

Spatial Prisoner's Dilemma in an Adaptive Network



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- * M. Zimmermann et al, Phys. Rev. E. 69, 065102-6 (2004)
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* IFISC



Paradigm for studies of cooperative behavior

Prisoner's Dilemma

		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Cooperate	1 year / 1 year	Free / 5 year
	Defect	5 year / Free	3 year / 3 year

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3 / 3	5 / 0
	Defect	0 / 5	1 / 1

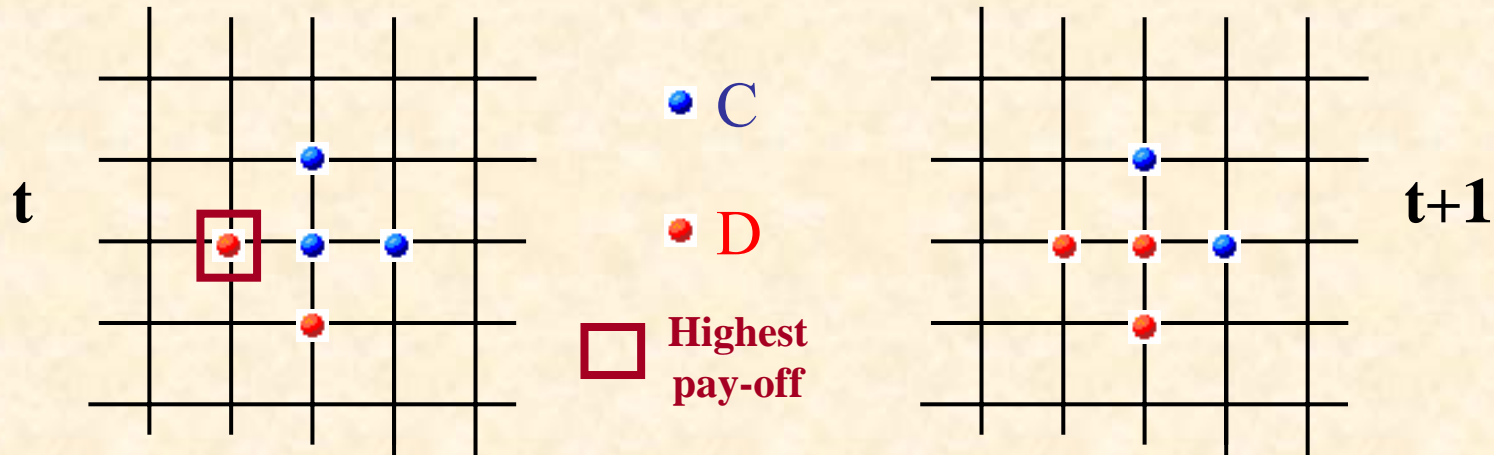
General problem:

- Whenever there is a conflict between self-interest and the common good.
- You are tempted to do something, but know it would be a great mistake if everybody did the same thing.

Alternative route to cooperation: Spatial games

Spatial effects can maintain coexistence between cooperators and defectors in a single non-repeated PD. A spatial game leads to results essentially different of those obtained in a global

game. (M. A. Nowak and R. M. May, *Nature* 359, 826 (1992); *Proc. Nat. Acad. Sci. USA* 91, 4877 (1994))

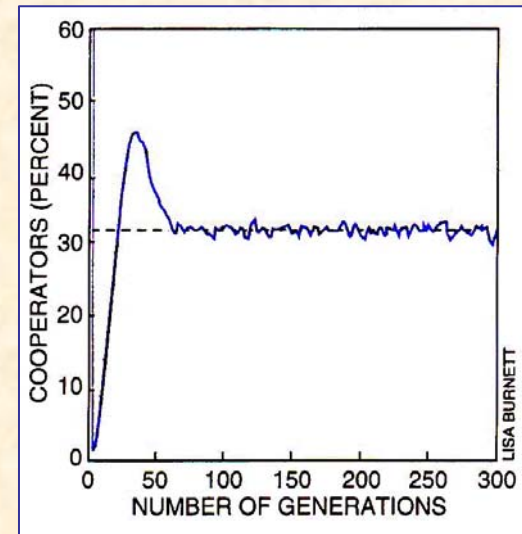
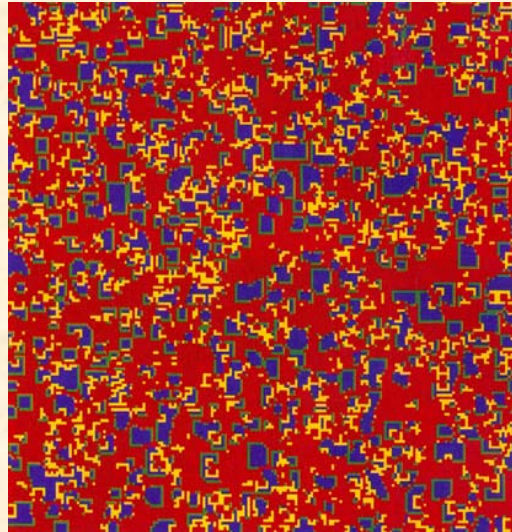
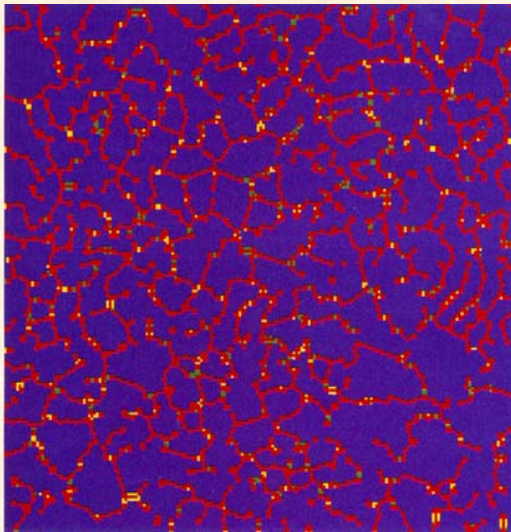


- *Players are only pure cooperators **C** or pure defectors **D**. No memory or strategy.*
- *They interact with neighbors in some spatial array.*
- *In each generation, players add up the scores from all encounters, and in the next generation a given cell is retained by its previous owner or taken over by a neighbor, depending on who has the largest pay-off.*

Spatial Prisoners' Dilemma

A simple spatial version of the PD, with no memories among players and no strategical elaboration can promote the coexistence of C and D in situations where one strategy would exclude all others if the interactions occurred randomly and homogeneously.

(M. A. Nowak and R. M. May, Nature 359, 826 (1992))



$1.75 < b < 1.8$

B=C (from C); R = D (from D)
Y=D (from C); G = C (from D)

$1.8 < b < 2$

Regular network,
Deterministic winning,
Discrete time: Synchronous update

Locality/clustering or Context Preservation (continuity of interactions) ?

(M. Cohen, R. Riolo and R. Axelrod, Rationality and Society 13, 5 (2001))

Co-evolution: PD game in a dynamic network

- **Motivation:** Adaptation of local neighborhoods: "*I no longer want to play with you!!*"
Example: scientific collaboration networks
- **Probabilistic:** Social plasticity p
(a) dependent (*coupled evolution*)
(b) non-dependent on strategies.
- **'Rational':** *A new link is created whenever both agents receive a benefit. If both do not benefit, the link is dismissed.*

Network adaptation is based on mutual benefit

- **C-C link:** Mutual benefit = $2R$ (2)
Both agents reinforce their relationship;
- **C-D link:** Mutual benefit = $T+S$ (b)
C-agent will try to dismiss, while D-agent will try to reinforce;
- **D-D link:** Mutual benefit = $2P$ (0)
Both agents try to dismiss the relationship. (**WEAKEST** link)

*Network adapts breaking the link **D-D** with probability p*

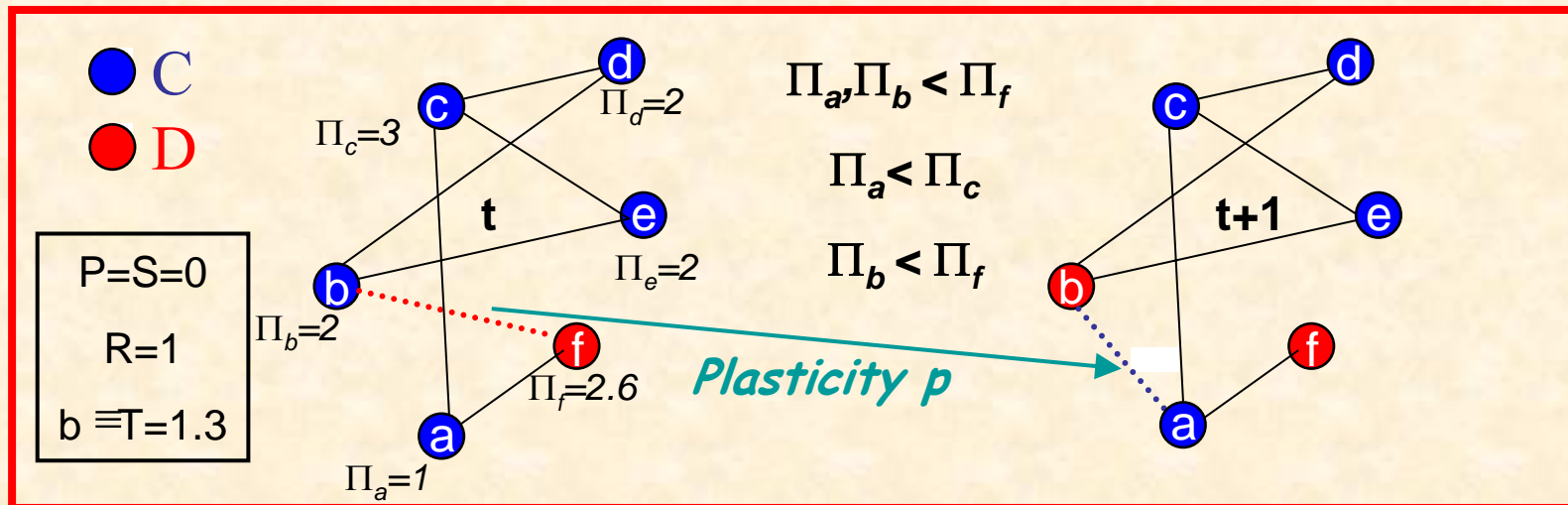
PD game with local interactions and co-evolving network

- Define a random *network* N , with an average number of links per site K .
- Each agent i plays a PD game with *all* its neighbors using the *same action* $s_i \in \{C, D\}$, and collects a total payoff Π_i .
- **Action update:** Each player i *imitates* the strategy of its neighbor (including himself) with the largest payoff, $l(i)$. Player i is **satisfied** if $i = l(i)$; otherwise **unsatisfied**

$$s_i(t+1) = s_{l(i)}(t) \quad \text{IMITATION BY SUCCESS}$$

- **Network Dynamics (Choosing partners):** If player i is an **unsatisfied D-agent**, it breaks **with probability p** any link with other **D-neighbour** and establishes randomly new links in the network.

EXIT FROM UNSATISFACTORY RELATION



Steady states

Actions (strategies) and network (links) do not change:

- All C-network is a steady state
- However an All D-network is NOT a steady state for $p \neq 0$

Conditions for a non-trivial steady state:

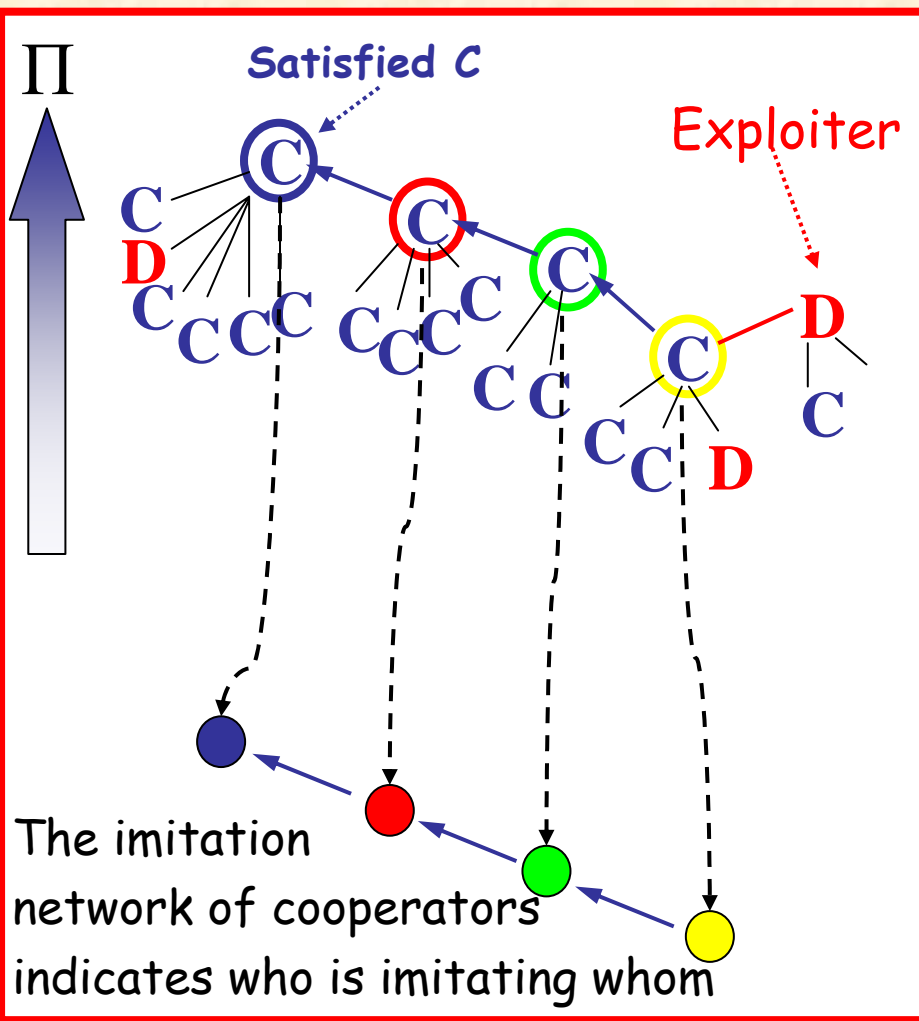
- No links between D-agents* so that *Network Dynamics* does not occur.
- If C-agent i interacts with D-agent d , $\Pi_{l(i)} > \Pi_d > \Pi_i$
 - ❖ *Chain of cooperators*: C-agents must imitate other C-agents (most C-agents are *Conformists*).
 - ❖ *D-agents have to be passive* local maxima (*Exploiters*), so that their strategy is not imitated.

SOCIAL DIFFERENTIATION:

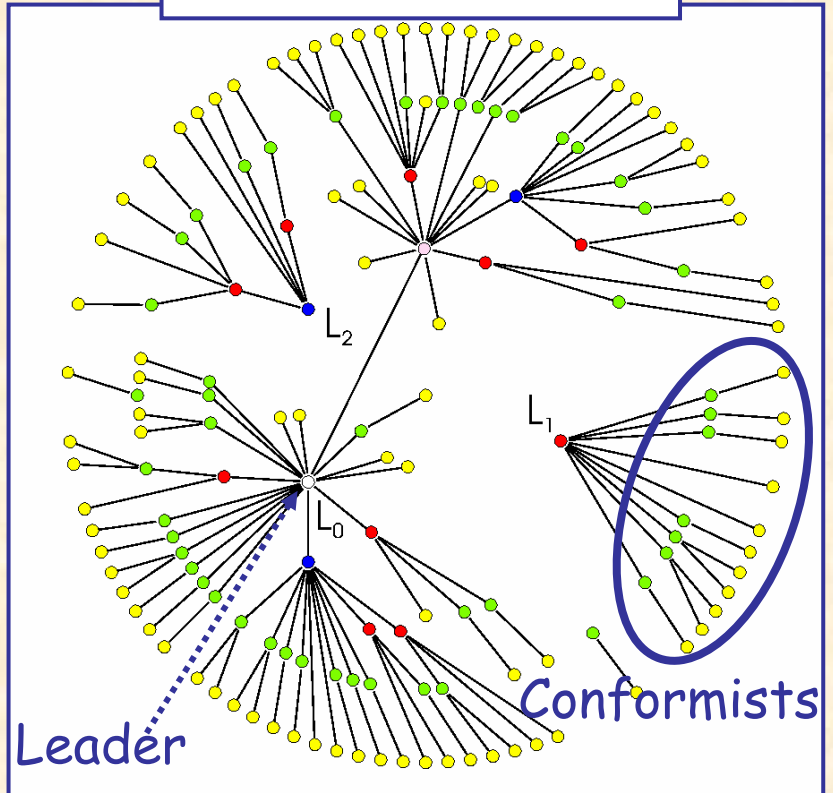
Leaders, Conformists, Exploiters

Leaders: Satisfied C-agents with maximum pay-off in the chain

Hierarchical Network



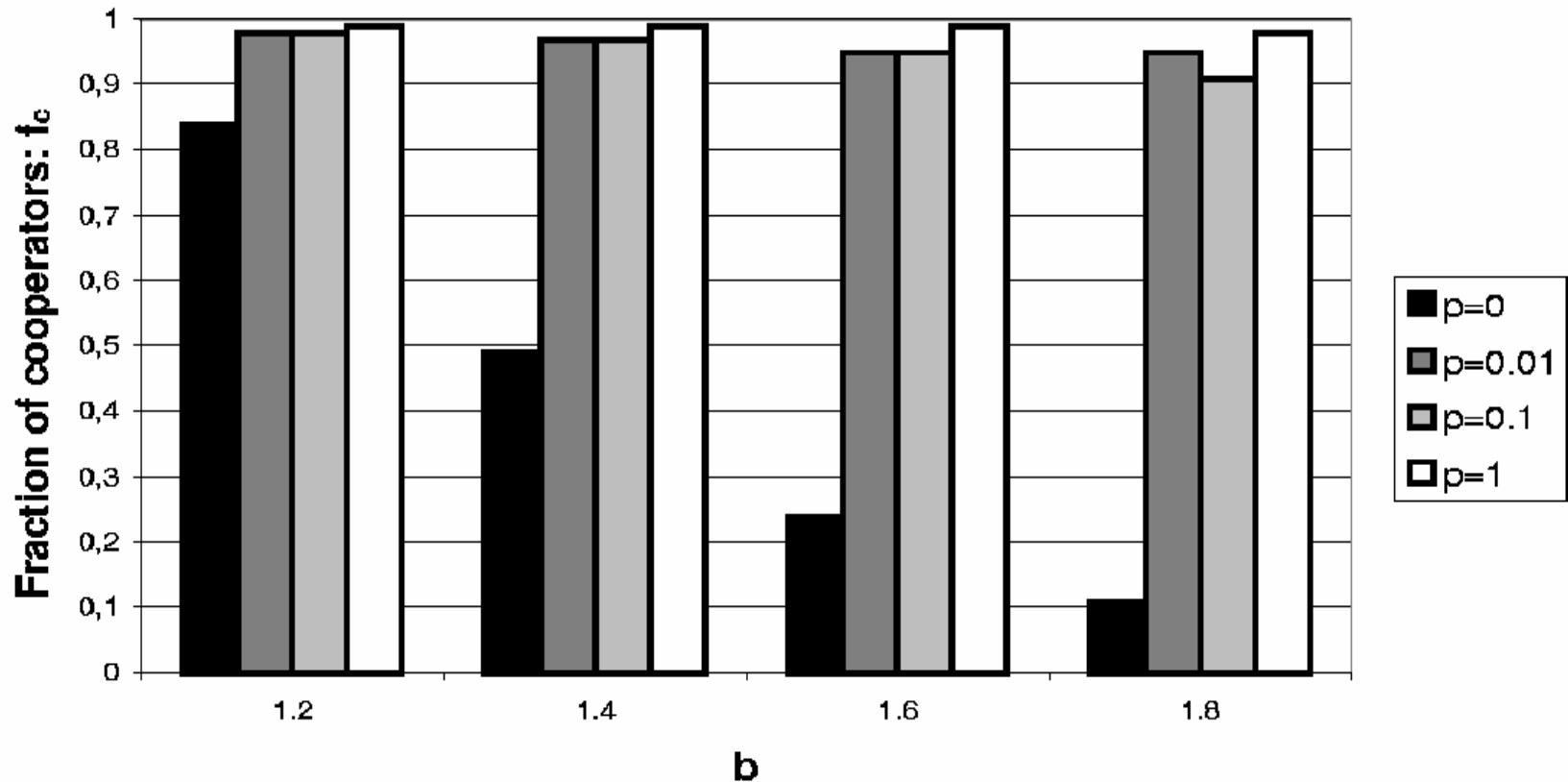
Imitation Network



L_0 : **absolute leader** with maximum payoff and largest number of links
 L_1, L_2 are leaders (satisfied C) with a payoff lower than the absolute leader.

Role differentiation, including spontaneous leader selection, emerges from stochastic dynamics of initially equivalent agents

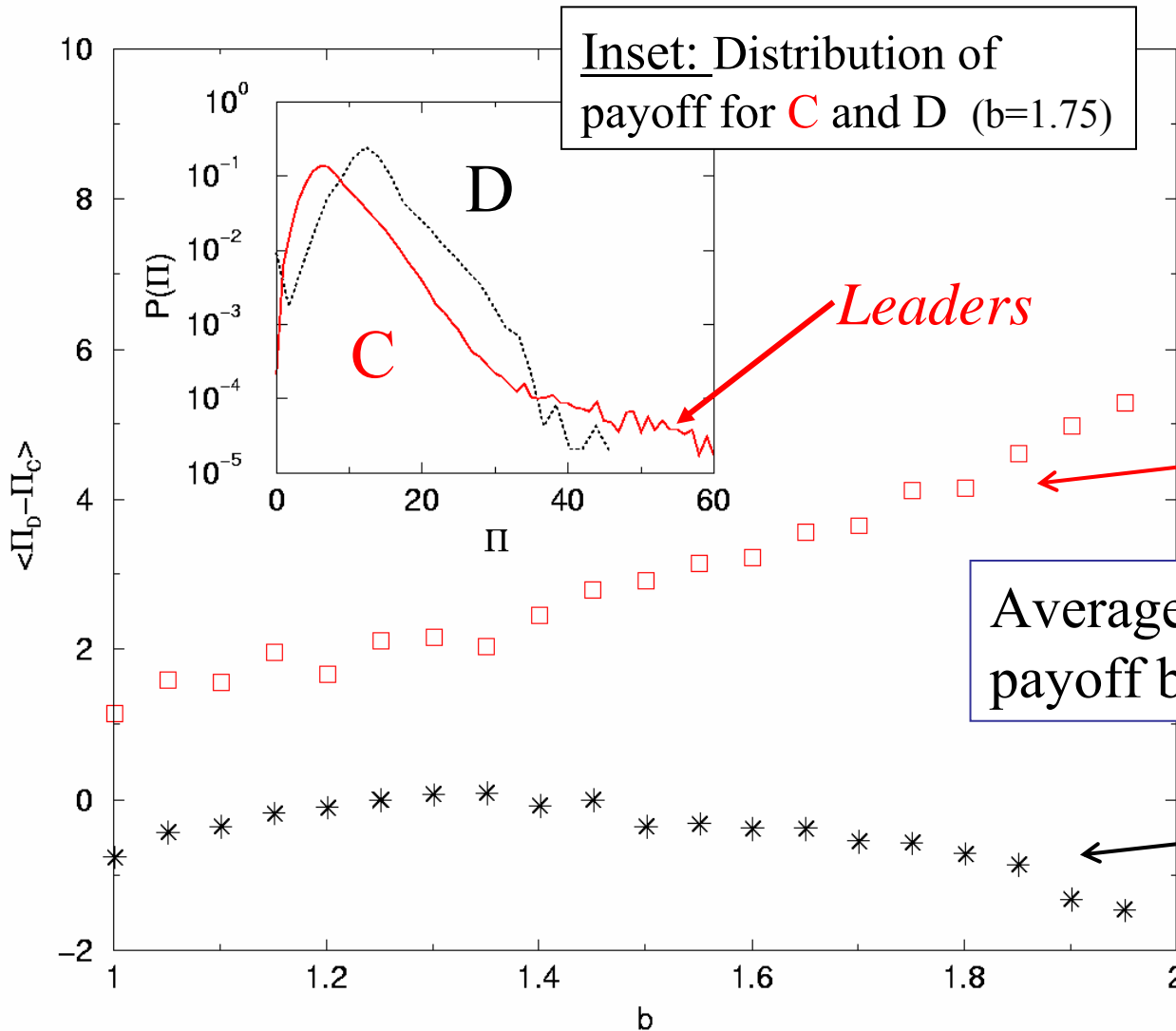
Fraction of cooperators



$N=10,000$ Averages over 100 runs

Initial condition: 60% C, random network $K=8$

Wealth distribution



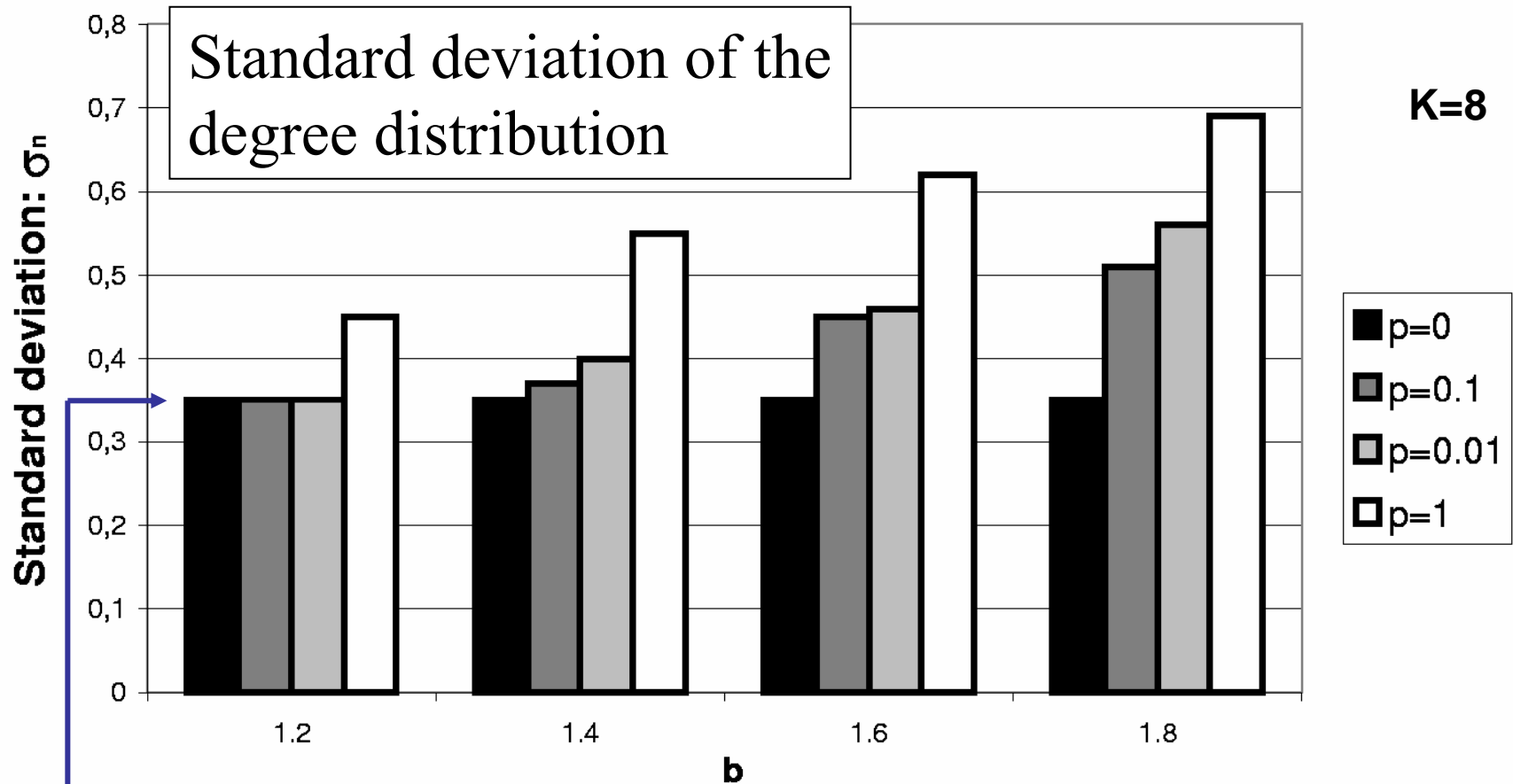
There are less Defectors but they get richer.

$p = 0.1$

Average difference of payoff between D and C.

$p = 0$

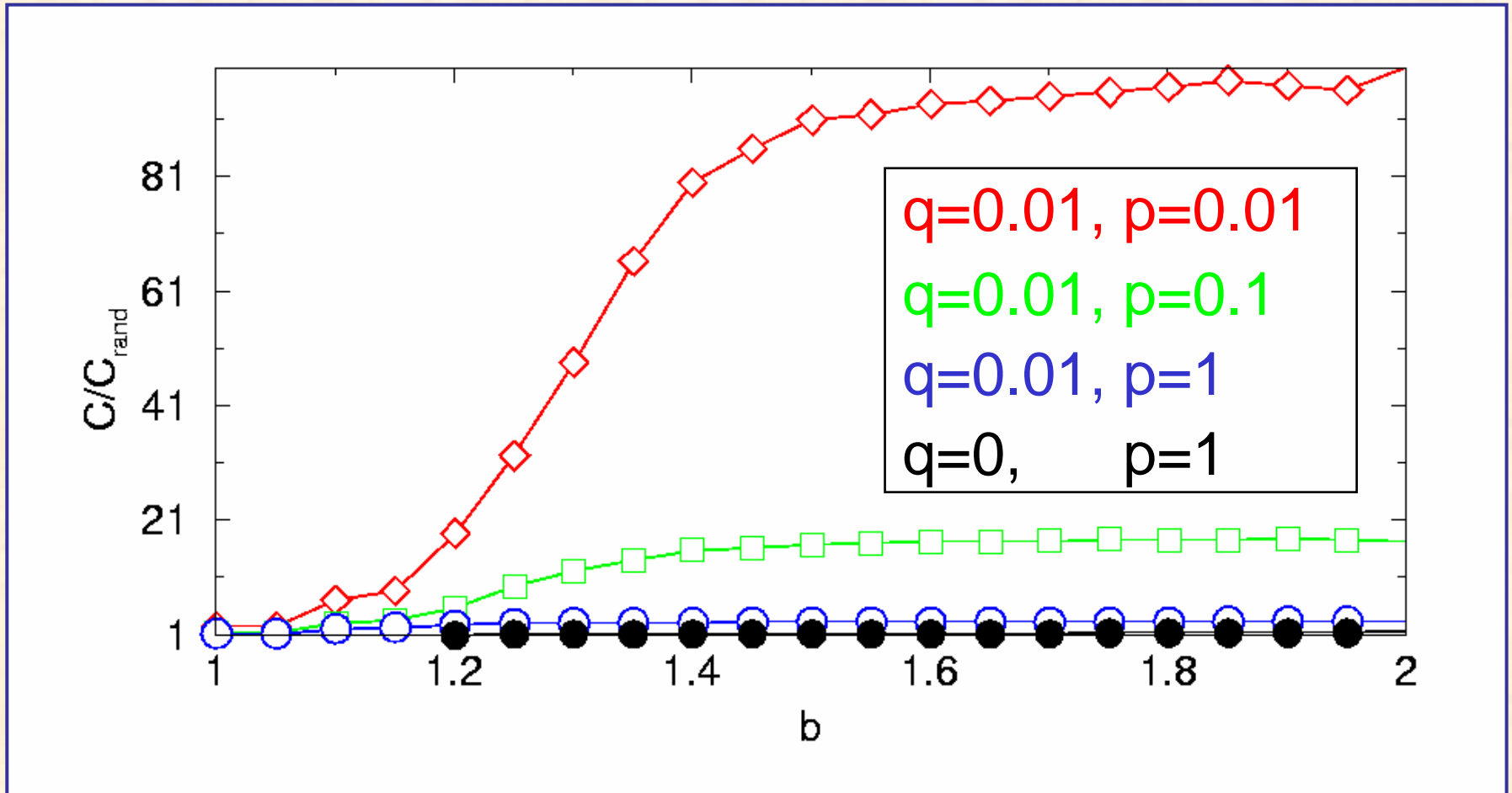
Structure of the social network



Poisson distribution
 $\sigma_n = 1$: Exponential

Gini coefficient: The social dynamics generates a flux of pay-off towards richer individuals

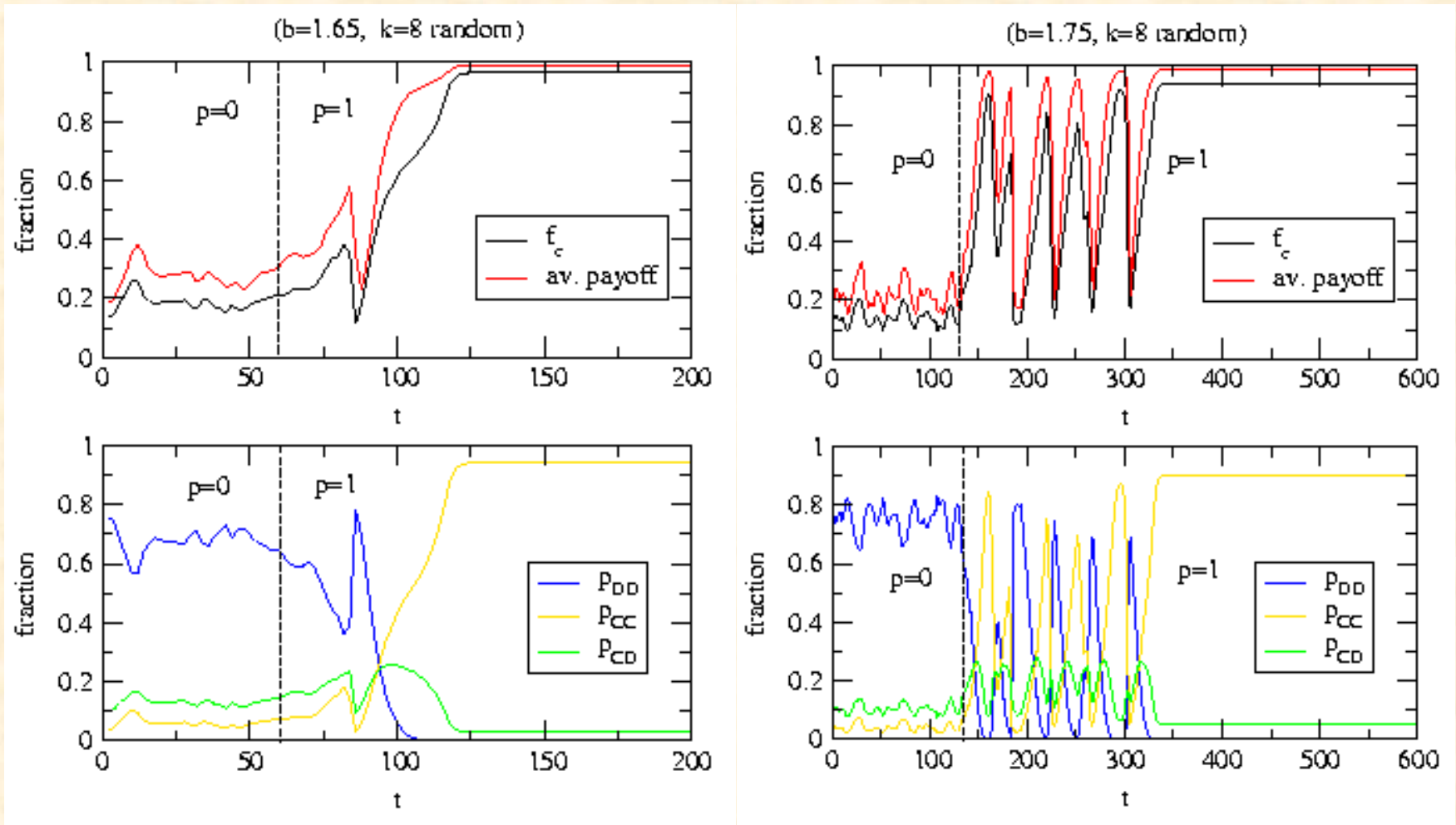
Clustering



q = probability that new partner is selected among neighbors of the neighbors

Small World connectivity emerges if local partner selection is allowed

Network evolution and large oscillations

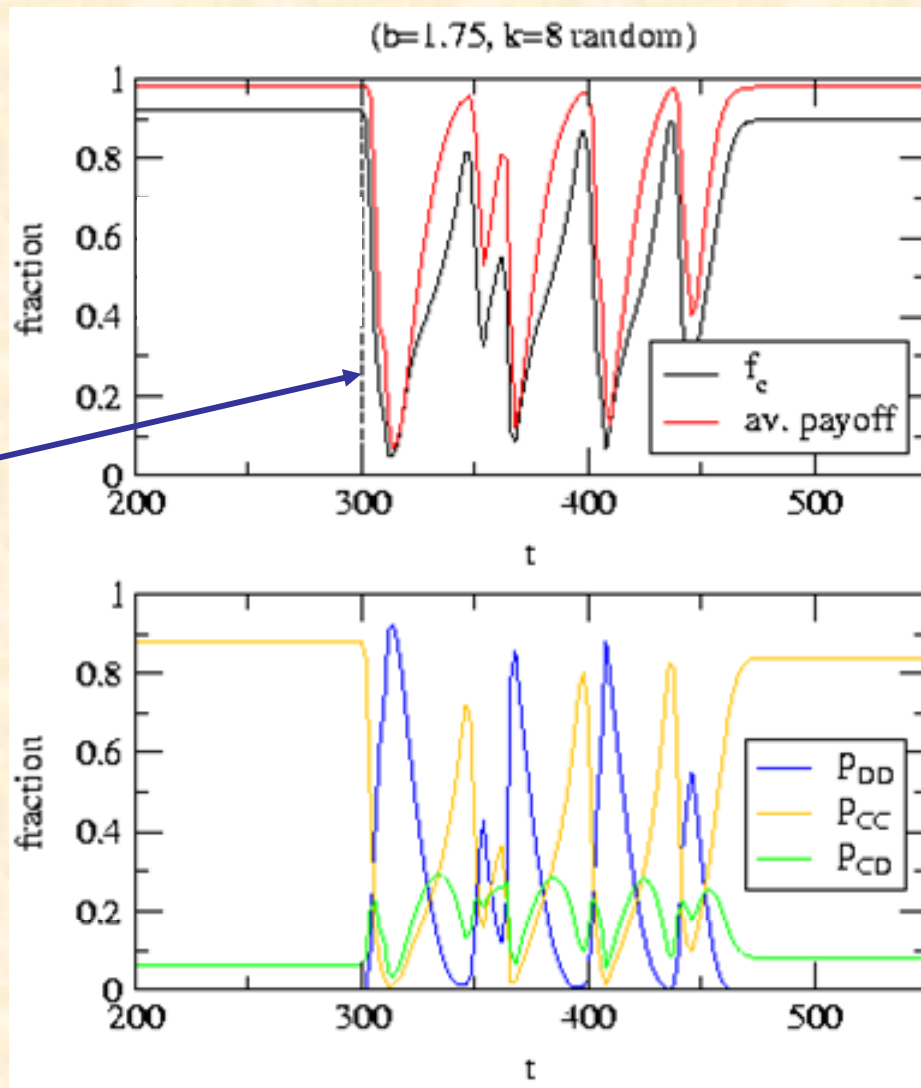


For high enough b , large transient oscillations occur

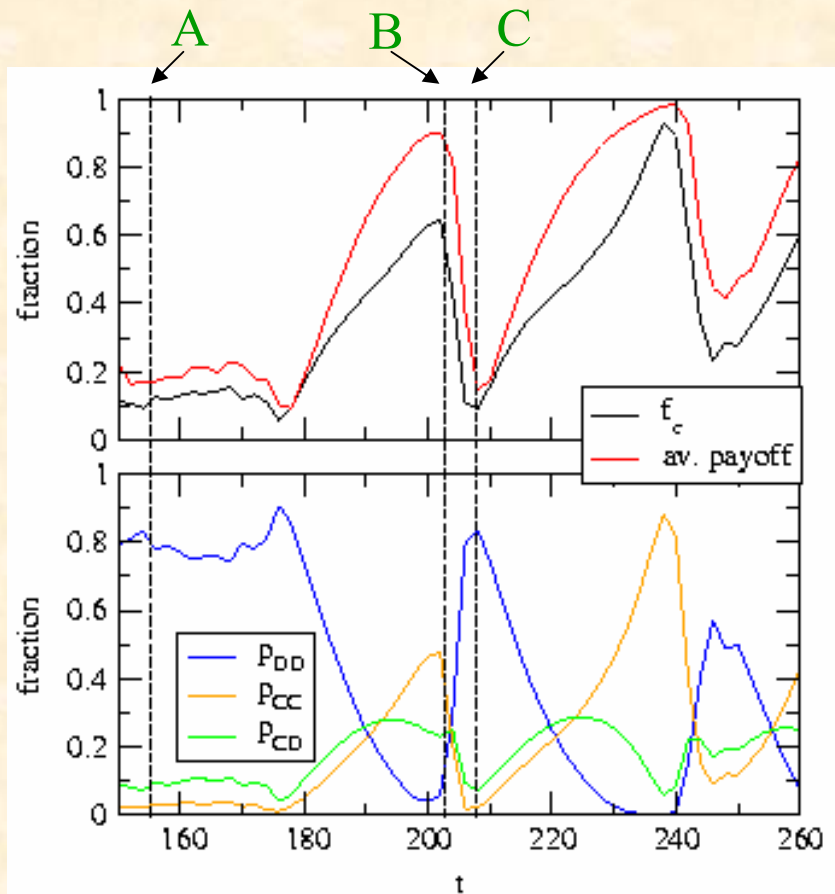


Social Crisis

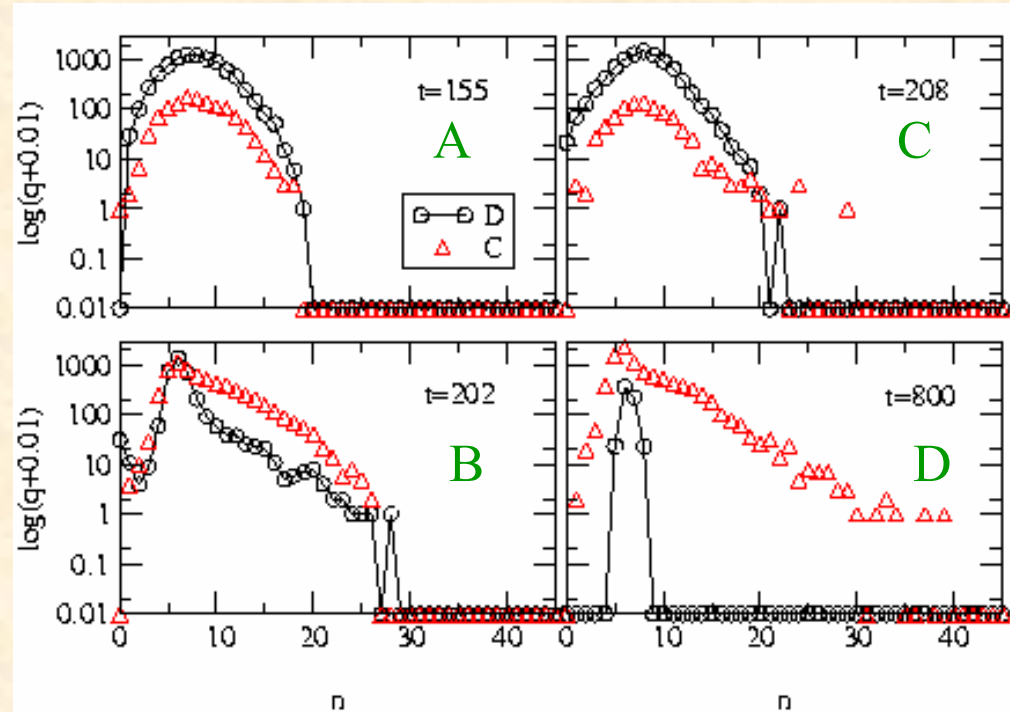
Perturbation
 $C \rightarrow D$
acting on the
leader



Large Oscillations



Links histograms



A, C: Gaussian

D: final state

B: Exponential

Conclusions

Structure of network is both cause and consequence of successful cooperation (Macy, 1991)

- Co-evolution (social plasticity) leads to:
 - > a highly **cooperative** steady state.
 - > the emergence of **social roles**: **leaders, conformists and exploiters**.
 - > hierarchical social networks with exponential tails in the connectivity and wealth distribution.
- Perturbations to leaders trigger large cascades (**social crisis**).
- A **small world** network can be obtained if we allow for local *partner selection*.