

Role of diversity in a neuronal model of wake-sleep cycle

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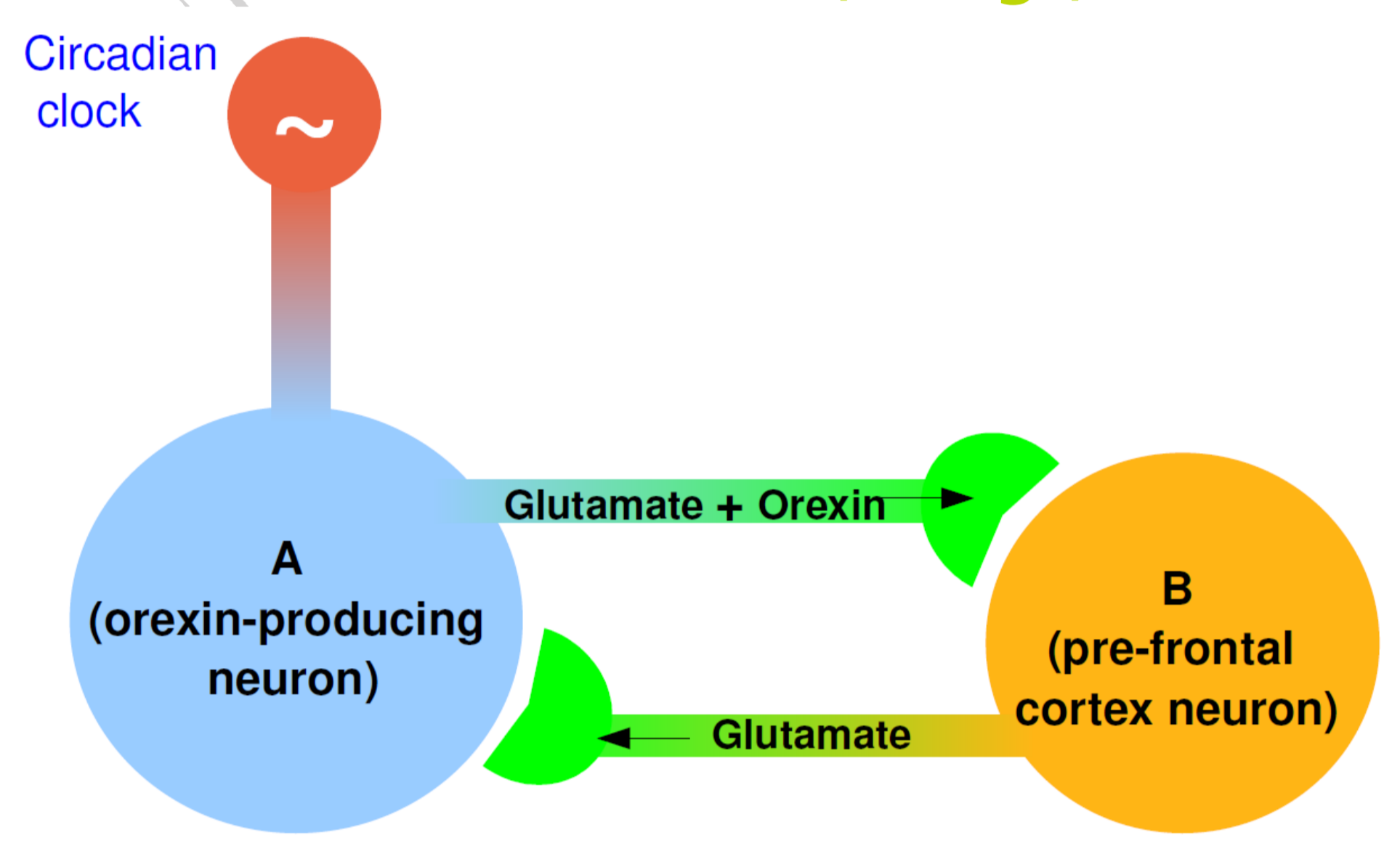
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ABSTRACT

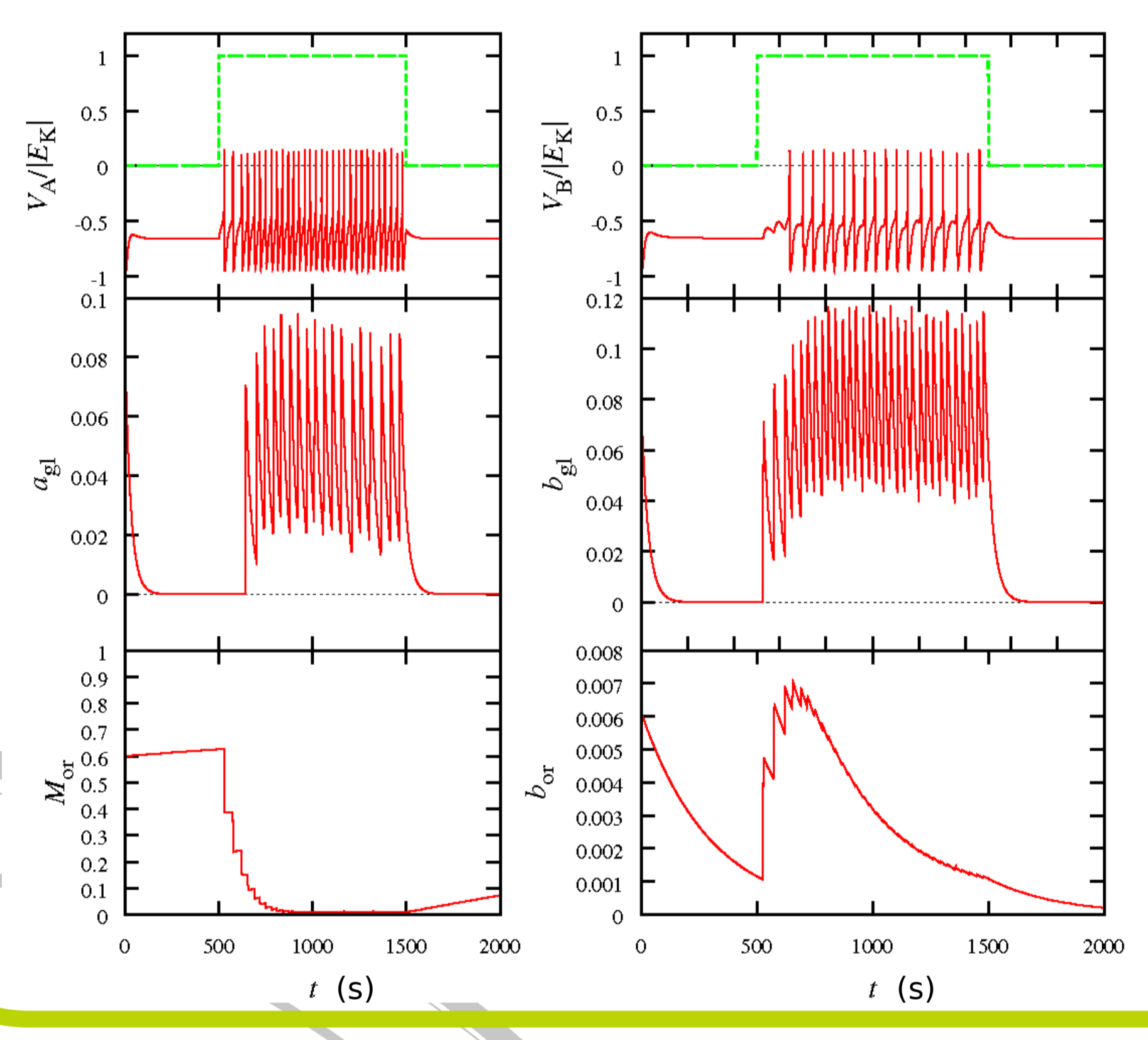
■ We study a heterogeneous generalization of the neuronal model for the wake-sleep cycle introduced by S. Postnova, K. Voigt, and H. A. Braun (PVB) [Preprint, submitted]. In that model one (A) orexin-producing neuronal unit interacts with another (B) unit representing e.g. one or a group of pre-frontal cortex neurons. The model reproduces the main features of the wake-sleep periodicity. We generalize the PVB model to an N_A -unit model (with N_A units interacting with one B-unit) in which the A-units are diversified from each other. The response of the system to a periodic circadian signal is studied for different degrees of diversity and it is shown that the model system exhibits collective effects, such as diversity-induced resonance.

MODEL

Two-Unit Model of Postnova, Voigt, and Braun



Response of the 2-unit system to a current pulse:



$(i) = (1, \dots, N_A)$

$$C \frac{dV_A^{(i)}(t)}{dt} = -g_L[V_A^{(i)}(t) - E_L] - g_{Na}[V_A^{(i)}(t) - E_{Na} a_{Na}^{(i)}(t)] - g_K[V_A^{(i)}(t) - E_K b_K^{(i)}(t)] - g_{gl}[V_A^{(i)}(t) - E_{gl} a_{gl}^{(i)}(t)] + I_{circ}(t)$$

$$C \frac{dV_B(t)}{dt} = -g_L[V_B(t) - E_L] - g_{Na}[V_B(t) - E_{Na} b_{Na}(t)] - g_K[V_B(t) - E_K b_K(t)] - g_{or}[V_B(t) - E_{or} b_{or}(t)] - g_{gl}[V_B(t) - E_{gl} b_{gl}(t)]$$

$$a_{Na}^{(i)} \equiv \Phi(S_{Na}(V_A^{(i)} - W_{Na}))$$

$$b_{Na} \equiv \Phi(S_{Na}(V_B - W_{Na}))$$

$$\frac{da_K^{(i)}}{dt} = -\frac{1}{\tau_K} [a_K^{(i)} - \Phi(V_A^{(i)}; S_K, W_K)]$$

$$\frac{db_K}{dt} = -\frac{1}{\tau_K} [b_K - \Phi(V_B; S_K, W_K)]$$

$$\frac{db_{or}}{dt} = -\frac{1}{\tau_{or}} \left[b_{or} - \frac{1}{N_A} \sum_{i=1}^{N_A} M_{or}^{(i)} \Phi(V_A^{(i)}; S_{or}, W_{or}^{(i)}) \right]$$

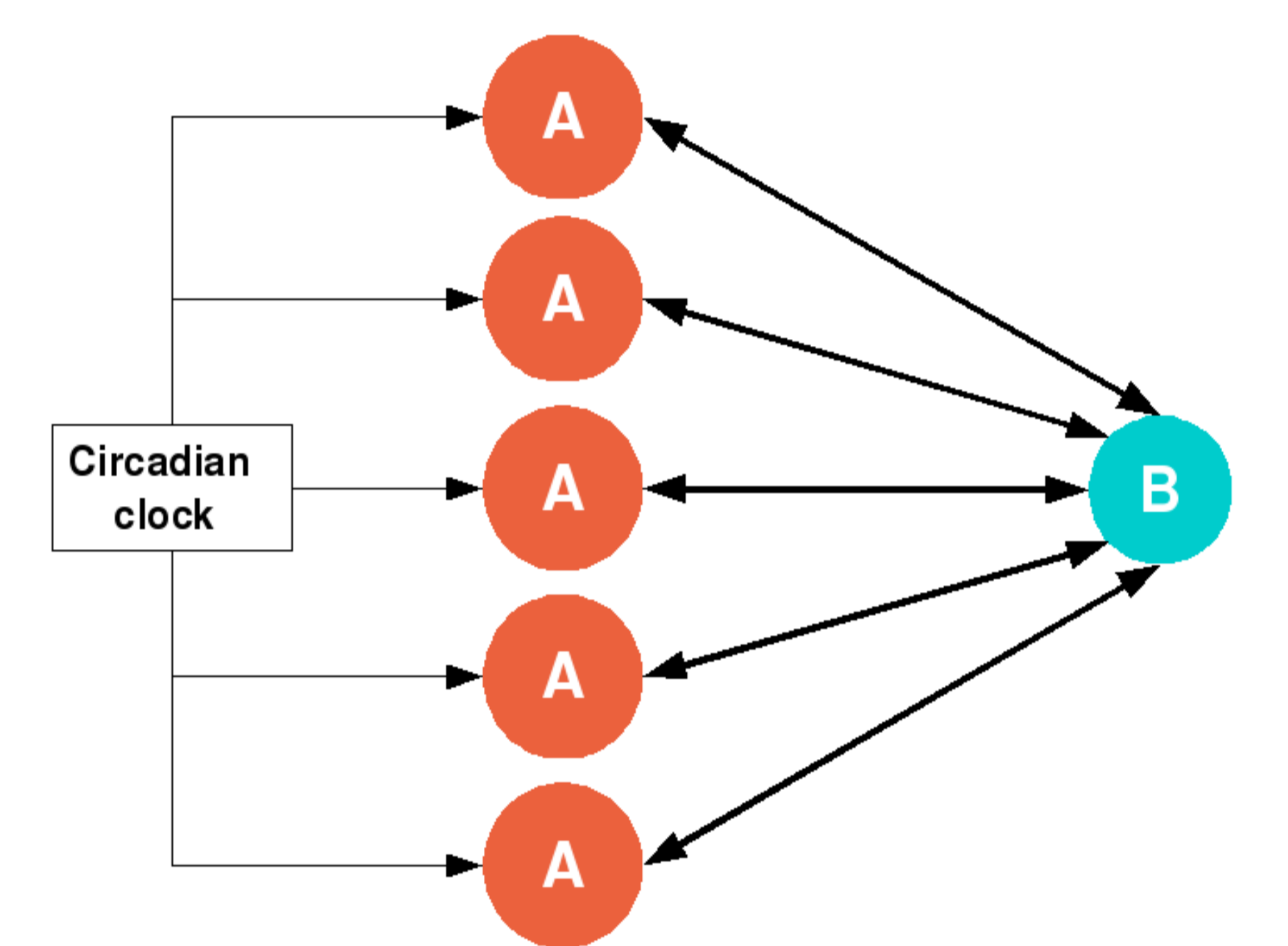
$$\frac{dM_{or}^{(i)}}{dt} = -\frac{1}{\tau_+} (1 - M_{or}^{(i)}) - \frac{1}{\tau_-} M_{or}^{(i)} \Phi(V_A^{(i)}; S_{or}, W_{or}^{(i)})$$

$$\frac{da_{gl}^{(i)}}{dt} = -\frac{1}{\tau_{gl}} [a_{gl}^{(i)} - \Phi(V_B; S_{gl}, W_{gl}^{(i)})]$$

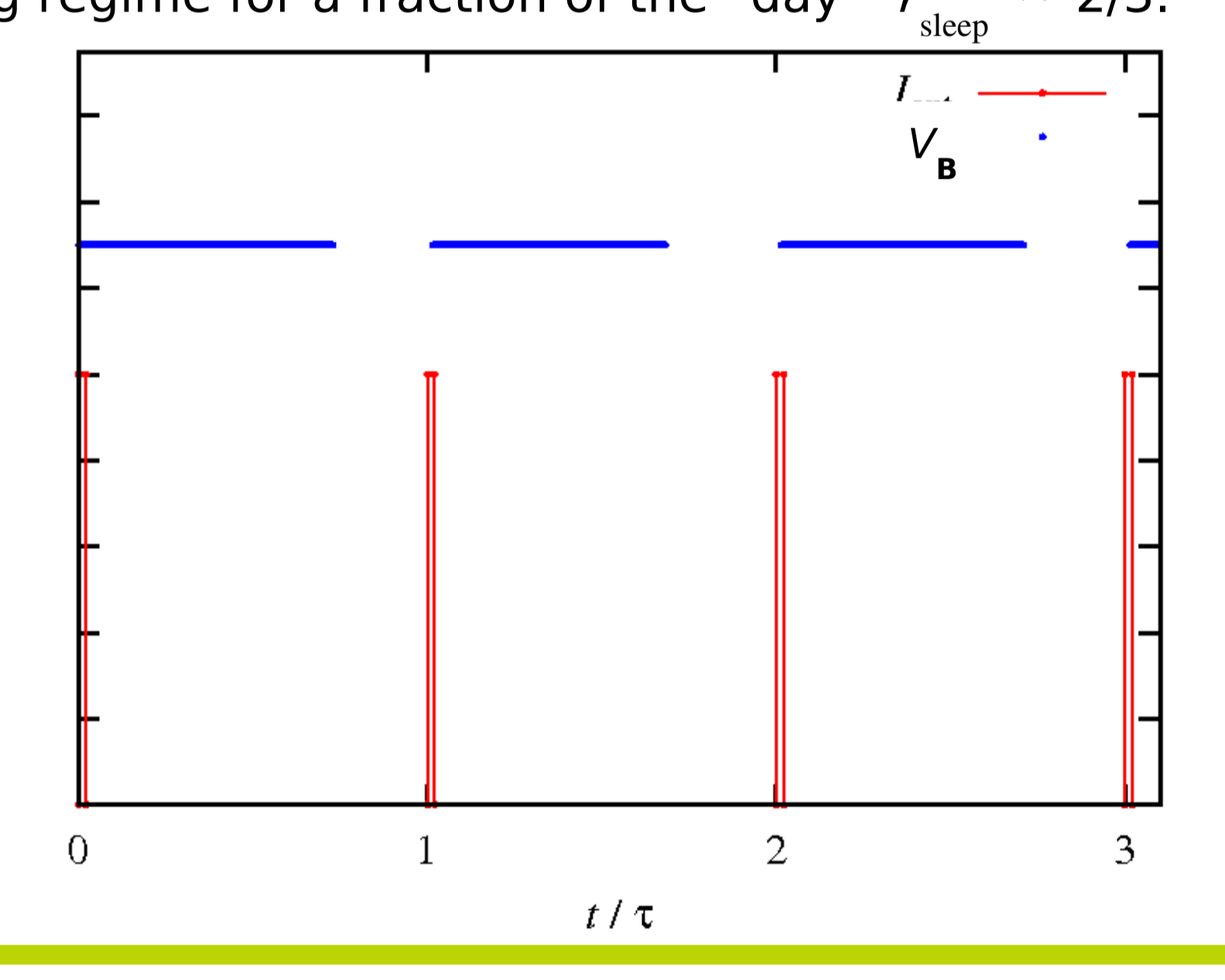
$$\frac{db_{gl}}{dt} = -\frac{1}{\tau_{gl}} \left[b_{gl} - \frac{1}{N_A} \sum_{i=1}^{N_A} \Phi(V_A^{(i)}; S_{gl}, W_{gl}^{(i)}) \right]$$

Sigmoid Function: $\Phi(S(V - W)) = \frac{1}{1 + \exp[-S(V - W)]}$

Generalized Model with N_A A-Units



Response of unit B to a periodic external current. The lower curve is the periodic external current, representing the circadian signal or a signal with daily periodicity, while the upper line is a raster plot of unit B, which is periodically in a spiking regime for a fraction of the "day" $r_{sleep} \sim 2/3$.



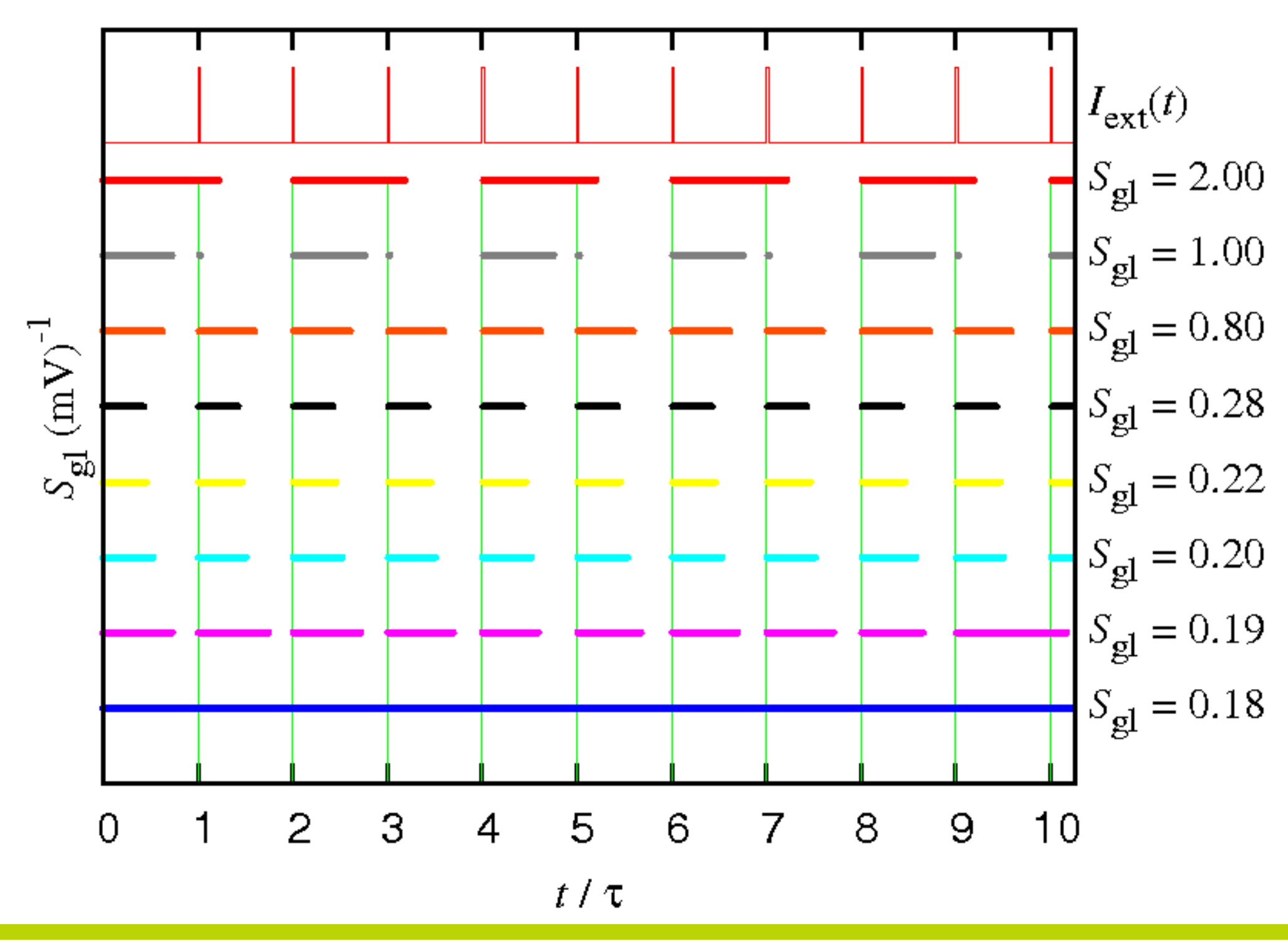
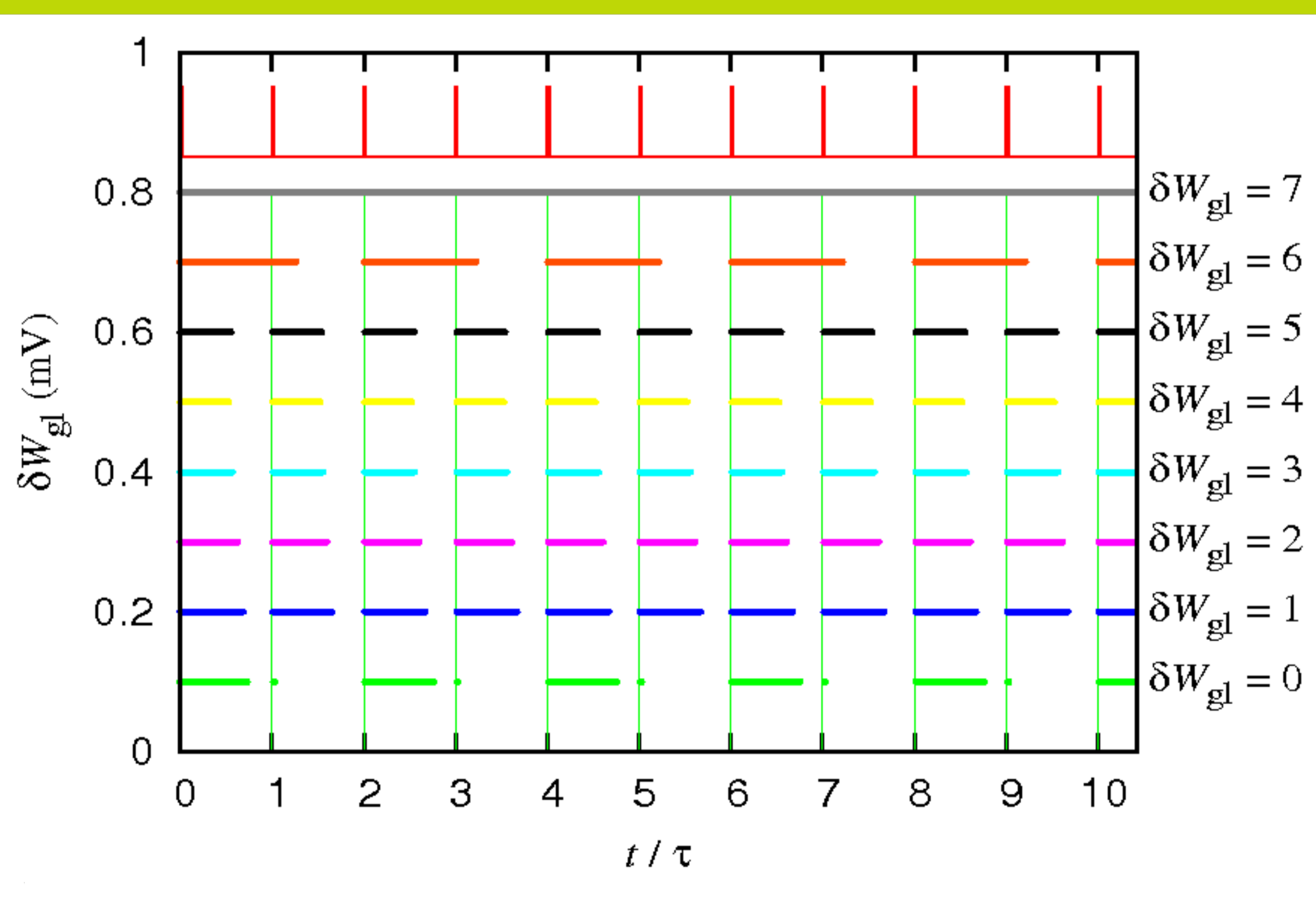
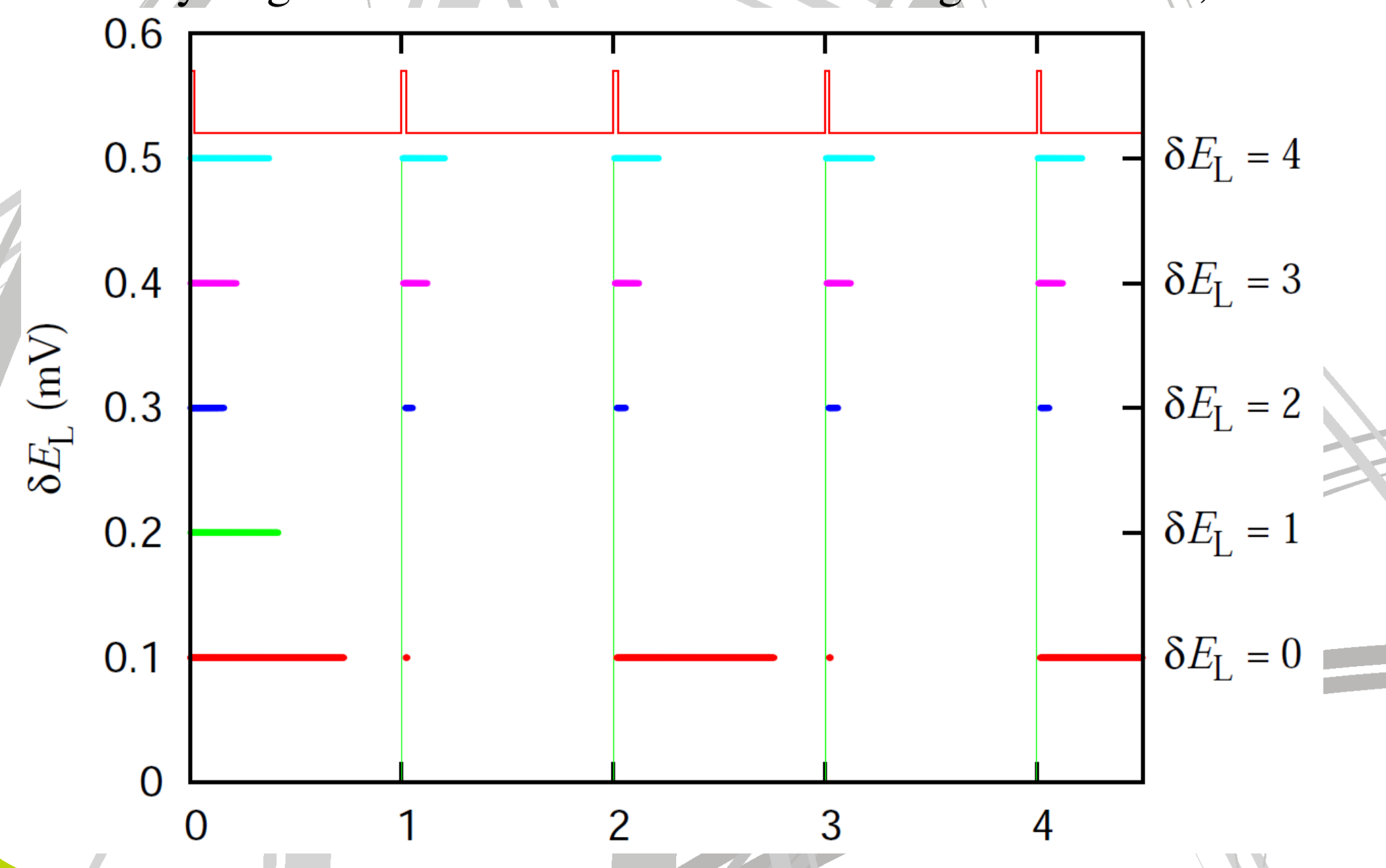
RESULTS

■ **Effect of diversity.** In general, diversifying even a little the parameters produces strong effects due to the nonlinearity of the dynamics. Three examples are shown here. In the figures below and on the right, the first curve from top is the normalized train of impulses of the circadian cycle (the period representing a rescaled "day" is here 24 s and the impulse length is 1/2 s). The other curves (segments) visible are raster plots of the potential V_B of the B unit.

■ **Below.** Diversifying the equilibrium leakage potentials E_L in each A-unit, increasing the standard deviation ΔE_L , the system can pass from double-periodic to periodic spiking regime

■ **Right-top.** Analogous behavior is observed varying the single glutamate thresholds W_{gl} of the A-units: from double- to single-periodicity and permanent spiking state.

■ **Right-bottom.** Also increasing the parameter S_{gl} in the B unit similar drastic changes are observed. In fact, S_{gl} represents the diversity of glutamate channel thresholds in a given neuron.



■ **Improvement in the response.** Diversifying the parameters does not simply produce deep changes in the spiking regime but, as shown in the example below, it can make the response of the system more robust against variations of the other parameters.

Let's define r_{sleep} as the average percentage of a "day" spent by unit B in a spiking regime, i.e. in the wakeful state, a number which can be obtained from the figures on the left. This should be $r_{sleep} \sim 2/3$ for an optimal wake state, but here we do not focus on the specific value of r_{sleep} . Rather, we explore the general type of spiking response: periodic, which is the expected one; the (unhealthy) double-periodic; and the permanent spiking regime, corresponding to insomnia. We varied the parameter S_{gl} for different degrees of diversity in the glutamate threshold W_{gl} , i.e. for different values of ΔW_{gl} . We found that there is an optimal diversity level ($\Delta W_{gl} \sim 4$ in the figure below) where the response remains periodic for most values of the parameter S_{gl} . For other values of ΔW_{gl} the appearance of double-periodicity or other non-periodic regimes is observed outside a relatively small range of S_{gl} .

