Dynamical properties of excitable dendritic trees

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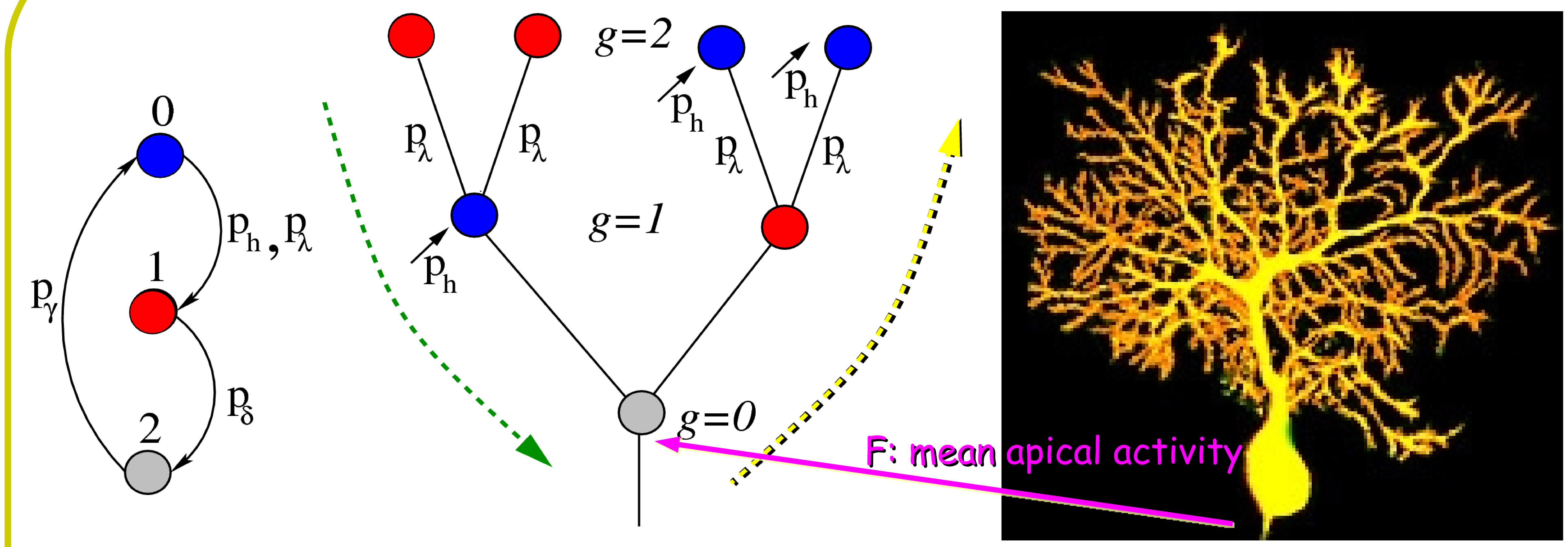
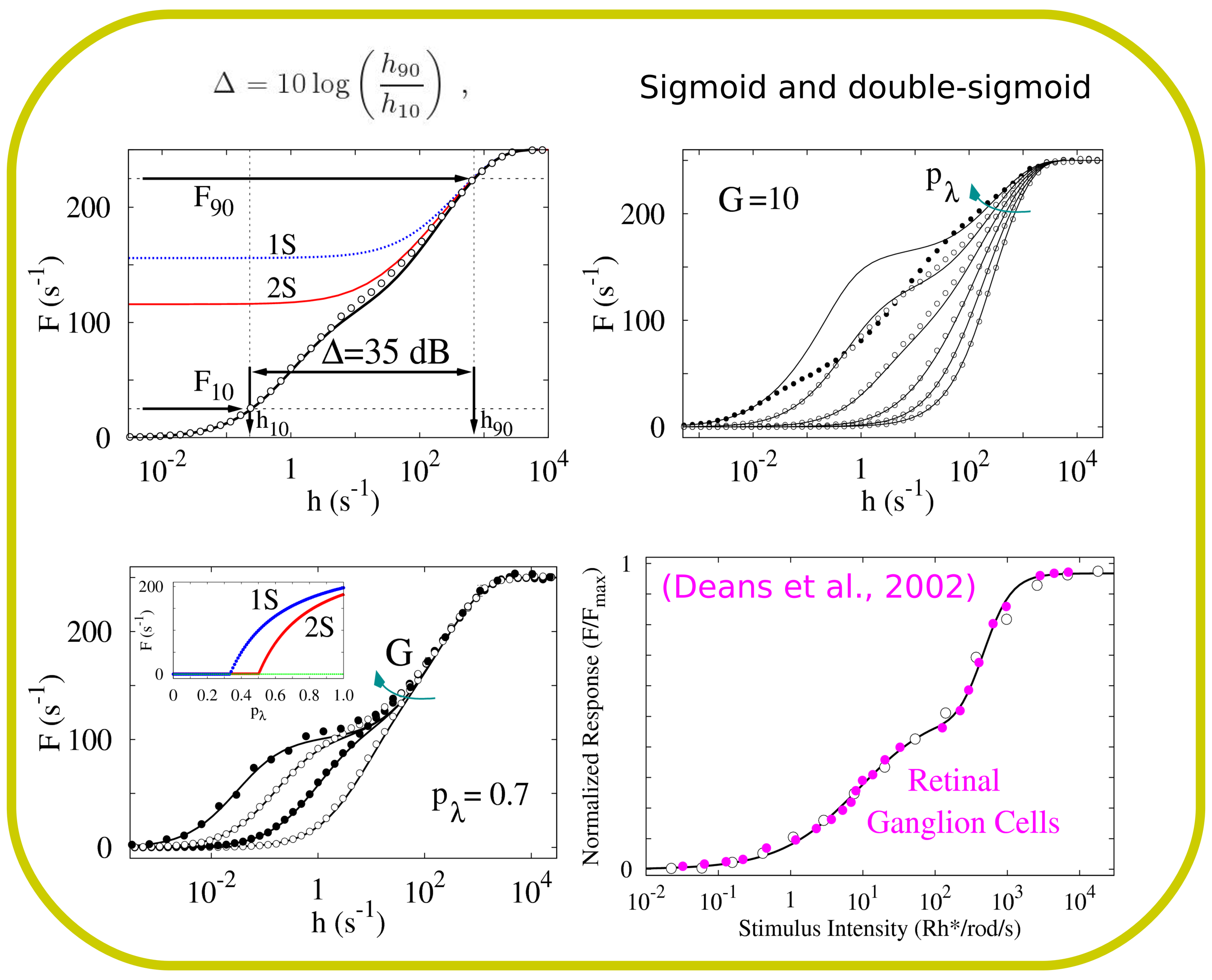
Abstract

Despite experimental and theoretical efforts, the role of dendritic excitability has remained elusive. Here we investigate a theoretical model of an active dendritic tree subjected to massive spatio-temporal stochastic input, so that the interplay between creation an annihilation of dendritic spikes can be assessed. Each branchlet is endowed with a simple excitable dynamics, but the dendritic topology is faithfully reproduced by means of a binary tree with a large number of excitable branchlets. We study analytically and through extensive simulations how collective dynamics of the branchlets give rise to a neuronal input-output response function F(h) with a much larger dynamic range than that of a passive tree. We write down the master equation and solve it in the mean field approximation. Both single site (1S) and pairs (2S) approximation present a nice description only for coupling values below a non-equilibrium critical point. Therefore we develop a direct mean field approximation which shows good agreement compared to the simulation results.

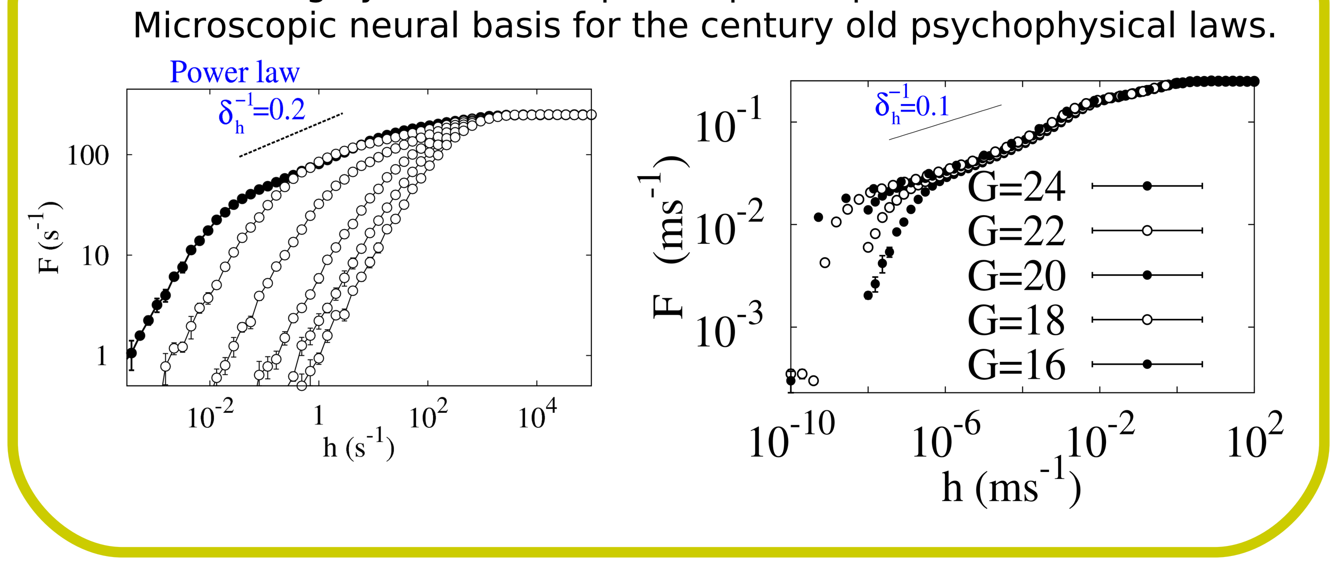
Active dendritic tree: Excitable Medium

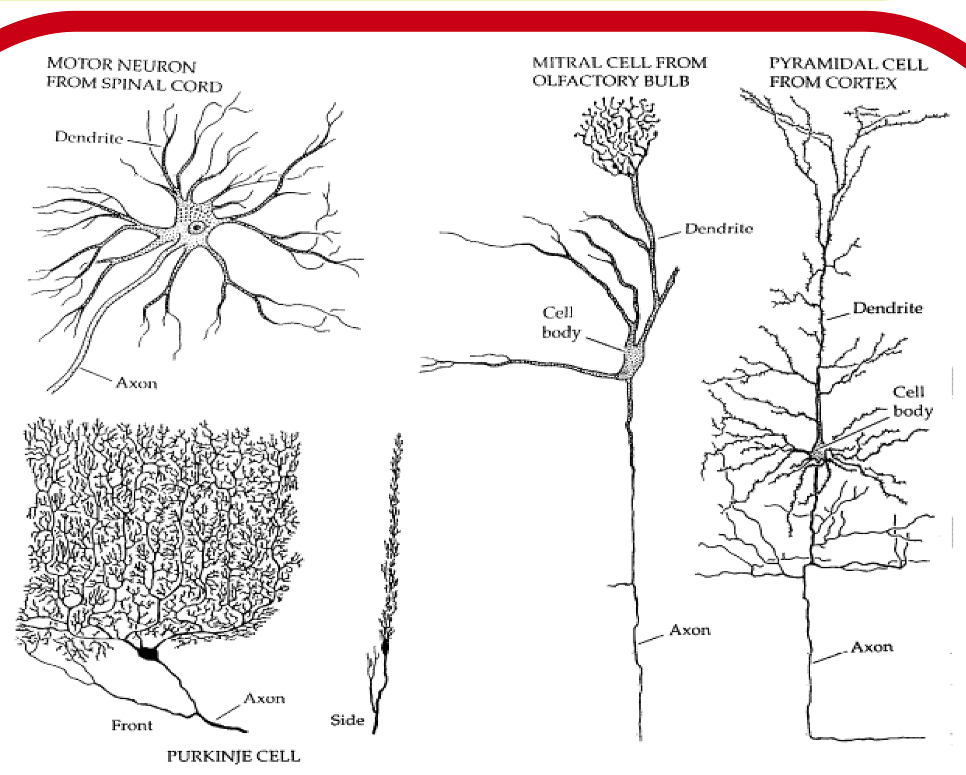


- Each active branchelet is modeled as a simple excitable cellular automaton;
- Real-like external stimulus- distributed along the whole tree, occurs stochastically and varies by orders of magnitude.
- Large tree topology reproduced;

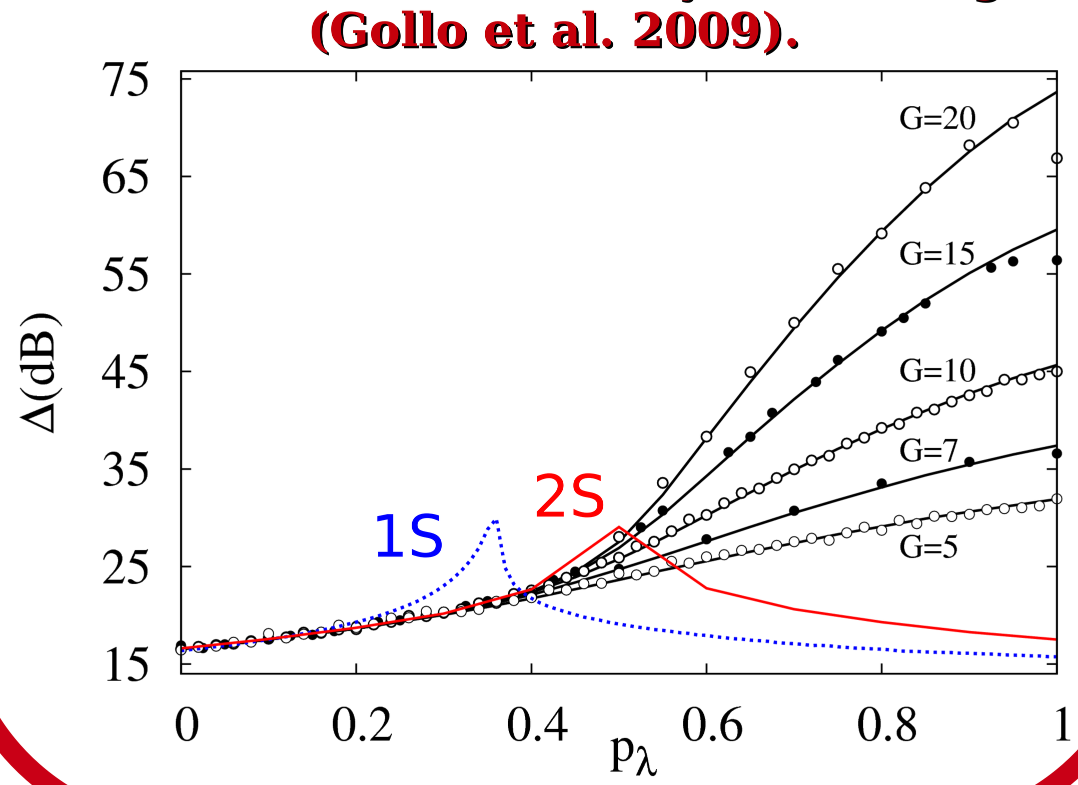


Highly nonlinear Input-output response functions.
Microscopic neural basis for the century old psychophysical laws.





What is the function of such elaborate and costly structures?
To enhance neuronal dynamic range (Gollo et al. 2009).



Master Equation

$$
\begin{aligned}
P^g_{t+1}(;1;) &= P^g_t(;0;) \\
&\quad -(1-p_h)\sum_{i=0}^{z-1}\left[p_\lambda^i\binom{z-1}{i}(-1)^i P^g_t\left(;0;1^{(i)}\right)\right. \\
&\quad \left.-\beta p_\lambda^{i+1}\binom{z-1}{i}(-1)^i P^g_t\left(1;0;1^{(i)}\right)\right] \\
&\quad +(1-p_\delta)P^g_t(;1;)\,, \\
P^g_{t+1}(;0;) &= 1-P^g_{t+1}(;1;)-P^g_{t+1}(;2;)\,, \\
P^g_{t+1}(;2;) &= p_\delta P^g_t(;1;)+(1-p_\gamma)P^g_t(;2;)\,,
\end{aligned}
$$

Single Site Mean Field Approximation

$$P^g_t(x|y) = P^g_t(;x;y)/P^{g+1}_t(;y;) \overset{(1S)}{\approx} P^g_t(x).$$

Pairs Approximation

$$P(x|y,z,w) \overset{(2S)}{\approx} P(x|y).$$

Directed Mean Field approximation

$$
\begin{aligned}
P^g_{t+1}(1A) &= P^g_t(0)\Lambda^g_A+(1-p_\delta)P^g_t(1A)\,, \\
P^g_{t+1}(1B) &= P^g_t(0)(1-\Lambda^g_A)\Lambda^g_B(t) \\
&\quad +(1-p_\delta)P^g_t(1B)\,, \\
P^g_{t+1}(1C) &= P^g_t(0)(1-\Lambda^g_A)\left(1-\Lambda^g_B(t)\right)\Lambda^g_C(t) \\
&\quad +(1-p_\delta)P^g_t(1C)\,.
\end{aligned}
$$

Where:

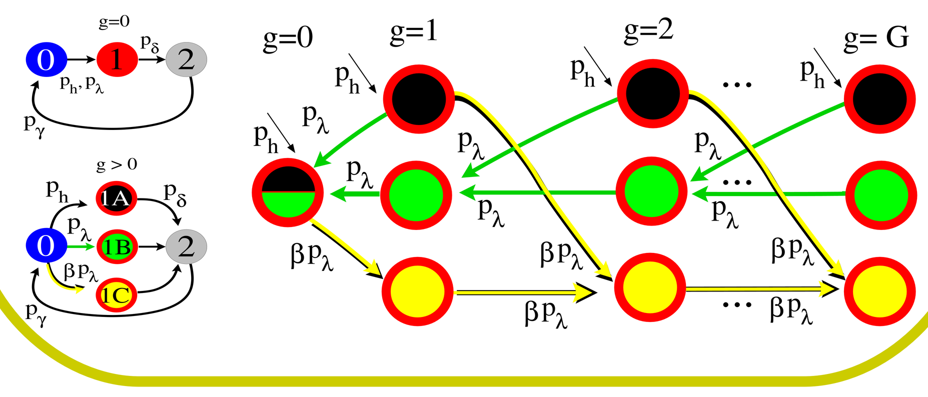
$$
\begin{aligned}
\Lambda^g_A &= p_h \\
\Lambda^g_B(t) &= 1-\left(1-p_\lambda\left(P^{g+1}_t(1A)+P^{g+1}_t(1B)\right)\right)^k, \\
\Lambda^g_C(t) &= \beta p_\lambda\left(P^{g-1}_t(1A)+P^{g-1}_t(1C)\right).
\end{aligned}
$$

It satisfies the normalization condition:

$$P^g_t(1) = P^g_t(1A)+P^g_t(1B)+P^g_t(1C)\,,$$

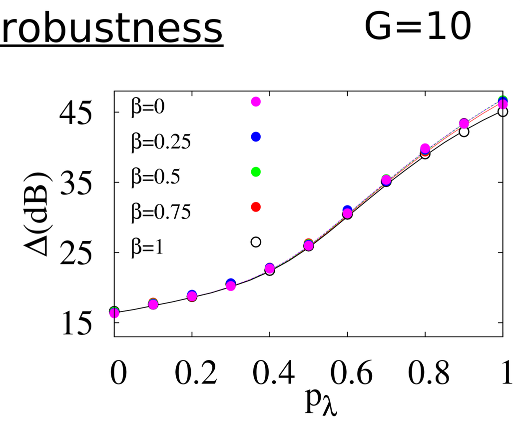
and

$F \equiv \lim_{t\to\infty} P^0_t(1).$ is used to compare with the simulation results.



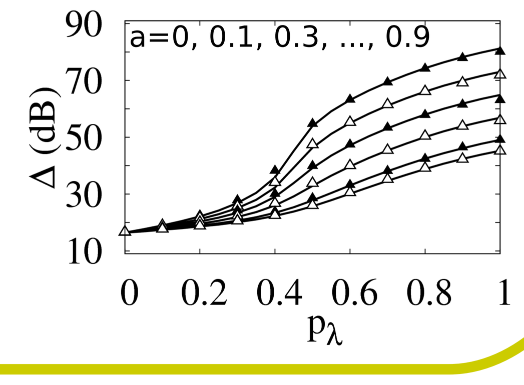
Model robustness G=10

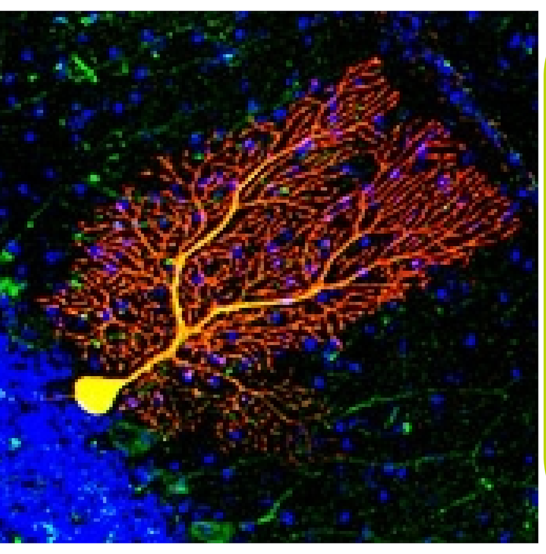
Asymmetrical propagation probability



Heterogeneous tree activation:

$h(g)=h_0 e^{ag}$





Conclusions

The model is capable to reproduce double-sigmoid response function.
Large dynamic range is obtained. The results are robust against variants of the model.
Our novel approximation correctly describes the system dynamics.

Gollo, L.L, Kinouchi, O., Copelli, M., 2009. Active dendrites enhance neuronal dynamic range. PloS Comput. Biol. 5(6) e10000402.
Deans MR, Volgyi B, Goodenough DA, Bloomfield SA, Paul DL (2002) Connexin36 is essential for transmission of rod-mediated visual signals in the mammalian retina. Neuron 36: 703–712.