Thematic Institute on Lyapunov analysis, ISC, Paris, 27 October 2009

Stretching structures from finite-size Lyapunov exponents:

Their impact across all biological scales

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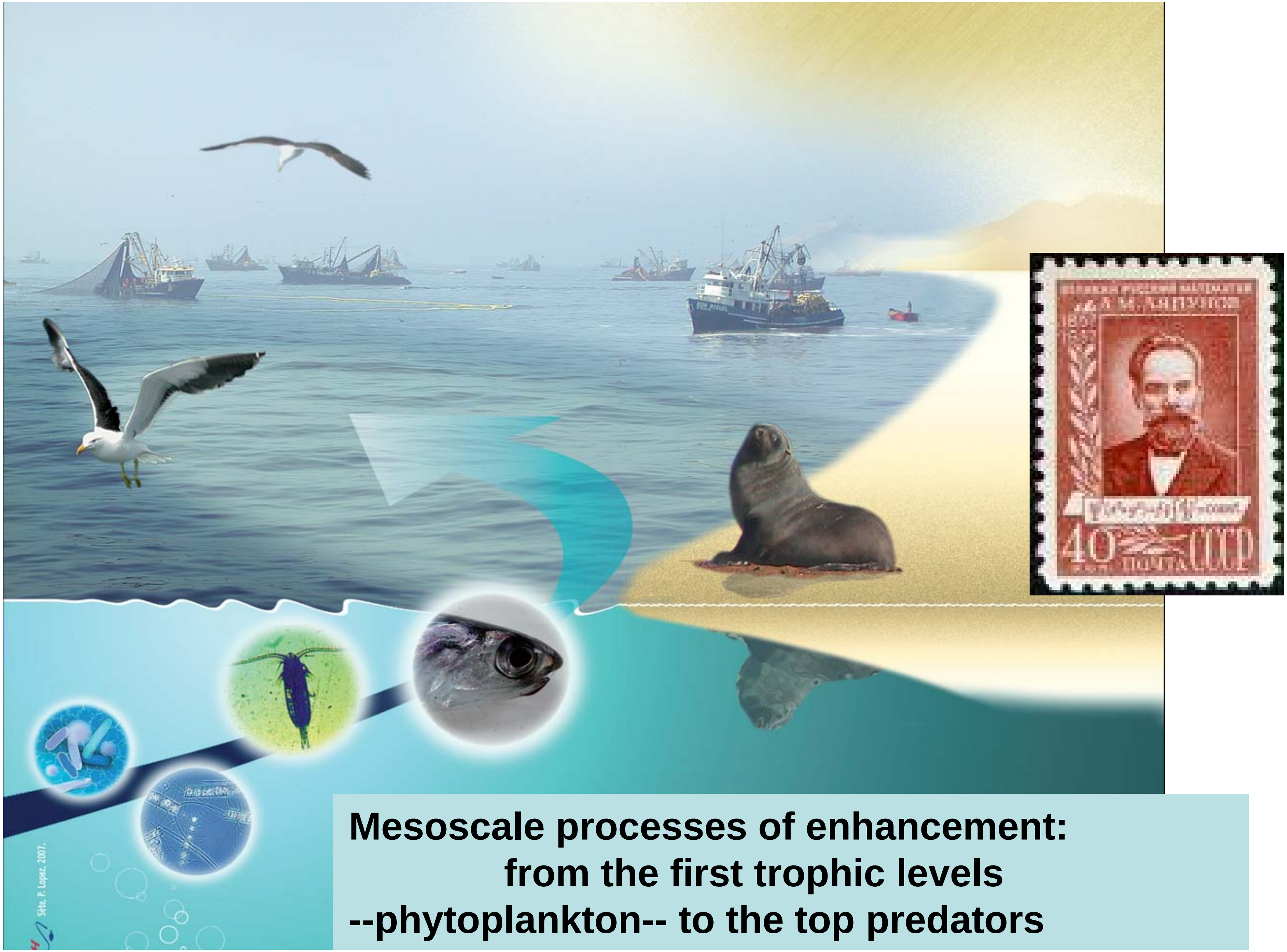
with C. López, F. d'Ovidio, V. Fernández, J. Isern-Fontanet,
E. García-Ladona, V. Rossi, V. Garçon, J. Sudre, S. Wiggins,
A.M. Mancho, E. Tew Kai, F. Marsac, H. Weimerskirch, …

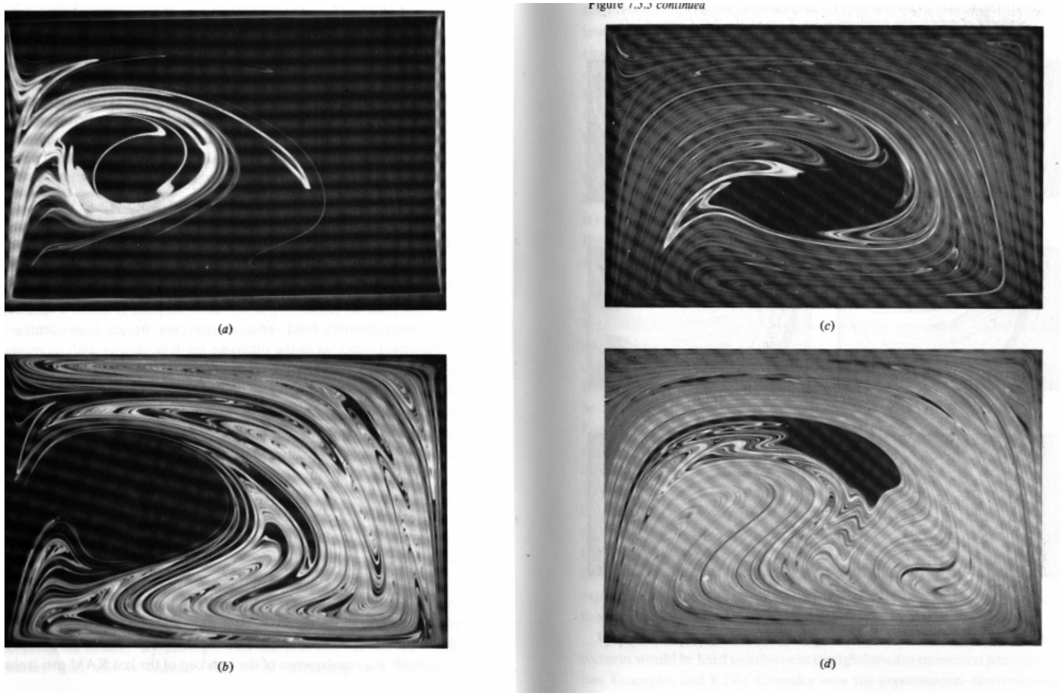


**Mesoscale processes of enhancement:
from the first trophic levels
--phytoplankton-- to the top predators**

OUTLINE

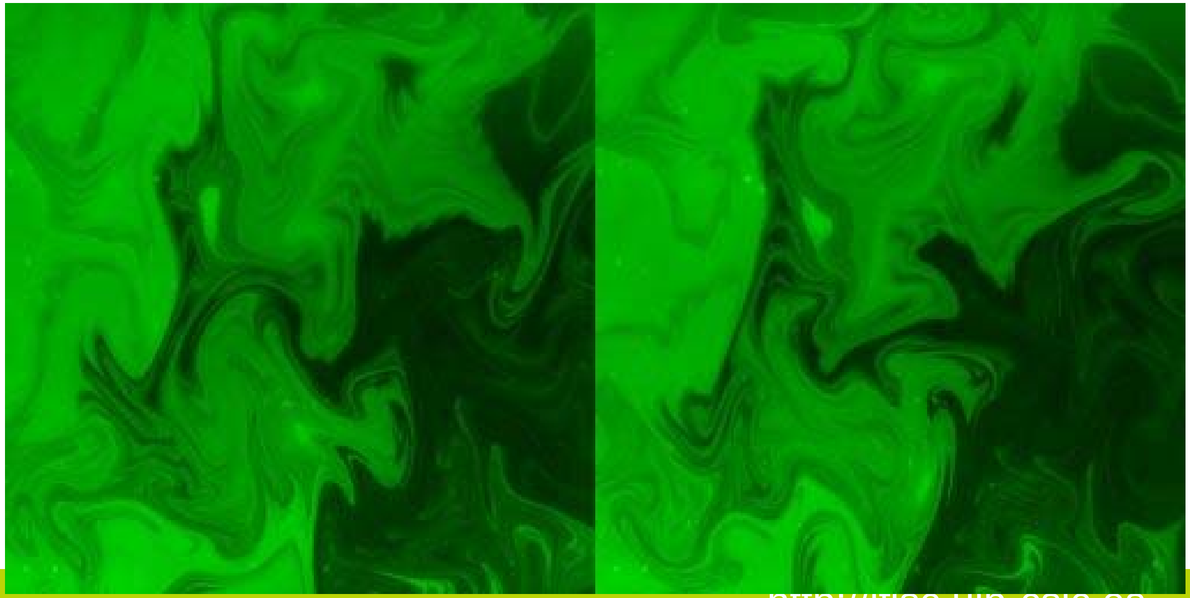
- The dynamical systems approach to fluid transport: hyperbolic points, manifolds, …
- Finite-size Lyapunov exponents
  - Abiotic tracers
  - Phytoplankton
  - Frigatebirds

The dynamical systems approach to fluid transport



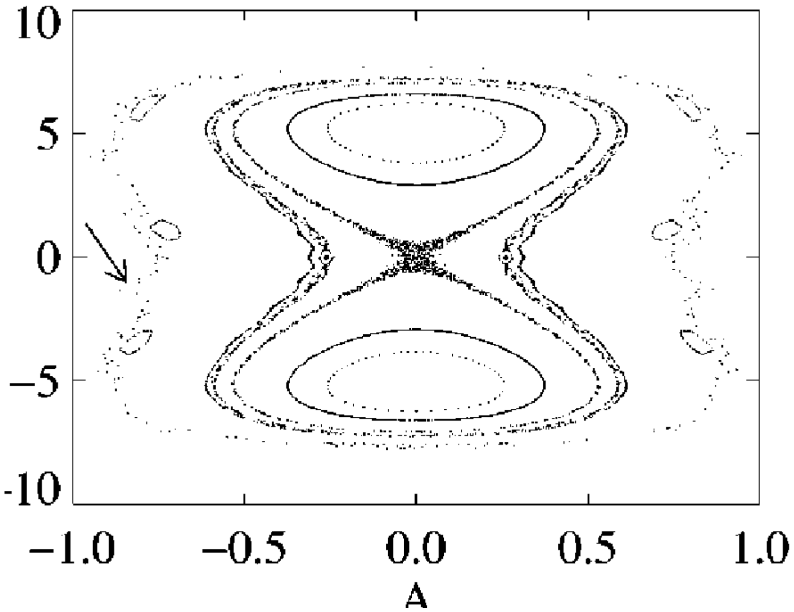
J.M. Ottino, The kinematics of mixing: stretching, chaos, and transport (Cambridge, 1989)

D. Rothstein, E. Henry, J. P. Gollub, Nature 401, 770 (1999)

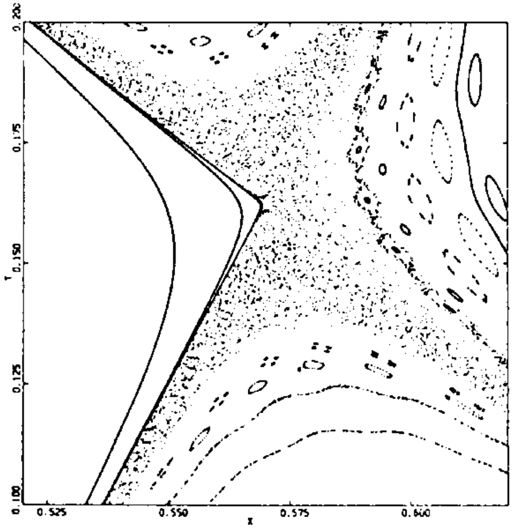


$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

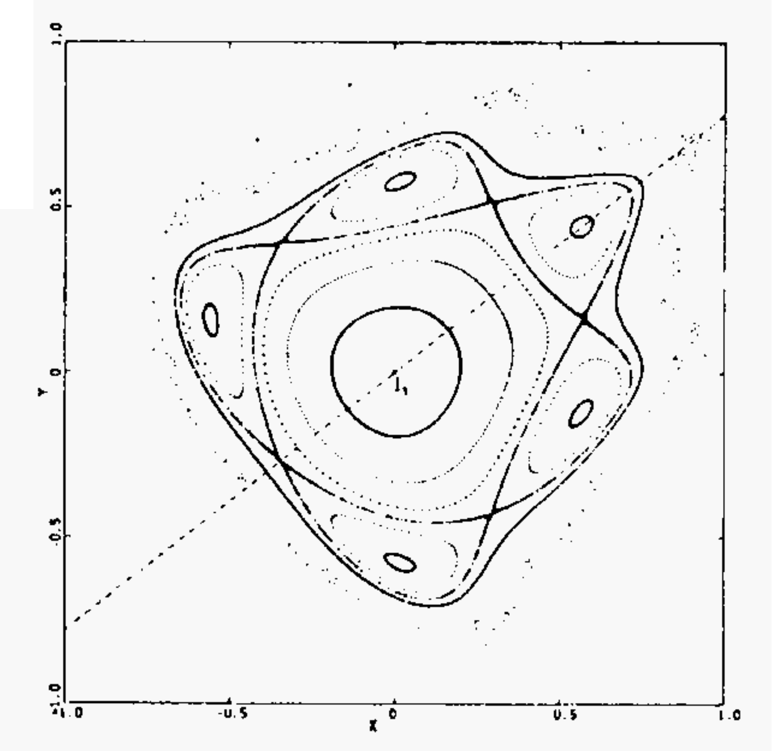
Twodimensional
time-periodic dynamical systems



Poincaré sections
or estroboscopic
maps



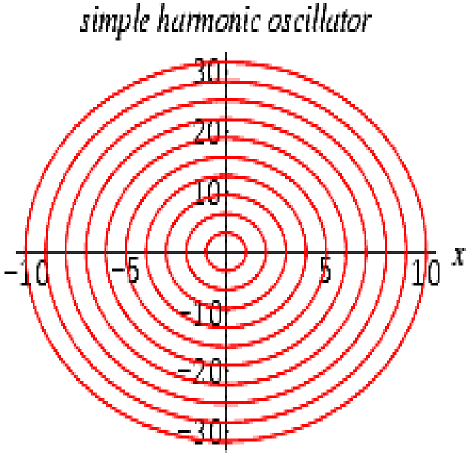
Chaotic seas,
KAM tori, …

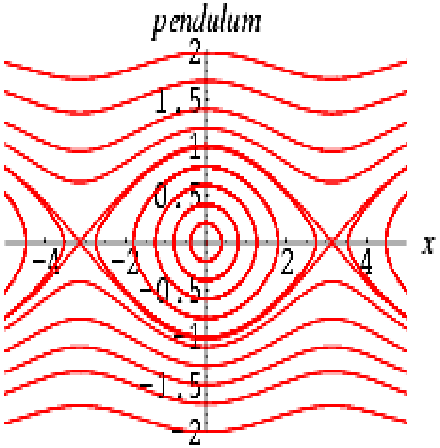


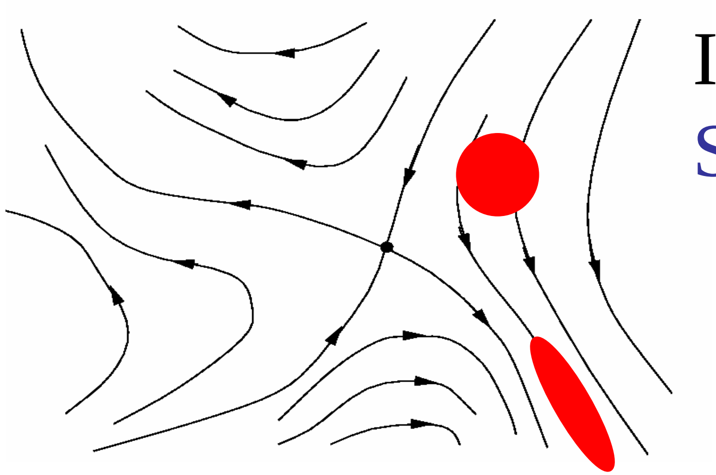
THE DYNAMICAL SYSTEMS APPROACH TO LAGRANGIAN FLUID TRANSPORT

Trajectories of two-dimensional steady or periodic flows are organized by the fixed points, or periodic orbits of the dynamical system

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t))$$





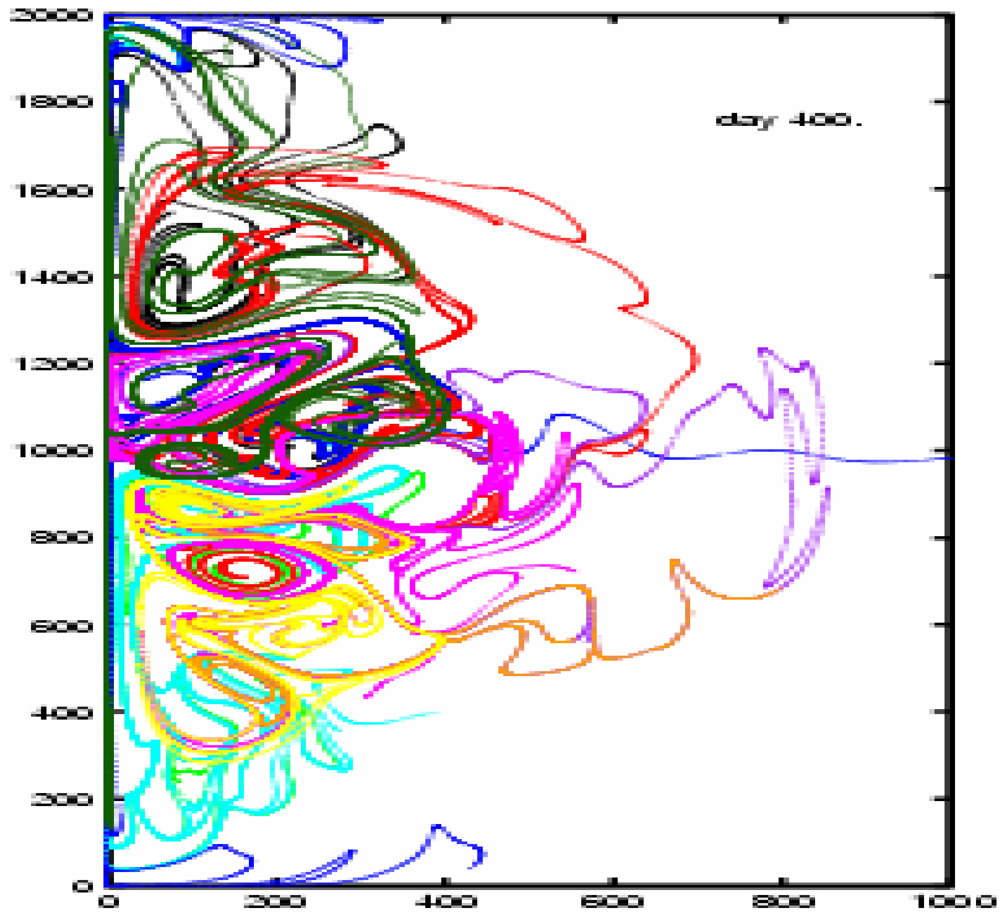


If hyperbolic:

Stable and unstable manifolds → separatrices

Tracers tend to approach unstable manifolds

But
unsteady flows …

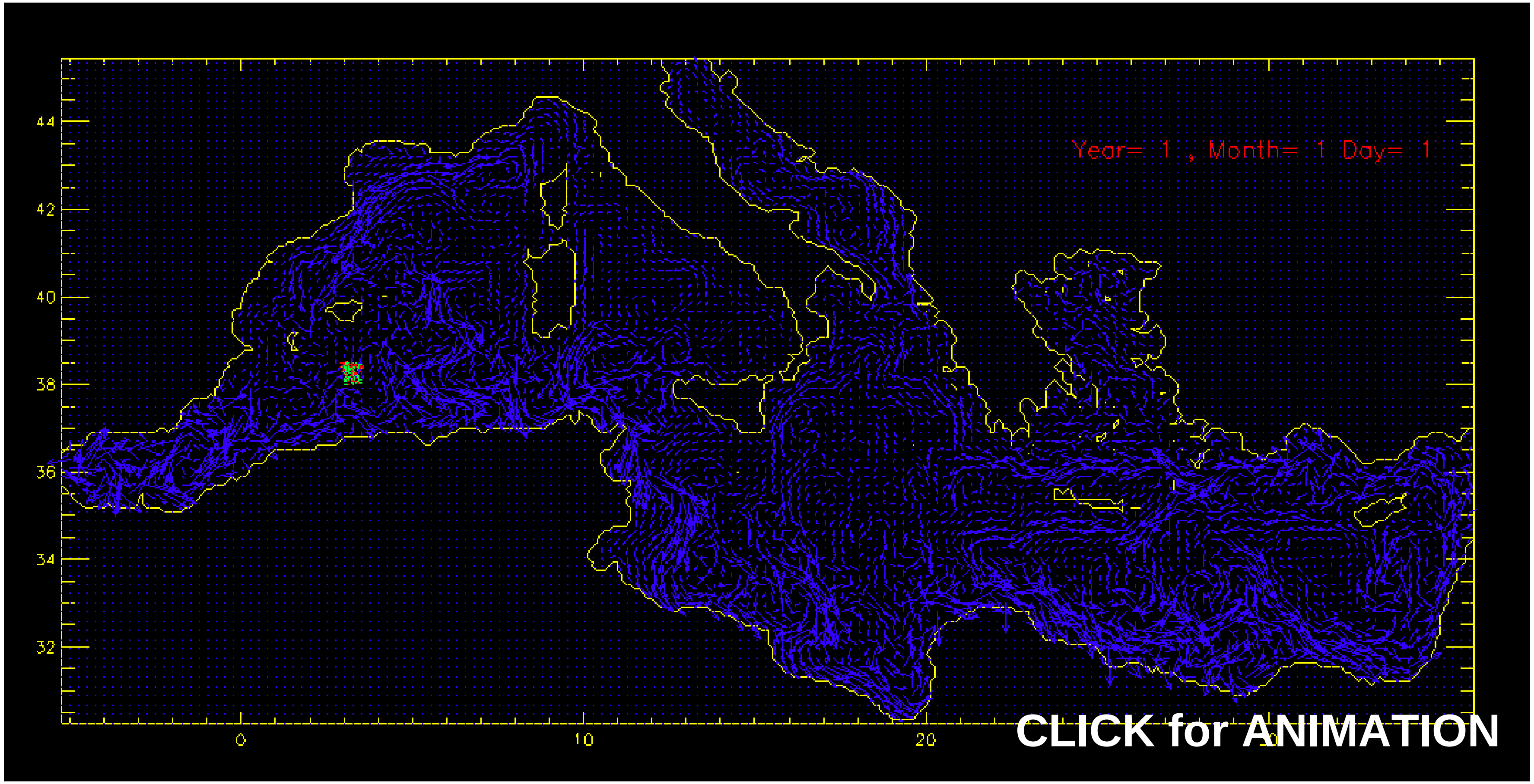


From Mancho, Small and Wiggins, 2005

Is there any particular subset of hyperbolic points and manifolds organizing the dynamics (the equivalent to the fixed points in autonomous systems) ? How to select them among this mess ?

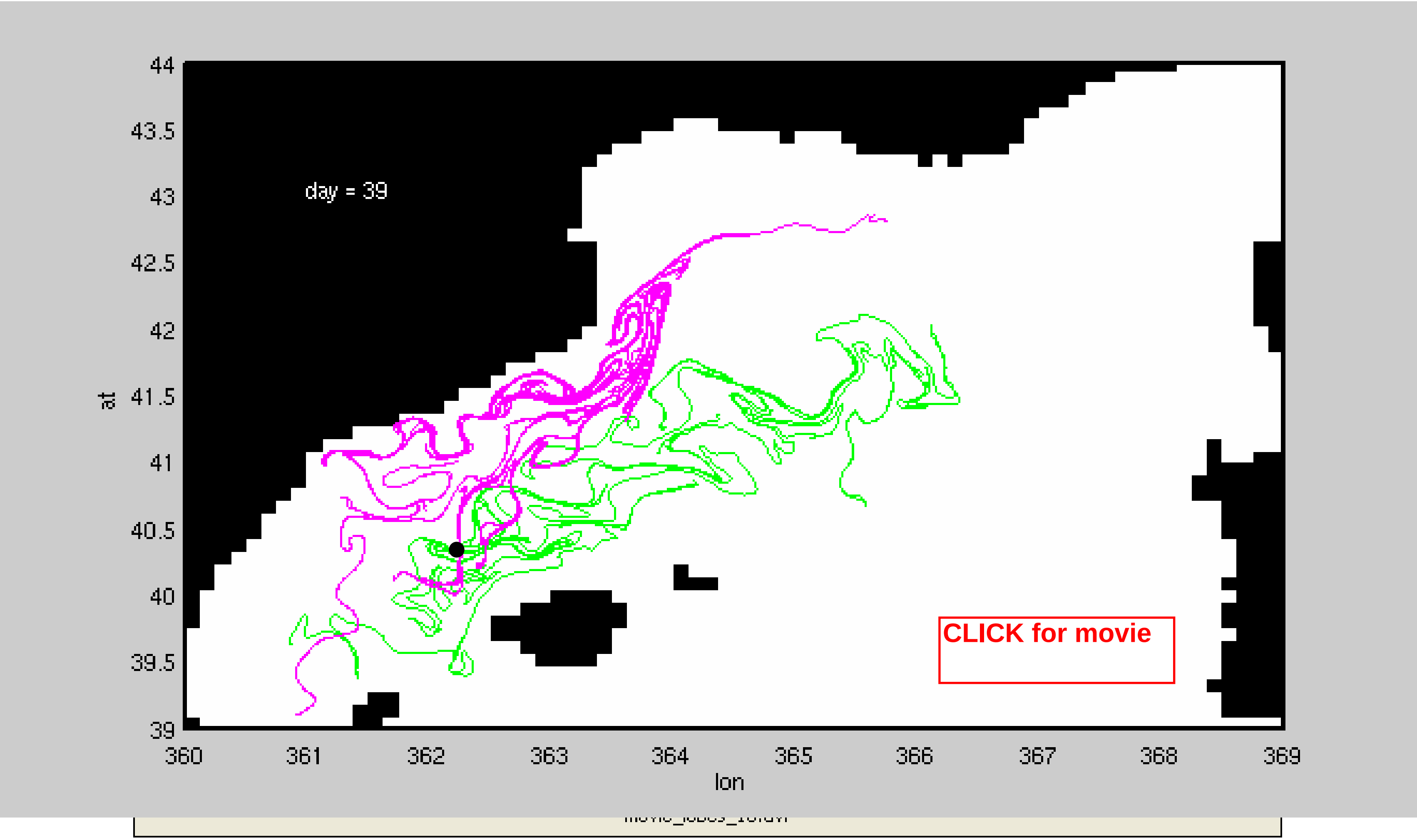
Identifying the relevant trajectories and manifolds in time-aperiodic dynamical systems

- Attracting or repelling material lines (Haller, Poje, …)
- Distinguished hyperbolic points (Wiggins, Ide, Mancho, …)
- Stretching-field methods: Finite-time Lyapunov exponents, Finite-size Lyapunov exponents, …(Vulpiani, Cencini, Legras, Artale, Haller, Lekien, …)
- Leaking (Tél, Schneider…)
- …



DieCAST model for the full Mediterranean
Primitive equations,
48 vertical levels, 1/8 º horizontal resolution,
climatological forcings … → 5 years of dayly velocity fields

Stable manifold ● Hyperbolic trajectory Unstable manifold



Mancho, Hernández-García, Small, Wiggins, Fernández, J. Phys. Ocean. 38, 1222 (2008)

Distinguished Hyperbolic Trajectories (DHTs)

(Mancho, Small, Wiggins, Ide, 2003; Mancho, Small, Wiggins, 2004, Physics Reports, 2006)

This methodology distinguishes hyperbolic points which are
‘more fixed than their neiborhood’
The analogous to fixed points in time-independent flows or to
periodic orbits in periodic flows.

$dx/dt=v(x) \longrightarrow$ Tentative trajectory $x_0(t)$

Expand around it: $x(t)= x_0(t)+\delta(t) \longrightarrow d\delta/dt = Dv(x_0(t))\,\delta + NL(\delta,t)$

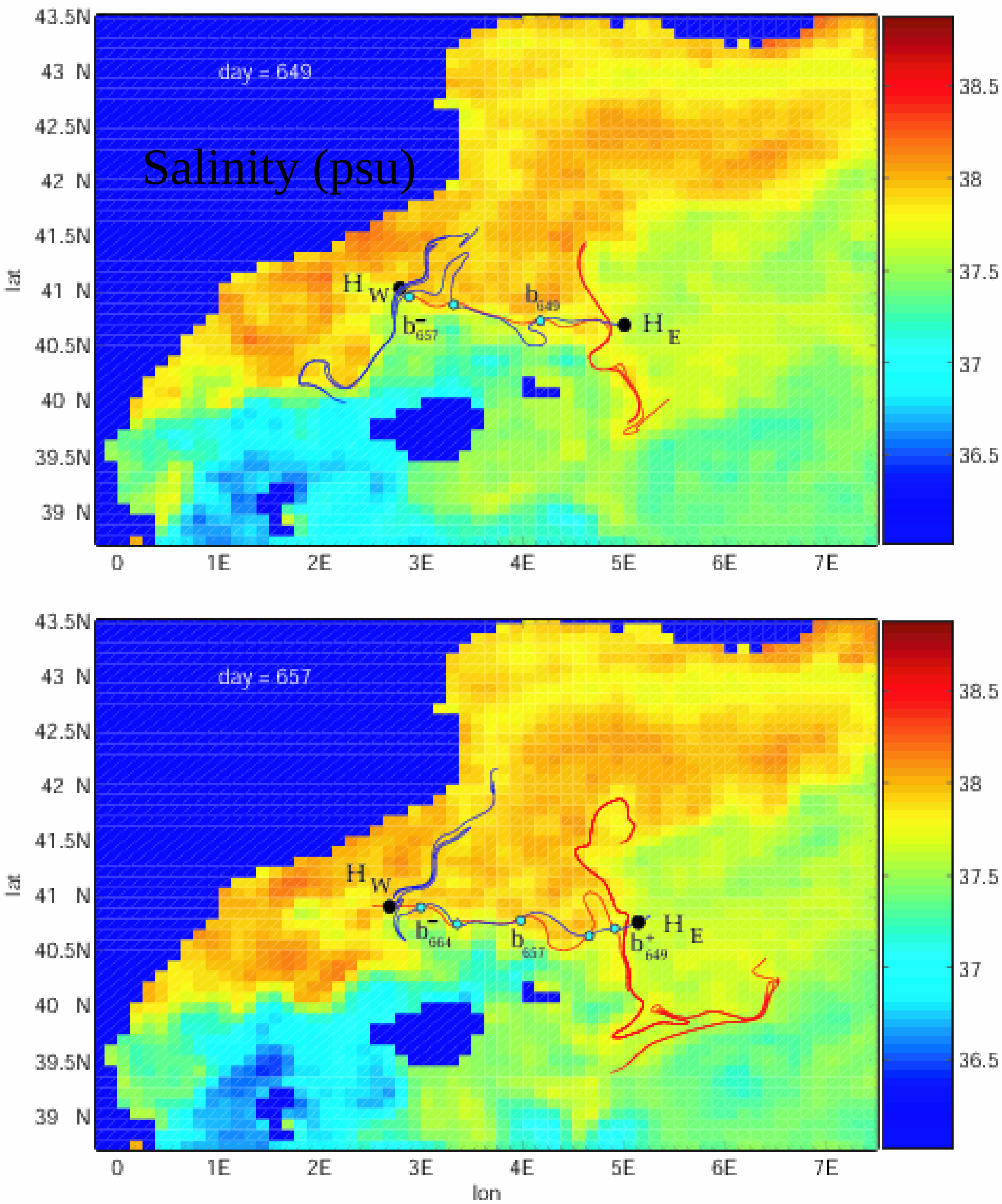
General solution: $\delta(t)$ = sum of ~~exponentials~~ + Integral(exp … $NL(\delta,t)$)

$\cancel{\delta_h(t)}$ $\delta_p(t)$

Choosing $x_0(t)$ in this way garantees a distinguished solution $x(t)= x_0(t)+\delta(t)$ which is not wandering arround exponentially fast, and neighbouring trajectories approach or escape from it exponentially

The stable and unstable manifolds of DHTs organize the flow

Manifolds in the Mediterranean



Unstable manifold of H_W

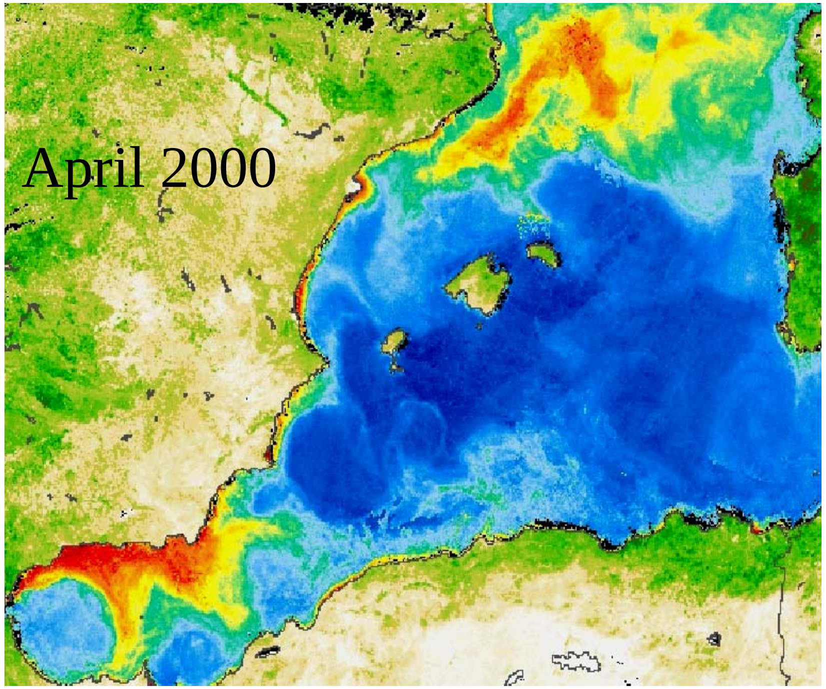
Stable manifold of H_E

Two DHT with manifolds pointing against each other, and intersecting in between:

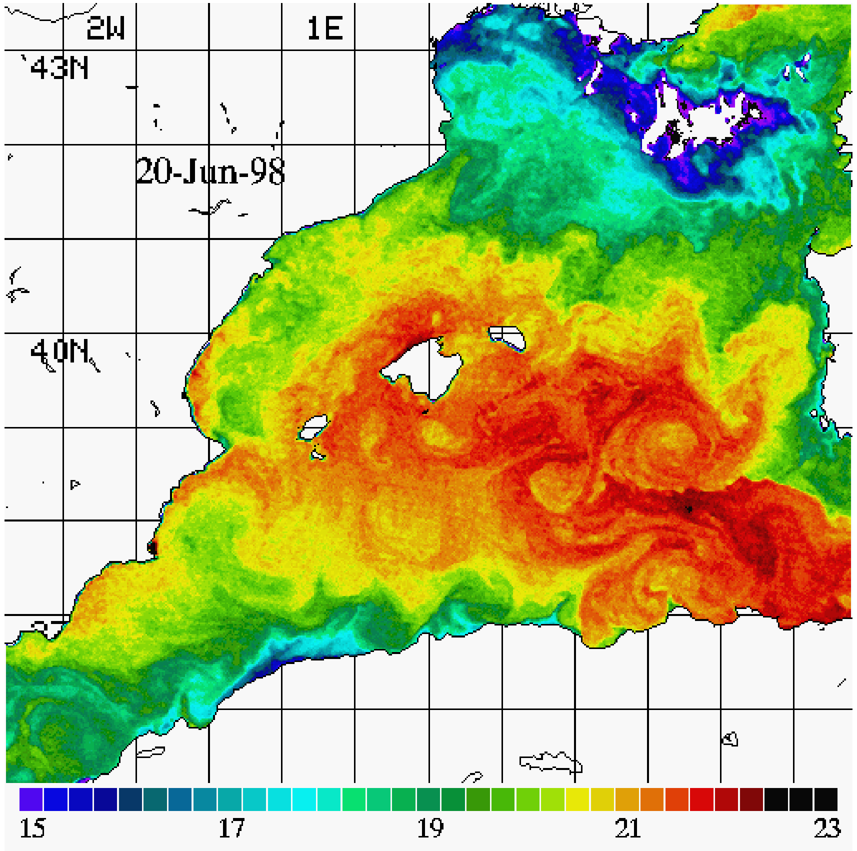
The turnstile mechanism

for transport across a Lagrangian line

The North-Balearic Front

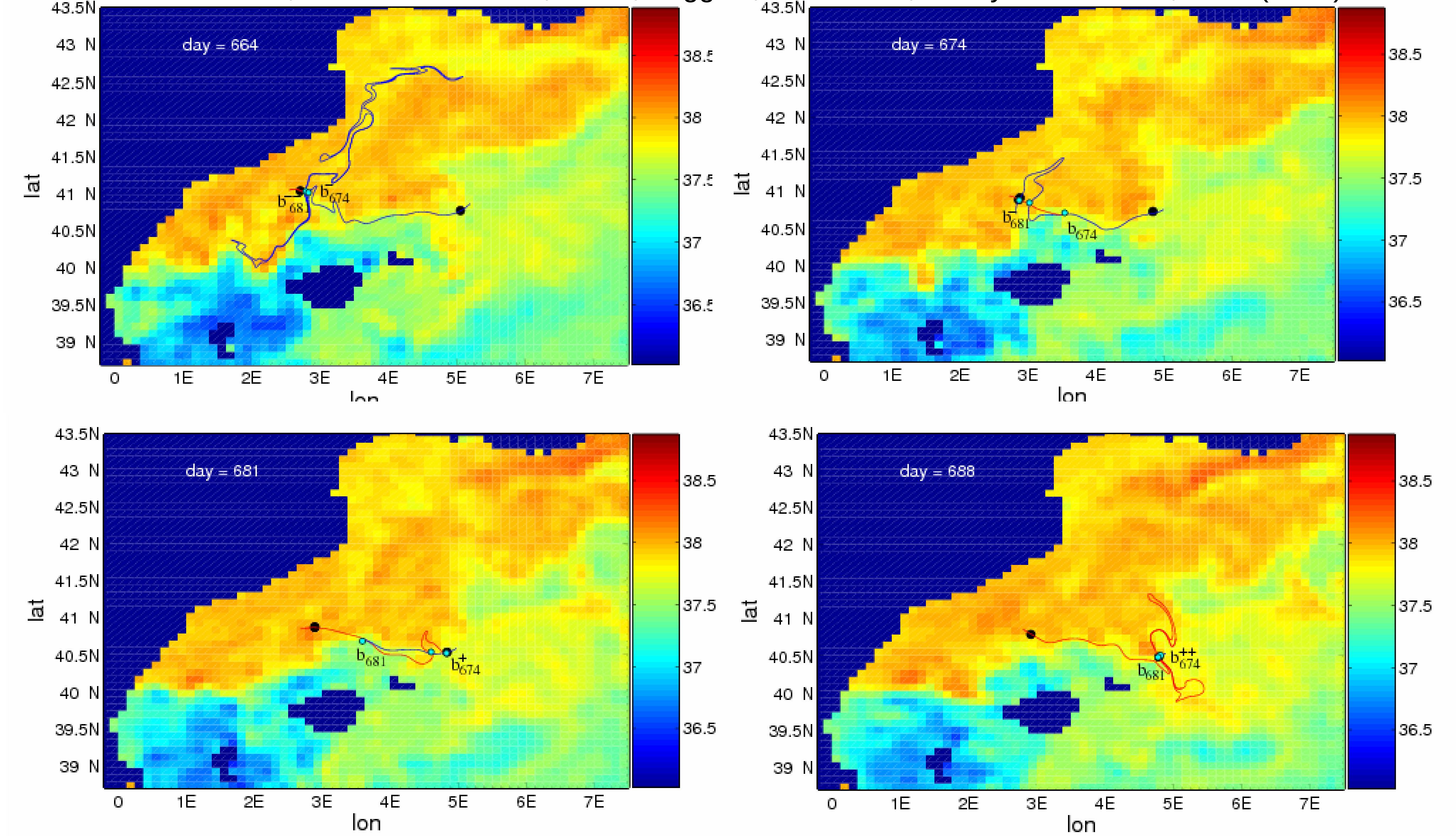


Chlorophyll



SST

Mancho, Hernández-García, Small, Wiggins, Fernandez, J. Phys. Ocean. 38, 1222 (2008)

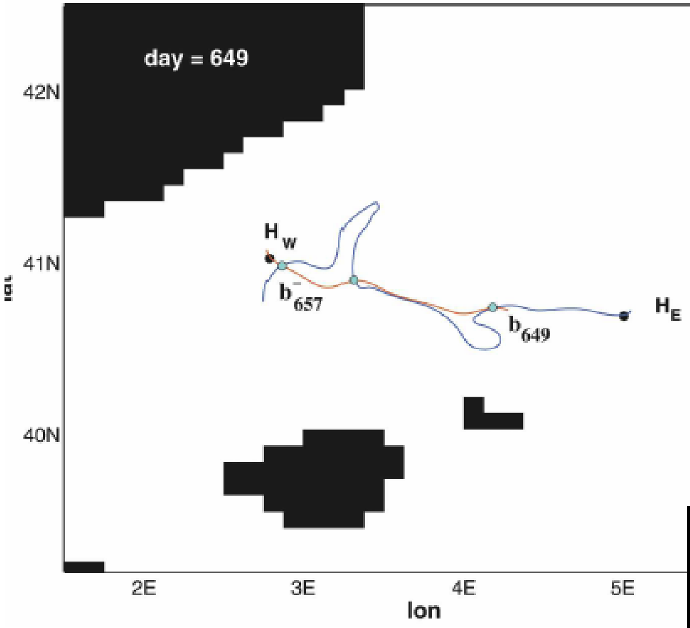


LOBE DYNAMICS, ROUTES OF TRANSPORT

The area of the lobes can be calculated:

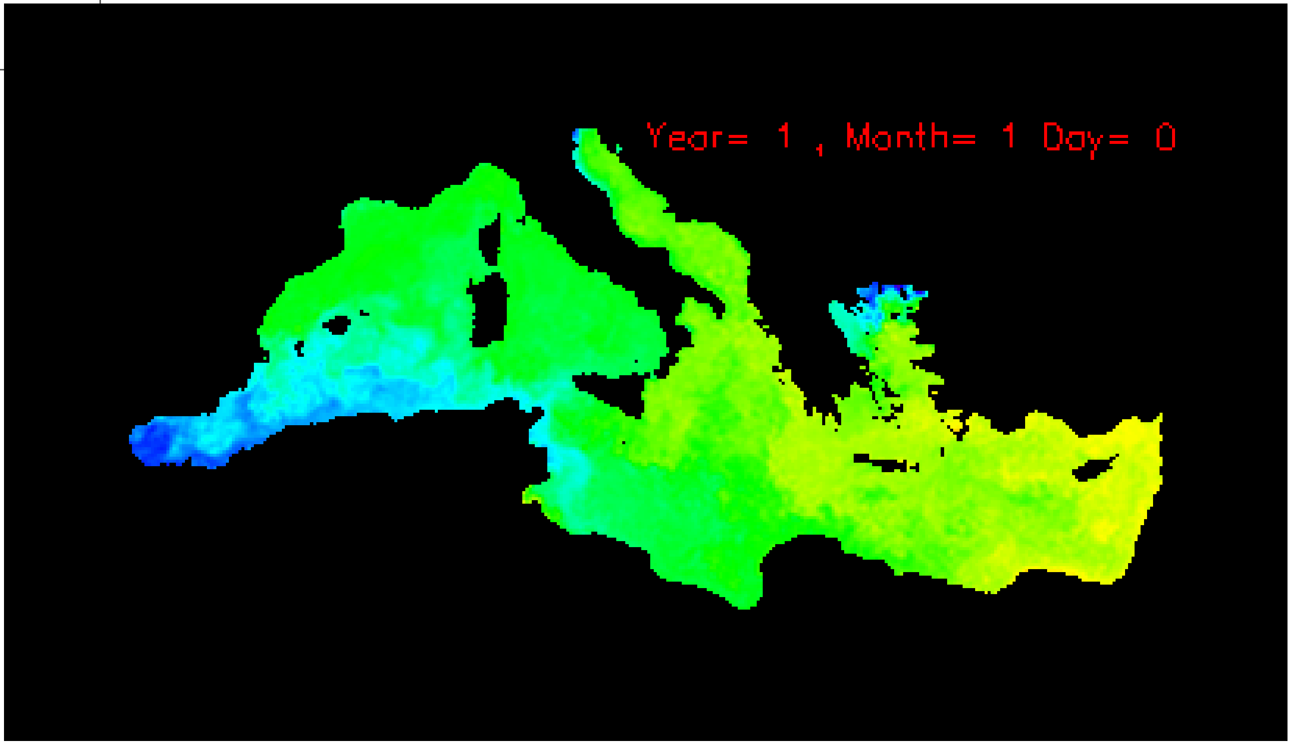
transport ≈ 0.02-0.04 Sv << 0.5 Sv (current)

BARRIER or FRONT



This gives information manifold by manifold

Can we have a more global view ?

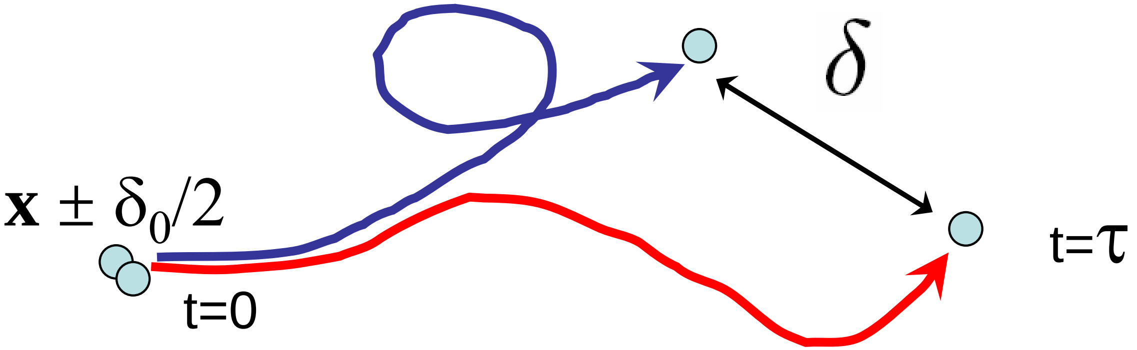


$$\lambda(t) = \lim_{\|\delta(0)\|\to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t\to\infty} \lambda(t)$$

Lyapunov exponent



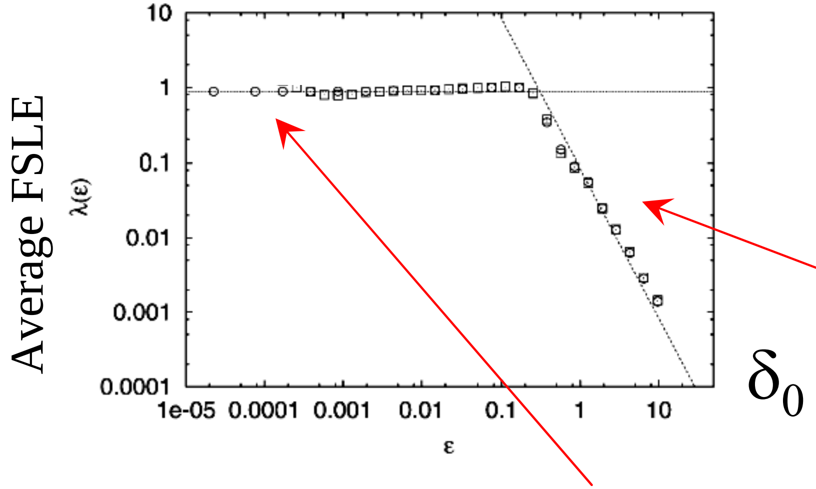
$$\boxed{\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}}$$

Finite-size Lyapunov exponent
FSLE

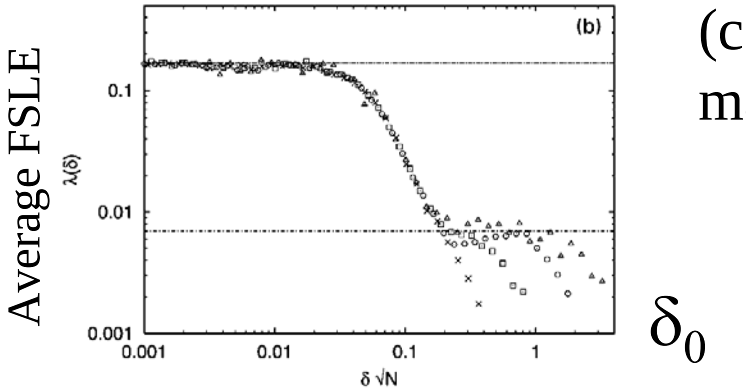
All the quantities are also functions of the initial position and time:

$$\lambda(\mathbf{x}, t, \delta_0, \delta_f)$$

A chaotic map



System with several time scales



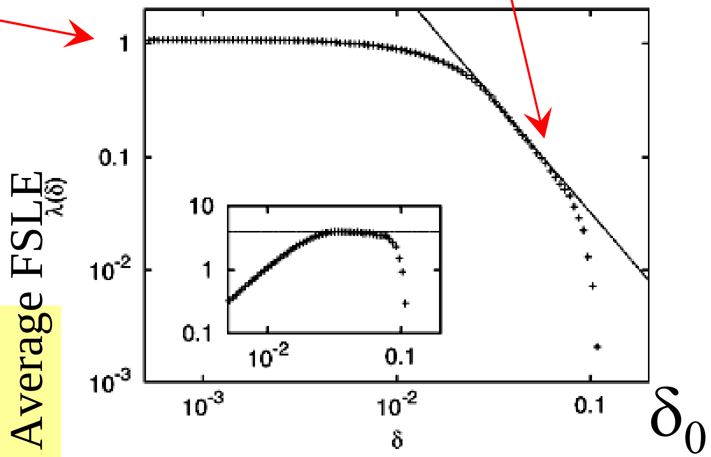
(coupled maps)

Subexponential growth (diffusion regime)

Exponential growth of separations (chaotic regime)

When $\delta_0 \to 0$,
FSLE→Lyapunov
and when $t \to \infty$,
FTLE→Lyapunov

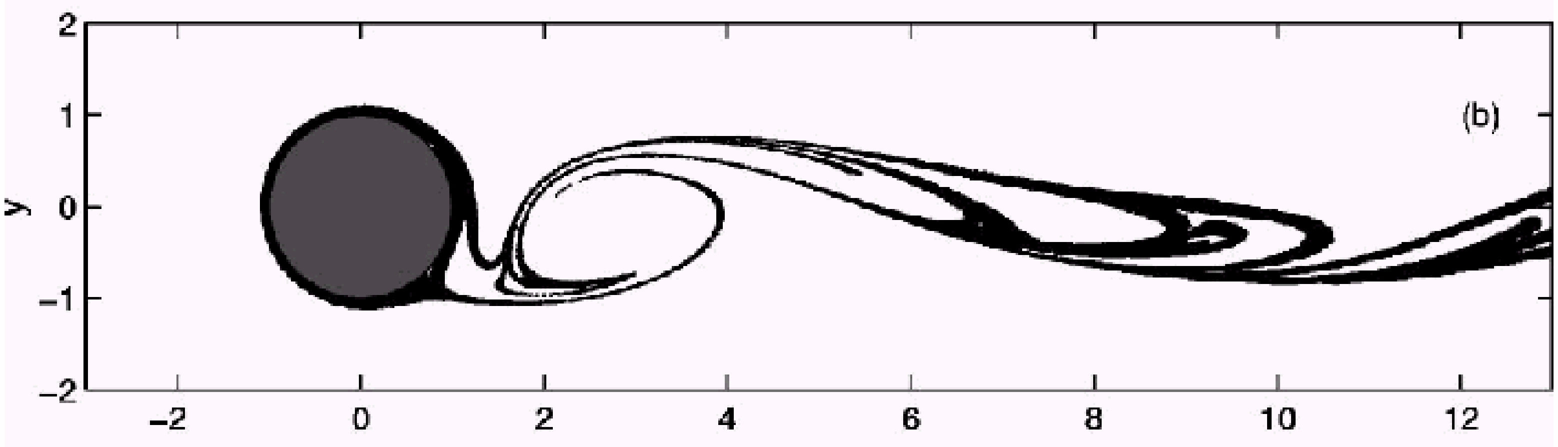
G. Boffetta et al. / Physics Reports 356 (2002) 367–474



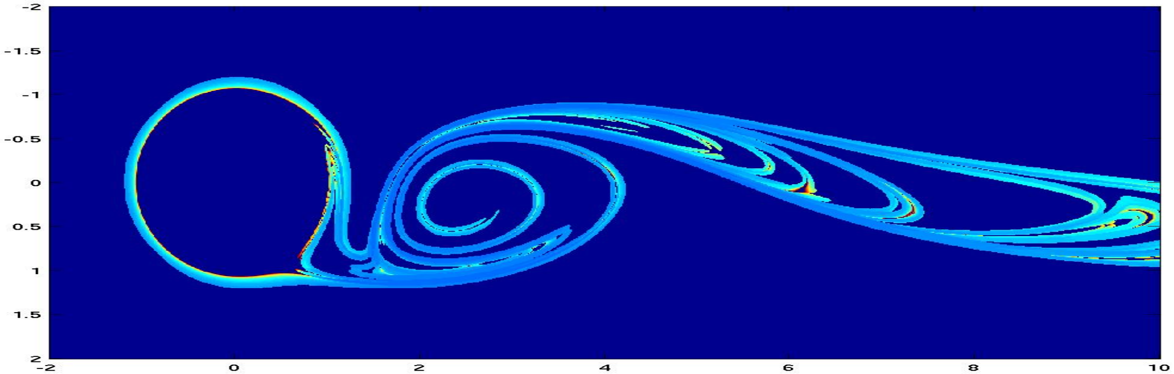
2D turbulence

The FSLE was originally introduced to quantify dispersion from non-infinitesimal initial separations

Characterizing transport with FSLEs

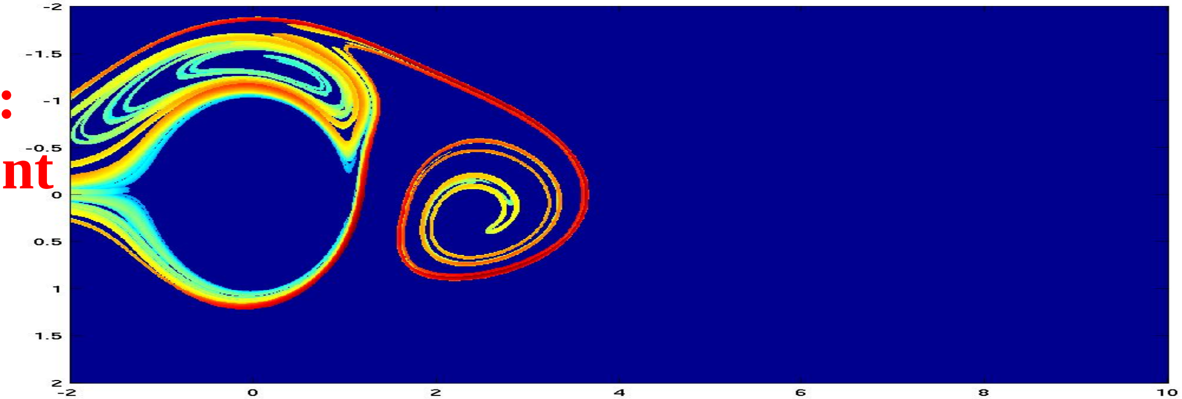


The spatial dependence of the FSLE allows the detection of stable and unstable manifolds of hyperbolic objects



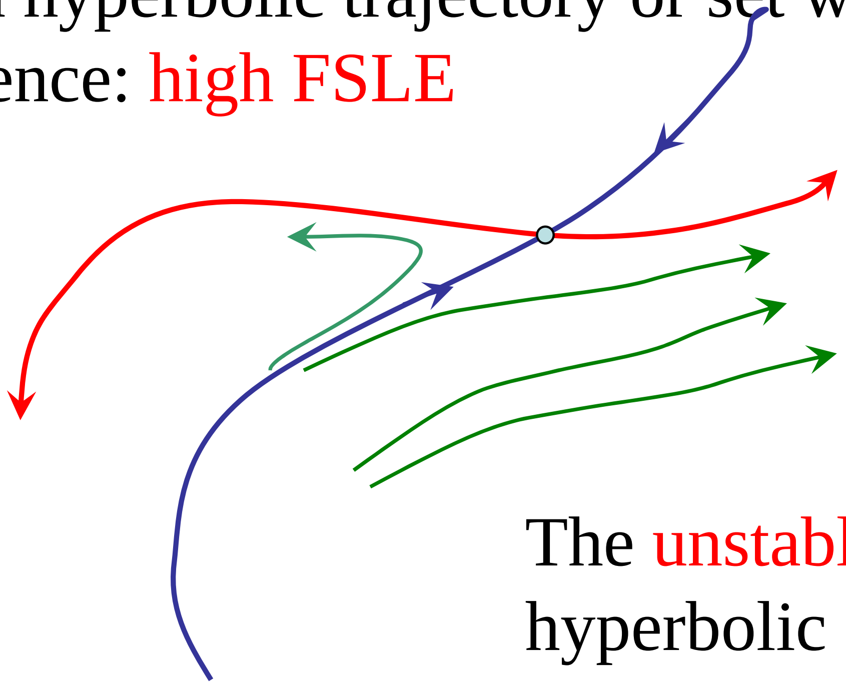
FSLE values from **time-backwards** trajectories

MAXIMA of FSLE: Lagrangian Coherent Structures (LCSs)



FSLE values from **time-forward** trajectories

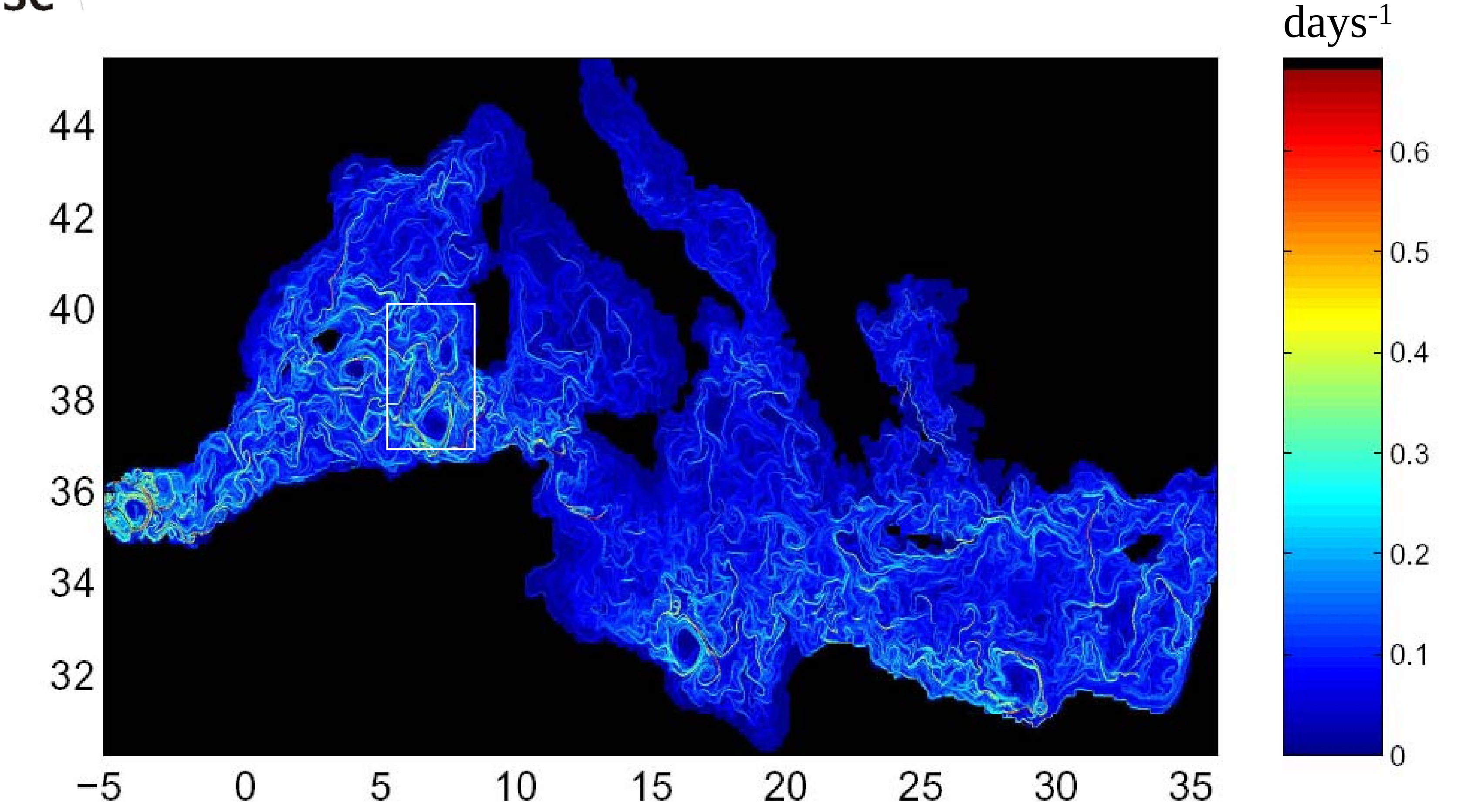
The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE



The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

Other types of Lyapunov exponents would display similar information, but FSLE is less afected by saturation

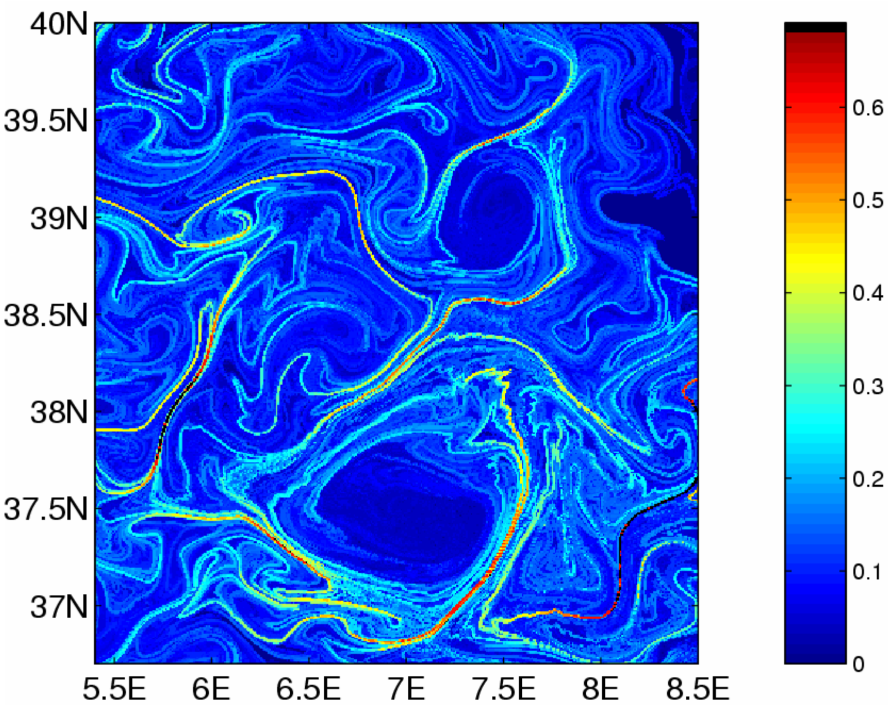
REMARK: these are heuristic consideration. Theorems needed (some available for FTLE)



$\delta_0 = 0.02\,^{\circ} \rightarrow \delta_f = 1\,^{\circ}$ (mesoscale transport)

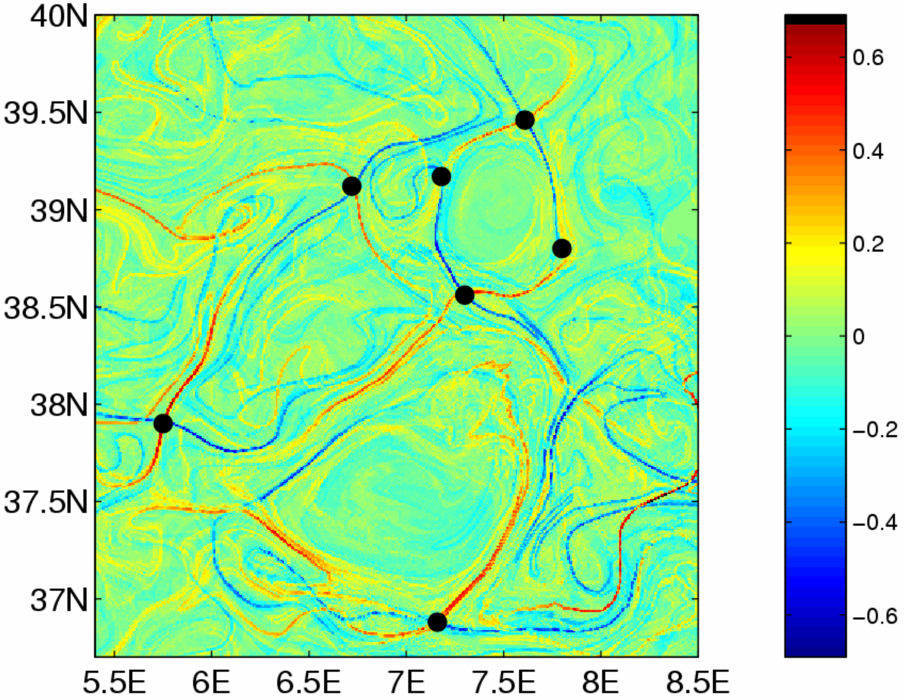
$\delta_0 \approx 2$ km $\rightarrow \delta_f \approx 110$ km twodimensional

d'Ovidio, Fernández, Hernández-García, López, Geophys. Res. Lett. 31, L17203 (2004)



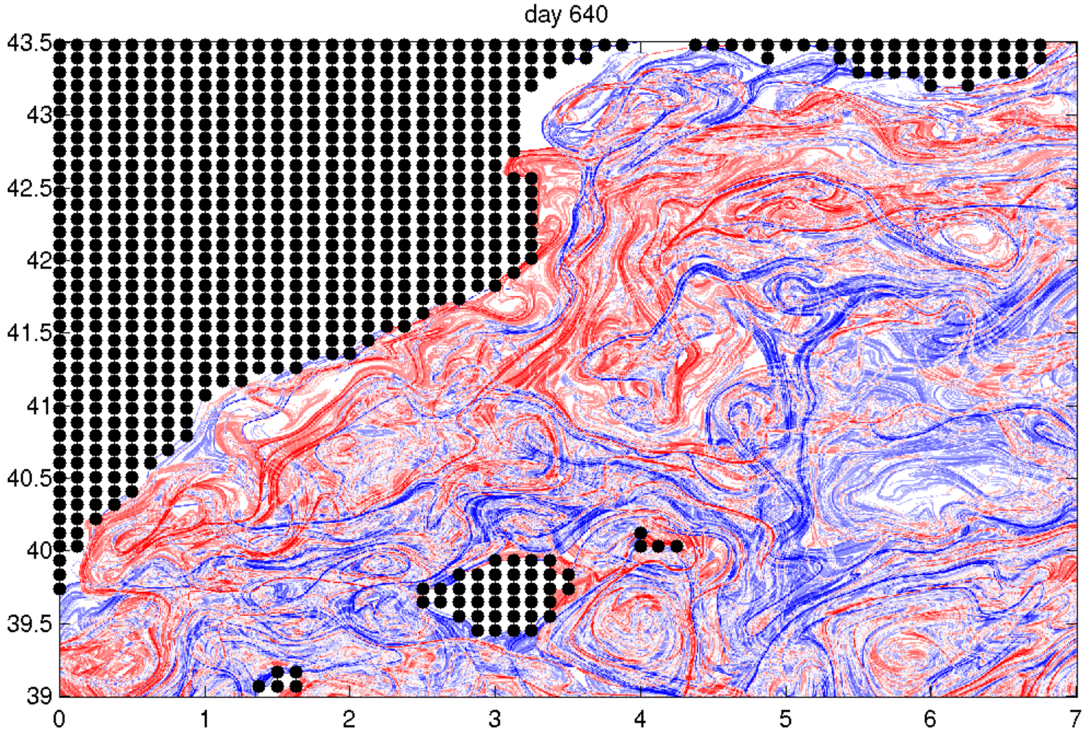
FSLE from time-backwards Integrations.
Are they really unstable manifolds of hyperbolic trajectories?

FSLE from forward and backwards integrations

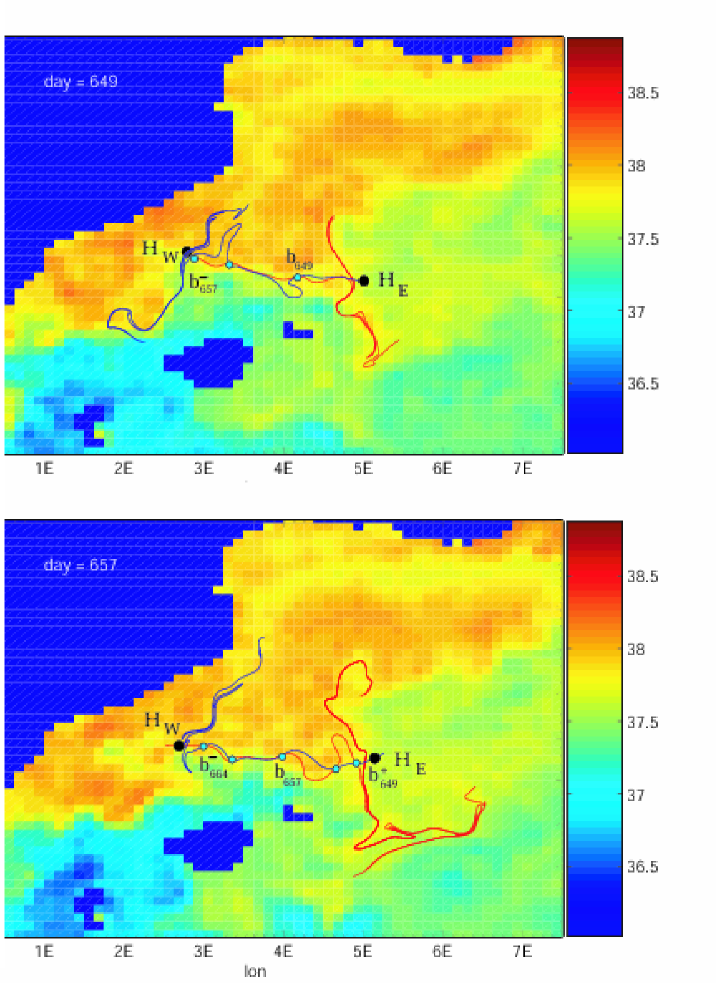


Click figures for movies

Characterizing transport with FSLEs



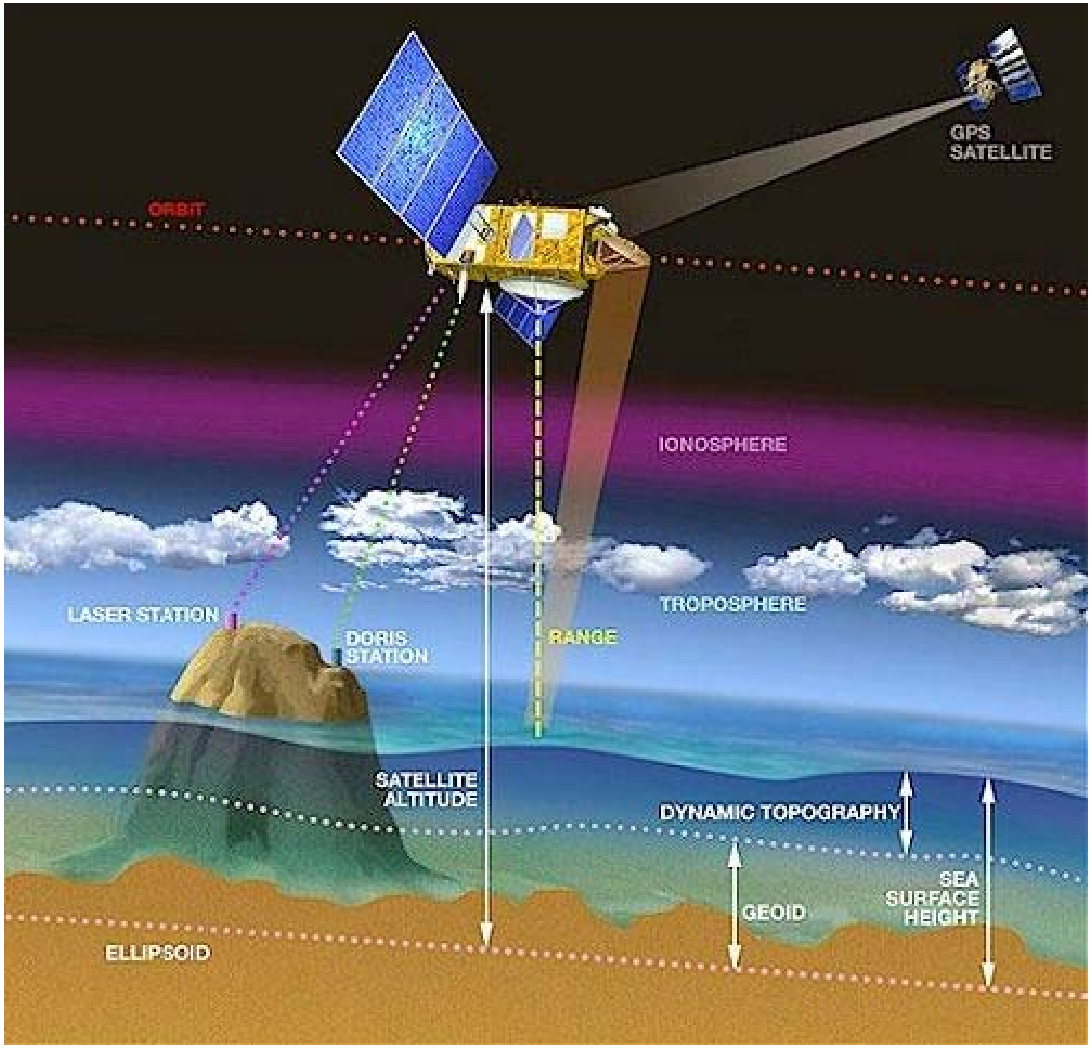
CLICK FOR MOVIE



High backwards Lyapunov values (unstable manifolds)

High forward Lyapunov values (stable manifolds)

SATELLITE ALTIMETRY FROM TOPEX/POSEIDON, ERS-2, JASON, ENVISAT, …



Dynamic Topography (DT)=
Sea Surface Heigh (SSH) – Geoid (G)

SSH ≈ 3 cm
G ≈ meters …

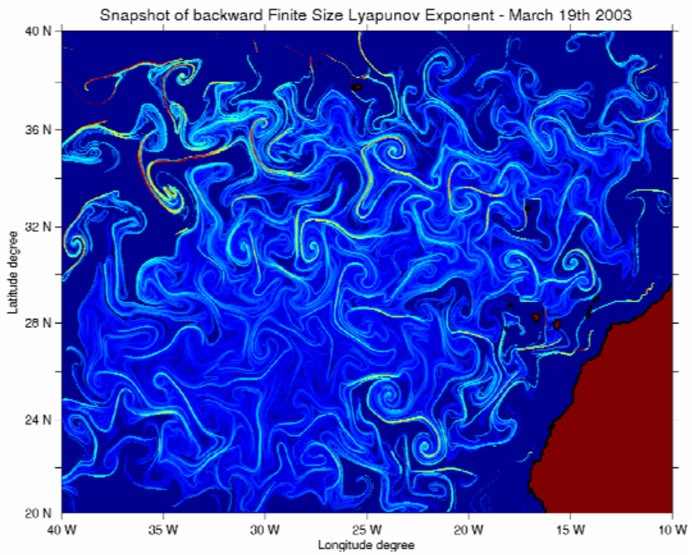
Sea Level Anomalies (SLA) =

SSH - $<SSH>_t$ = DT - $<DT>_t$

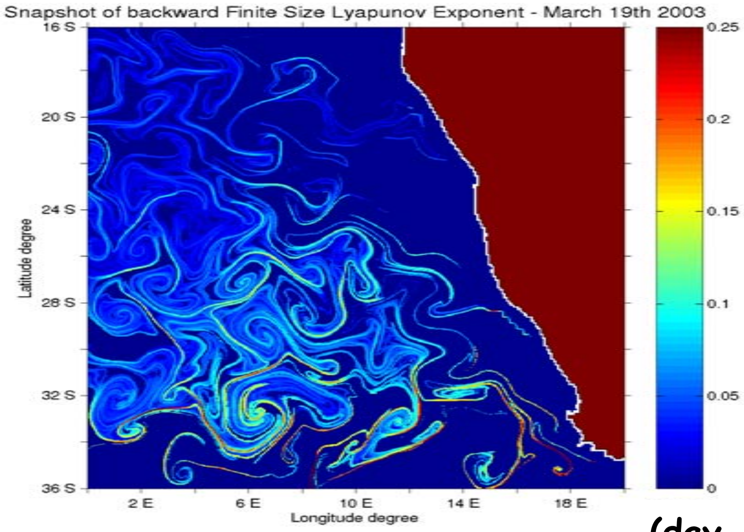
Dynamic topography determines, via the Colioris force, the velocity field (at large scales, geostrophic approximation)

FROM ALTIMETRY DATA

BENGUELA

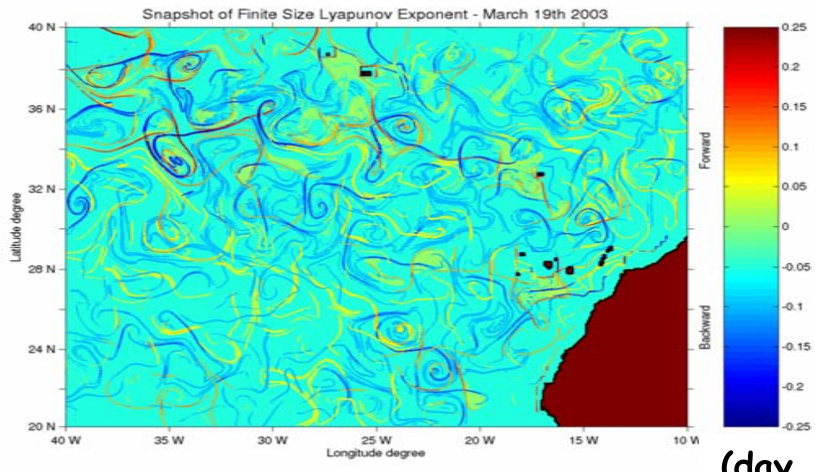


(day s^{-1})

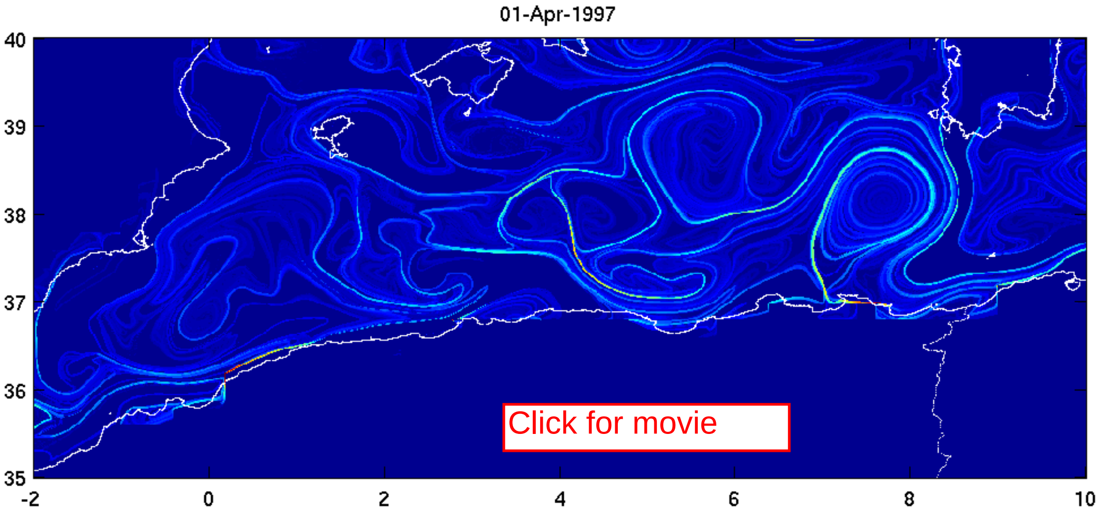


(day s^{-1})

March 19
2003
snapshots



(day s^{-1})



CANARY

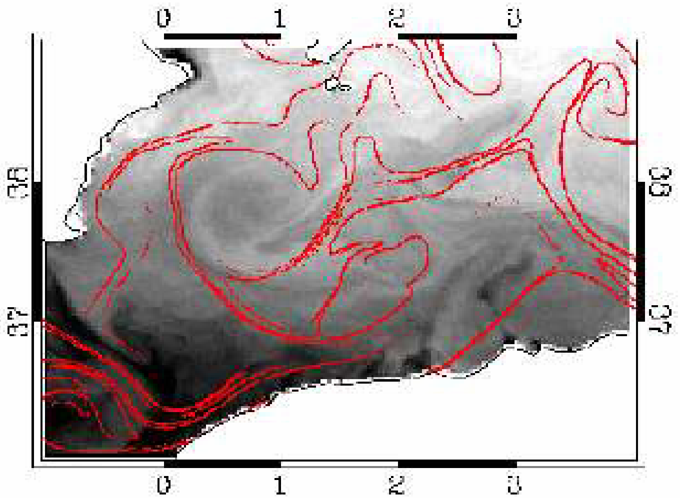
Note the presence of SUB-MESOSCALE detail

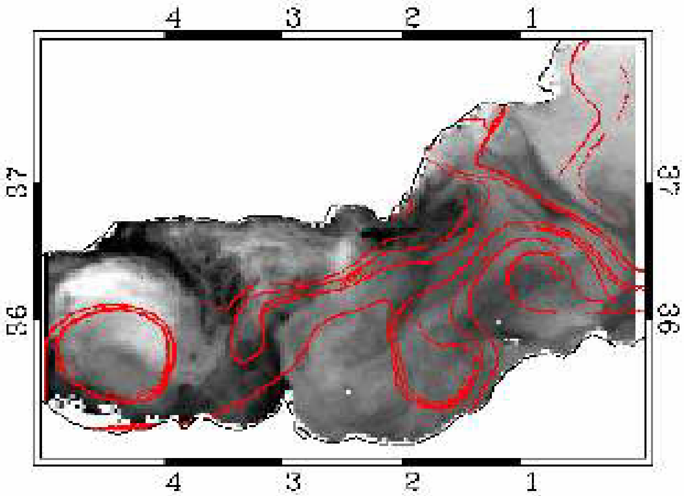
d’Ovidio et al. Deep-Sea Res. I 56, 15 (2009)
V. Rossi et al. Nonlin. Proc. Geophys. 16, 557 (2009)

Sea Surface Temperature vs lines of FSLE > 0.1 day^{-1} (LCSs)

d’Ovidio et al.
Deep-Sea Res. I (2009)

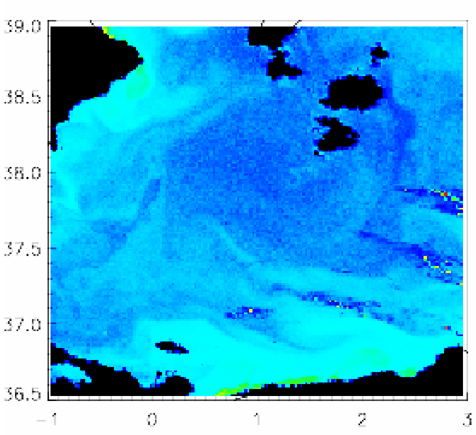
July 9 2003



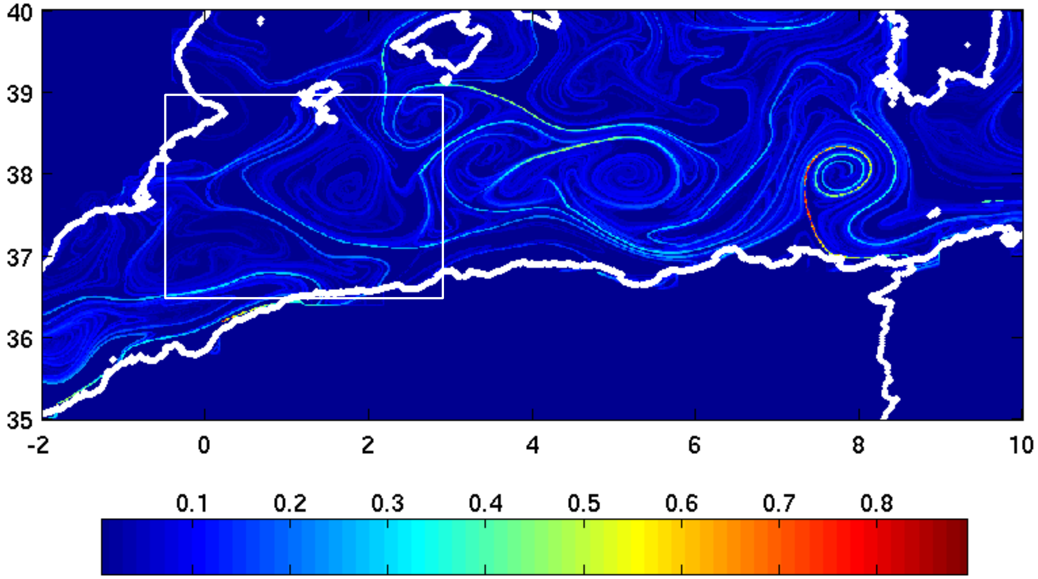


April 6 2004

Chlorophyill

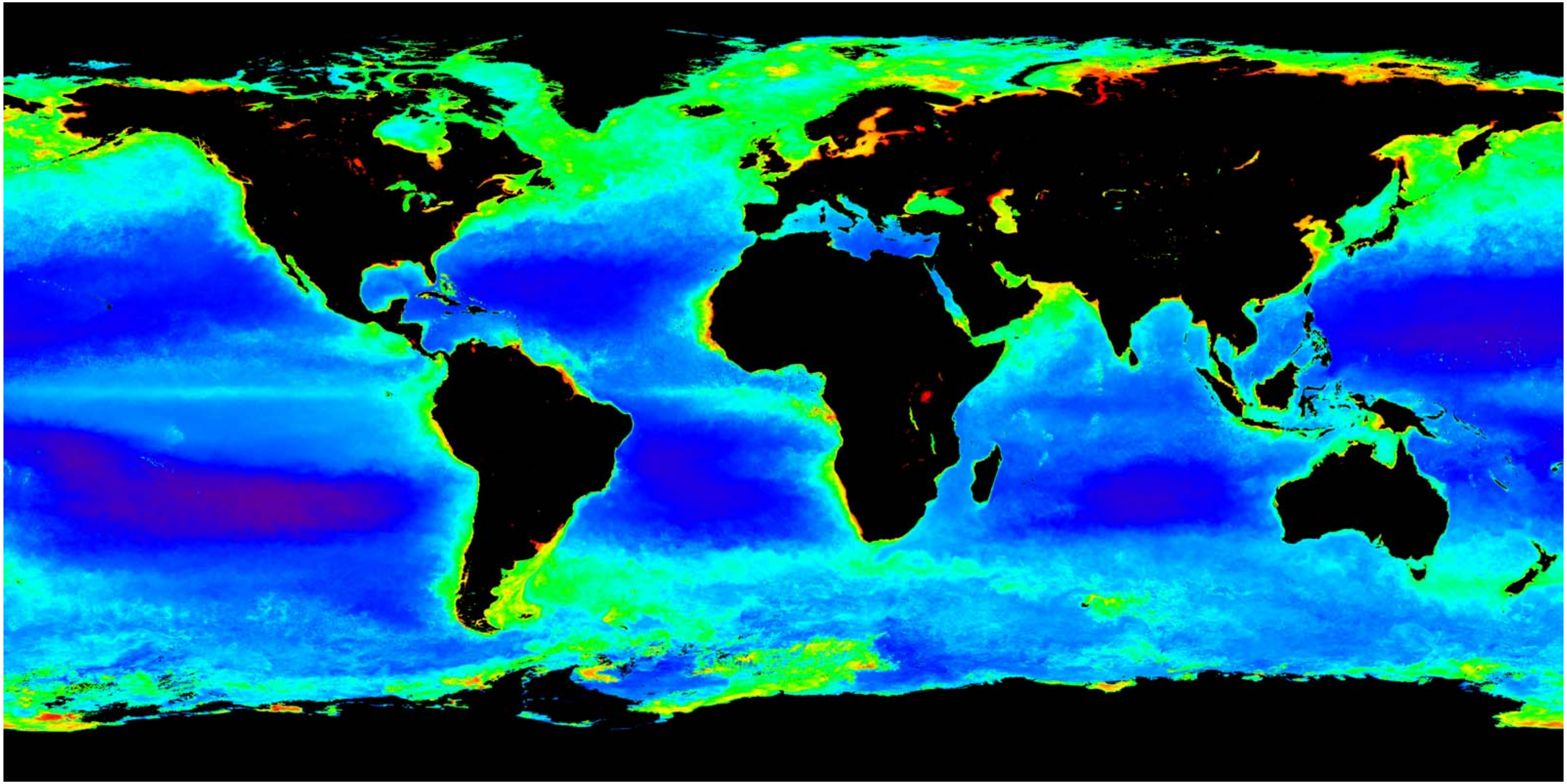


18 May 1998



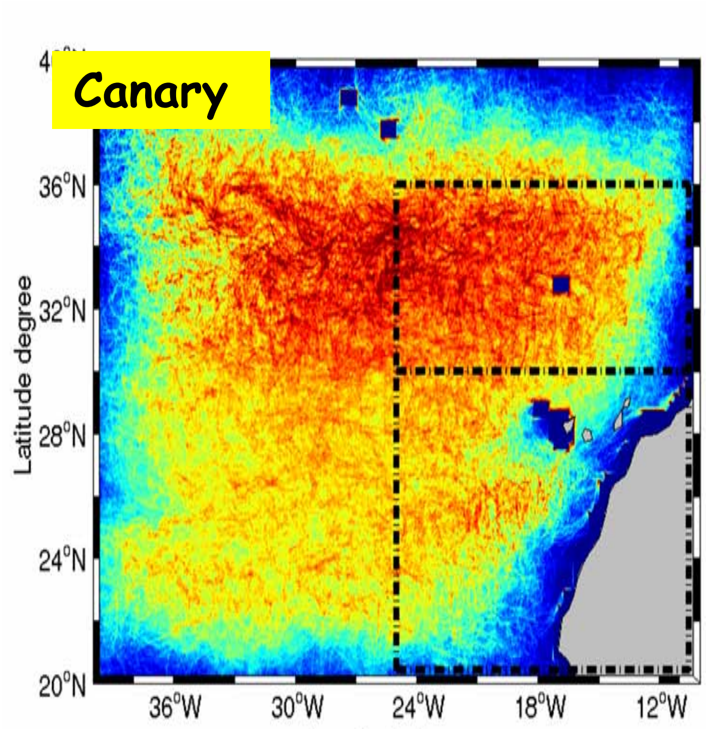
days^{-1}

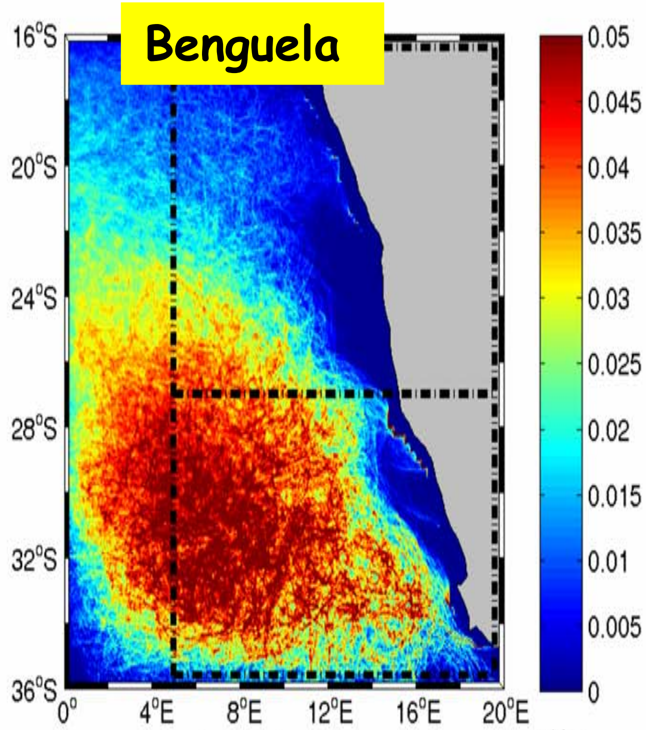
Chlorophyll-a (≈ phytoplankton) from space



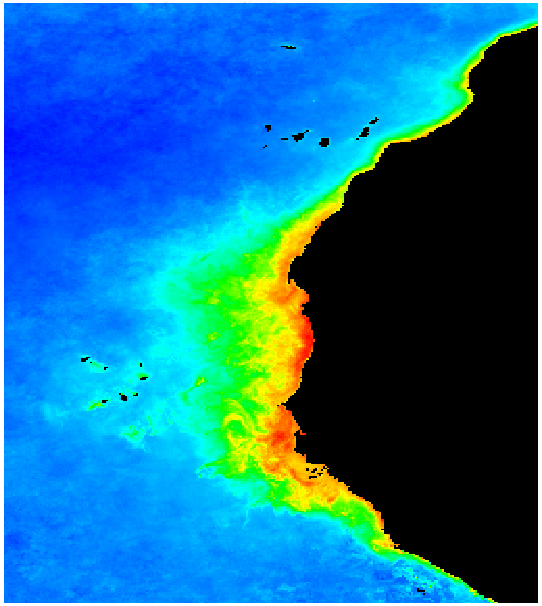
MODIS Image

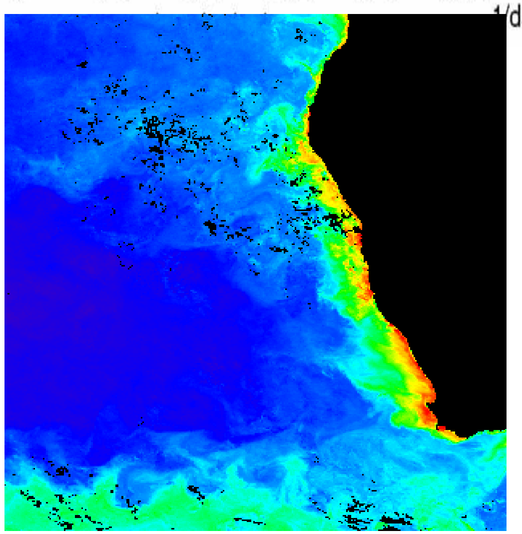
Phytoplankton and FSLE in the world major upwelling areas





Backward FSLE (λ-):
Temporal average
(a measure of horizontal MIXING)
from June 2000 till
June 2005



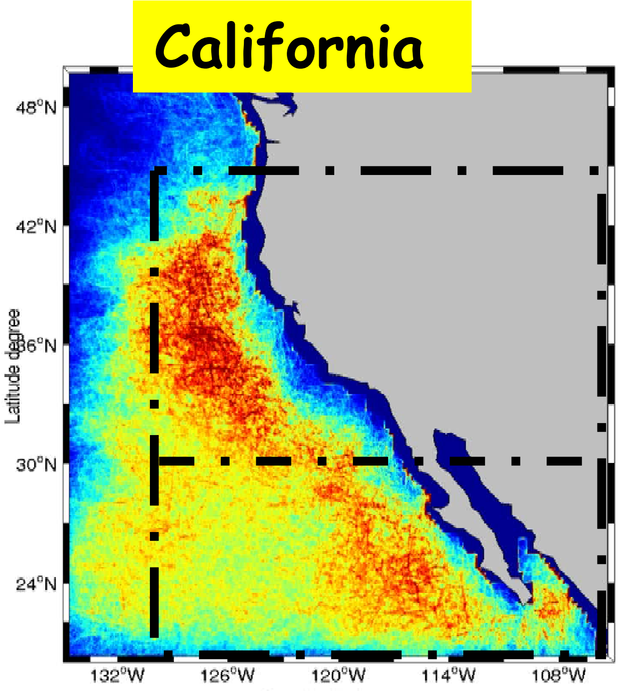


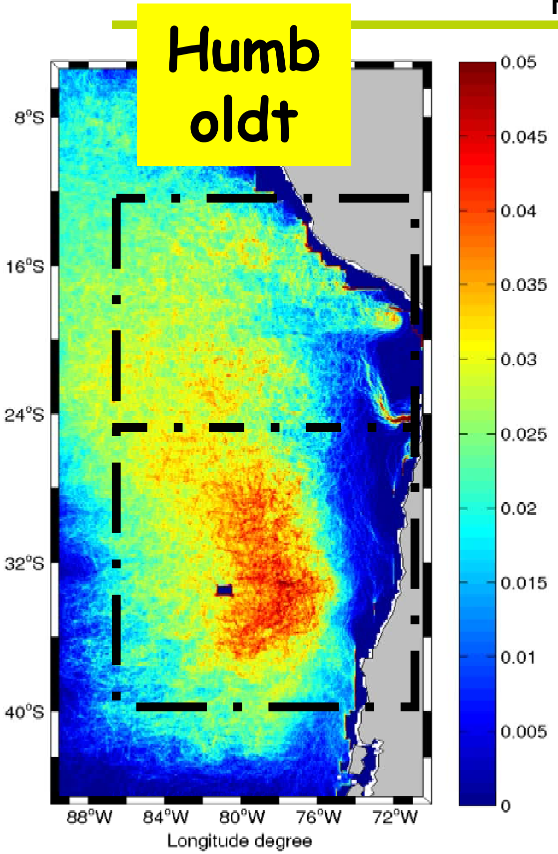
Rossi et al.,

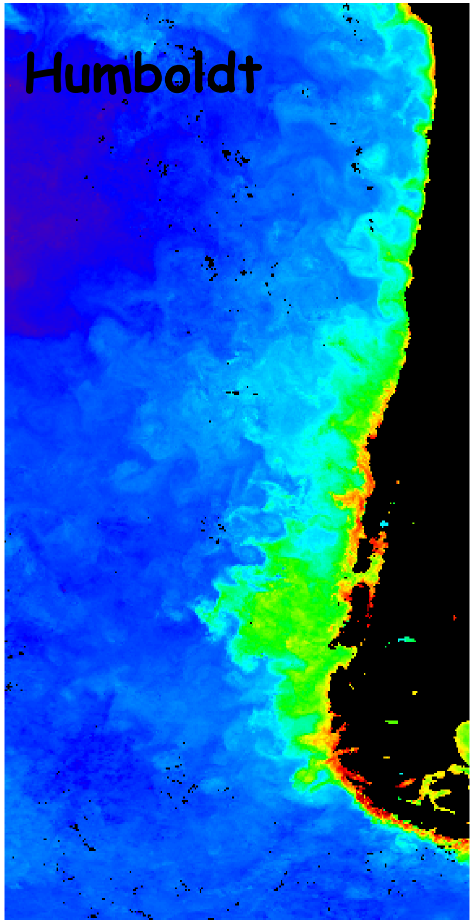
Geophys, Res. Lett. 2008

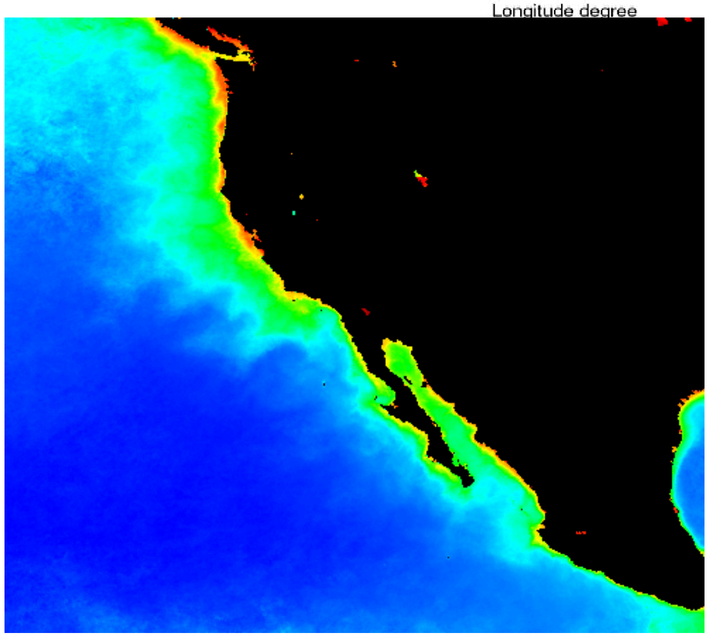
Nonlin. Proc. Geophys. 2009

FSLE and phytoplankton



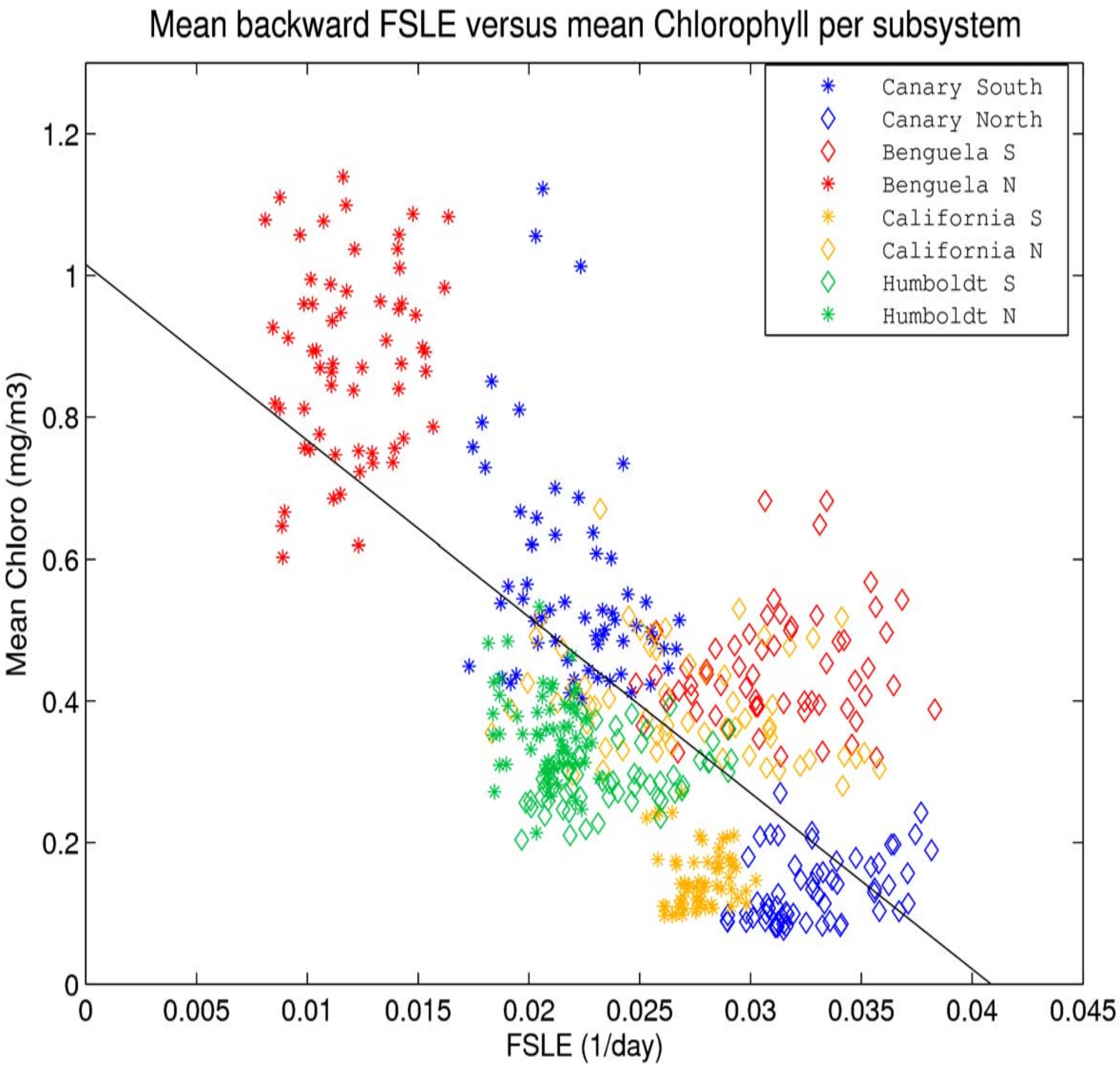






Backward FSLE $(\lambda\text{-})$:
Temporal average
(a measure of horizontal MIXING)
from June 2000 till June 2005

Rossi et al.,
Geophys, Res. Lett. 2008
Nonlin. Proc. Geophys. 2009

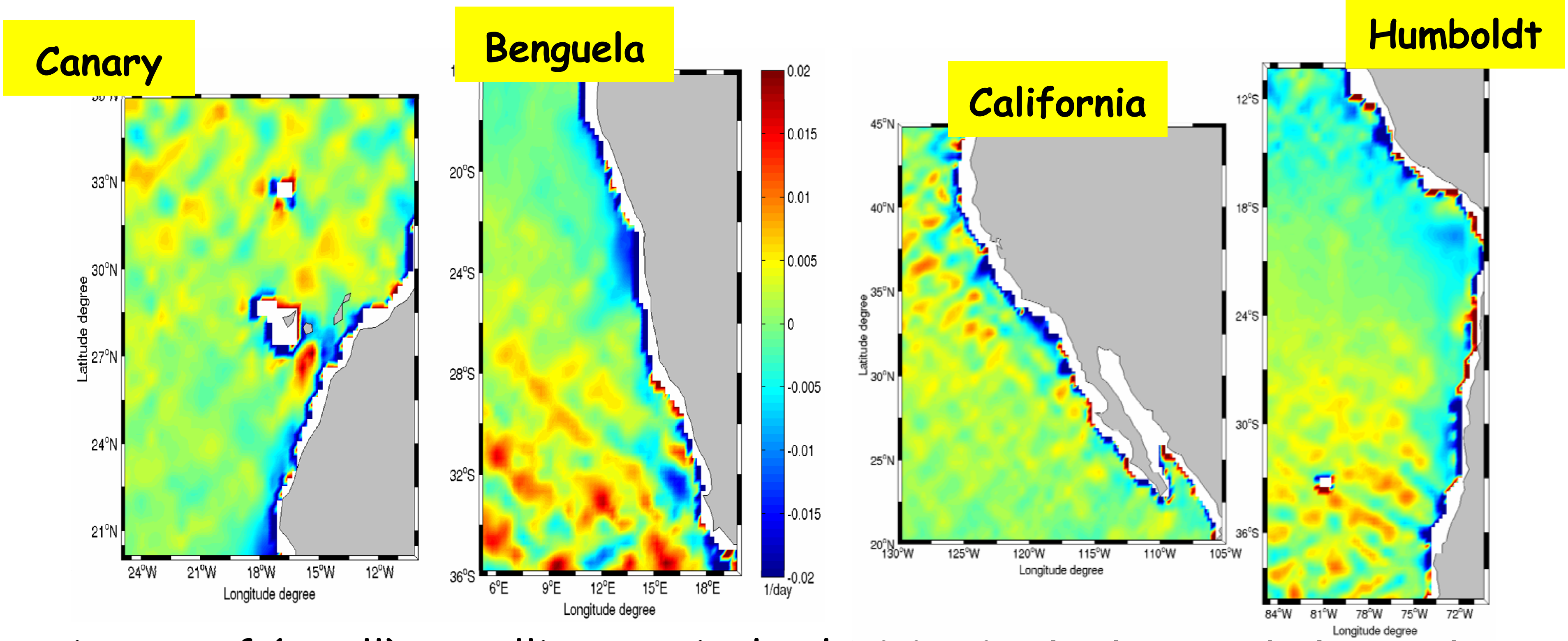


- Negative correlation
- Clustering
- Less turbulent systems are characterized by: LOW FSLE / HIGH CHLOROPHYLL.
- Most turbulent systems: HIGH FSLE / LOW CHLOROPHYLL.

Opposite to behavior seen in less enriched systems

Temporal averages of vertical velocities from incompressibility condition

$$\Delta(x,y,t) \equiv \partial_z V_z = -(\partial_x V_x + \partial_y V_y)$$



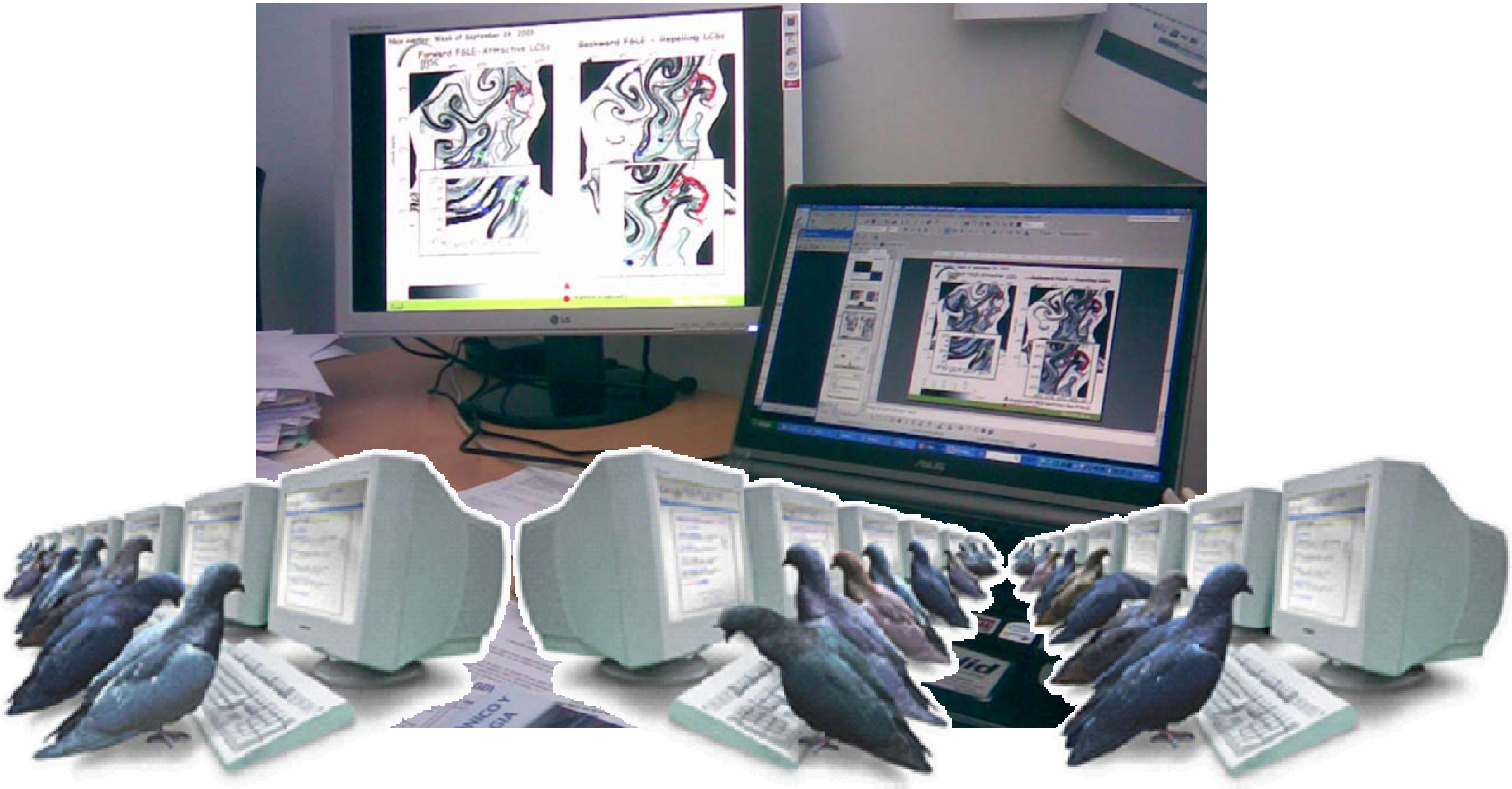
- Dominance of (small) upwelling vertical velocities in the less turbulent subsystem.

- Thus, probably the influence of horizontal stirring on plankton is only indirect: need to understand the 3d flow structure: high FSLE associated to low Eckman transport.

Rossi et al., Geophys, Res. Lett. 2008, Nonlin. Proc. Geophys. 2009

- Lagrangian Coherent Structures give the skeleton of horizontal transport
- This certainly influences abiotic quantities: temperature, nutrients, …
- This certainly influences plankton distribution
- From there, impact is expected in plankton consumers, their predators, … cascades up along the food chain …

Do birds know about FSLE calculations?



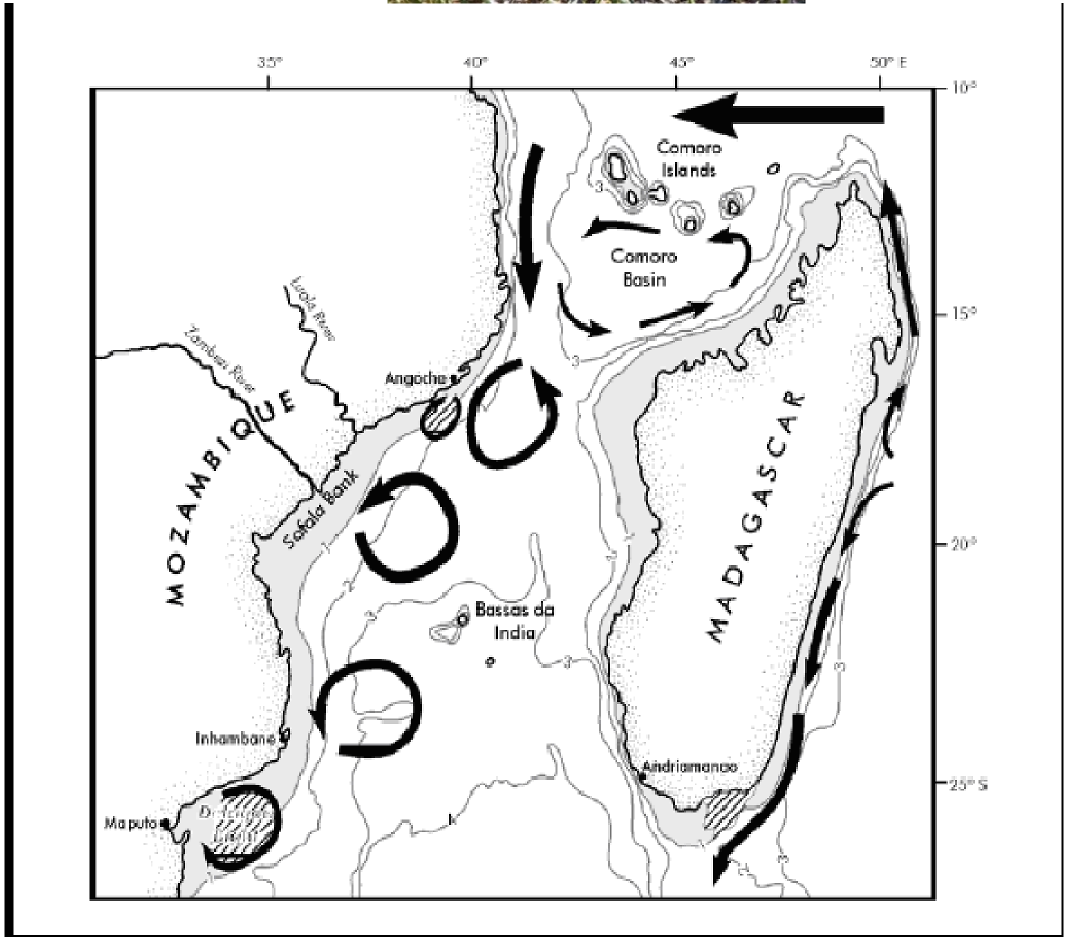
Tew Kai, Rossi, Sudre, Weimerskirch, Lopez, Hernandez-Garcia, Marsac, Garçon,
PNAS 106, 8245 (2009)

FRIGATEBIRDS in the MOZAMBIQUE CHANNEL

Particular topography (channel/islands) linked with strong mesoscale activity:

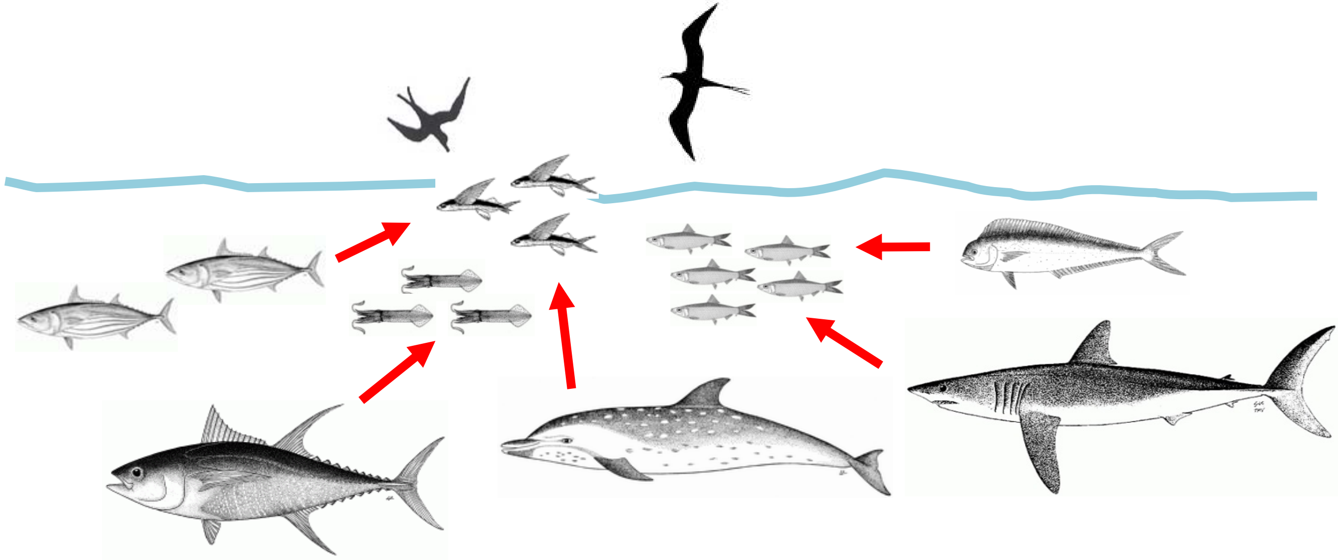
- Large anticyclonic cell at the north
- Local upwellings
- Anticyclonic and cyclonic mesoscale eddies moving southward permanently.

(De *Ruijter et al., 2004*)



Great frigatebird (*fregata minor*):

- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**



Weimerskirch et al. 2004; Jaquemet et al. 2005

Satellite transmitter and altimeter
(total weight : 1 to 3% mass of adults,
max 45g)

8 birds (from Europa Island community) fitted with satellite transmitter and altimeter.

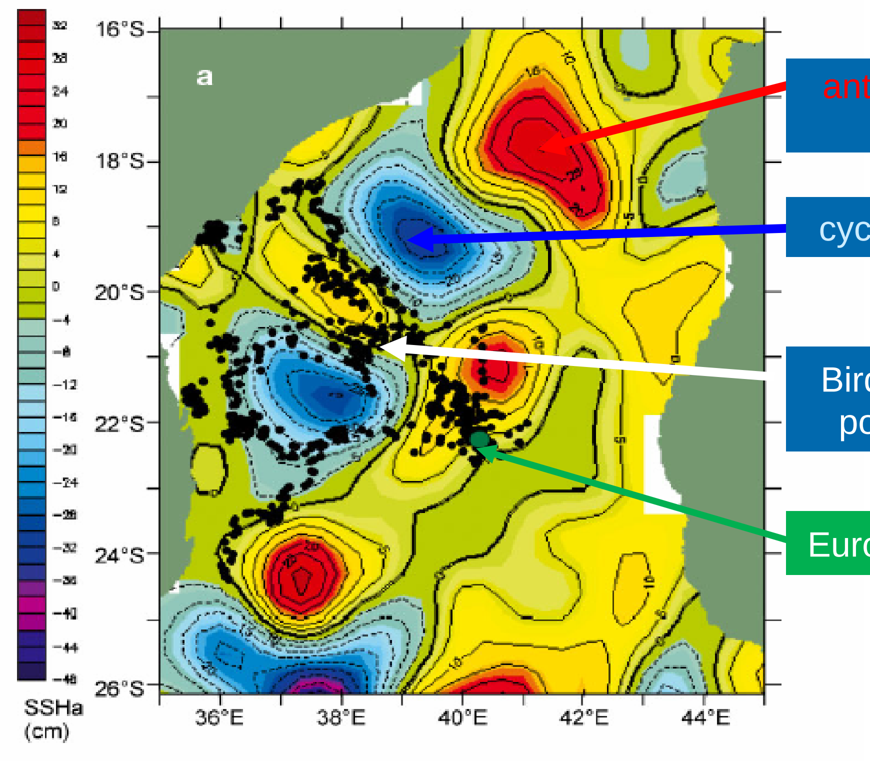
Followed for their foraging trips from August 18 to September 30, 2003.

1600 Argos from 50 trips positions, distributed into 17 long trips (> 614 km) and 33 short trips.

(Weimerskirch et al., 2004)

SSH (cm): Eulerian view

Lagrangian FSLEs versus SSH



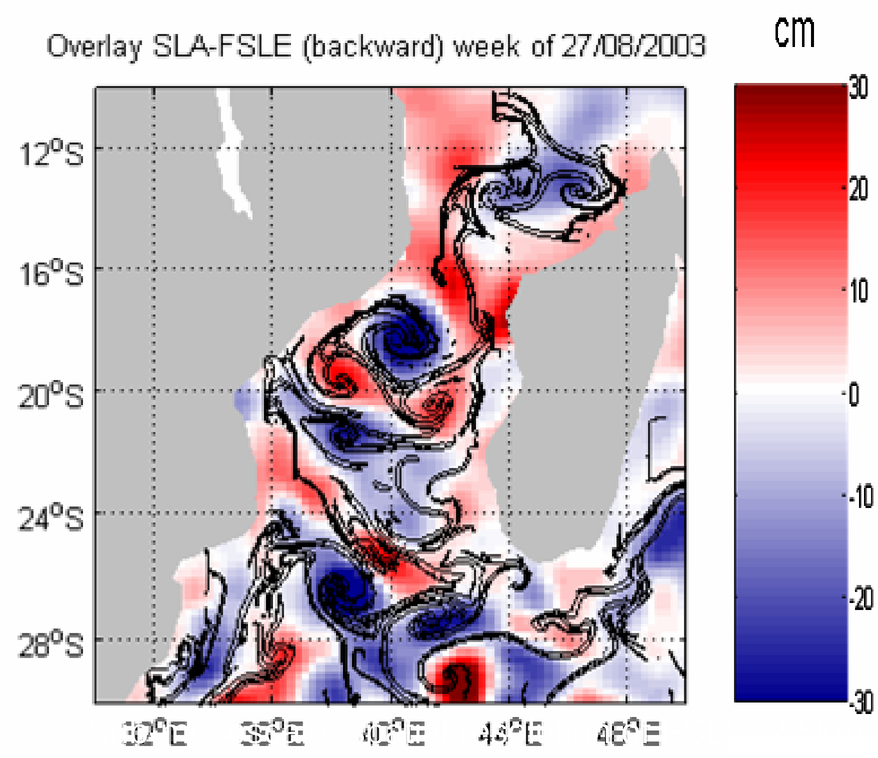
anti-cyclonic eddy

cyclonic eddy

Birds GPS positions

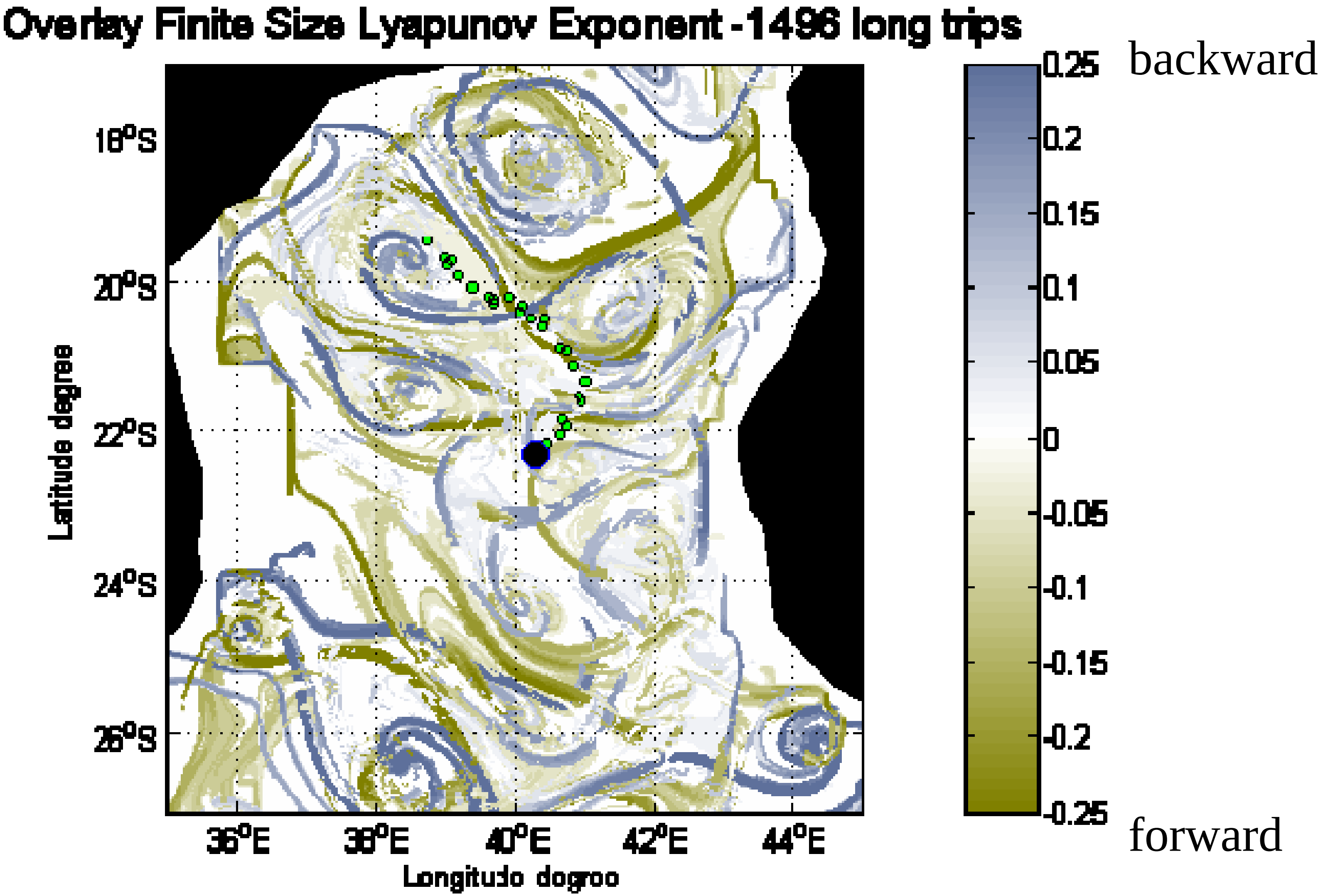
Europa Island

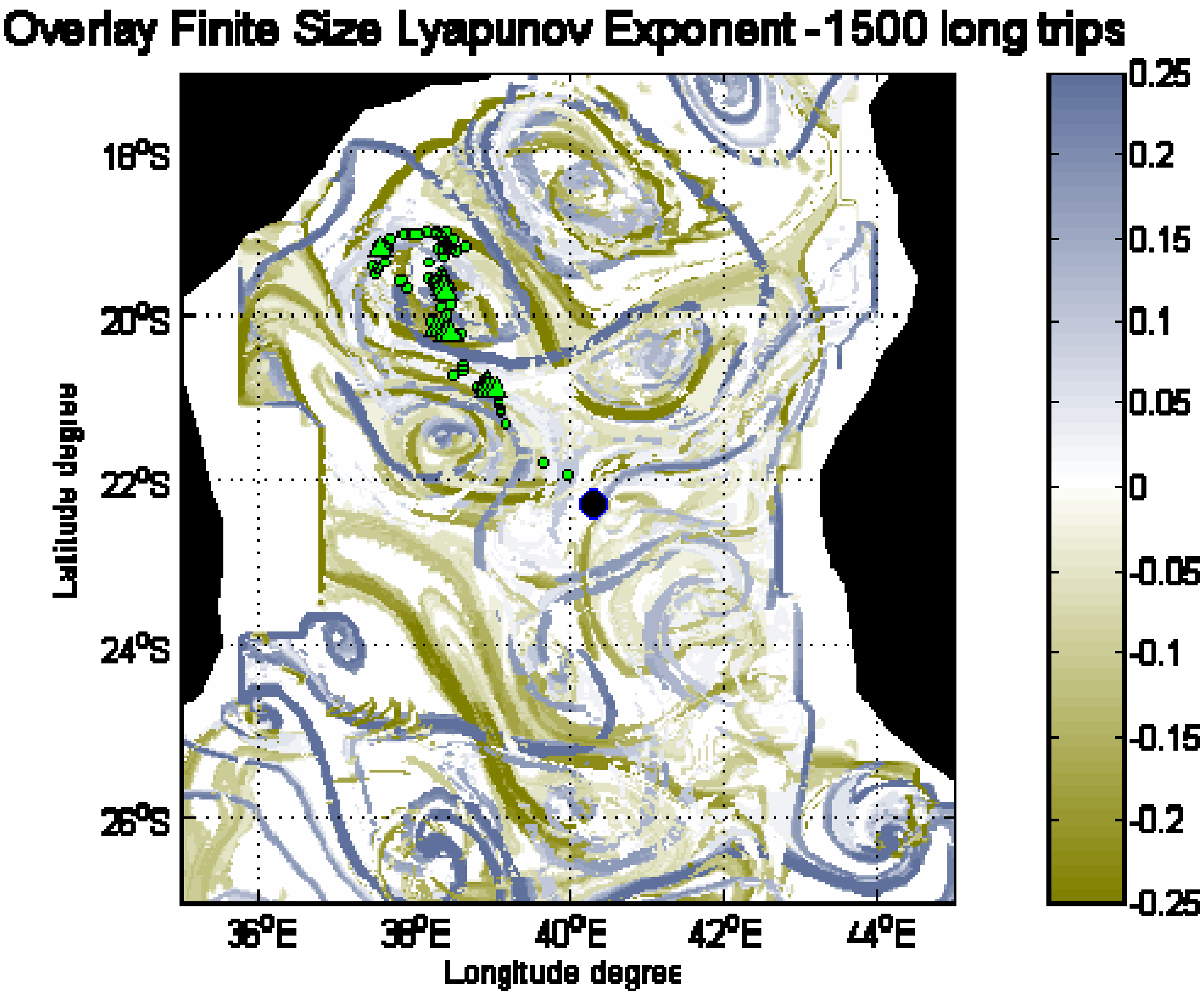
Weimerskirch et al,2004



The Lagrangian FSLE gives access to submesoscale structures

Lagrangian Coherent Structures: |FSLE| > 0.1 day^{-1}


Overlay Finite Size Lyapunov Exponent -1496 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
backward
forward


Overlay Finite Size Lyapunov Exponent -1500 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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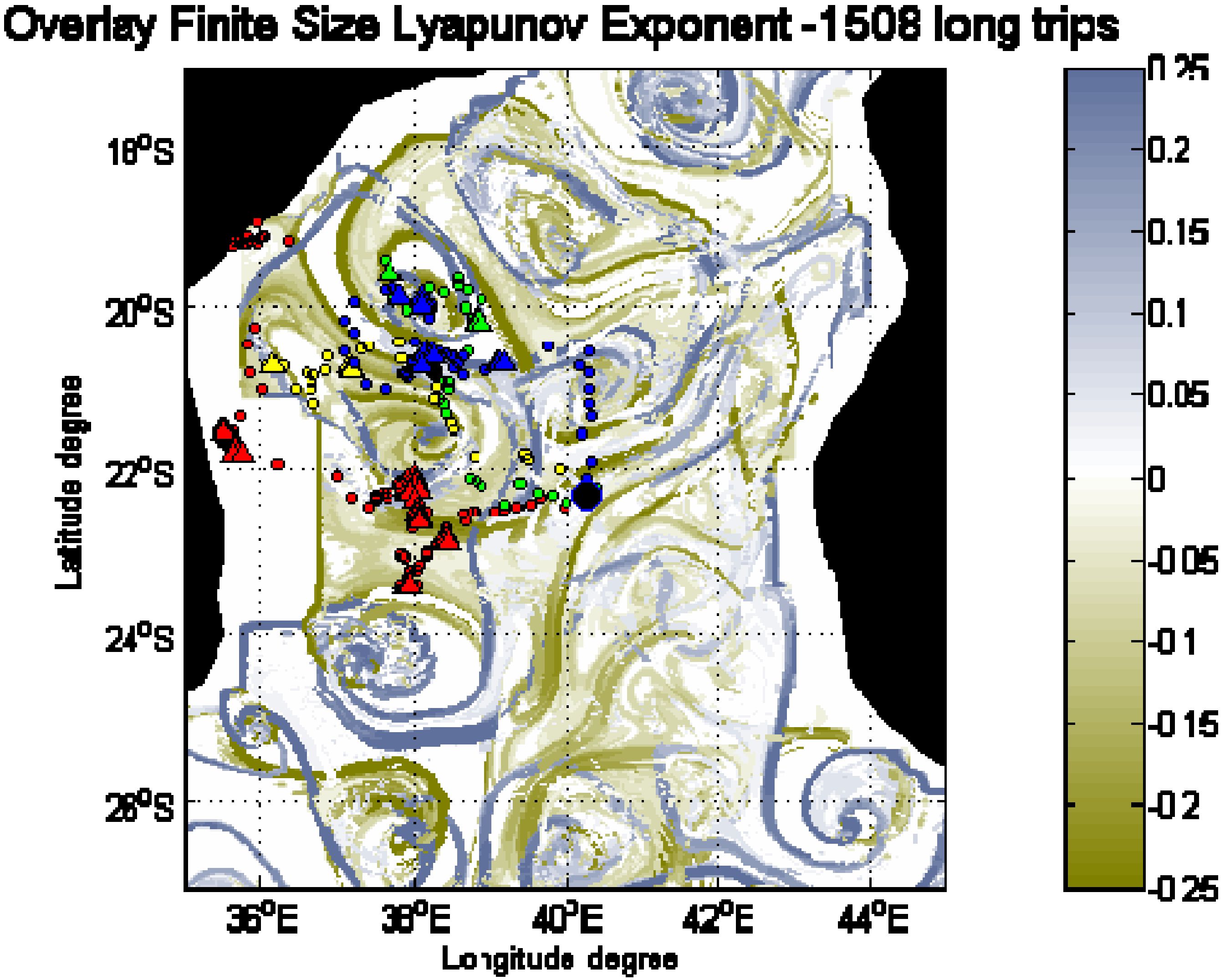
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1508 long trips
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22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Latitude degree
Longitude degree
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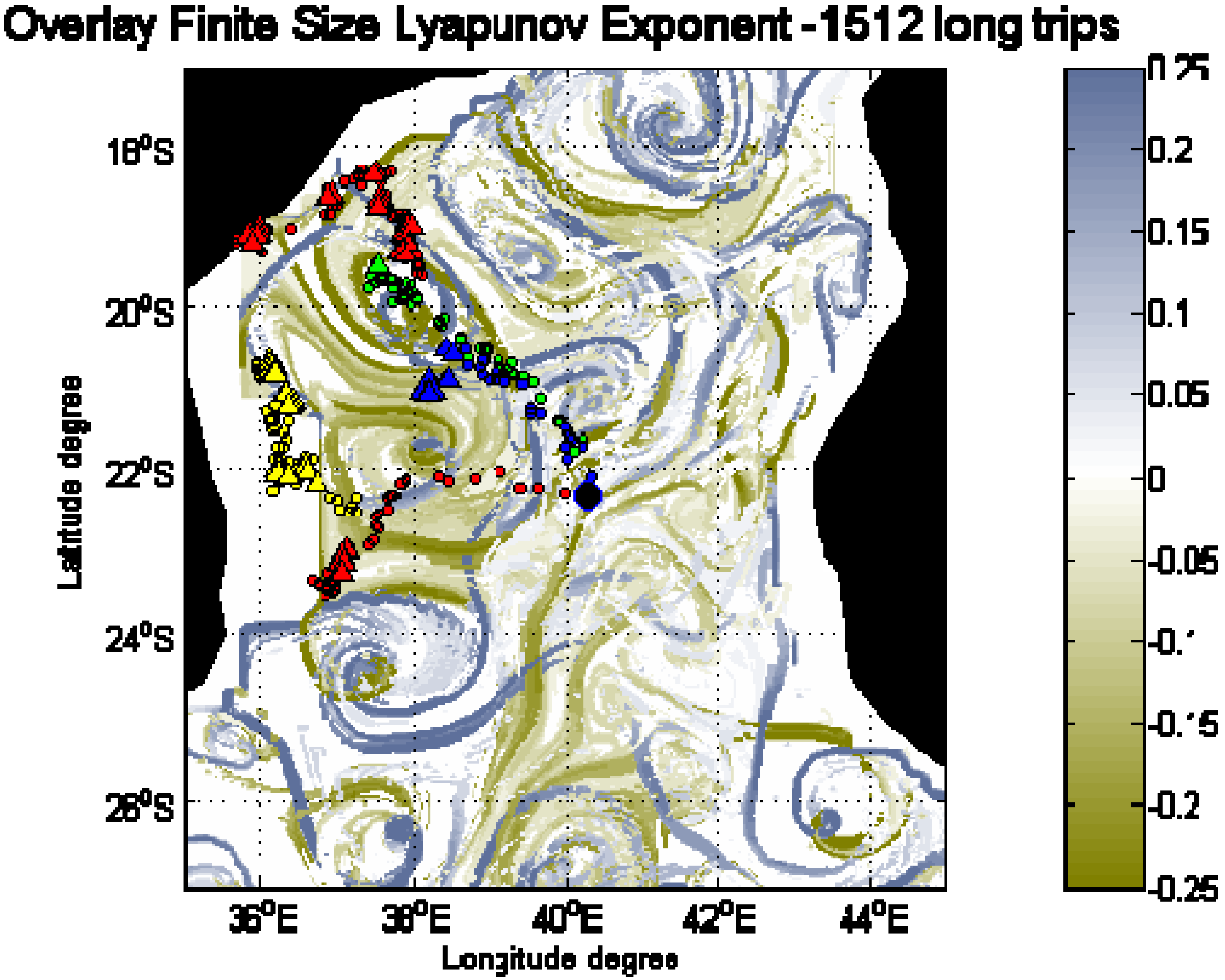
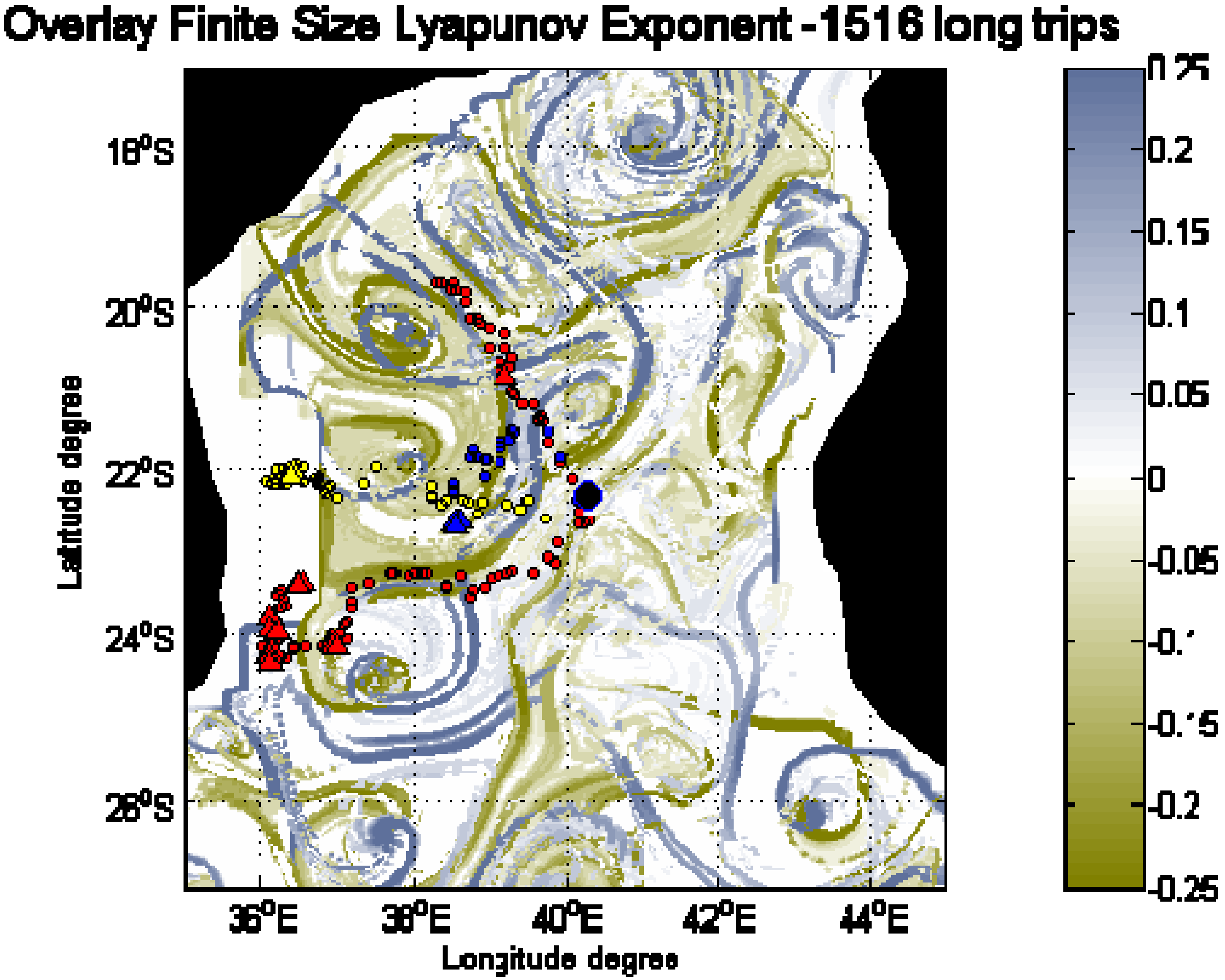
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25



Overlay Finite Size Lyapunov Exponent -1512 long trips


Overlay Finite Size Lyapunov Exponent -1516 long trips
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22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
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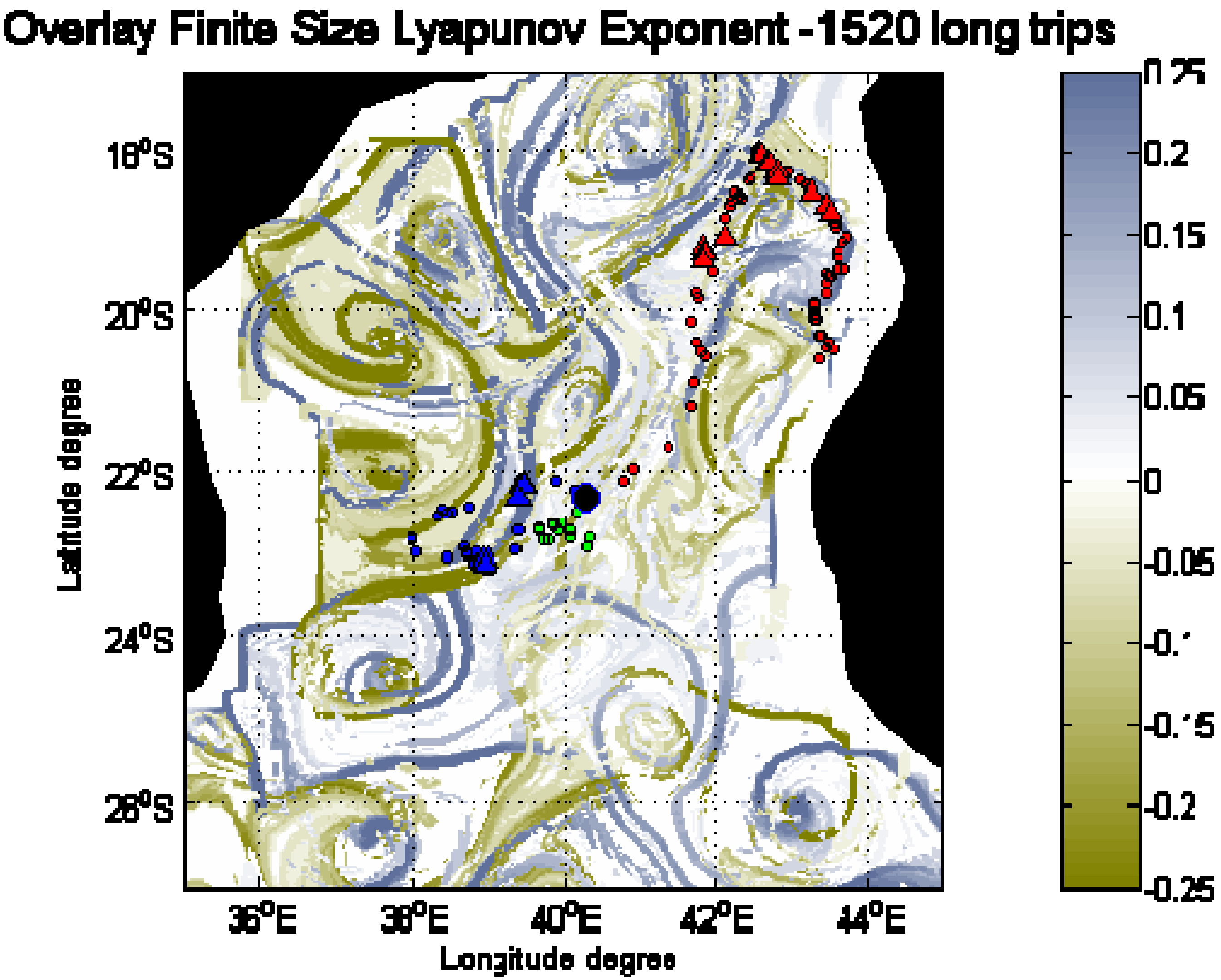
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1520 long trips
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22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Latitude degree
Longitude degree
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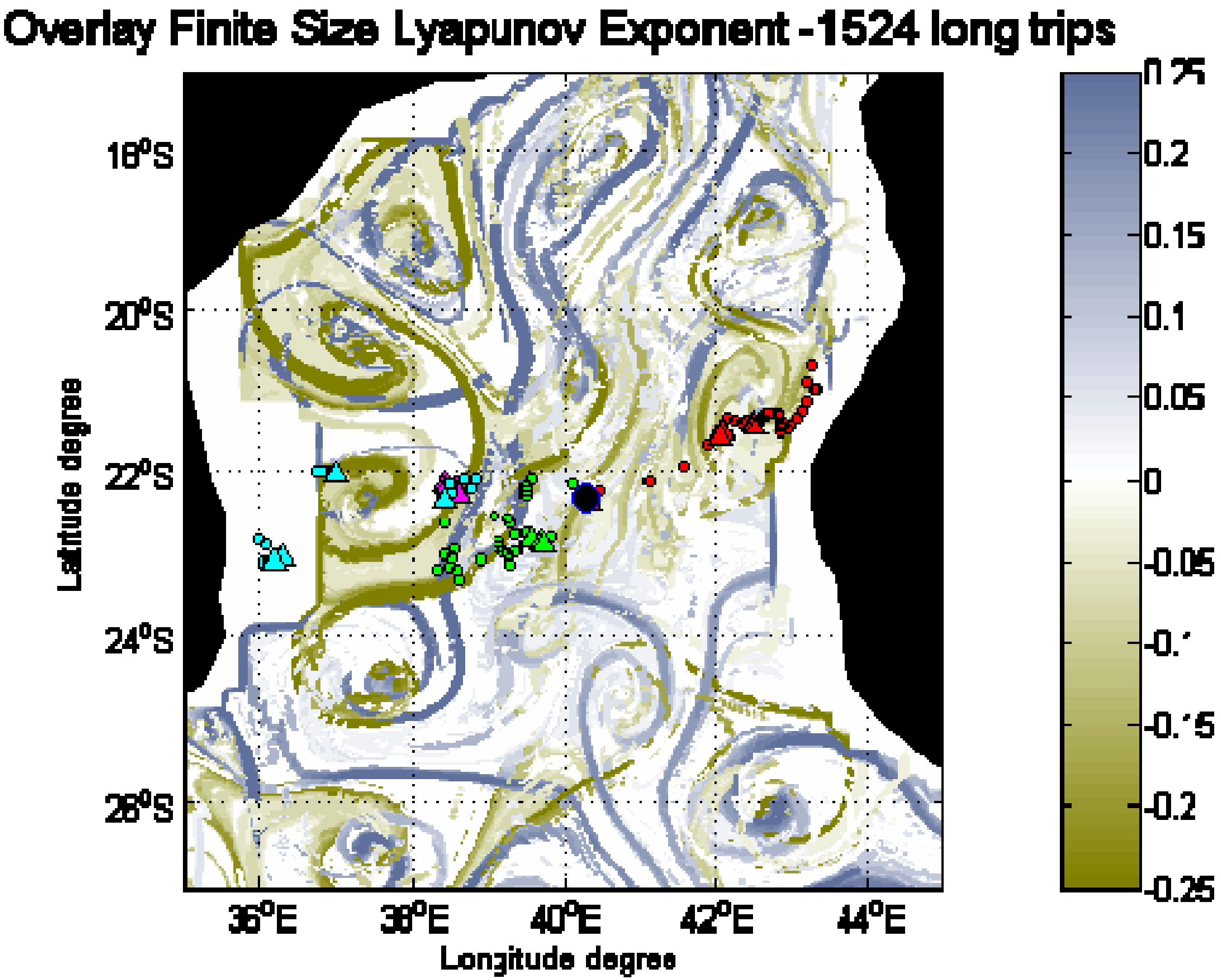
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1524 long trips
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24°S
26°S
Latitude degree
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38°E
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42°E
44°E
Longitude degree
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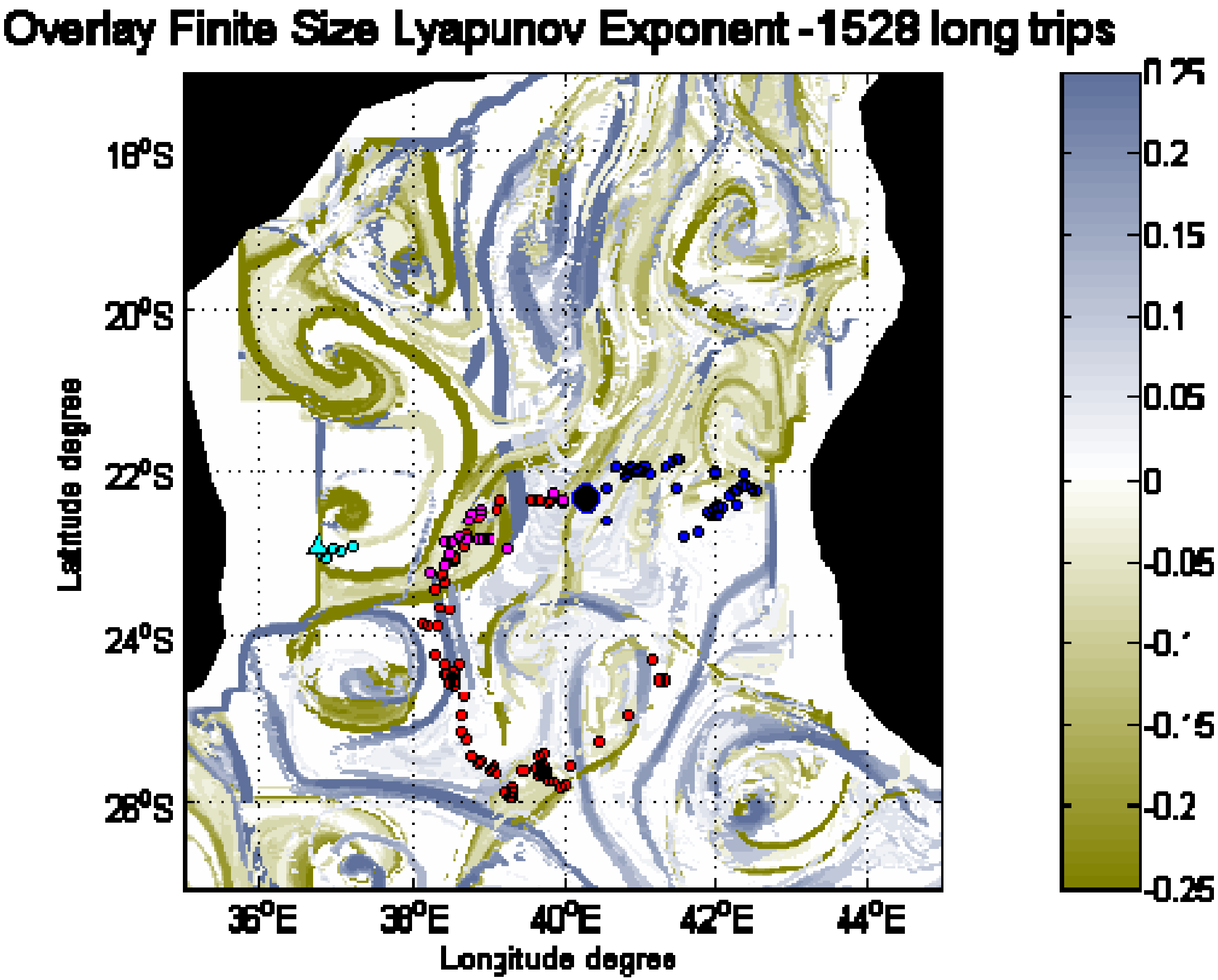
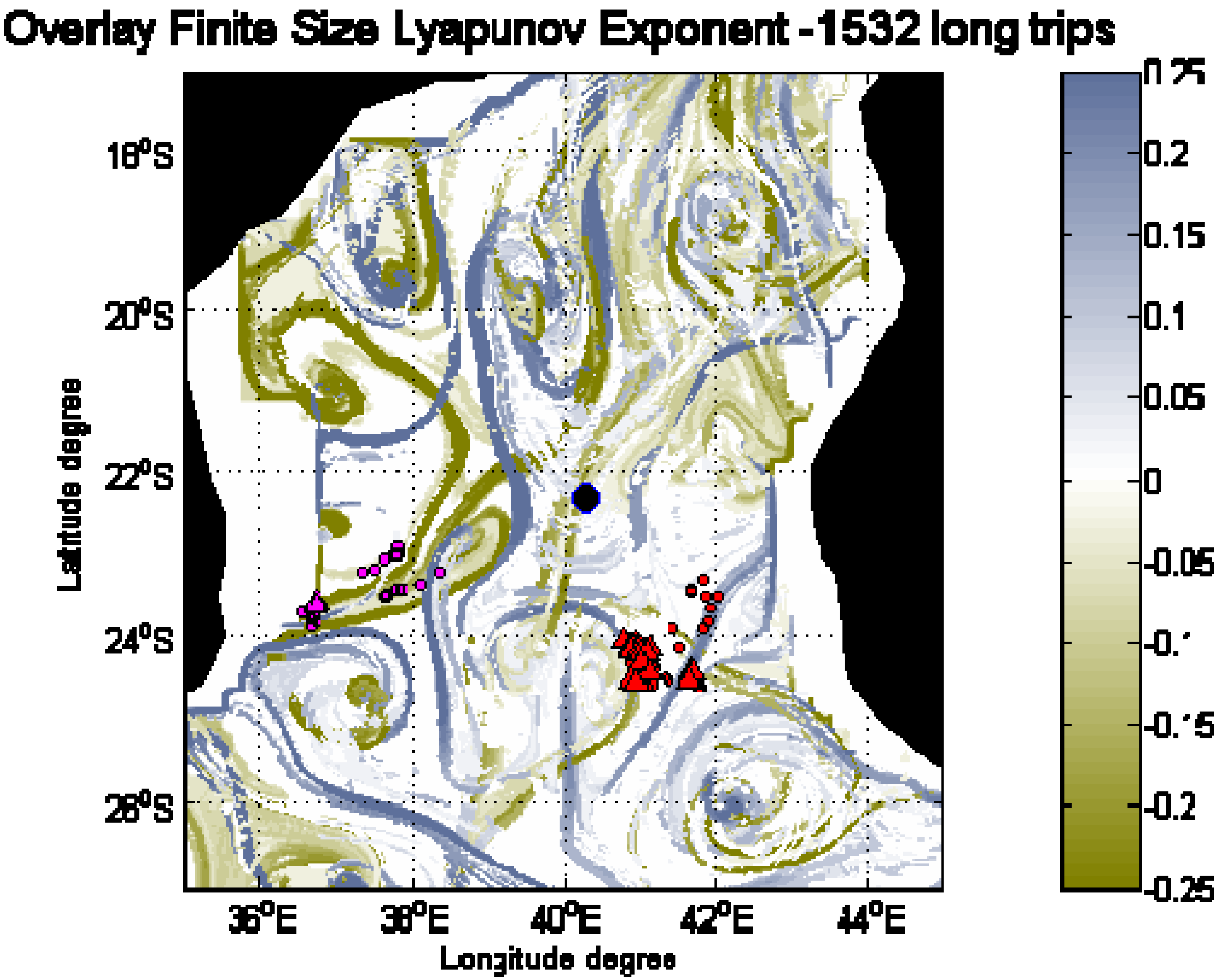
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Frigatebirds and FSLE




Overlay Finite Size Lyapunov Exponent -1532 long trips
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22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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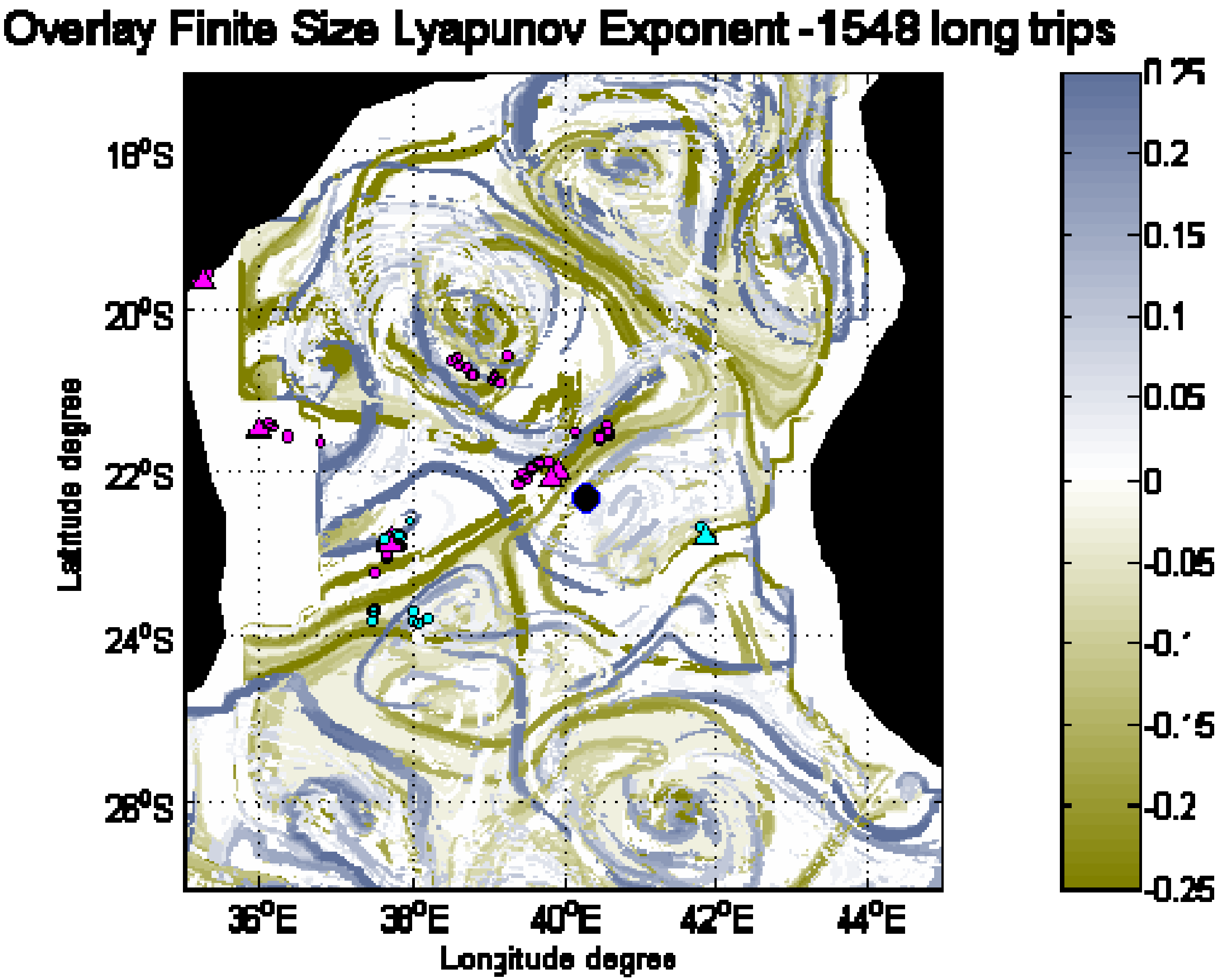
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1548 long trips
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22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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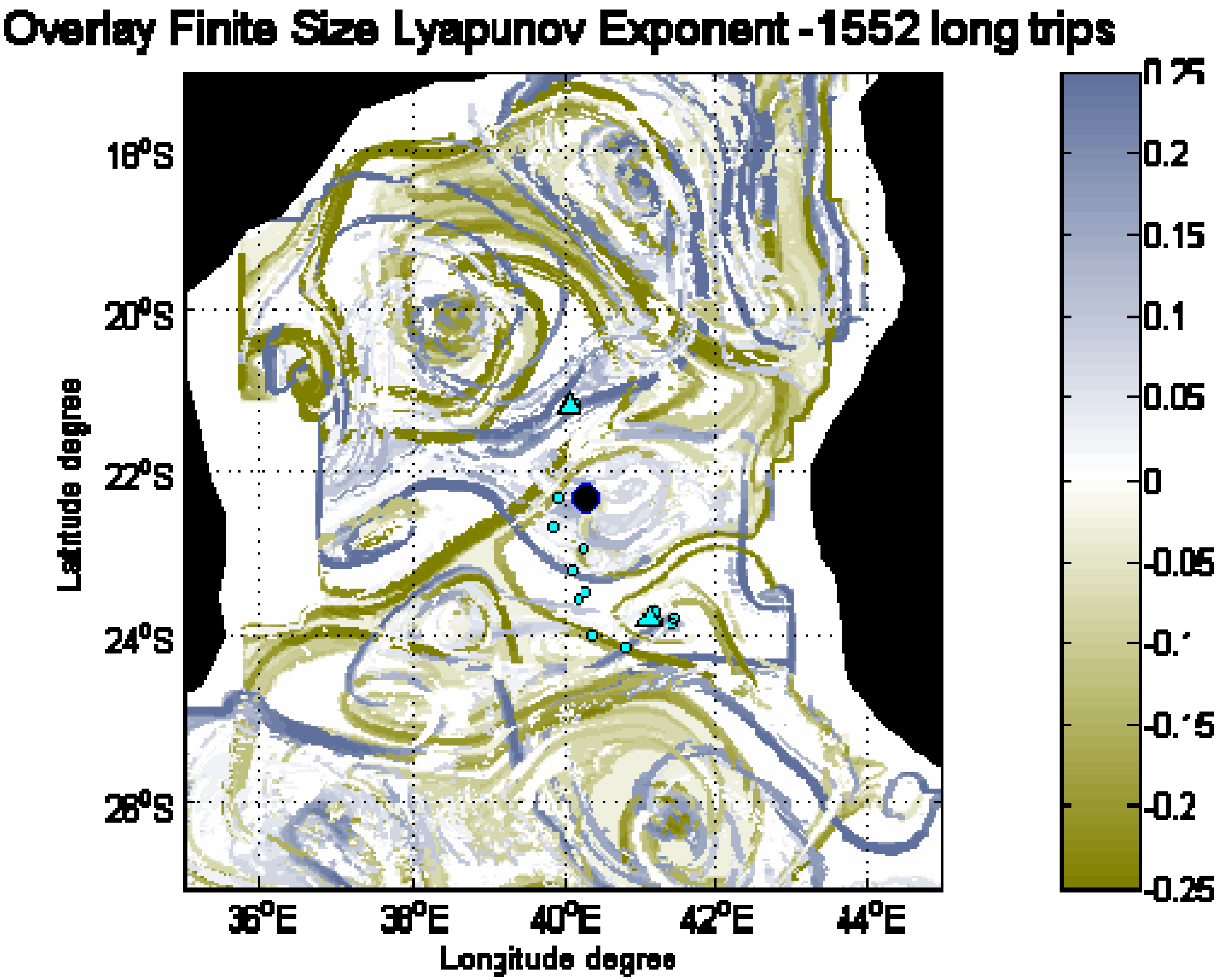
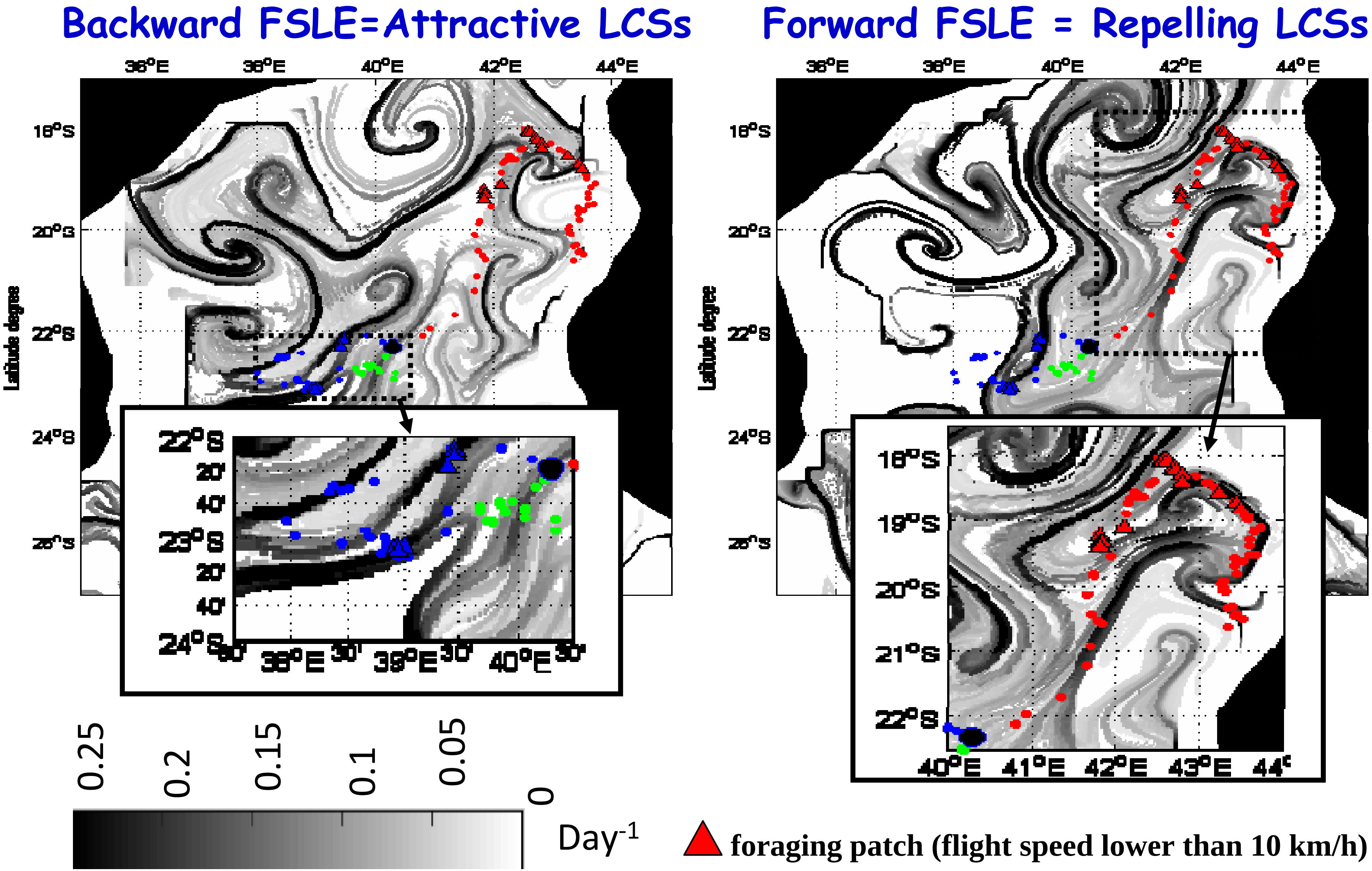
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25



Overlay Finite Size Lyapunov Exponent -1552 long trips

Week of September 24, 2003



Backward FSLE=Attractive LCSs

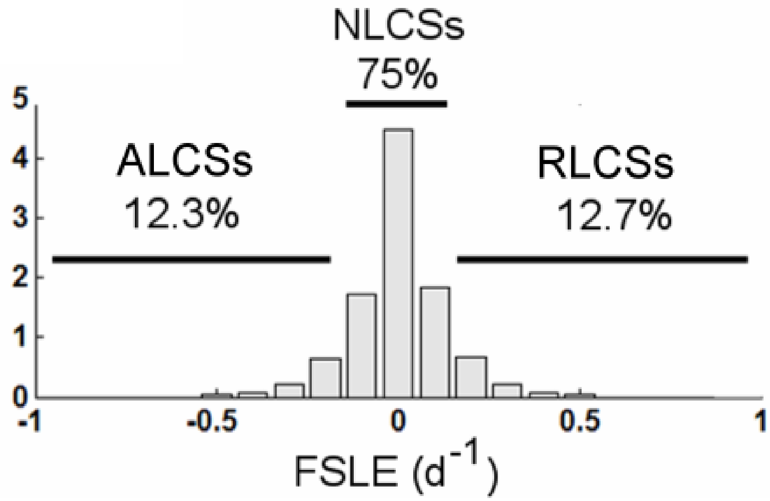
Forward FSLE = Repelling LCSs

▲ **foraging patch (flight speed lower than 10 km/h)**

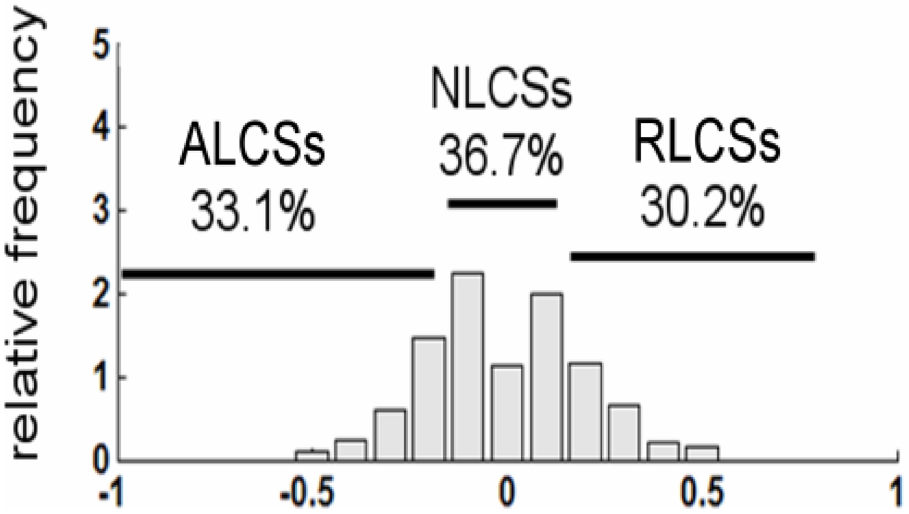
● **seabird trajectory**

Histograms of FSLE values

On the whole area



On the birds positions



ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day^{-1}

RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day^{-1}

NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird’s positions are on them

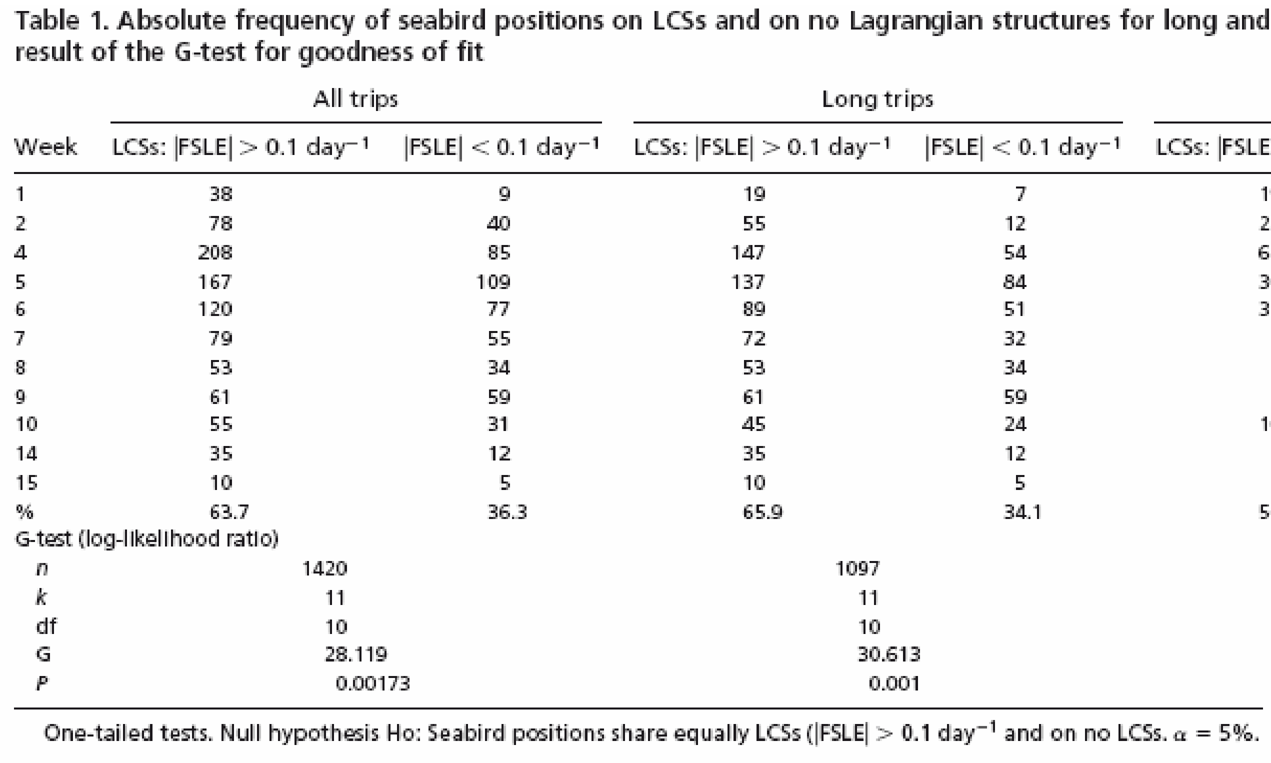
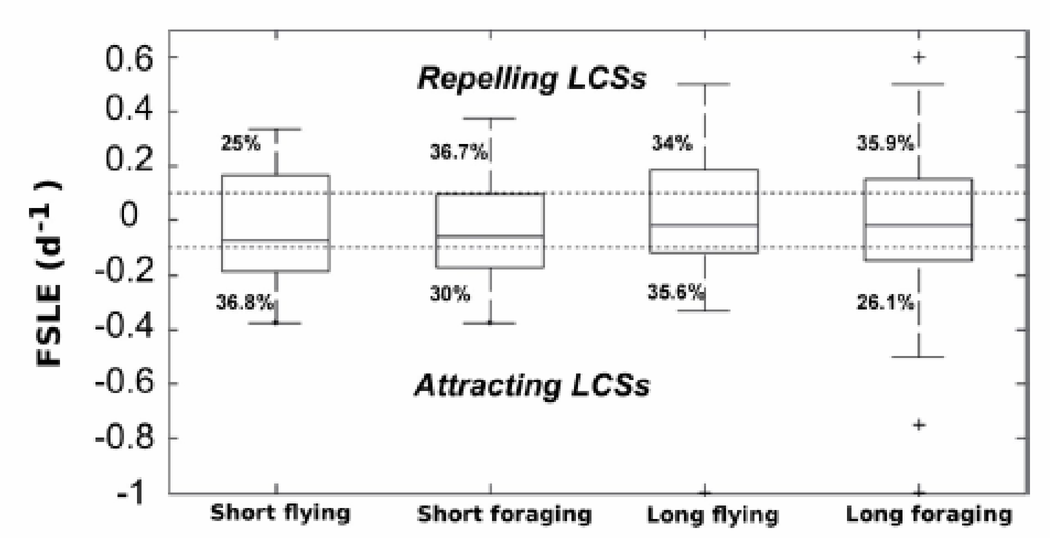


Table 1. Absolute frequency of seabird positions on LCSs and on no Lagrangian structures for long and short trips per week and result of the G-test for goodness of fit

	All trips		Long trips		Short trips	
Week	LCSs: $\lvert FSLE\rvert > 0.1$ day^{-1}	$\lvert FSLE\rvert < 0.1$ day^{-1}	LCSs: $\lvert FSLE\rvert > 0.1$ day^{-1}	$\lvert FSLE\rvert < 0.1$ day^{-1}	LCSs: $\lvert FSLE\rvert > 0.1$ day^{-1}	$\lvert FSLE\rvert < 0.1$ day^{-1}
1	38	9	19	7	19	2
2	78	40	55	12	23	28
4	208	85	147	54	61	31
5	167	109	137	84	30	25
6	120	77	89	51	31	26
7	79	55	72	32	7	23
8	53	34	53	34	—	—
9	61	59	61	59	—	—
10	55	31	45	24	10	7
14	35	12	35	12	—	—
15	10	5	10	5	—	—
%	63.7	36.3	65.9	34.1	56.0	44.0
G-test (log-likelihood ratio)						
n	1420		1097		323	
k	11		11		7	
df	10		10		6	
G	28.119		30.613		32.057	
P	0.00173		0.001		0.000	

One-tailed tests. Null hypothesis Ho: Seabird positions share equally LCSs ($\lvert FSLE\rvert > 0.1$ day^{-1} and on no LCSs. $\alpha = 5\%$.

STATISTICAL TESTS



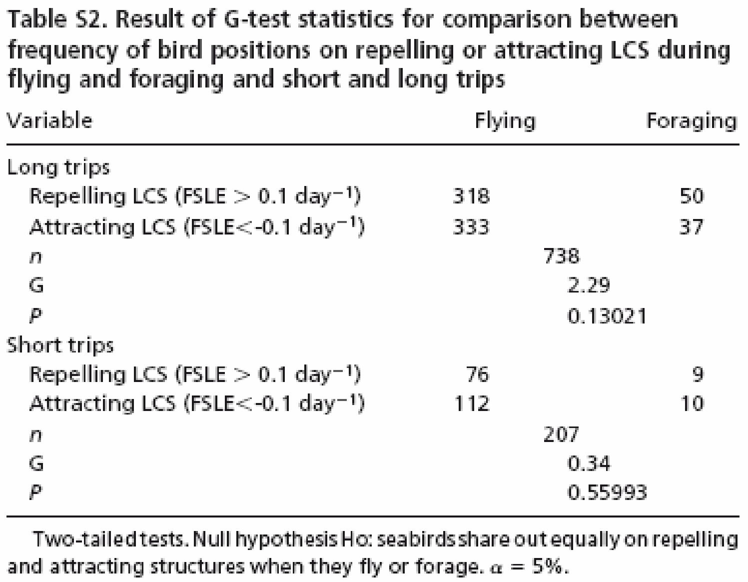


Table S2. Result of G-test statistics for comparison between frequency of bird positions on repelling or attracting LCS during flying and foraging and short and long trips

Variable	Flying		Foraging
Long trips			
Repelling LCS (FSLE $>$ 0.1 day^{-1})	318		50
Attracting LCS (FSLE$<$-0.1 day^{-1})	333		37
n		738	
G		2.29	
P		0.13021	
Short trips			
Repelling LCS (FSLE $>$ 0.1 day^{-1})	76		9
Attracting LCS (FSLE$<$-0.1 day^{-1})	112		10
n		207	
G		0.34	
P		0.55993	

Two-tailed tests. Null hypothesis Ho: seabirds share out equally on repelling and attracting structures when they fly or forage. $\alpha = 5\%$.

Results of statistical tests:

- Frigate birds fly on top of LCCs both for travelling as for foraging
- No significant difference between day and night positions
- No significant difference between come and return trip

Frigatebirds 'follow' LCSs not only to find there prey, but as biological corridors which bring them to foraging places

Aggregation of prey on LCSs? or aggregation of subsurface predators?

Olfatory clues (DMS produced by zooplankton) ? thermal air currents?

Puzzling issue: no significant difference between attracting and repelling LCSs

- Tangencies between manifolds?
- Interleaving between them?
- 3d dynamics associated both to ALCS and RLCS?
- Do they simply avoid low FSLE regions?

Tew Kai et al. PNAS (2009)

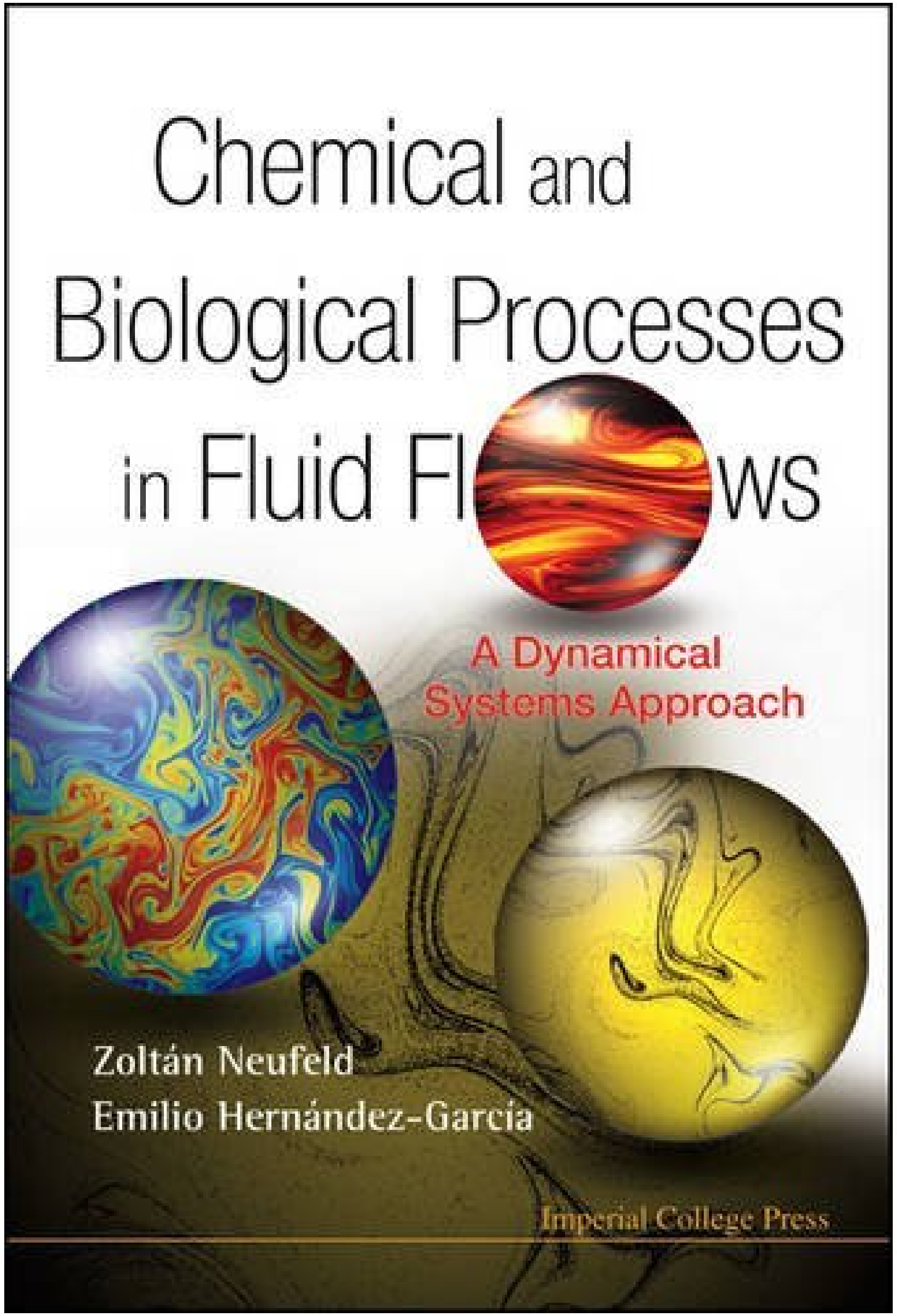
DISTINGUISHED HYPERBOLIC POINTS AND THEIR MANIFOLDS

- Rigorously identifies hyperbolic structures in the flow
- Identifies observed structures such as 'barriers to transport' known to oceanographers
- Allows the calculation of lobe areas and transport, and traces the origin of water masses
- On the negative side: hyperbolic points should be found one by one: no global view. The numerics is rather delicate.

FINITE-SIZE LYAPUNOV EXPONENT FIELDS

- Able to reveal globally the dynamical structures in the flow: main hyperbolic trajectories, their manifolds, …
- Simple enough to be applied in a practical way to real and complex ocean velocity fields.
- On the negative side: relationship with material lines is not rigorous, and lobe areas not easy to compute.
- Reveals impact of fluid flow on biological dynamics at all scales: from plankton to top predators
- Relationship with 3d dynamics desirable

http://ifisc.uib-csic.es/publications.php



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