From microscopic to macroscopic dynamics in systems with two symmetric absorbing states

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Abstract

The study of systems with absorbing states has become one of the fundamental topics in modern non-equilibrium statistical physics. During the last decade, special attention was given to systems with two symmetric absorbing states, due to the emergence of a large amount of new models coming form researchers working in interdisciplinary physics, such as, ecological modeling, opinion dynamics and language evolution. Even though some of these models are very different in nature, they exhibit similar dynamical properties, such as coarsening and critical exponents, suggesting that they belong to the same universality class. The question that rises is: is it possible to classify these models based on their microscopic rules only?

In this work, we answer this question by developing an approach to study general spin models with two symmetric absorbing states. Starting from the microscopic dynamics on a square lattice, we derive a Langevin equation for the time evolution of the magnetization field, that successfully explains coarsening properties of a wide range of nonlinear voter models and systems with intermediate states. We find that the macroscopic behavior only depends on the first derivatives of the spin-flip probabilities. Moreover, an analysis of the mean-field term reveals the three types of transitions commonly observed in these systems -generalized voter, Ising and directed percolation-.

Spin systems with two symmetric absorbing states

Systems with two symmetric (equivalent) states represented by S=-1,1 are called Z_2-symmetric. Example: Ising model.

Absorbing state (AS): any state in a statistical system that has no microscopic fluctuations.

Consequence: once the AS is reached, the system cannot scape from it (non-equilibrium).

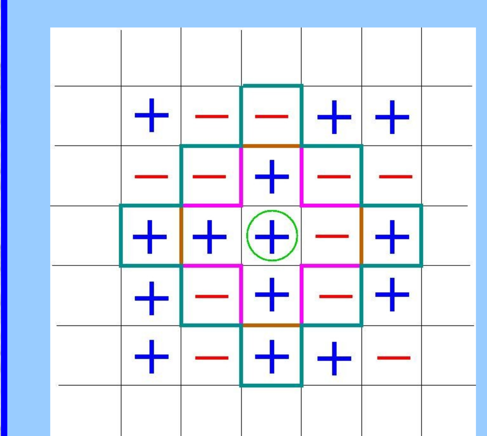
Example: Fully ordered state in T=0 Ising model.

Z_2 AS systems: a general approach

Universality: Many models with different dynamical rules but same macroscopic behavior (coarsening, critical exponents).

- Question: Can we classify models by their microscopic dynamics ?

Generic lattice model:



- S_r = -1, 1 (spin at site r).

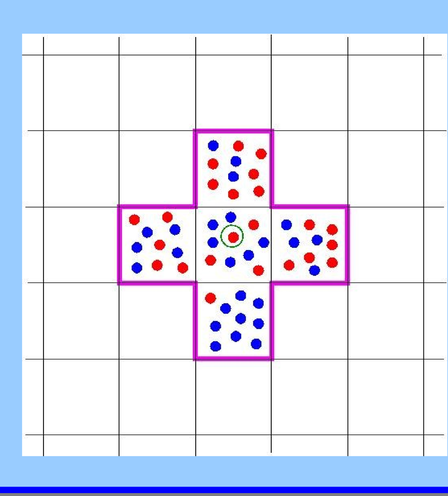
r =(r_1, r_2, ..r_d), d = space dimension.

$$\psi_{\mathbf{r}} \equiv \frac{1}{z}\sum_{\mathbf{r}'/\mathbf{r}} S_{\mathbf{r}'}$$

- $f(-S_r\,\psi_r)$ (spin-flip probability)
- $f(-1) = 0$ (absorbing condition)

Field approximation

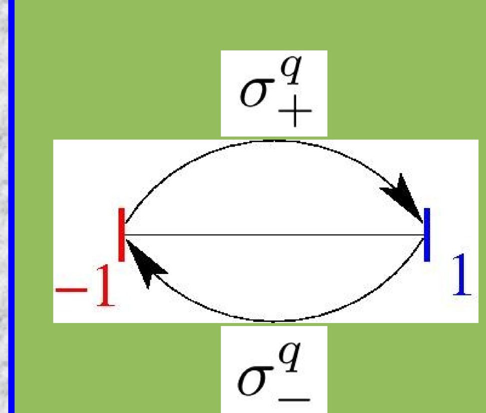
$\Phi_r(t)$ = *magnetization field* at site **r** at time t (continuous spin)



$$\phi_{\mathbf{r}}(t) \rightarrow \frac{1}{\Omega}\sum_{j=1}^{\Omega} S_{\mathbf{r}}^{j} \qquad \psi_{\mathbf{r}} \rightarrow \frac{1}{z}\sum_{\mathbf{r}'/\mathbf{r}} \phi_{\mathbf{r}'}(t)$$

- Choose a site **x** at random.
- Choose one particle from **x** at random.
- Flip its spin S_x with probability $f(-S_x\,\psi_x)$.
- Repeat.

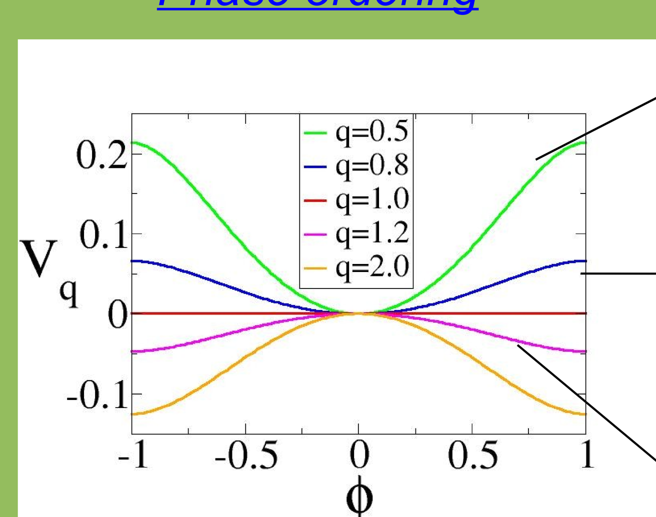
Example: Abrams-Strogatz model for language evolution.

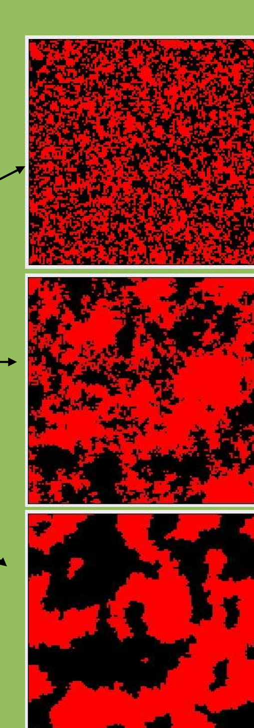


-1 = Language A.
1 = Language B.

$$f(\psi) = \left(\frac{1+\psi}{2}\right)^q$$

Phase ordering





q = 0.5 Disordered active state.

q = 1.0 Ordering without surface tension.

q = 2.0 Ordering by surface tension.

■ down spins
■ up spins

$$\frac{\partial\phi}{\partial t} = \frac{(q-1)}{3\times 2^q}(1-\phi^2)\left[6\phi+(q-2)(q-3)\phi^3\right] + \frac{q}{2^q}\left[2+(q-1)(q-4)\phi^2\right]\Delta\phi+\eta$$

$$V_q(\phi) = -\frac{(q-1)}{3\times 2^q}\left\{3\phi^2+\left[(q-2)(q-3)-6\right]\frac{\phi^4}{4}-(q-2)(q-3)\frac{\phi^6}{6}\right\}$$

Fokker-Planck equation

$$\frac{\partial}{\partial t}\mathcal{P}(\{\phi\},t) = \sum_{\mathbf{x}} -\frac{1}{\Omega}\frac{\partial}{\partial\phi}\Big\{2\left[W^+(\phi,\mathbf{x},t)-W^-(\phi,\mathbf{x},t)\right]\mathcal{P}(\{\phi\},t)\Big\} + \frac{1}{\Omega^2}\frac{\partial^2}{\partial\phi^2}\Big\{2\left[W^+(\phi,\mathbf{x},t)+W^-(\phi,\mathbf{x},t)\right]\mathcal{P}(\{\phi\},t)\Big\}$$

Langevin equation

$$\frac{\partial\phi_{\mathbf{r}}(t)}{\partial t} = \left[1-\phi_{\mathbf{r}}(t)\right]f(\psi_{\mathbf{r}}) - \left[1+\phi_{\mathbf{r}}(t)\right]f(-\psi_{\mathbf{r}}) + \eta_{\mathbf{r}}(t)$$

Noise:

$$\langle\eta_{\mathbf{r}}(t)\eta_{\mathbf{r}'}(t')\rangle = \Big\{\left[1-\phi_{\mathbf{r}}(t)\right]f(\psi_{\mathbf{r}}) + \left[1+\phi_{\mathbf{r}}(t)\right]f(-\psi_{\mathbf{r}})\Big\}\delta_{\mathbf{r},\mathbf{r}'}\delta(t-t')/\Omega^{1/2}$$

Expansion around $\psi_r = 0$, up to 4^{th} order.

$$f(\psi_{\mathbf{r}}) = \frac{1}{2}(1+\psi_{\mathbf{r}})\left(c+a\psi_{\mathbf{r}}+d\psi_{\mathbf{r}}^2-b\psi_{\mathbf{r}}^3\right)$$

$$c \equiv 2f(0),\quad a \equiv 2f'(0)-c,\quad d \equiv f''(0)-a,\quad b \equiv -\frac{f'''(0)}{3}+d$$

$$\Delta\phi_{\mathbf{r}} \equiv \frac{1}{z}\sum_{\mathbf{r}'/\mathbf{r}}(\phi_{\mathbf{r}'}-\phi_{\mathbf{r}}) = \psi_{\mathbf{r}}-\phi_{\mathbf{r}}$$ (Laplacian operator)

Langevin equation for Φ

$$\frac{\partial\phi}{\partial t} = (1-\phi^2)(a\phi-b\phi^3) + \left[a+c+(d-2a-3b)\phi^2\right]\Delta\phi+\eta$$

Noise:

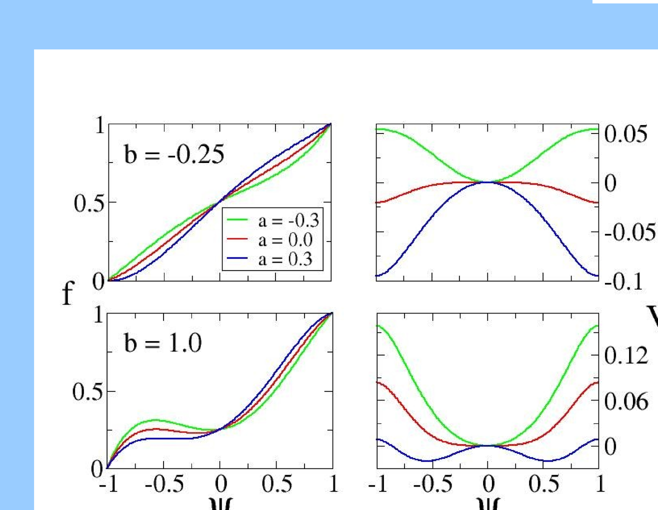
$$\langle\eta_{\mathbf{r}}(t)\eta_{\mathbf{r}'}(t')\rangle = \Big\{(1-\phi^2)(c+d\phi^2)+(a-c+2d)\phi\Delta\phi\Big\}\delta_{\mathbf{r},\mathbf{r}'}\delta(t-t')$$

Phase ordering

$$\frac{\partial\phi}{\partial t} = D\Delta\phi - \frac{\partial V}{\partial\phi}$$

Time-dependent Ginzburg-Landau equation with potential

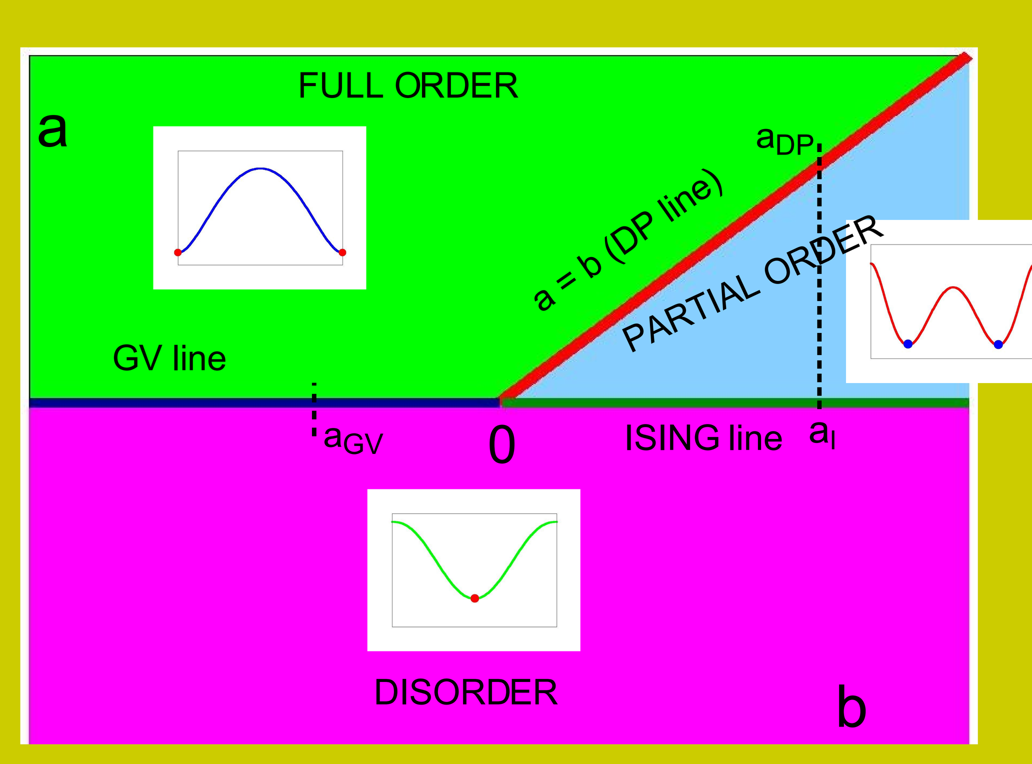
$$V(\phi) = -\frac{a}{2}\phi^2 + \frac{a+b}{4}\phi^4 - \frac{b}{6}\phi^6$$



f '(0) < f(0) : disordered active state

f '(0) > f(0) : coarsening by surface tension

Classes of transitions:
b ≤ 0: Generalized Voter
b > 0: Ising and Directed Percolation



Summary

- Starting from the microscopic dynamics, we derived a Langevin equation for the macroscopic evolution of general spin systems with two symmetric absorbing states.
- The equation allows to predict the macroscopic behavior (ordering dynamics, critical properties) of models, by knowing the first derivatives of the transition probability.
- Open problem: more than two symmetric states ?

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