

Effects of the topology and delay in a neuronal network

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Outline

- Introduction
 - Importance of delay in neuronal system
 - Network topology
 - Synchronization measure
- Homogeneous delay
 - Rewiring and neighbors dependence
 - Frequency locking
 - Natural frequency dispersion
- Heterogeneous delay
 - Preliminary results
- Anatomical network
- Conclusions & future work

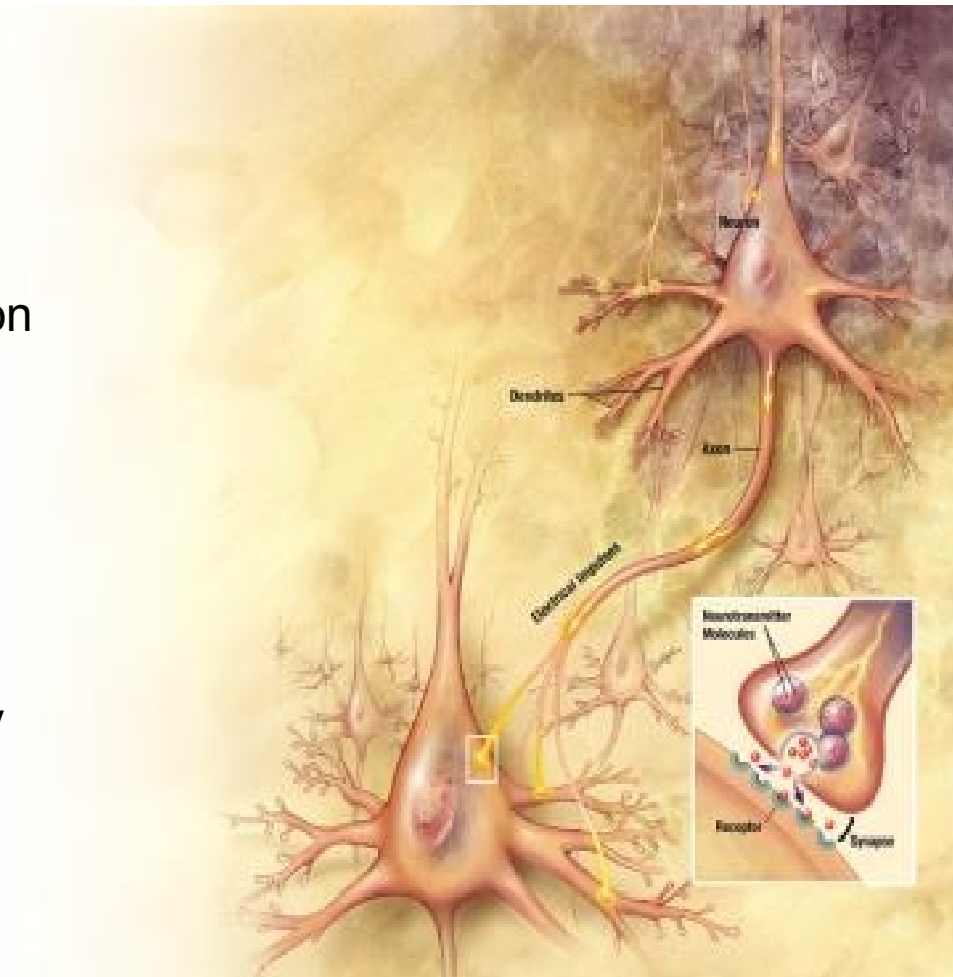
Introduction

> Why consider delay in Neuronal Systems?

- Finite propagation speed
- Chemical reactions
- Latency times of neural excitation
- Integration of information from multiple sources

> Interested in:

- Understand the role of the delay in synchronization



Introduction

> Our system

- 10^3 neurons (Hodking-Huxley model)

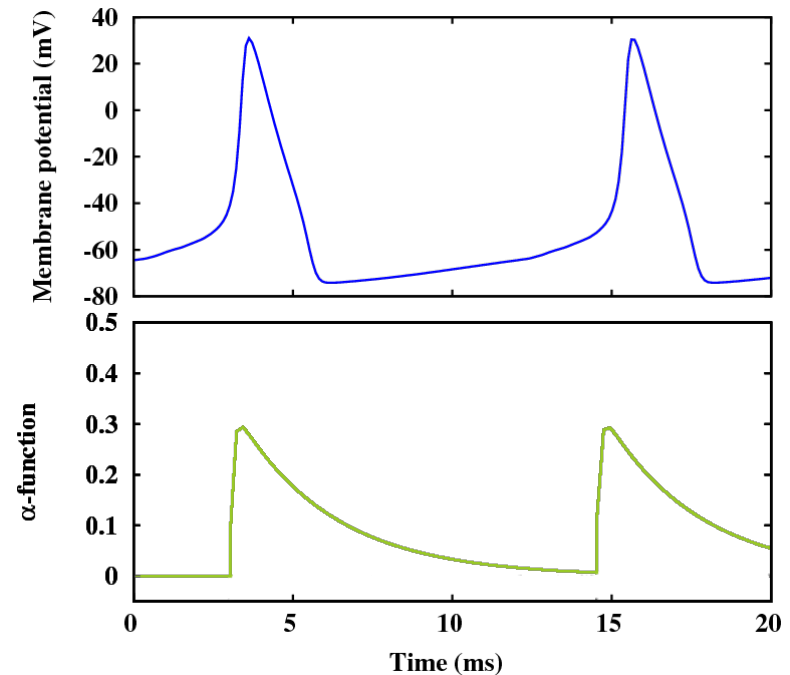
$$C_m \dot{V}_i = I_i - I_i^{ion} - I_i^{syn}$$

- Synaptically connected (α -functions)
- Delay in the connections (τ)

$$I_i^{syn} = - \frac{g_{max}}{n_i} \sum_{spikes(j)} \alpha(t - t_{spikes(j)} - \tau) (V_i(t) - E_{syn})$$

$$\alpha(t) = \frac{1}{\tau_d - \tau_r} (\exp(-t/\tau_d) - \exp(-t/\tau_r))$$

with $\tau_d = 3 \text{ ms}$; $\tau_r = 0.1 \text{ ms}$; $E_{syn} = 0 \text{ mV}$



[Destexhe, Neural. Comp. 6, 1418 (1994)]

> Model details

[Hodgkin-Huxley, *J. Physiol.* 117, 500 (1952)]

$$C_m \dot{V}_i = I^{app} - I_i^{ion} - I_i^{syn}$$

$$I_i^{ion} = -g_{Na} m^{\gamma} h (V_i - E_{Na}) - g_K n^{\xi} (V_i - E_K) - g_L (V_i - E_L)$$

with gating variable dynamics

$$\dot{m} = \alpha_m(V)(1-m) - \beta_m(V)m$$

$$\dot{n} = \alpha_n(V)(1-n) - \beta_n(V)n$$

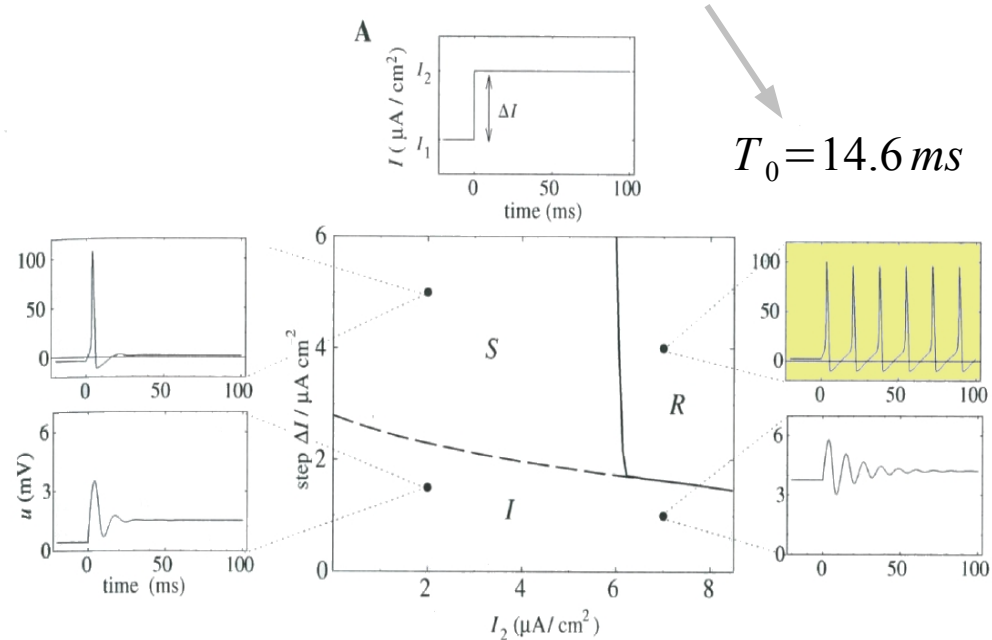
$$\dot{h} = \alpha_h(V)(1-h) - \beta_h(V)h$$

> Parameters

x	E_x	g_x
Na	115 mV	120 mS/cm ²
K	-12 mV	36 mS/cm ²
L	10.6mV	0.3mS/cm ²

x	$\alpha_x(u / mV)$	$\beta_x(u / mV)$
n	$(0.1 - 0.01 u) / [\exp(1 - 0.1 u) - 1]$	$0.125 \exp(-u / 80)$
m	$(2.5 - 0.1 u) / [\exp(2.5 - 0.1 u) - 1]$	$4 \exp(-u / 18)$
h	$0.07 \exp(-u / 20)$	$1 / [\exp(3 - 0.1 u) + 1]$

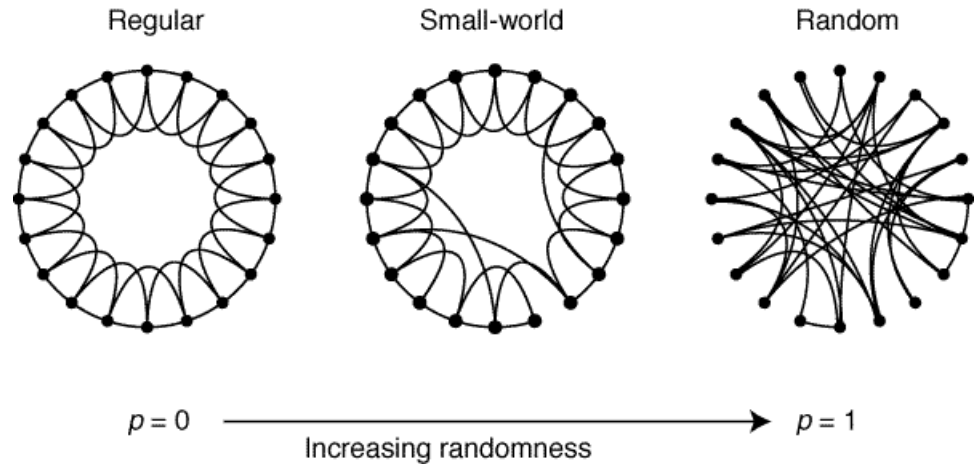
> Operation regime $I^{app} = 10 \mu A/cm^2$



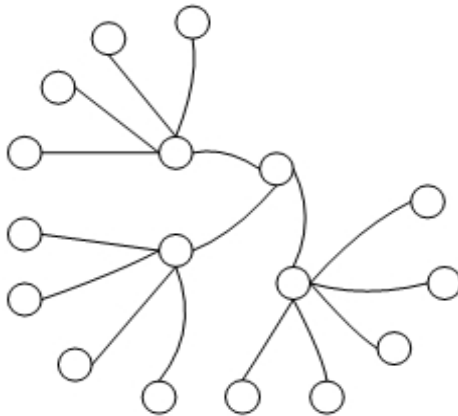
Introduction

> Network topology

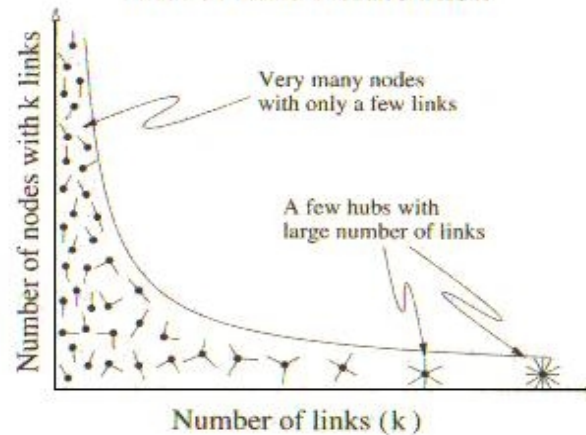
- Regular
- Small-world
- Random
- Scale-free
- Mean-field (all2all)



Scale-Free Network



Power Law Distribution



Introduction

> Synchronization measure

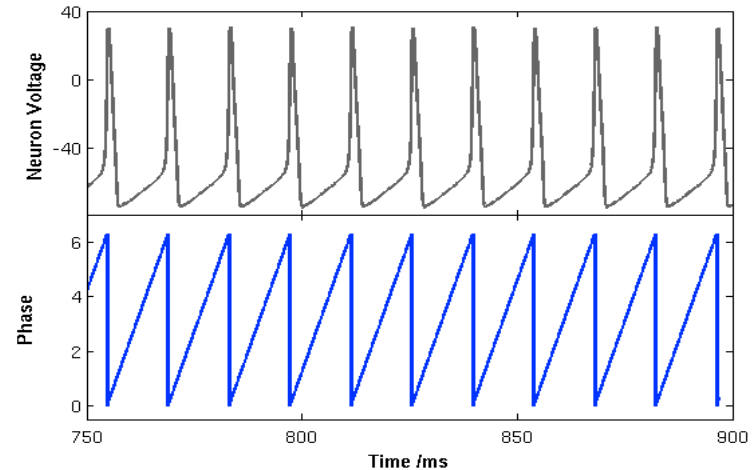
- Phase of each neuron

$$\phi_i(t) = 2\pi \frac{t - \tau_k}{\tau_{k+1} - \tau_k}$$

- Local order parameter

$$s_i(t) = \frac{1}{n_i} \sum_{j \in \text{neigh}(i)} \sin^2 \left(\frac{\phi_i(t) - \phi_j(t)}{2} \right)$$

$$S^{loc} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left(\frac{1}{N} \sum_{i=1}^N s_i \right) dt$$



- Global order parameter

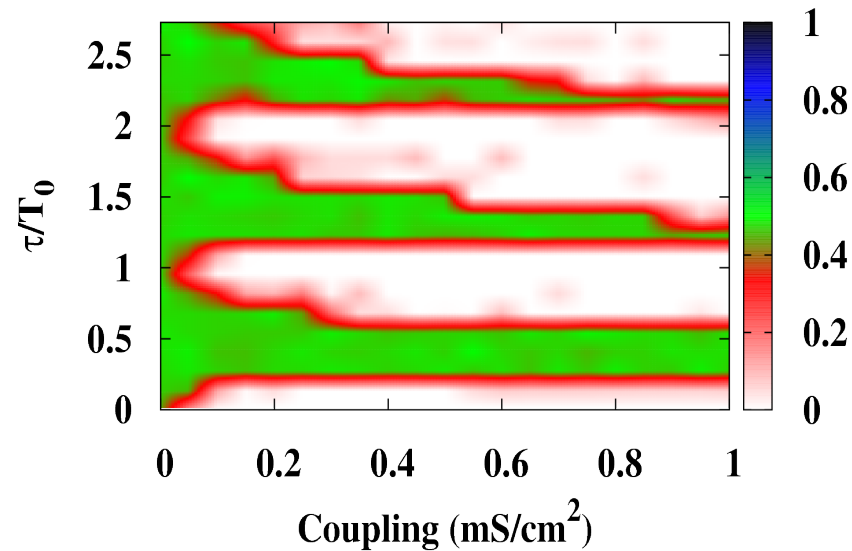
$$s_i'(t) = \frac{1}{N} \sum_{j=1}^N \sin^2 \left(\frac{\phi_i(t) - \phi_j(t)}{2} \right)$$

$$S^{glob} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left(\frac{1}{N} \sum_{i=1}^N s_i' \right) dt$$

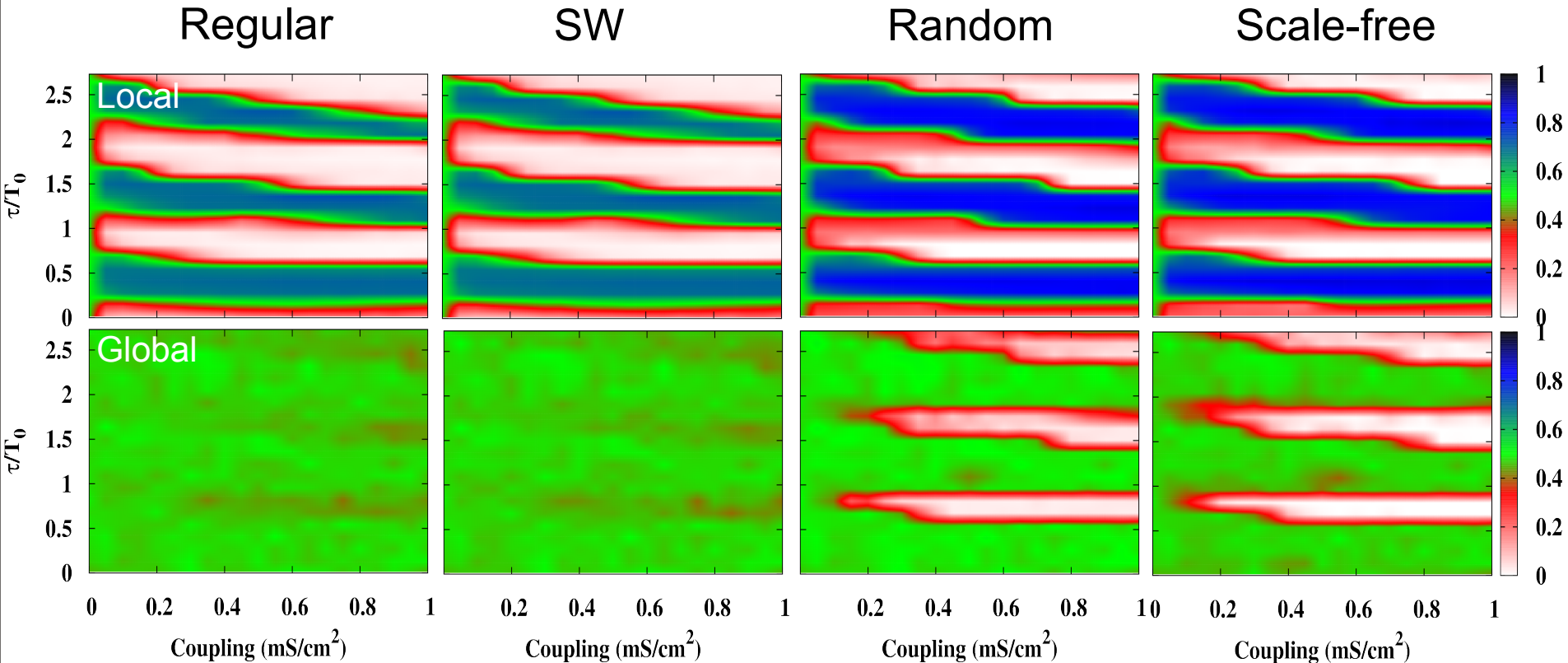
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Synchronization regions

Mean-field (all-to-all)



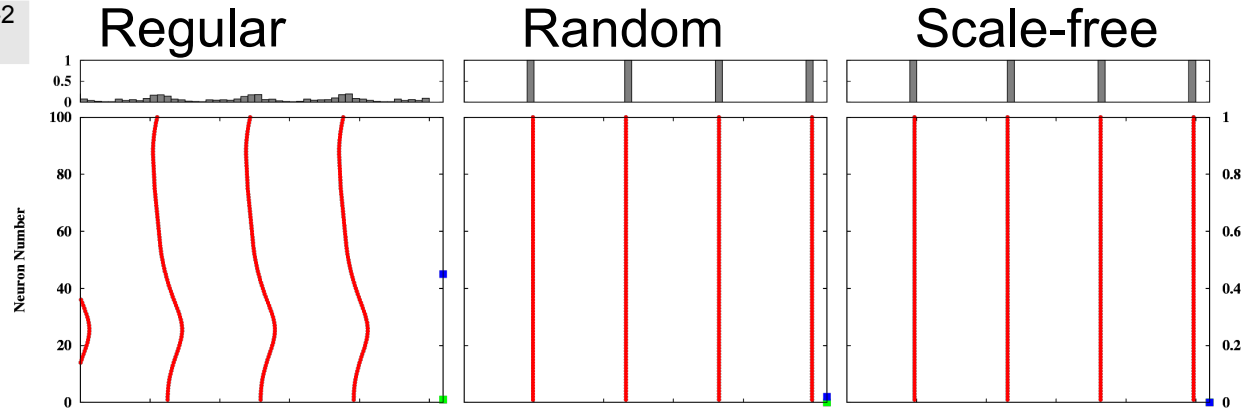
Local and global synchronization regions



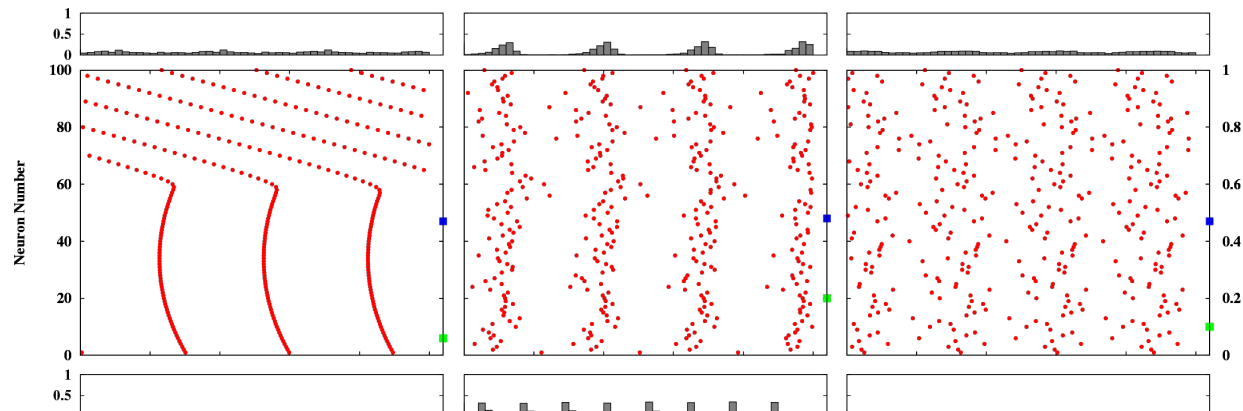
Raster plots and firing histograms

coupling: 0.8 mS cm^{-2}

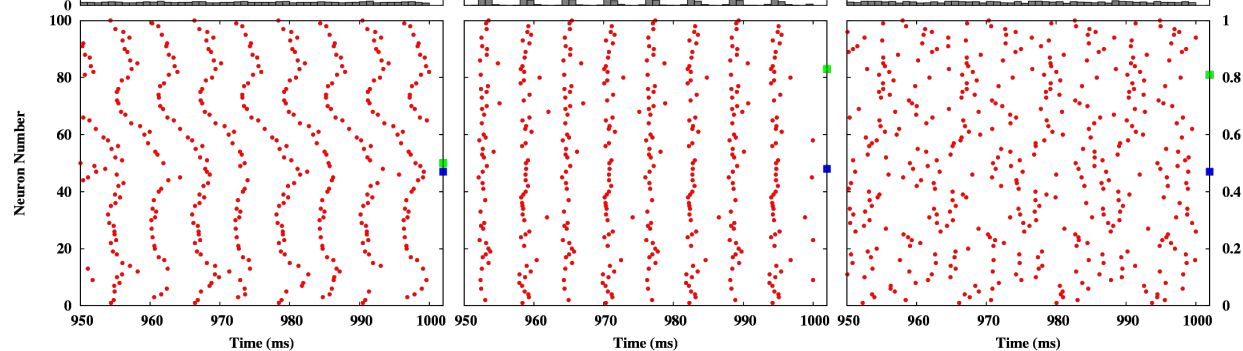
$\tau: 12 \text{ ms}$



$\tau: 14 \text{ ms}$



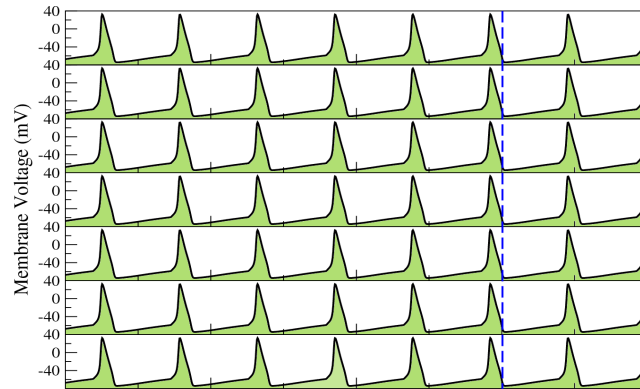
$\tau: 16 \text{ ms}$



Membrane voltage traces (Random Network)

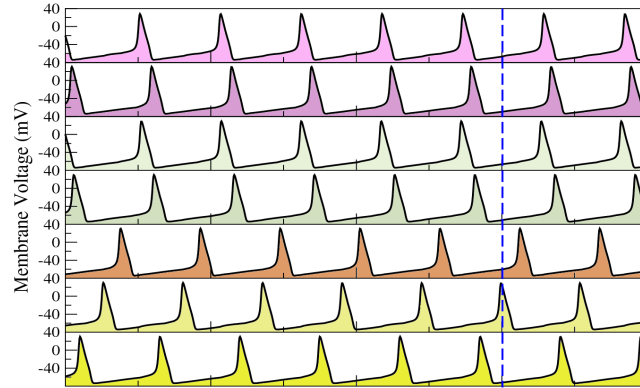
coupling: 0.8 mS cm^{-2}

τ : 12 ms



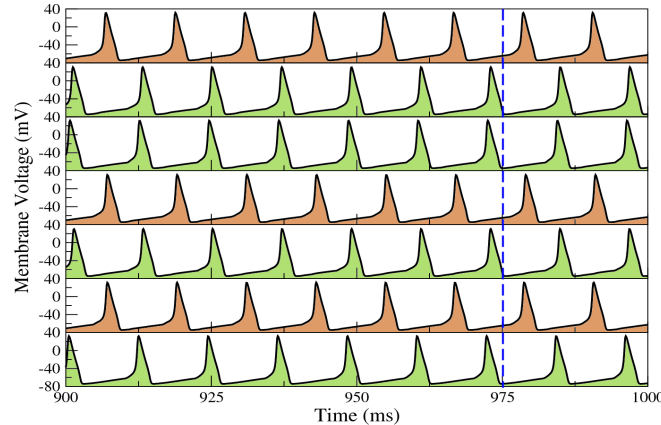
in-phase state

τ : 14 ms



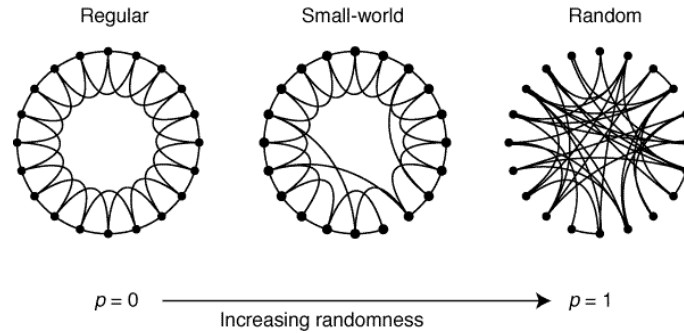
out-of-phase state

τ : 16 ms



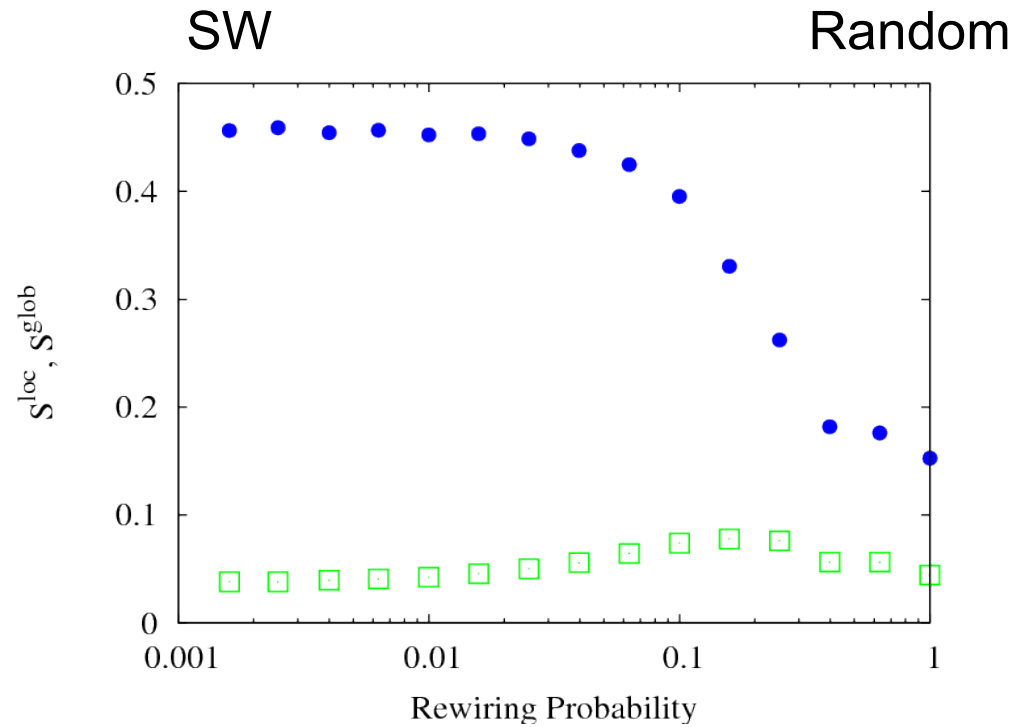
anti-phase state

Rewiring dependence



coupling: 0.2 mS cm^{-2}

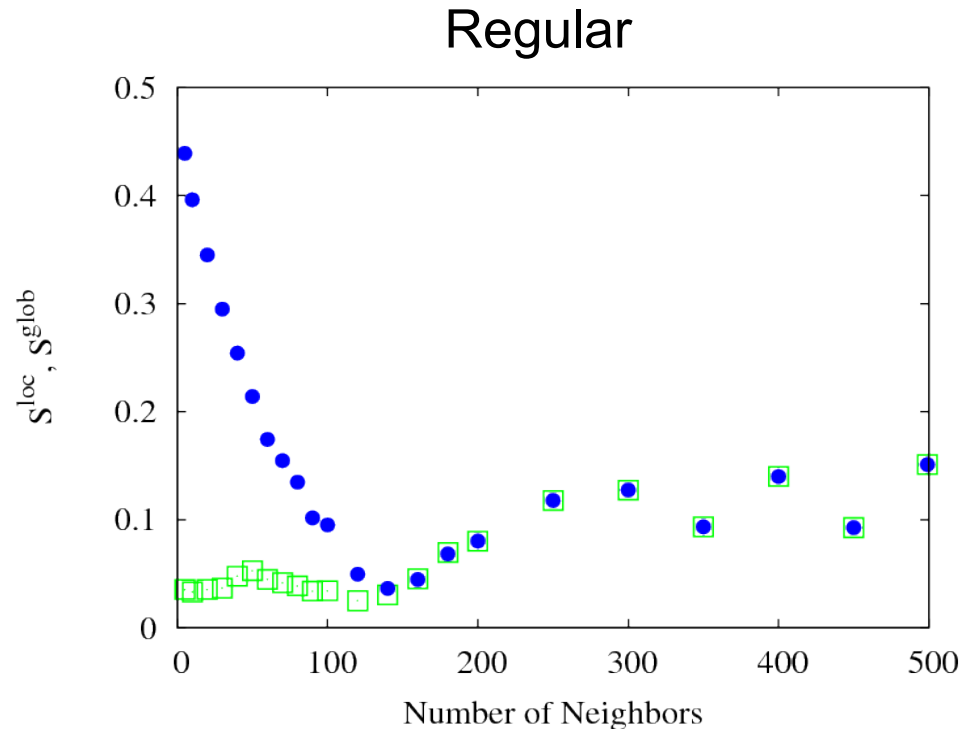
delay: 12 ms



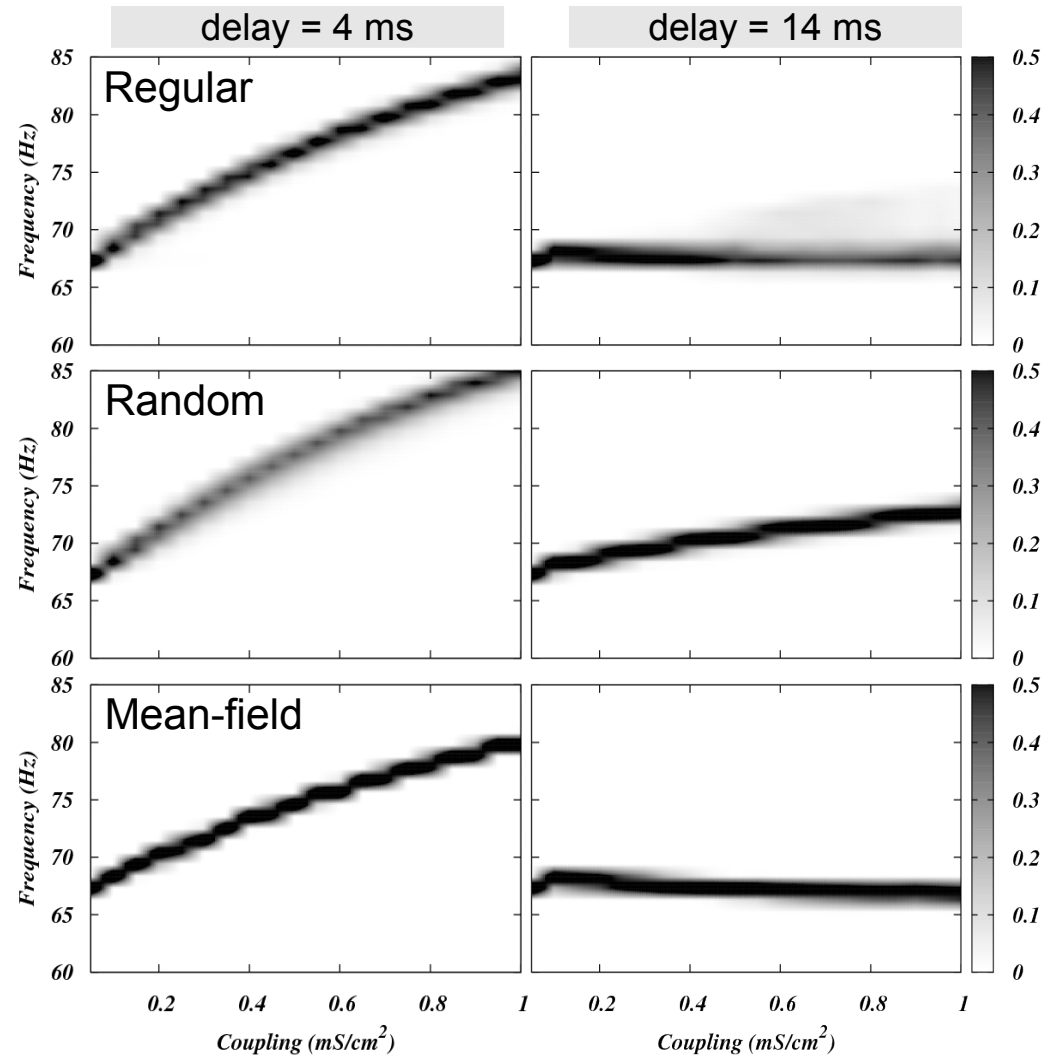
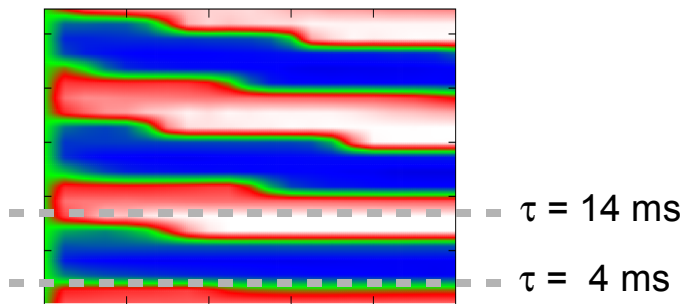
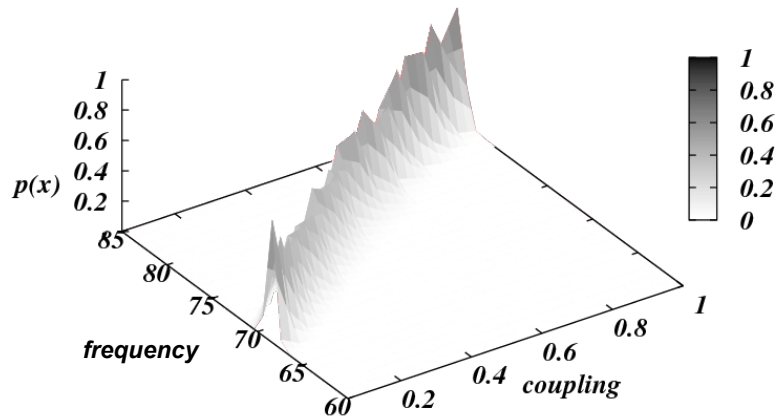
Effect of the neighbors

coupling: 0.2 mS cm^{-2}

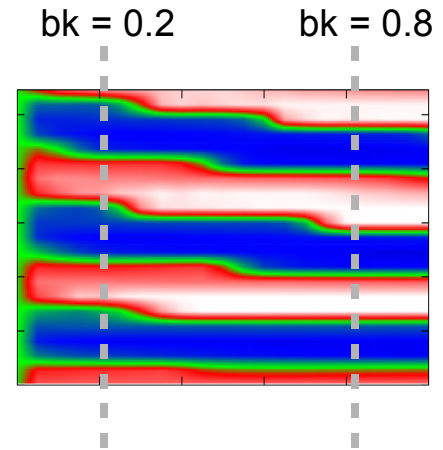
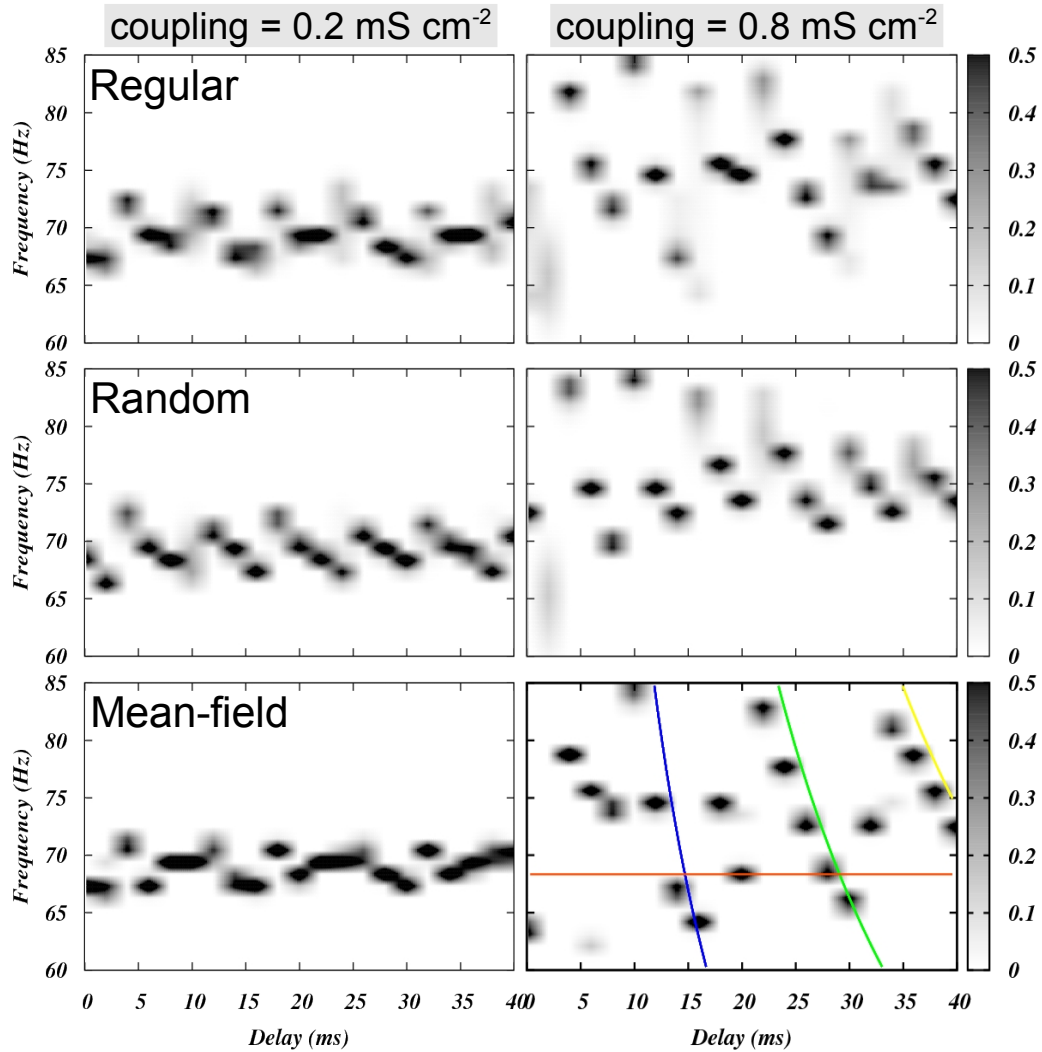
delay: 12 ms



Frequency locking

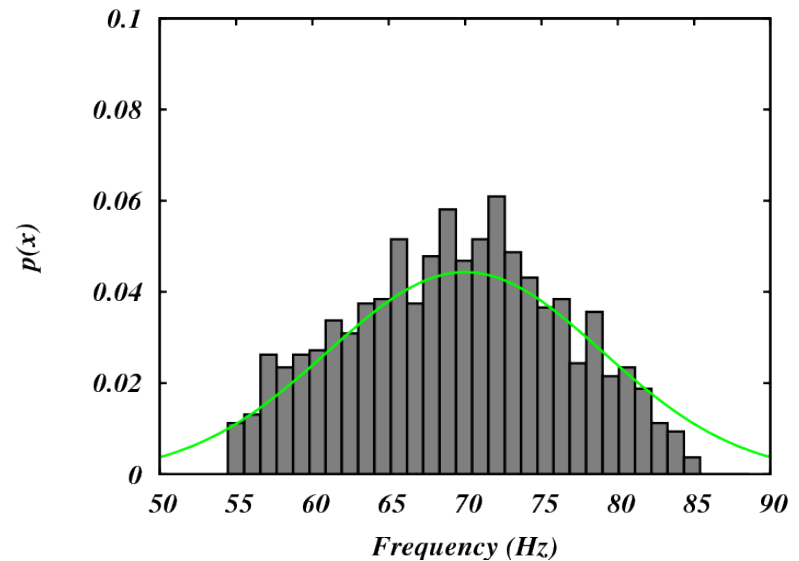
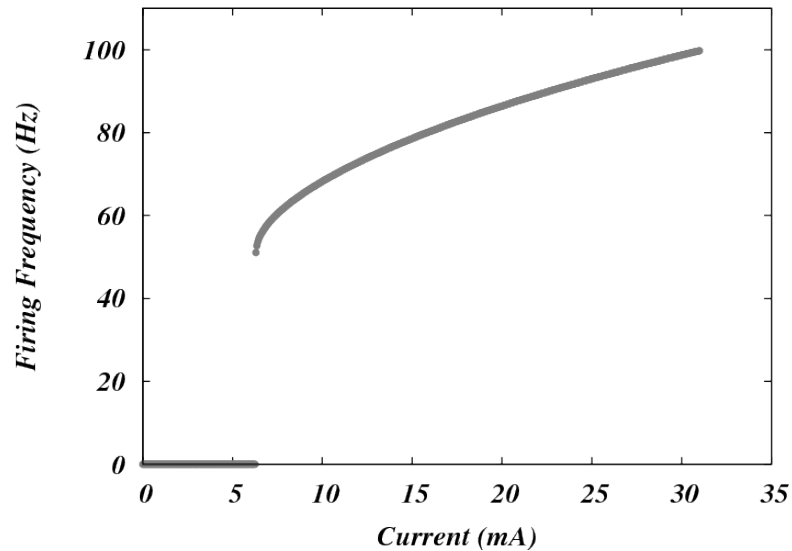


Frequency locking



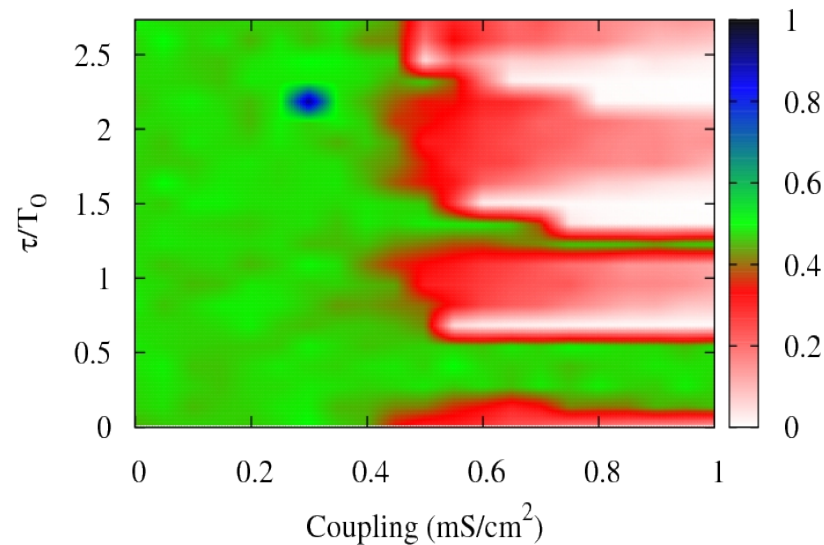
- > Gaussian distribution of natural frequencies ($f_m = 70 \text{ Hz}$; $\sigma^2 = 9 \text{ Hz}^2$)

Excitable type II

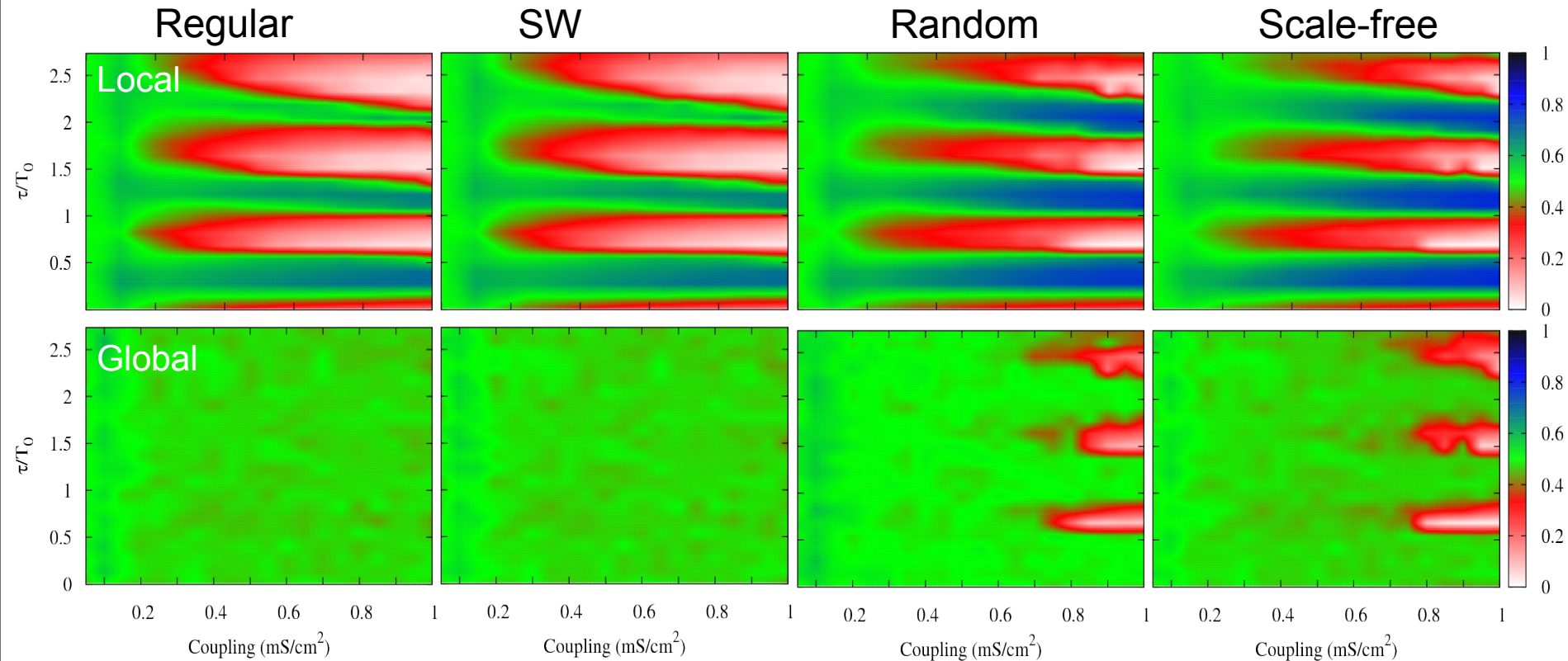


- > Gaussian distribution of natural frequencies ($f_m = 70 \text{ Hz}$; $\sigma^2 = 9 \text{ Hz}^2$)

Mean-field (all-to-all)



> Gaussian distribution of natural frequencies ($f_m = 70$ Hz ; $\sigma^2 = 9$ Hz²)



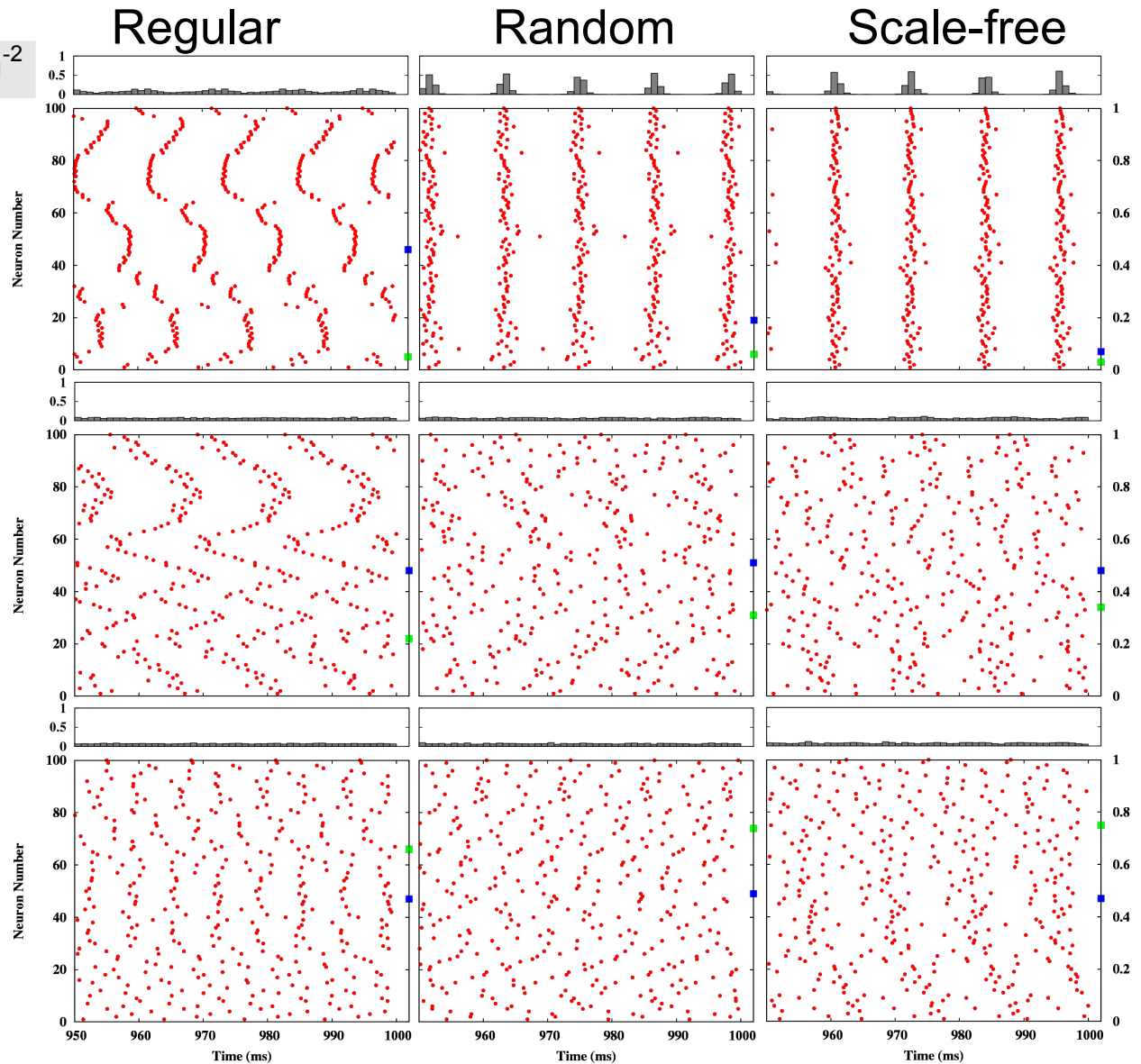
Raster plots and firing histograms

coupling: 0.8 mS cm^{-2}

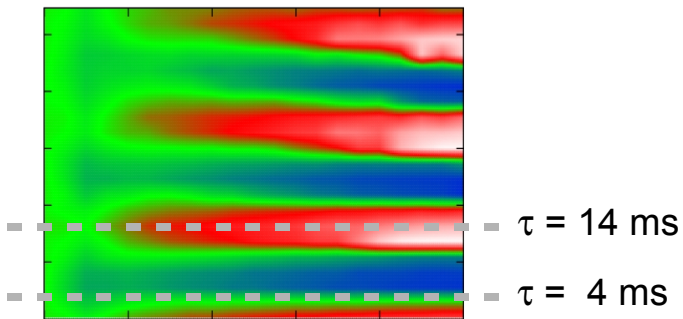
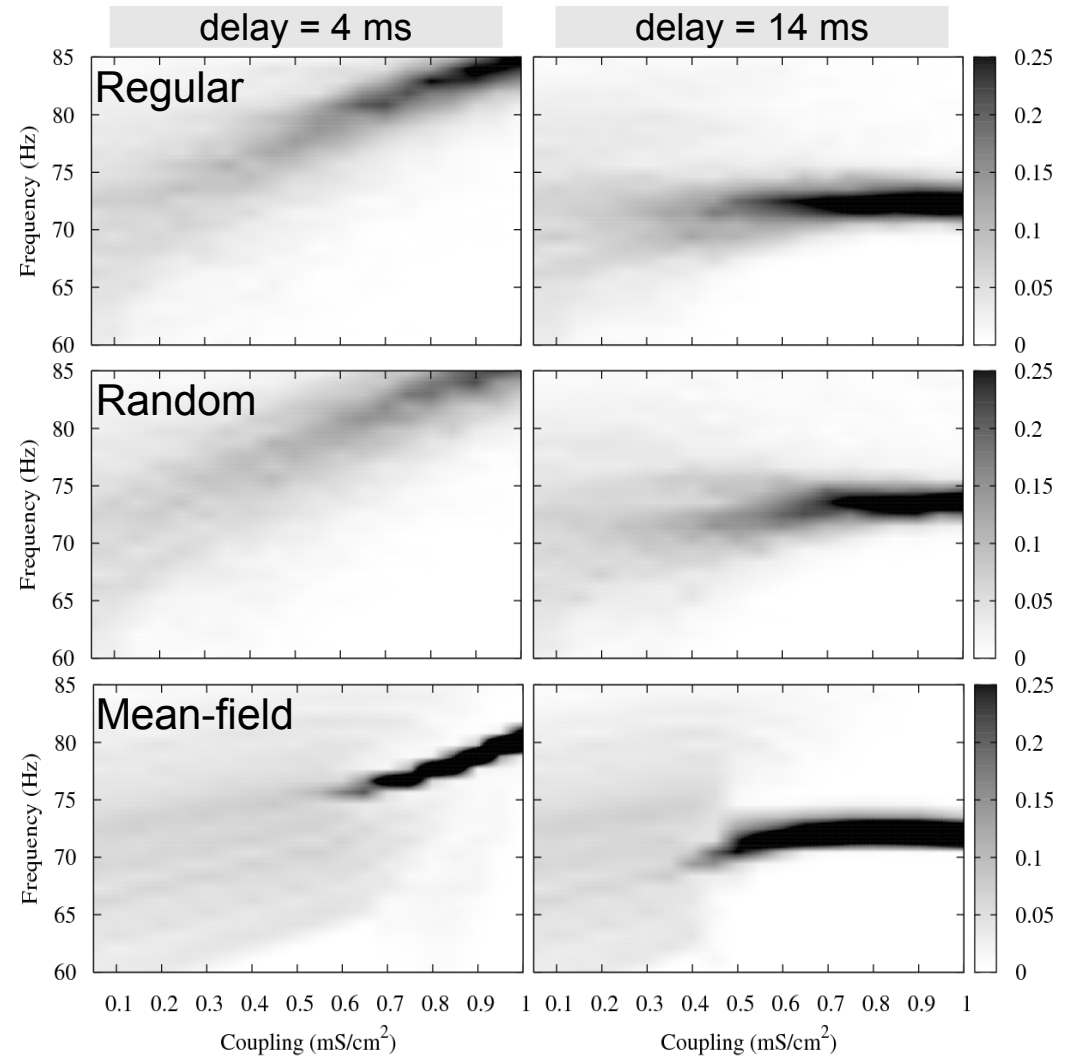
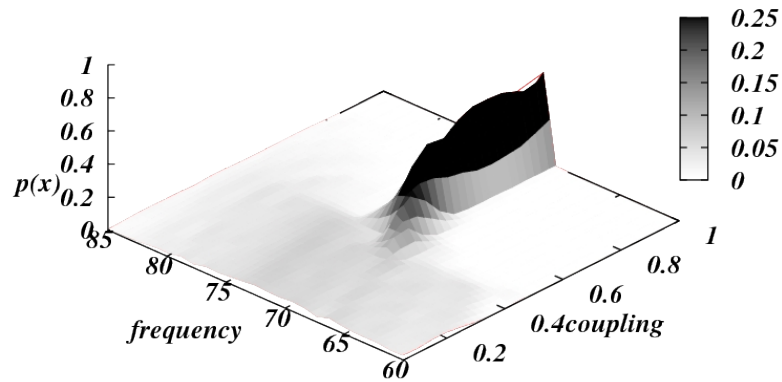
$\tau: 10 \text{ ms}$

$\tau: 14 \text{ ms}$

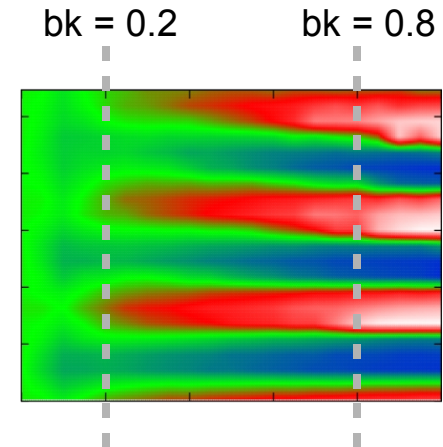
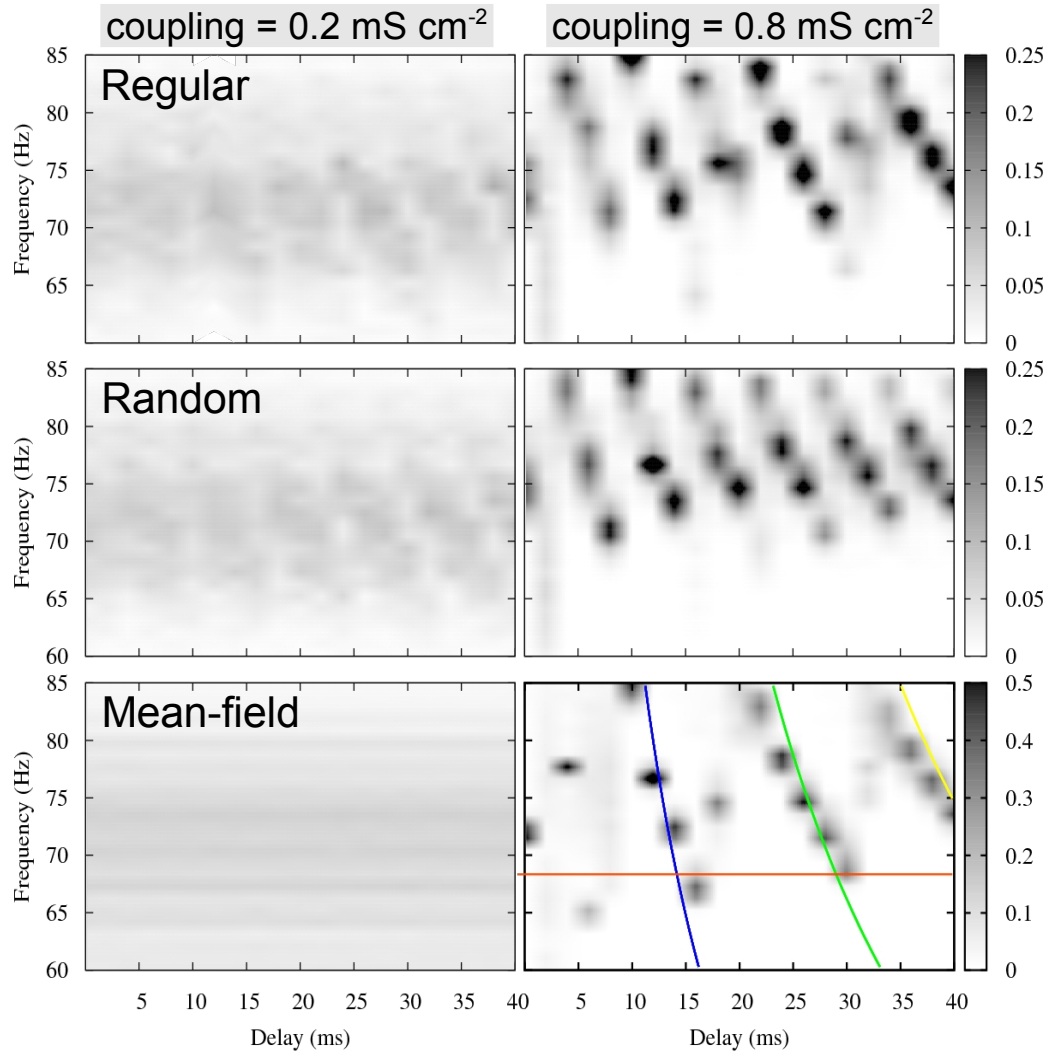
$\tau: 18 \text{ ms}$



Frequency locking (Gaussian distribution of natural frequencies)



Frequency locking (Gaussian distribution of natural frequencies)

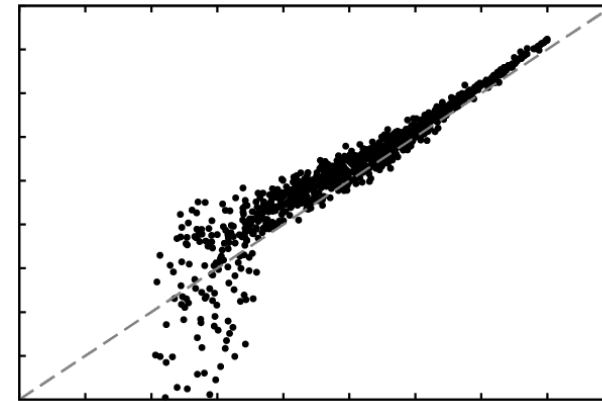
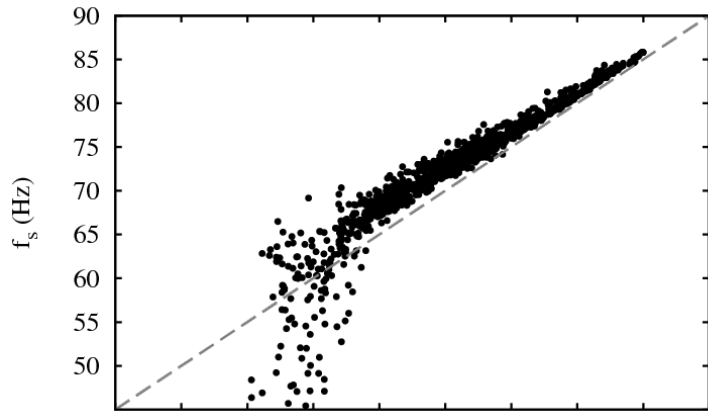


Frequency distribution

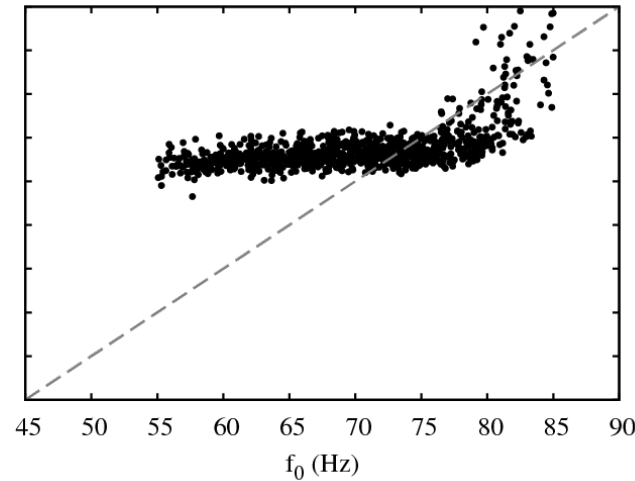
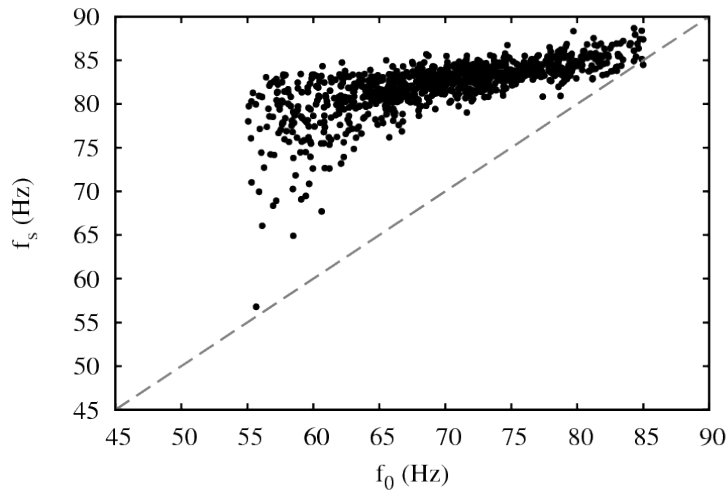
Random

delay: 4 ms

delay: 14 ms



couplig: $0.2 \text{ mS}\cdot\text{cm}^{-2}$

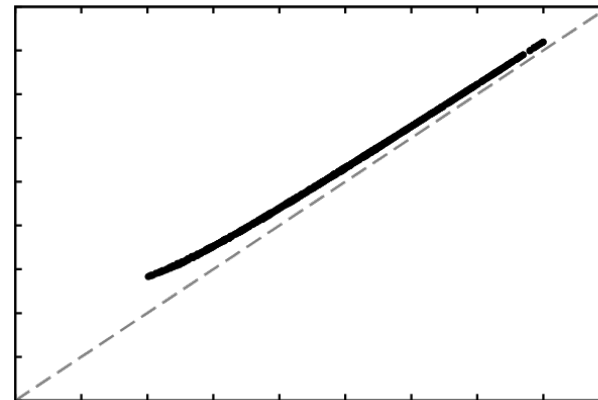
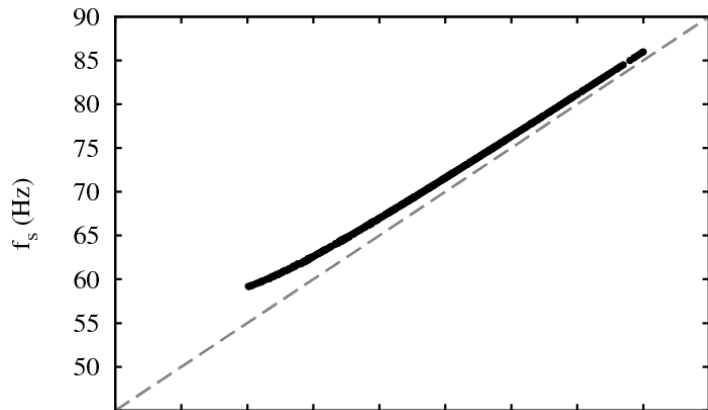


couplig: $0.8 \text{ mS}\cdot\text{cm}^{-2}$

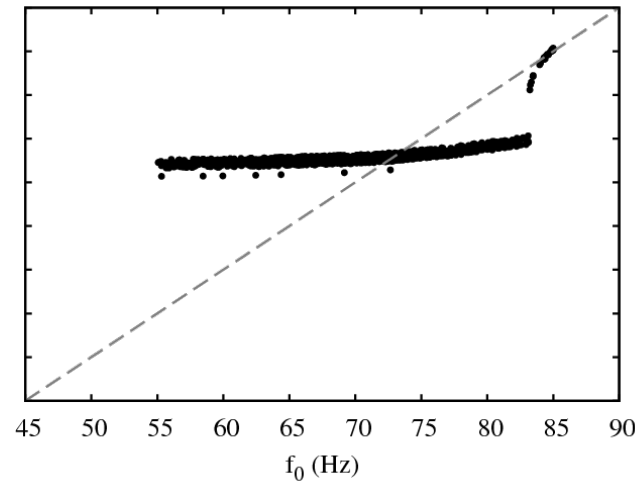
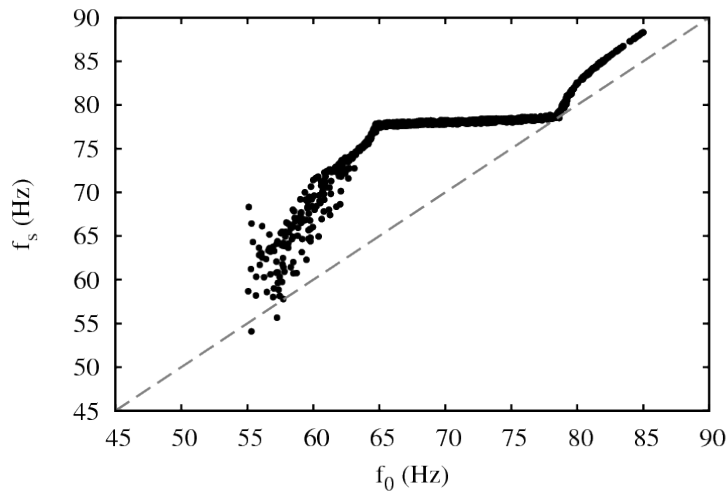
Frequency distribution Mean-field (all-to-all)

delay: 4 ms

delay: 14 ms



couplig: $0.2 \text{ mS}\cdot\text{cm}^{-2}$



couplig: $0.8 \text{ mS}\cdot\text{cm}^{-2}$

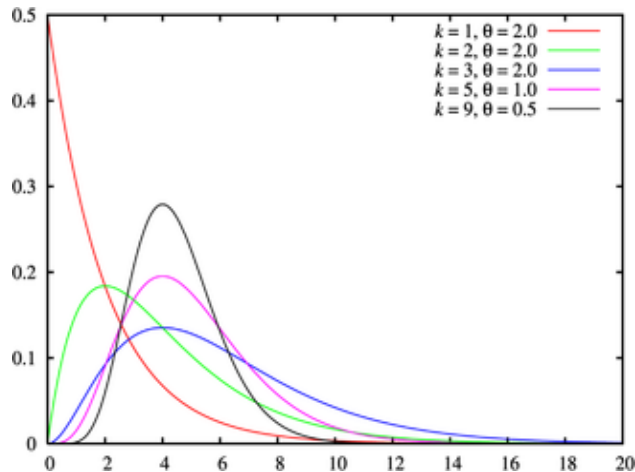
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- **Heterogeneous delay**
 - **Preliminary results**
 - Anatomical network
 - Conclusions & future work

Heterogeneous delays

> Gamma distribution

$$f(x; k; \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)}$$

for $x > 0$; $k, \theta > 0$



- Mean:

$$\langle \tau \rangle = k \theta$$

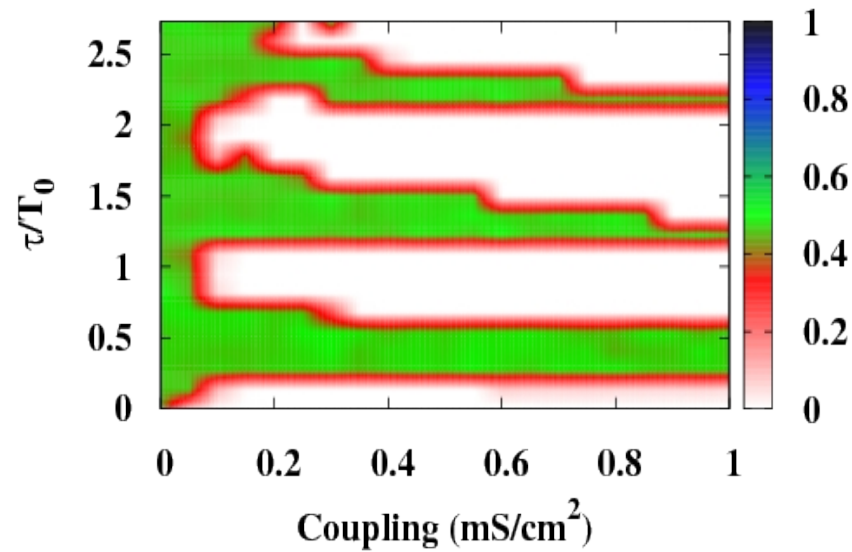
- Variance:

$$\sigma^2 = k \theta^2$$

Synchronization regions

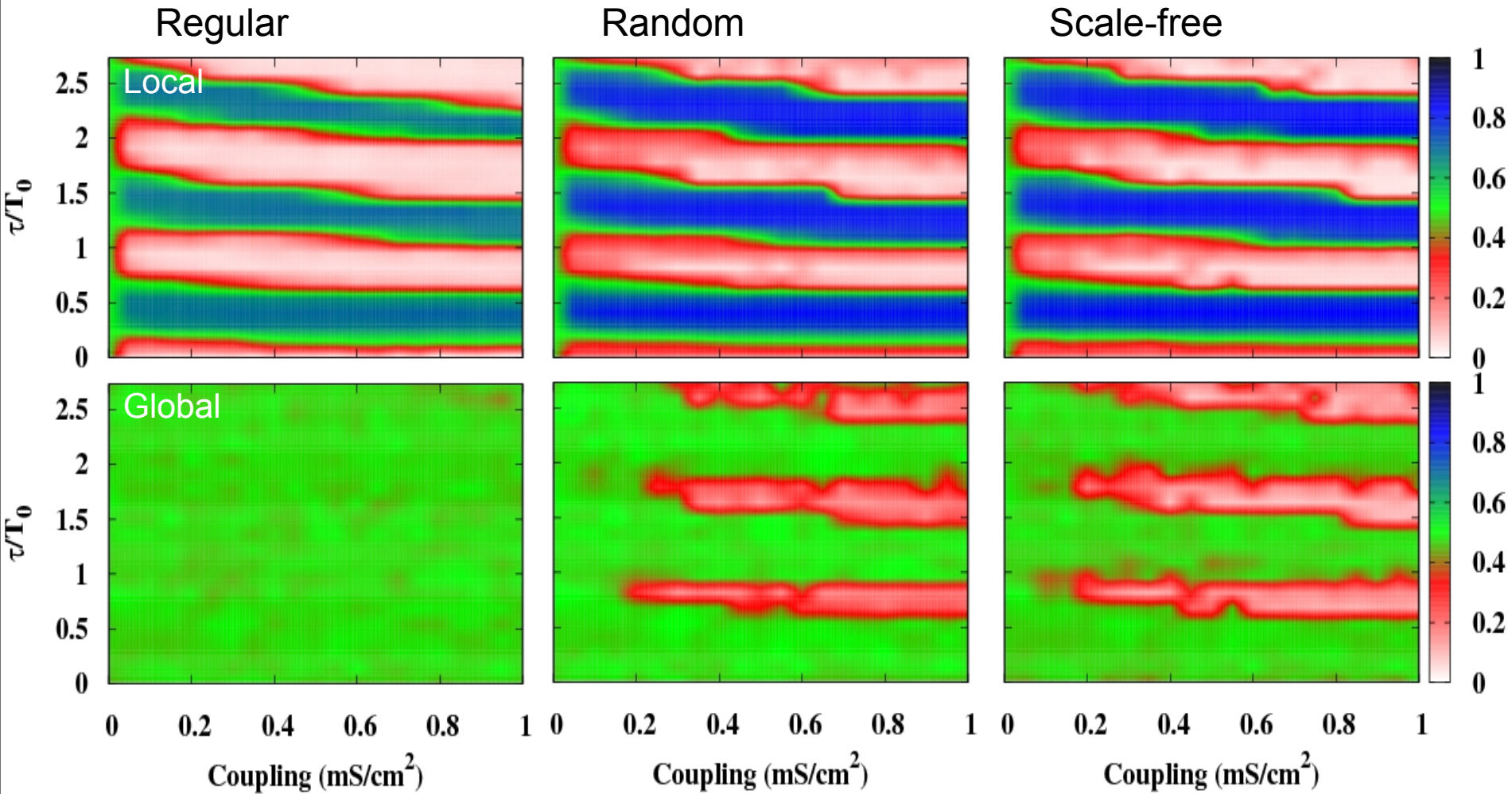
$\sigma^2 = 0.5 \text{ ms}^2$

Mean-field (all-to-all)



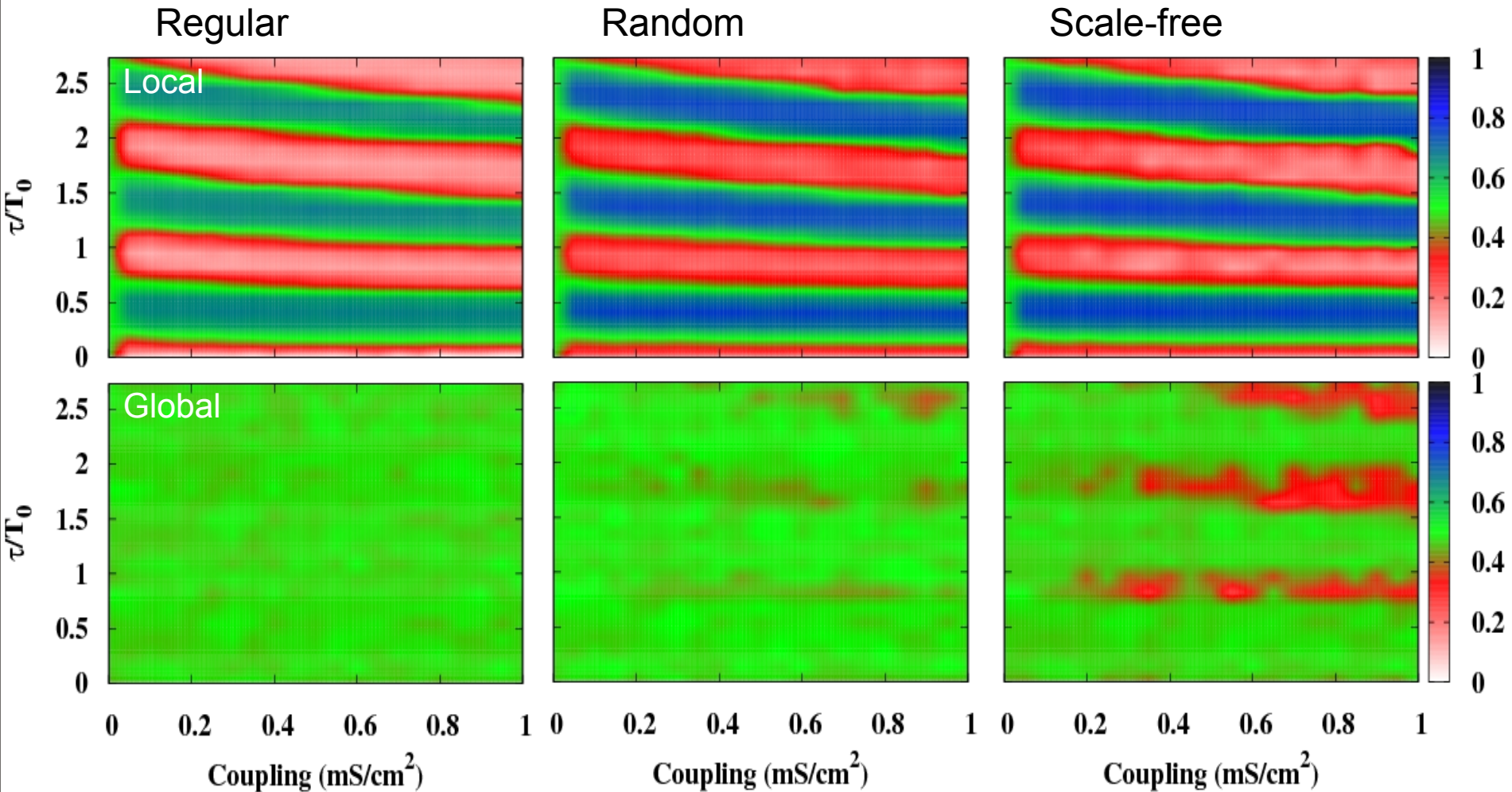
Local versus global synchronization

$\sigma^2 = 0.5 \text{ ms}^2$

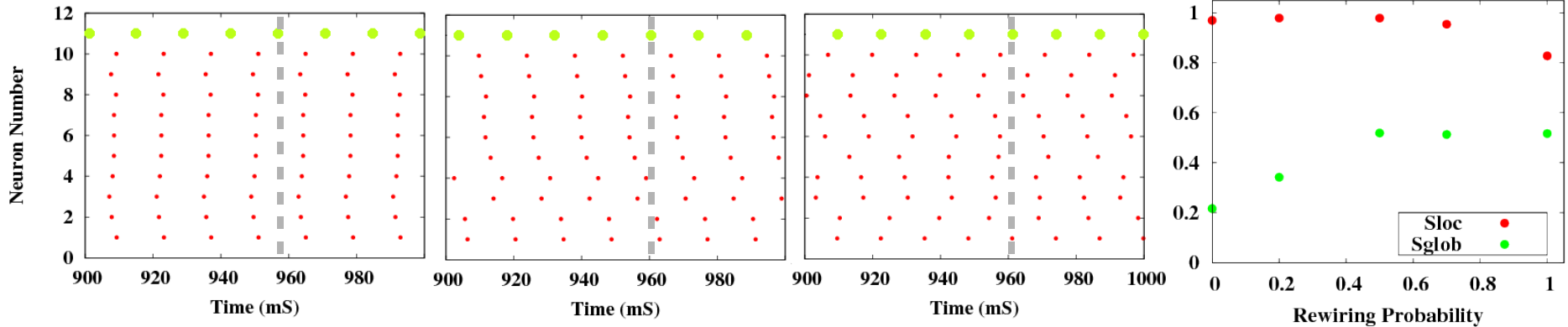
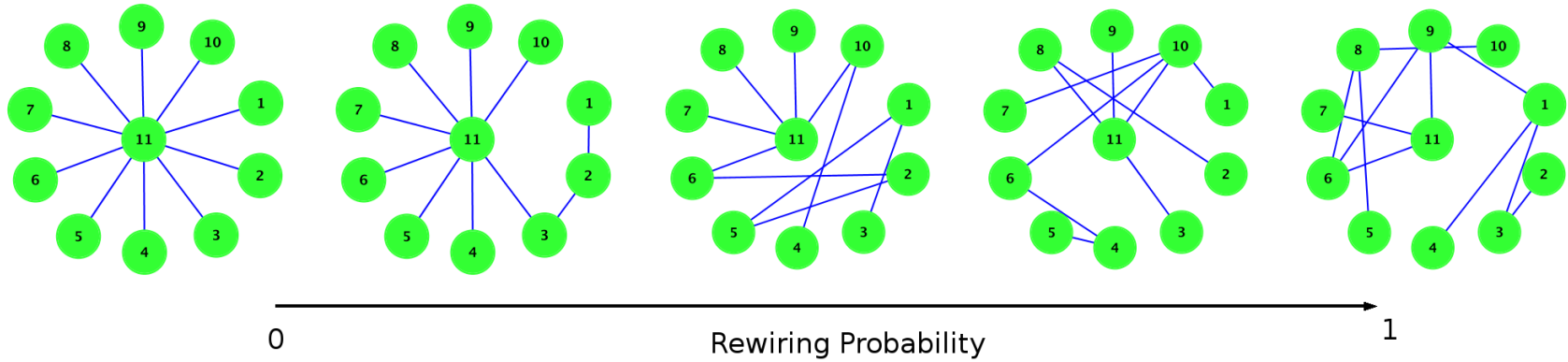


Local versus global synchronization

$$\sigma^2 = 2 \text{ ms}^2$$

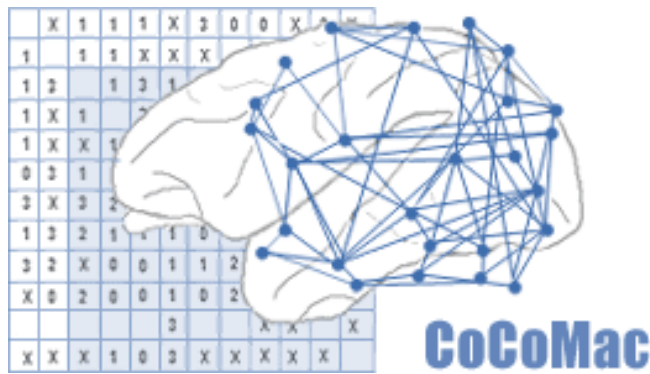


Simple example



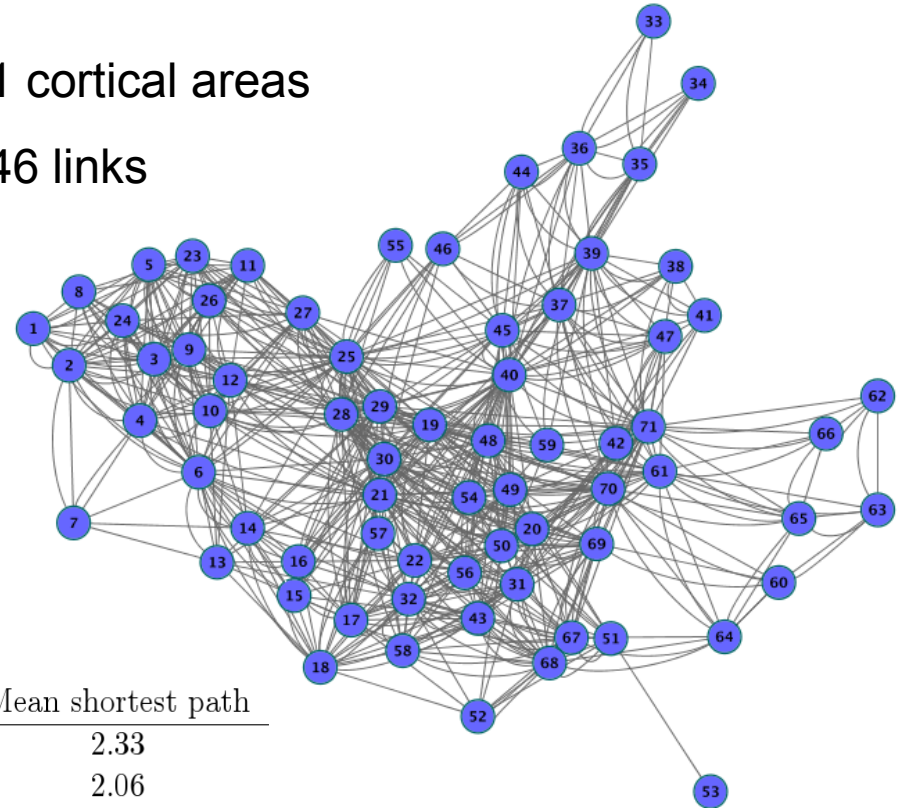
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> Macaque cortico-cortical network



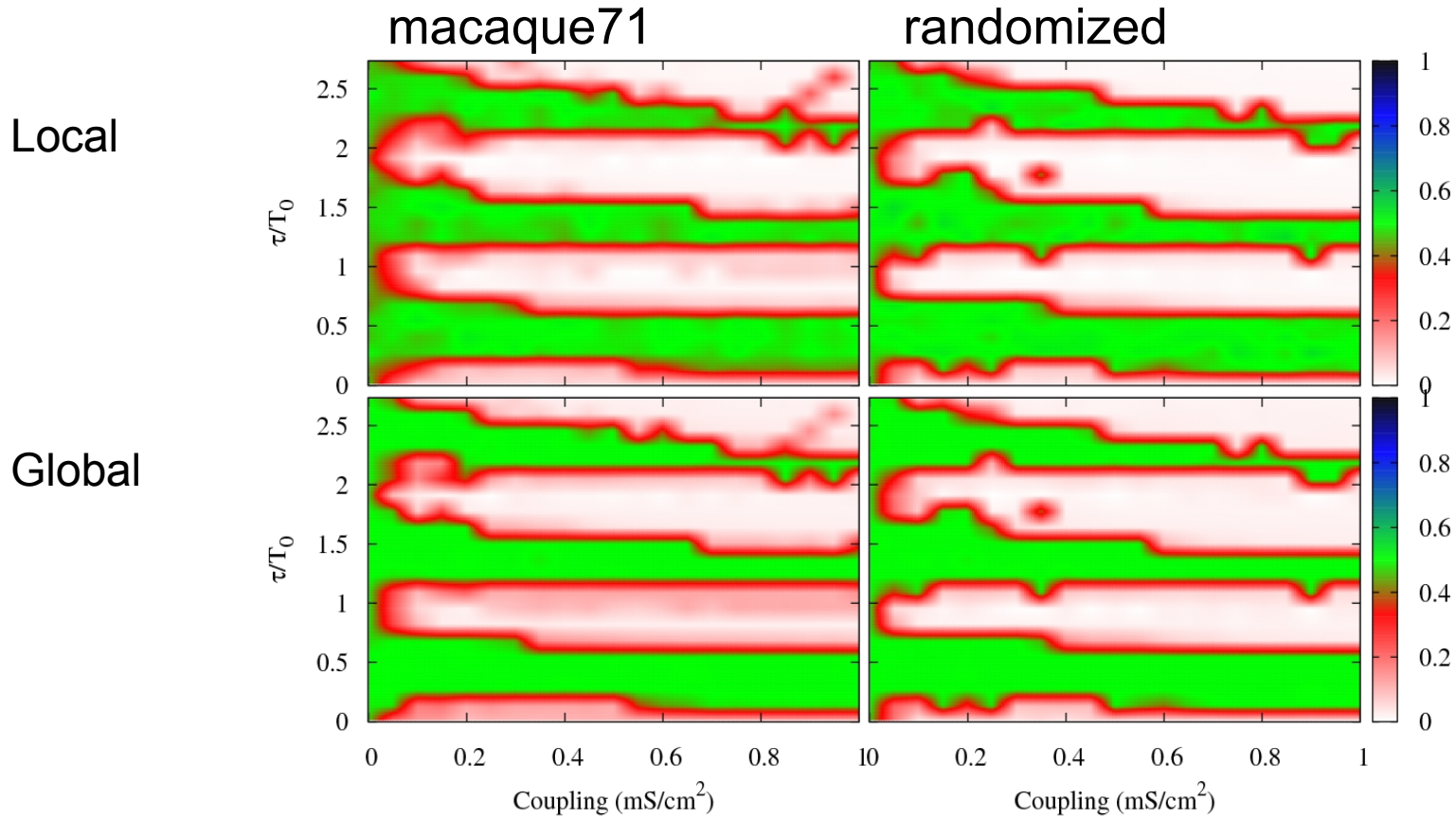
www.cocomac.org

- 71 cortical areas
- 746 links



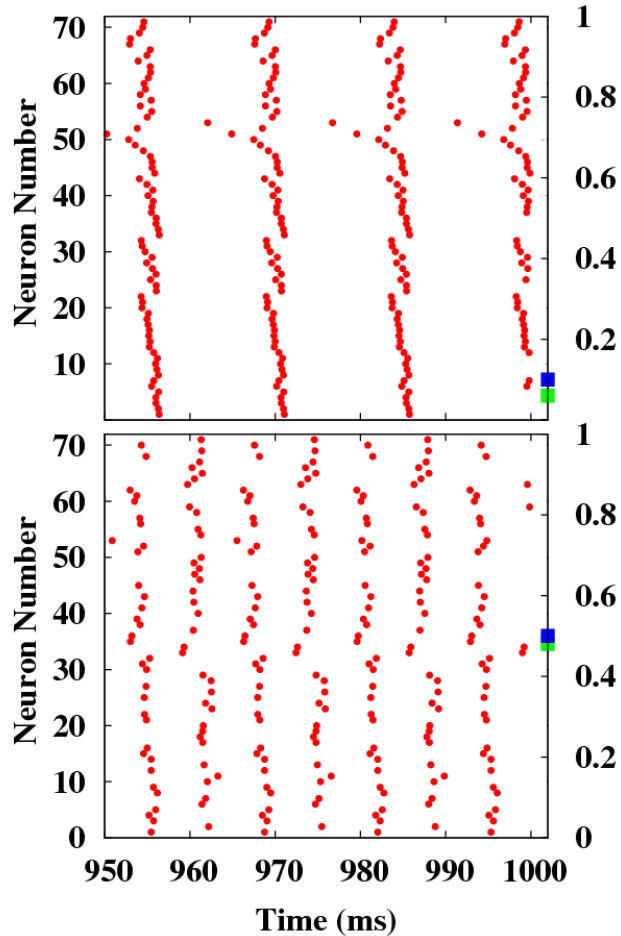
Metric	Clustering coefficient	Average degree	Mean shortest path
Macaque Network	0.46	10.5	2.33
Randomized version	0.24	10.5	2.06

> Local and global synchrony

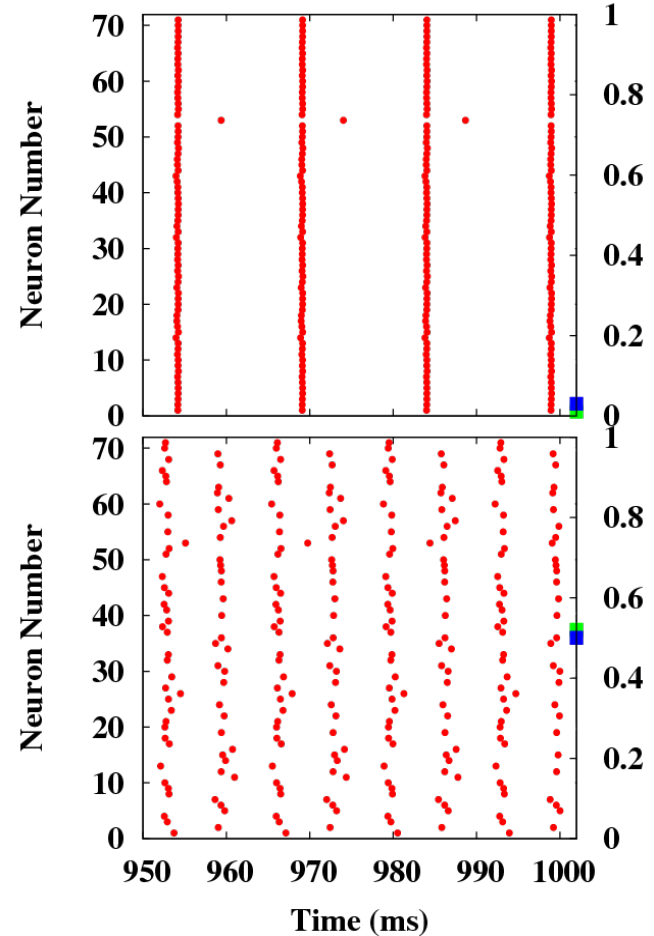


> Raster plots

macaque71



randomized



Conclusions & future work

- > NN with different topologies and delayed connections
 - Single delay
 - Distribution of natural frequencies
 - Heterogeneous delay
 - Preliminary results
 - Anatomical network
- > Next
 - Extend heterogeneous delay
 - Study the effect of inhibition
 - Include plasticity rule

Thank you for your attention!