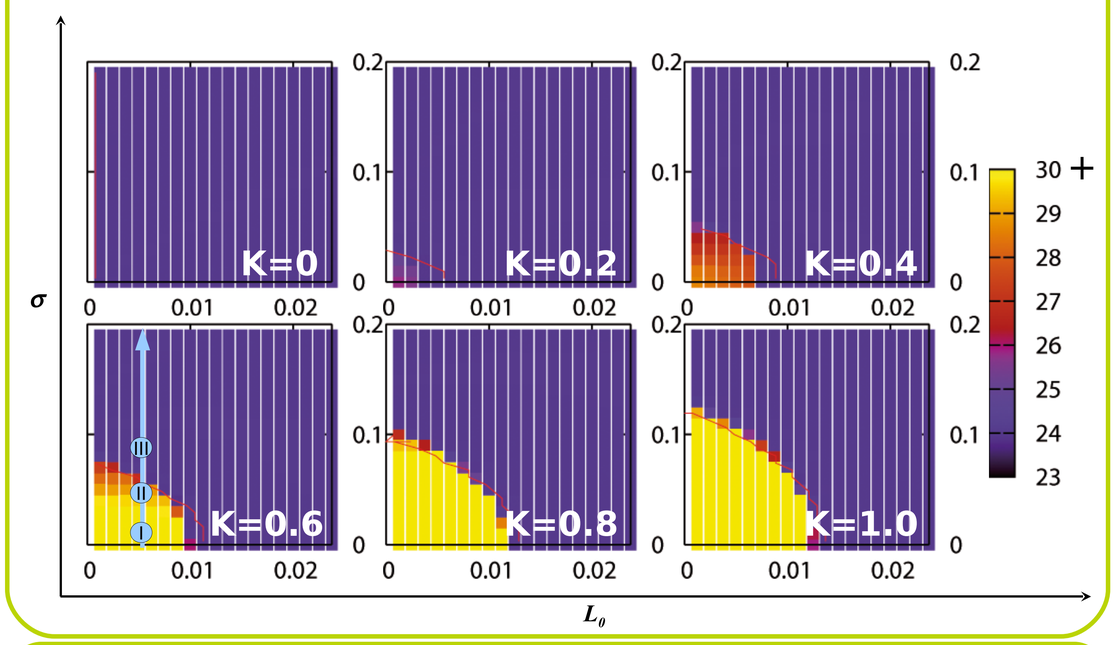
Power of driving frequency in output spectrum:

$$R = \frac{4}{L_0^2} \left| \langle e^{-i\omega t} F(t) \rangle \right|^2$$

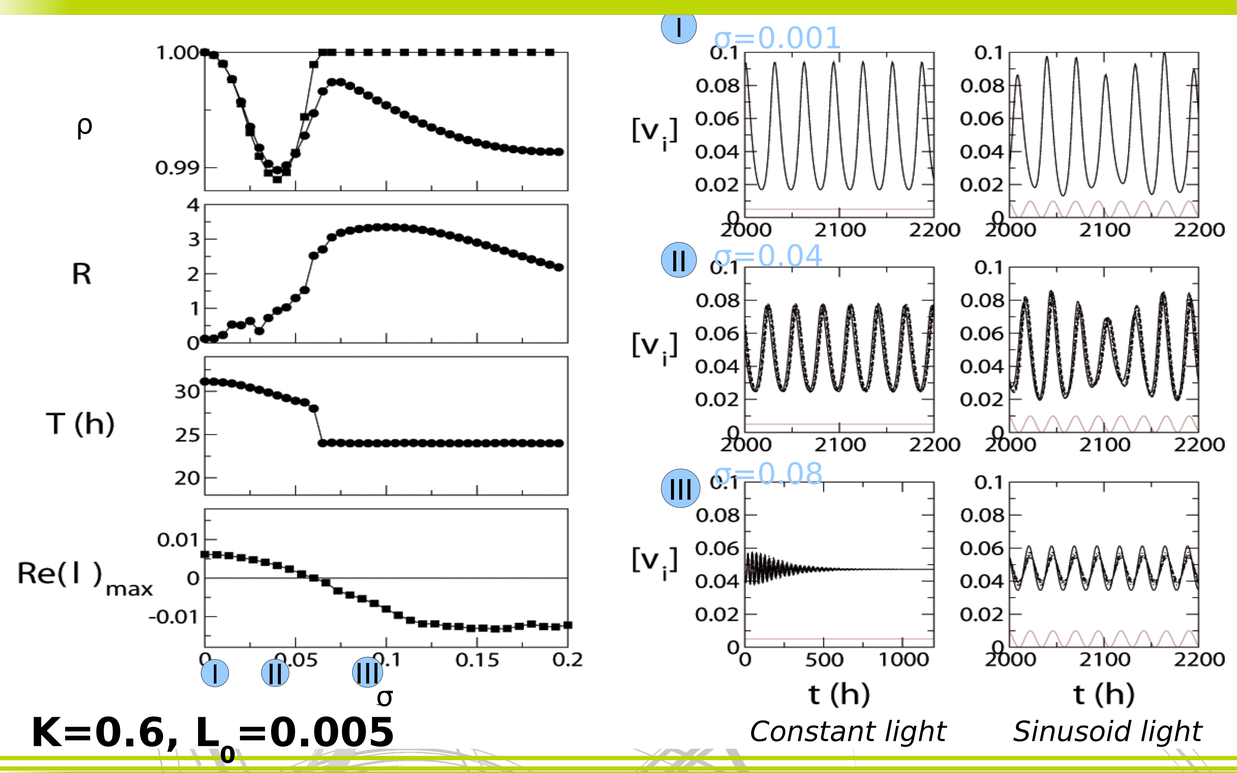
Synchrony order parameter ρ



Mean of individual periods

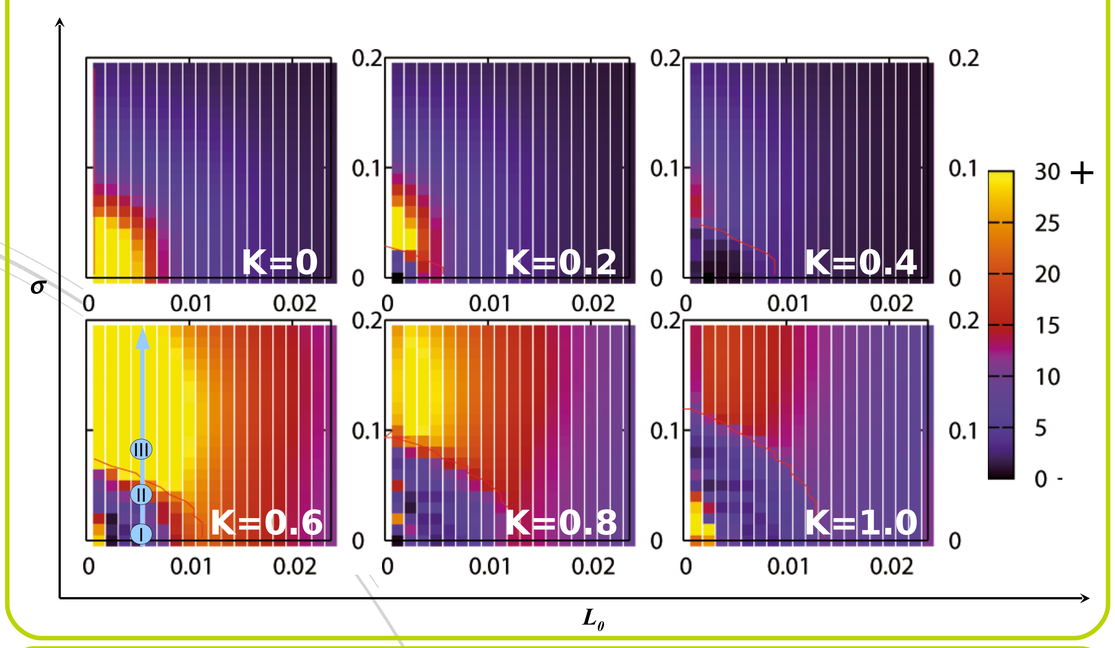


Simulation results



K=0.6, L_0=0.005

Response R to external frequency



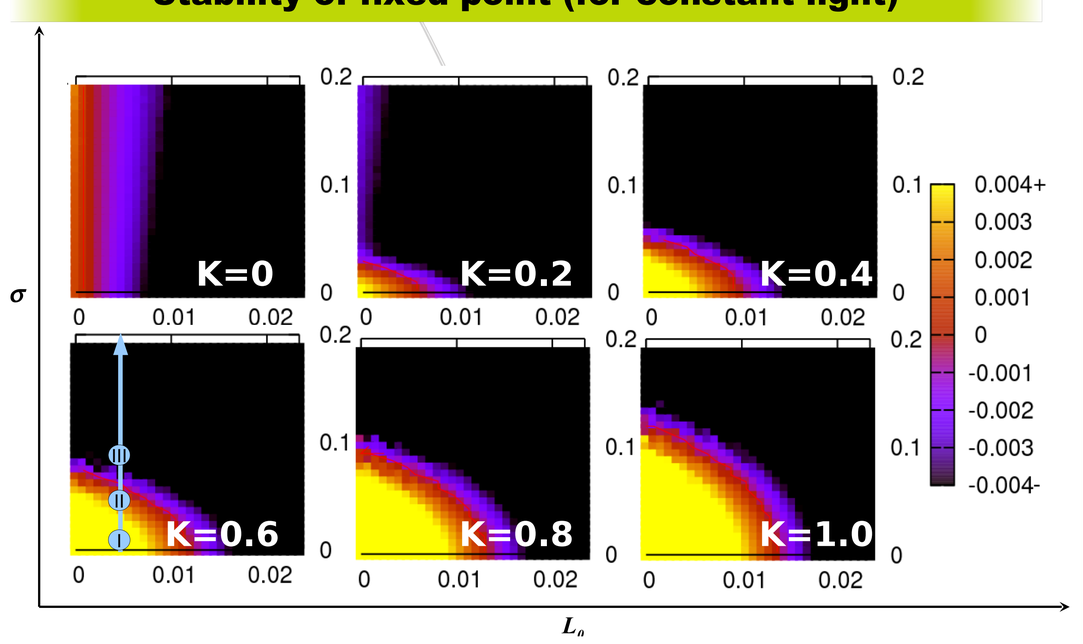
Conclusions

Diversity (at optimal levels; not too large, not too small) is able to improve the collective response of the neuron ensemble to the 24h cycle.
The mechanism is related to the oscillator death it produces: the damped oscillators follow better the external signal than the ones self-oscillating with different periods (which in the strong coupling regime leads to fast oscillations).

References

Model equations as in:
[1] *Gonze et.al., Biophys J, Vol 89, July 2005, 120-129*
Order parameters from:
[2] *Garcia-Ojalvo et.al.,PNAS USA, 101, (2004), 10955–10960*
[3] *Toral et.al.,to appear in Int. J. Bif. Chaos*

Stability of fixed point (for constant light)



maximal real part of eigenvalue (mean) for random matrices with N=100