EFFECTS OF TOPOLOGY AND DELAYED CONNECTIONS IN THE SYNCHRONIZATION PROPERTIES OF A NEURONAL NETWORK

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Abstract

In this study we investigate the local and global synchronization of an ensemble of delayed coupled neurons. We begin by investigating a circuit composed of neurons described by the Hodgkin and Huxley model [1] with reciprocal delayed chemical connections, modeled as alpha function [2]. We study the influence of different topologies on the local and global synchronization of the network. Five different types of topologies are analysed: regular, small-world, random, scale-free and all-to-all [3].

To gain insight into the effects of the delay in the synchronization properties, we consider two different configurations: one in which the delays of all the connections are homogeneous and another in which heterogeneous delays are considered. To characterize the synchronization we use an order parameter based on the phase difference between elements [4]. We compute the local synchronization using the phase difference of all pairs of connected neighbours, and the global synchronization using the phase difference between all the neurons in the network. We find that synchronized firing activity between one neuron and its neighbours (local synchronization) is achieved in all the networks that we have considered while a high randomness in the connections is required for a global synchronized activity. While varying the coupling delay we find a resonant-like effect in the synchronization even in the all- to-all topology.

Introduction

> Our system

> 10^3 neurons (Hodgkin-Huxley model)

$$C_m \dot{v}_i = I_i - I_i^{ion} - I_i^{syn}$$

$$I_i^{ion} = -g_{Na} m^3 h (v_i - E_{Na}) - g_K n^4 (v_i - E_K) - g_L (v_i - E_L)$$

$$\dot{m} = \alpha_m(v)(1-m) - \beta_m(v) m$$
$$\dot{n} = \alpha_n(v)(1-n) - \beta_n(v) n$$
$$\dot{h} = \alpha_h(v)(1-h) - \beta_h(v) h$$

Parameters	Value
g_K	36 mS/cm^2
g_{Na}	120 mS/cm^2
g_L	0.3 mS/cm^2
E_K	−12 mV
E_{Na}	115 mV
E_L	10.6 mV
I_{app}	10 mA
T_o	14.67 ms
τ_d	0.3 ms
τ_r	3 ms
E_{syn}	0 mV

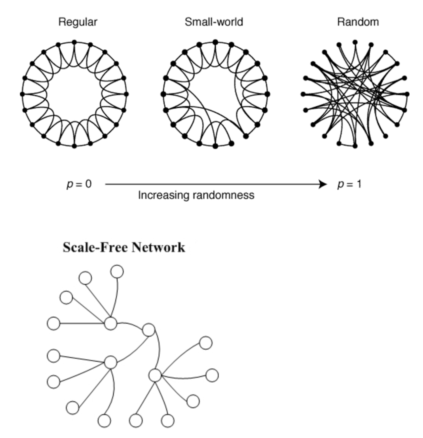
> Reciprocal delayed chemical connections (alpha functions)

$$I_i^{syn} = -\frac{g_{max}}{N_i} \sum_{spikes\, nn} \alpha(t - t_{spikes} - \tau)(v_i(t) - E_{syn})$$

$$\alpha(t) = \frac{1}{\tau_d - \tau_r}\left(\exp(-t/\tau_d) - \exp(-t/\tau_r)\right)$$

> Network Topology

- Regular
- Small Word
- Random
- Scale-Free
- All to all

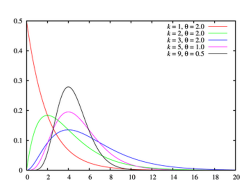


> Delay

- Homogeneous distribution
- Heterogeneous distribution (Gamma function)

mean: $\langle \tau \rangle = k\,\theta$

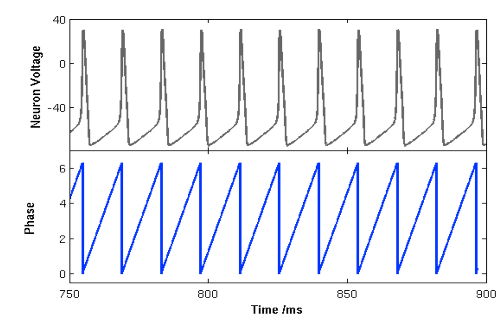
variance: $\sigma^2 = k\,\theta^2$



> Synchronization measure

- Phase of the neuron

$$\varphi_i(t) = 2\pi \frac{t - \tau_k}{\tau_{k+1} - \tau_k}$$



- Local and Global synchrony

$$s_i(t) = \frac{1}{n} \sum_{j \in neigh(i)} \sin^2\left(\frac{\varphi_i(t) - \varphi_j(t)}{2}\right) \qquad s_i'(t) = \frac{1}{N} \sum_{j=1}^{N} \sin^2\left(\frac{\varphi_i(t) - \varphi_j(t)}{2}\right)$$

$$S^{loc} = \lim_{T\to\infty} \frac{1}{T} \int \left(\frac{1}{N}\sum_{i=1}^{N} s_i\right) dt \qquad S^{glob} = \lim_{T\to\infty} \frac{1}{T} \int \left(\frac{1}{N}\sum_{i=1}^{N} s_i'\right) dt$$

Results

Homogeneous delays

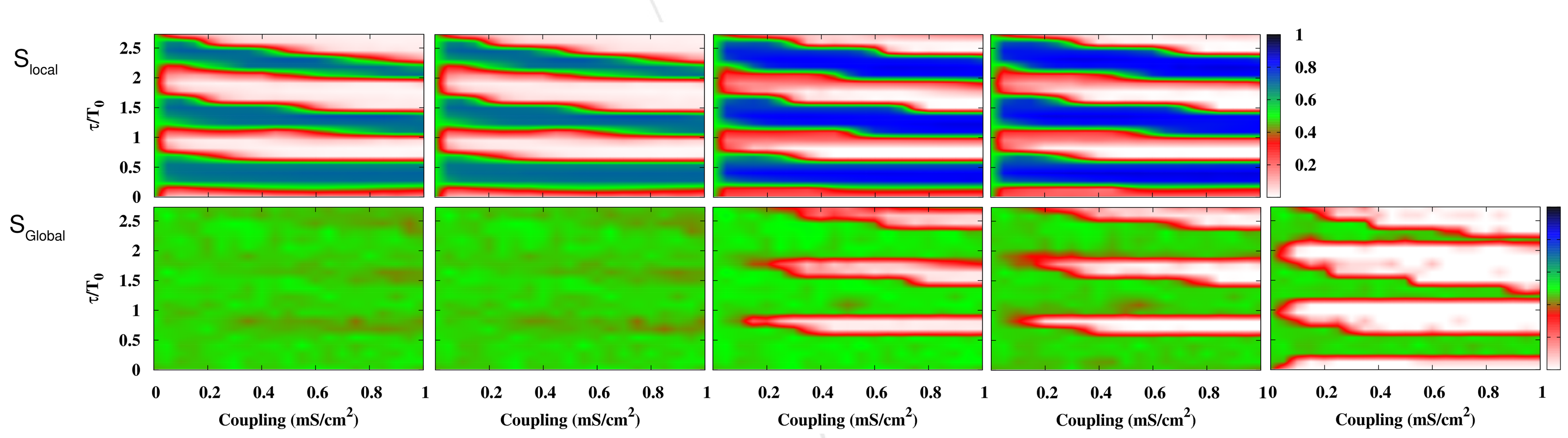


Figure 1. Local and global synchronization index for the different network topologies as function of both the coupling strength and the delay of the connections. An index value of zero means synchronization, in-phase state, whereas a value of one represent an anti-phase state.

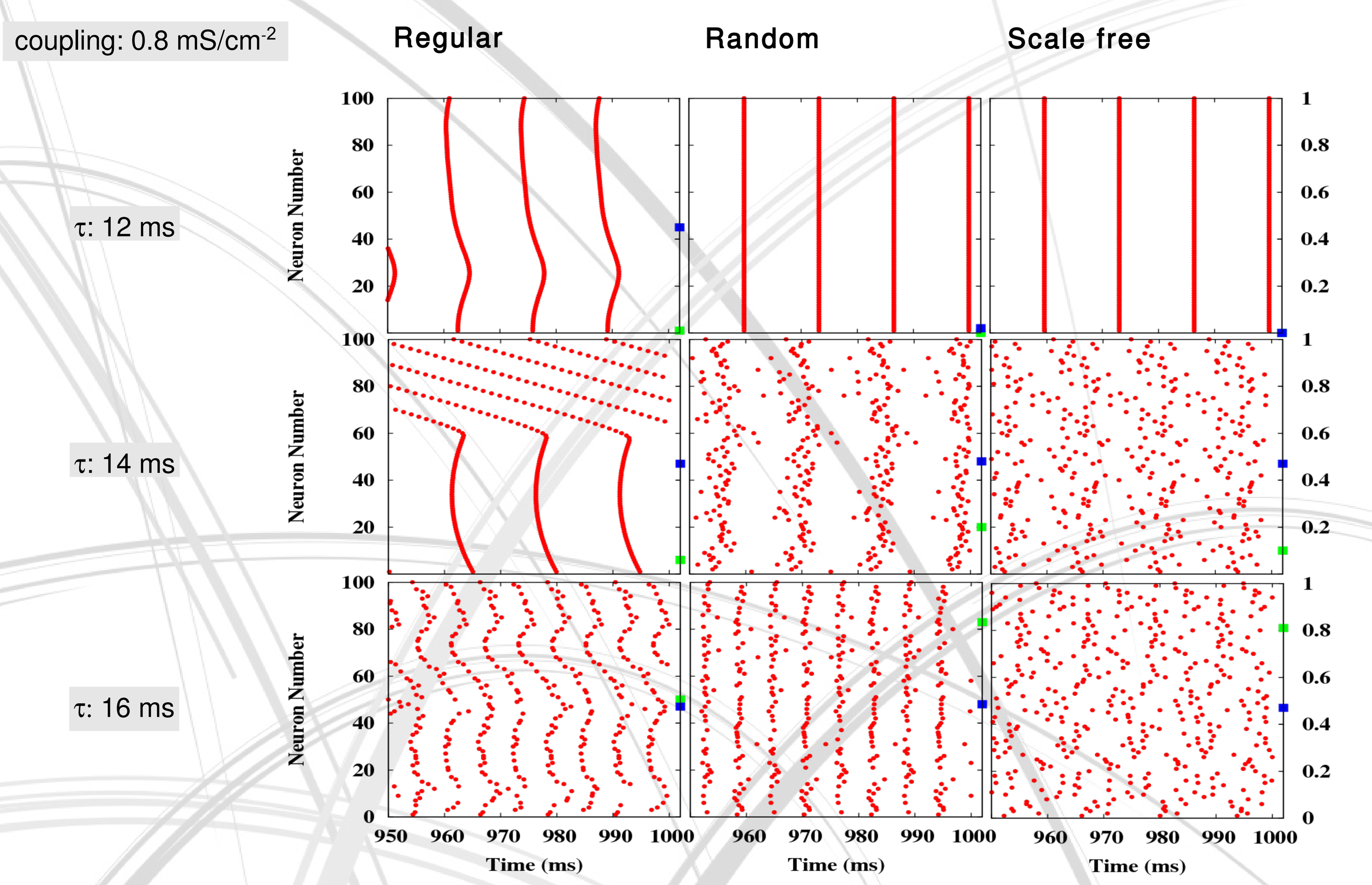


Figure 2. Raster plots of spikes for the first 100 neurons for different network topologies. For a coupling strength of 0.8 mS/cm² and delay: 12-14-16 ms. On the right y-axis is plotted the value of the local (green) and global (blue) synchronization index.

Heterogeneous delays

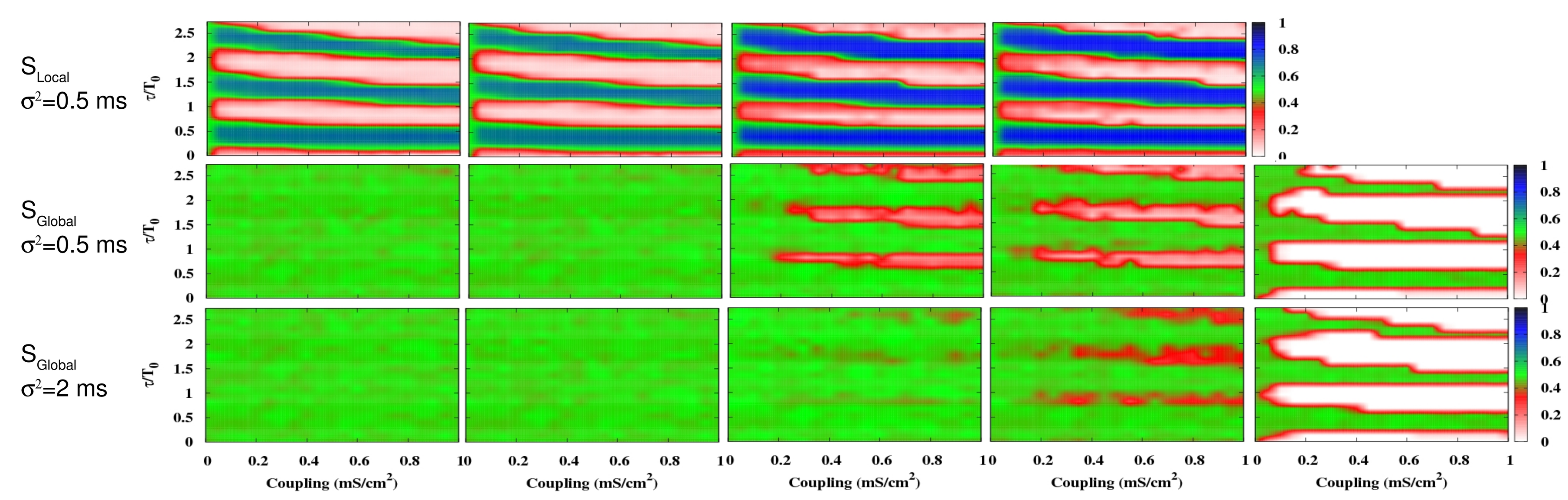


Figure 3. Local and global synchronization index for different network topologies as function of both the coupling strength and the mean delay in the connections. Delays were generated according to a gamma distribution. The mean value was varied between 0.2 to 40 ms, and the variance was maintained constant at 0.5 ms in the middle panel and 2 ms in the lower one.

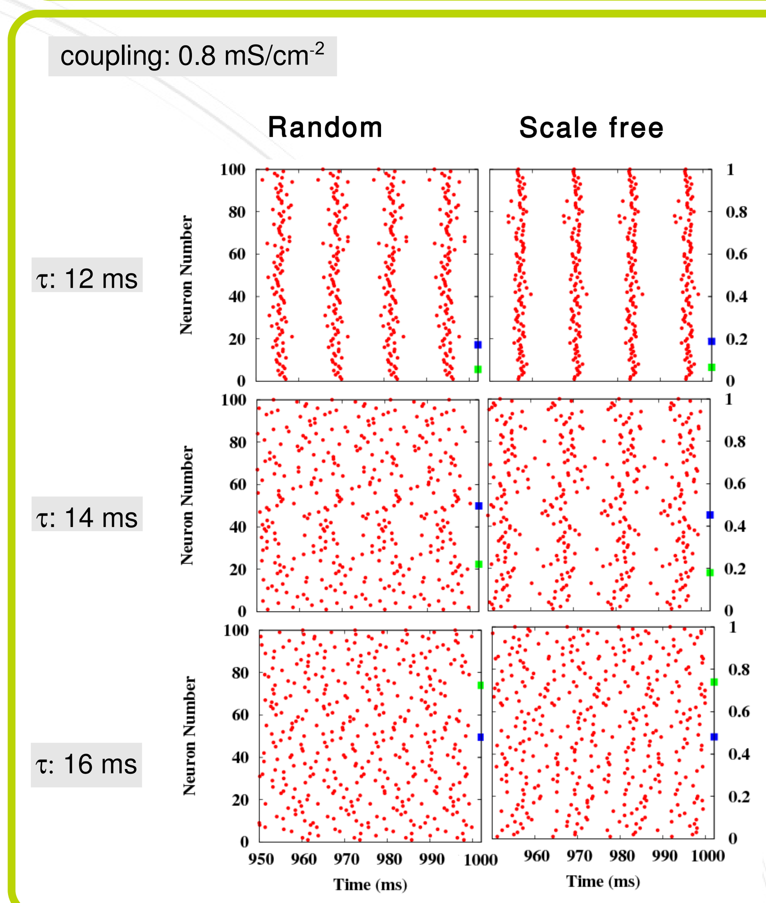


Figure 4. Raster Plots of spikes for the first 100 neurons. For a coupling strength of 0.8 mS/cm² and delay:12-14-16 ms. On the right y-axis is plotted the value of the local (green) and global (blue) synchronization index.

Why is the scale free topology more robust than the random one?

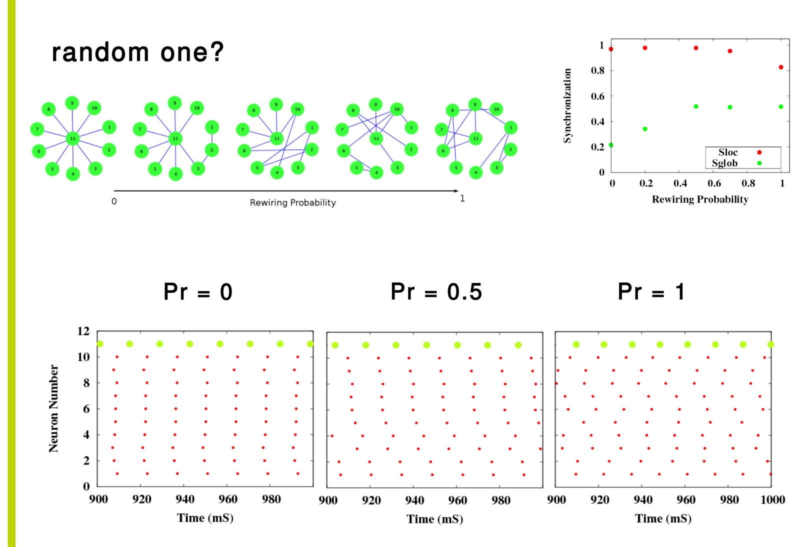


Figure 5. Raster plot for eleven neurons in a star-like network, for a coupling delay of 0.7mS/cm², mean delay 6 ms and variance 0.5 ms. The rewiring probability was varied between zero and one. In this way we change the topology from a star-like network (pr=0) to a random one (pr=1). This simple example allowed us to understand the robustness of the scale-free topology against the delay dispersion.

Conclusions

- For all the network topologies considered we observed that the variation of the delays reveals a resonant-like effect in the synchronization index (even in the all-to-all topology).
- We found that synchronized firing activity between one neuron and its neighbours (local synchronization) is achieved in all the networks while a high randomness in the connections is required to reach a global synchronized state.
- The effect of the heterogeneous delay on the dynamics of coupled neurons was explored. We observed that the global synchronization is lost when the dispersion of the delay distribution is large (s²~2 ms) for a random network. On the contrary, the scale free topology is more robust and maintain globally synchronized regions even for large variances in the distribution of delays.

[1] Hodgkin, A., and Huxley, A. (1952): A quantitative description of membrane current and its application to conduction and excitation in nerve. J. Physiol., 117: 500-544. [2] Destexhe A., Mainen Z.F., Sejnowski T.J. (1994) An efficient method for computing synaptic conductances based on a kinetic model of receptor binding Neural Comput, 6:14-18. [3] Albert, R. and Barabasi, A.L.(2002) Mechanics of complex networks, Rev. Mod. Phys., 74,47. [4] Grigori V. Osipov, Arkady S. Pikovsky, Michael G. Rosenblum, and Jürgen Kurths (1995) Phase synchronization effects in a lattice of nonidentical Rössler oscillators Phys. Rev. E, 55: 2353-2361.

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