Spontaneous ordering against an external field in nonequilibrium systems.

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Abstract

We study the collective behavior of nonequilibrium systems subject to an external field with a dynamics characterized by the existence of non-interacting states. Aiming at exploring the generality of the results, we consider two types of models according to the nature of their state variables: (i) a vector model, where interactions are proportional to the overlap between the states, and (ii) a scalar model, where interaction depends on the distance between states. In both cases the system displays three phases: two ordered phases, one parallel to the field, and another orthogonal to the field; and a disordered phase. The phase space is numerically characterized for each model in a fully connected network. By placing the particles on a small-world network, we show that, while a regular lattice favors the alignment with the field, the presence of long-range interactions promotes the formation of the ordered phase orthogonal to the field.

Motivation

A general question in framework of statistical physics of interacting (particles, spins, agents) is the competition between local-particle interaction and particle interaction with a externally applied field.

Common answer: strong external field dominates over local particle-particle interaction and orders these systems by aligning particles with the broken symmetry imposed by the field.

Is this valid in systems with non-potential interactions?.

Definition of Vector model

Axelrod's model
(J. Conflict. Res. 41,203 (1997))

Question: *"If people tend to become more alike in their beliefs, attitudes and behavior when they interact, why do not all differences eventually disappear?"*

Proposal: Simple model to explore mechanisms of competition between consensus and cultural diversity (*nonequilibrium transition order-disorder*)

Definition of culture: *"Set of individual attributes that are subject to social influence."*

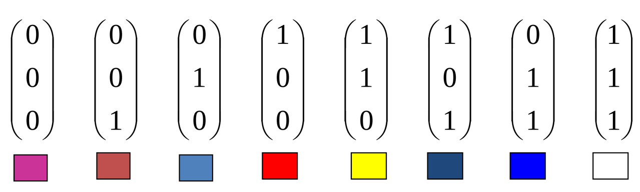
Premises: 1) The probability of interaction between individuals is proportional to the number of attributes (cultural features) that they share.

2) The interaction increases cultural similarity.

Cultural vector of i: $C_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{if}, \ldots, \sigma_{iF})$

F = # Features; q = # Traits per feature; $\sigma_{if} \in \{0, \ldots, q-1\}$

q^F = # equivalent cultural states.



Example: F=3; q=2 8 different cultural states.

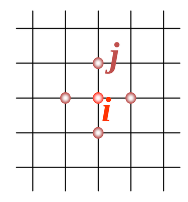
Dynamics of interaction in the Vector model

Dynamics of interaction

Castellano, Marsili, Vespignani, Phys. Rev. Lett. 85, 3536 (2000)
San Miguel et al., Computing in Science and Engineering 7, 67 (2005)

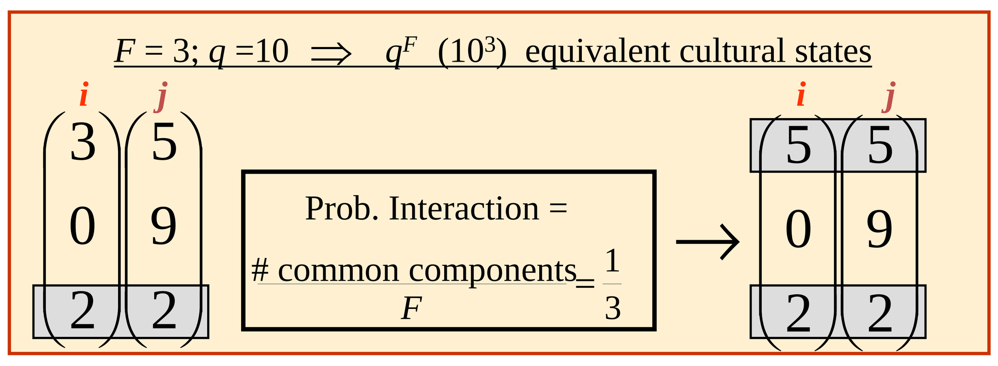
Start from a random initial condition

At each time step:
• Pick an individual *i* at random.
• Pick a one of its neighbor (j at random.

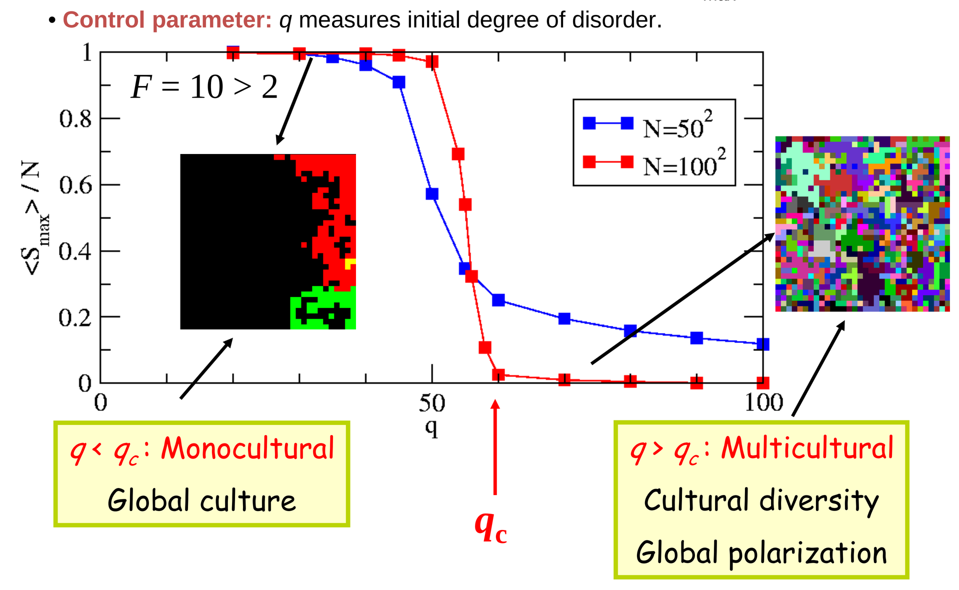


They interact with probability equal to the fraction of shared features (components).

In case of interaction, an unshared feature is selected at random and *i* copies *j*'s value for this feature.



• **Order parameter:** size of the largest homogeneous domain S_{max}
• **Control parameter:** q measures initial degree of disorder.

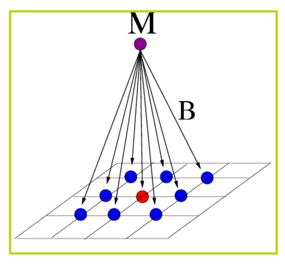


Vector model with external field interaction.

Definition of external field: *an external cultural vector that can interact with all the elements in the system. (mass media or propaganda)*

$M = (\mu_1, \mu_2, \ldots, \mu_f, \ldots, \mu_F)$

$\mu_f \in \{0, \ldots, q-1\}$



- Uniform for all agents i
- Fixed for all times

Assumptions:

- *All elements in the system have the same probability of being affected by the external field at any time (is uniform and constant in time).*
- *Probability of interaction of an element with the field is proportional to the number of common component).*

Parameter B: *probability to interact with the field.* $B \in [0,1]$

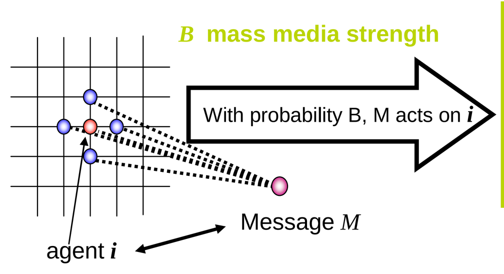
Dynamics of interaction with external field

Agent i: $C_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{if}, \ldots, \sigma_{iF})$ ⟷ External Field: $M = (\mu_1, \mu_2, \ldots, \mu_f, \ldots, \mu_F)$

Parameter $B \in [0, 1]$: probability that M acts on i in one time step: "*strength*" of field interaction.

$1-B$: probability to interact with j selected at random among nearest neighbors of i.

⇨ *M* acts as a additional effective neighbor of *i*.



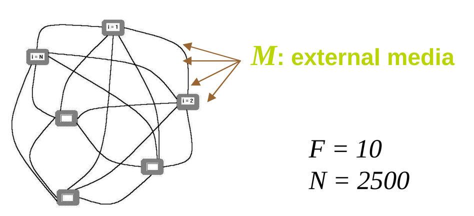
1) If M acts on agent i, the probability of interaction p_{iM} is proportional to the overlap between i and M

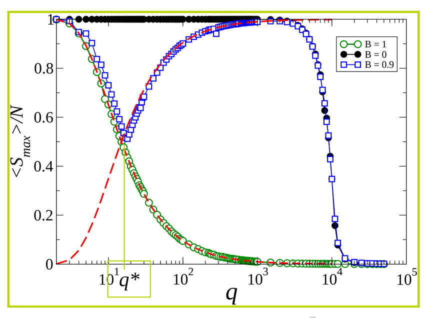
2) Agent- Ext. Field interaction results in agent i adopting a cultural feature of M

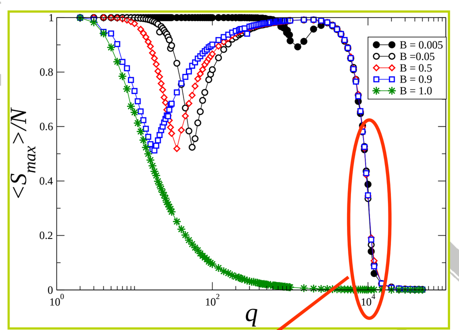
Results.

Global coupling: all-to-all

Globally coupled network







$q_{c(G)}$ is independent of B.

• Competition between the order induced by an external field and spontaneous order.

• For B → 1: where:

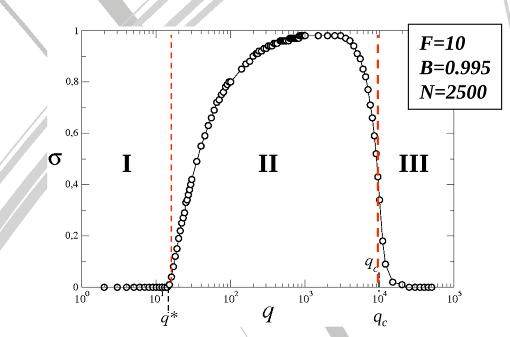
$$\frac{\langle S_{max}\rangle}{N} = \begin{cases} 1-\left(1-\frac{1}{q}\right)^F, & \text{if } q \le q^* \\ \left(1-\frac{1}{q}\right)^F, & \text{if } q > q^* \end{cases} \qquad q^* = \left[1-\left(\frac{1}{2}\right)^{1/F}\right]^{-1}$$

$$\sigma = \frac{\langle S_{max} - S_M\rangle}{N}$$

External Field

S_{max}: size of largest domain

S_M: size of domain having state equal to M



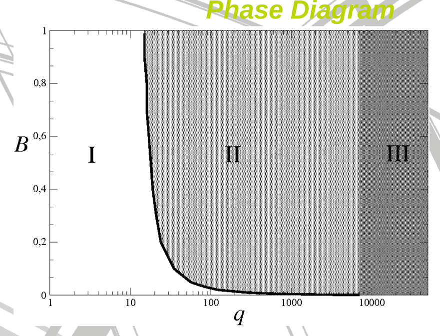
Phases:

I: homogeneous, ordered = external field
$S_{max} = S_M \neq 0$ for $q < q^*(B)$

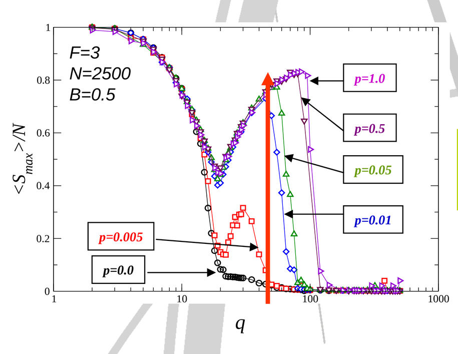
II: alternative ordering state ≠ external field
$S_{max} > S_M$ for $q^*(B) < q < q_c$

III: disordered
$S_{max} \to 0$, $S_M \to 0$ for $q > q_c$

Phase Diagram



Rol of topology



The emergence of a self-organized group opposed to the external field is possible because of the existence of long-range links.

Scalar model with external field interaction.

• *Bounded confidence model (Deffuant G et al. Adv. Complex System.* **3** *87 (2000)):*

• The system consists of a population of N nodes where the state of a node i is given by a real number $C_i \in [0,1]$.

• We introduce an external field $M \in [0,1]$ that can interact with any of the particles in the system.

• The strength of the field is again described by a parameter $B \in [0,1]$ that measures the probability of particle-field interactions, as in the vector model.

Dynamics:

We start from a uniform random initial distribution of the states of the particles.

i) With probability B, the node i interacts with the field M:

If $|C_i - M| < d$ then

$$C_i^{t+1} = \frac{1}{2}(M + C_i^t)$$

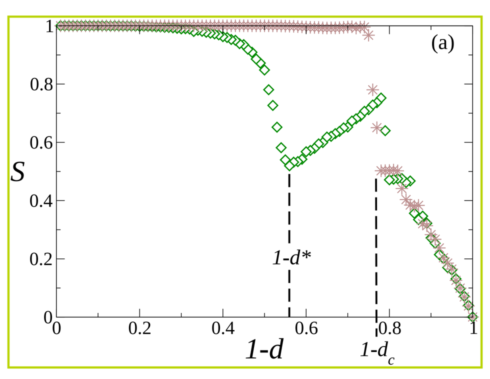
ii) Otherwise, a nearest neighbor j is selected at random:

if $|C_i - C_j| < d$ then

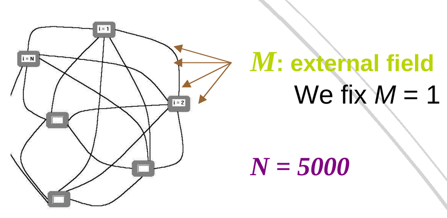
$$C_i^{t+1} = C_j^{t+1} = \frac{1}{2}(C_i^t + C_j^t)$$

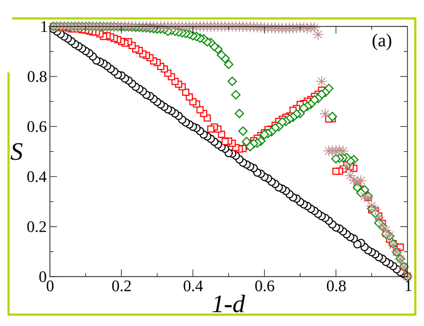
The parameter d defines a threshold distance for interaction.

Globally coupled network



Global coupling: all-to-all





B=0
B=0.5
B=0.8
B=1.0

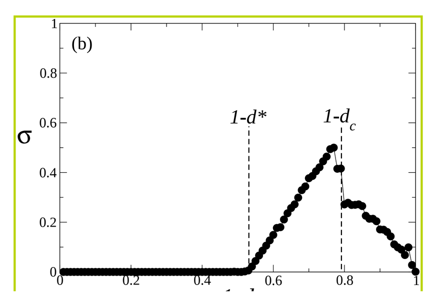
Transitions in globally coupled society

$$\sigma = \frac{\langle S_{max} - S_M\rangle}{N}$$

External Field

S_{max}: size of largest domain

S_M: size of domain having state equal to M



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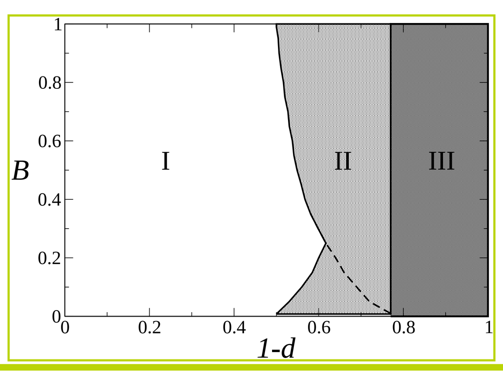
III: disordered
$d > 1\text{-}d_c$



Summary

• We have studied the collective behavior of nonequilibrium systems subject to an external field.

• We found three phases depending on parameter values:

• *Two ordered phases:* one having a state equal to the external field, another where the state of the largest domain is orthogonal to the field.

• *A disordered phase*

• The occurrence of an ordered phase with a state orthogonal to the field is enhanced by the presence of non-interacting state and long-range connections in the underling network.

• The generality of the results have been explored in two different models.