Workshop on Challenges and Visions in the Social Sciences, ETH August 08

Group Formation:
Fragmentation Transitions
in Network Coevolution Dynamics

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CO-EVOLUTION

Dynamics of Networks:

1. Dynamics **OF** network formation: Structure created by individual choices/actions

Rightwing view

2. Dynamics **ON** the network: Actions of individuals constrained by the social network

Leftwing view

3. Co-evolution of agents and network :

Circumstances make men as much as men make circumstances

..new research agenda in which the structure of the network is no longer a given but a variable.....explore how a social structure might evolve in tandem with the collective action it makes possible (Macy, Am. J. Soc. 97, 808 (1991))

Final Goal: Understanding *dynamical* processes of group formation and social differentiation: Emergence of social dynamical networks with

-Social structure

-Weak links (Granovetter)

-Community structure

NETWORK CO-EVOLUTION MODELS

Review paper: *T. Gross and B. Blasius, J. R. Soc. Interface* ***5****, 259 (2008)*

Key ingredients.

a) Going beyond dynamical models in which:

- Network evolution is decoupled from the evolution of agents actions
- Complete network redefined at each time step

b) Social plasticity as ratio of time scales of evolution of network and action

→ **Generic result:** **Network fragmentation transition**

(Independent of link conservation, rewiring rule, interaction….)

Zachary´s karate club

Two examples in model of consensus dynamics:

- **Voter model:** Minimal model

  F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. 100, 108702 (2008)

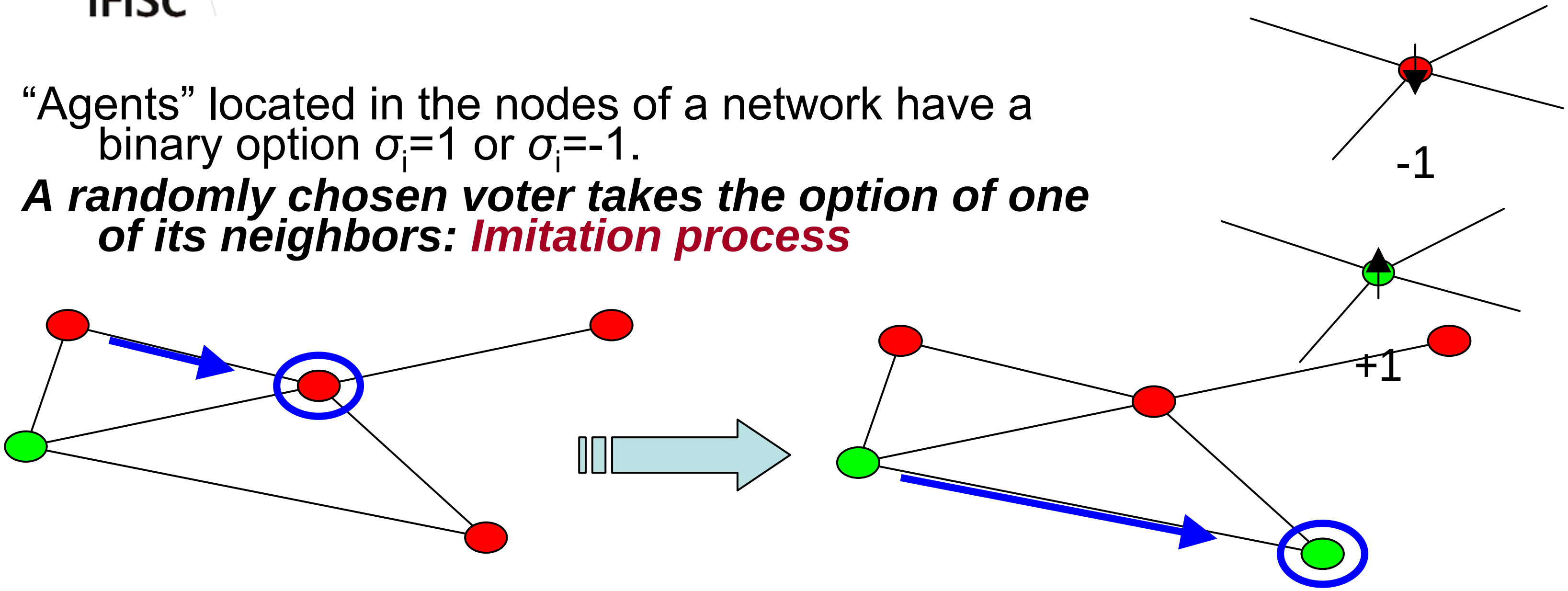
- **Axelrod´s cultural model:** Robustness of globalization-polarization transition

  F. Vazquez et al. Physical Review E, 76, 046120 (1-5) (2007)
  D. Centola et al. Journal of Conflict Resolution, 51, 905-929 (2007)

Voter Model

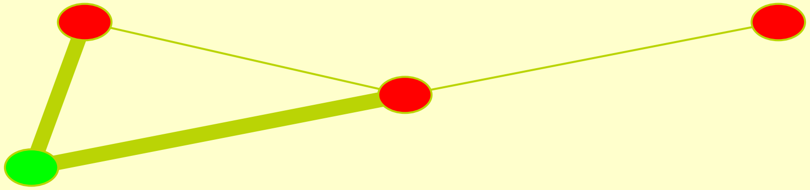
"Agents" located in the nodes of a network have a binary option $\sigma_i=1$ or $\sigma_i=-1$.

A randomly chosen voter takes the option of one of its neighbors: Imitation process



Qs?: When and how one of the two absorbing states (consensus) is reached? Effect of network of interactions?

Order Parameter: Average interface density (measure of ***active links***)



$$\rho = \frac{1}{2N\langle k\rangle}\left(1-\sum_{i=1}^{N}\sum_{j\in\nu(i)}\sigma_i\sigma_j\right)$$

ρ=0 in absorbing state

Interface or active link: a link connecting nodes with different states.

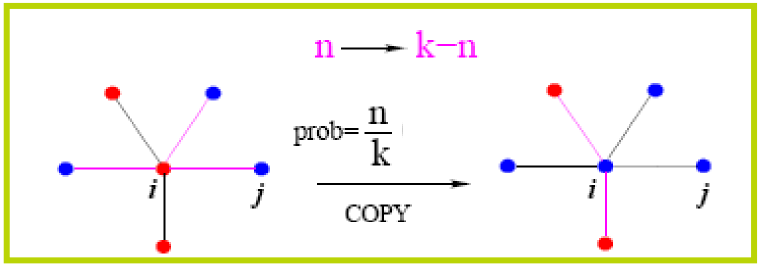
Mean Field Voter Model

F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. ***100****, 108702 (2008)*

- ***Mean Field Node Dynamics:*** $$\frac{d<\sigma>}{dt}=0$$

- ***Mean Field Link Dynamics:***

ρ = global density of active links
n = active links
k-n = inert links
k= degree of node *i*



$$\Delta\rho=\frac{2(k-2n)}{<k>N}$$

Node i of degree k:

$$\left.\frac{d\rho}{dt}\right|_k=\frac{1}{1/N}\sum_{n=0}^{k}B(n,k)\frac{n}{k}\frac{2(k-2n)}{<k>N}$$

$$B(n,k)=\text{Prob. that node } i \text{ has } n \text{ active links}$$

$$B(n,k)\approx\frac{k!}{n!(k-n)!}\rho^n(1-\rho)^{k-n}$$

Mean Field: ρ~ prob that a link from node *i* is active

$$\frac{d\rho}{dt}=\sum_k P_k\left.\frac{d\rho}{dt}\right|_k=\frac{2\rho}{<k>}\left[(<k>-1)(1-2\rho)-1\right]$$

$$\rho^s=\frac{<k>-2}{2(<k>-1)}$$

Voter Model in Random Networks

F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. ***100****, 108702 (2008)*

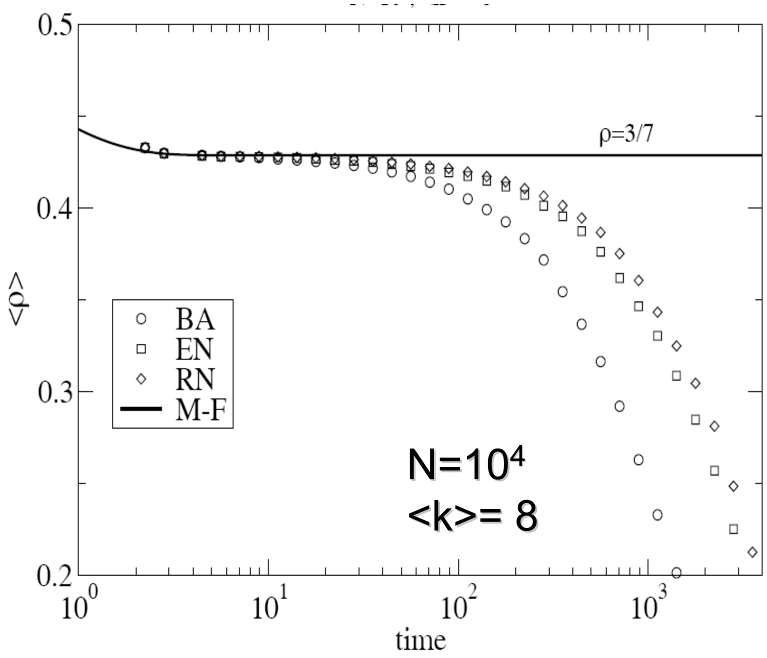
- ***Mean Field Node Dynamics:*** $\frac{d<\sigma>}{dt} = 0$

Mean Field Link Dynamics:

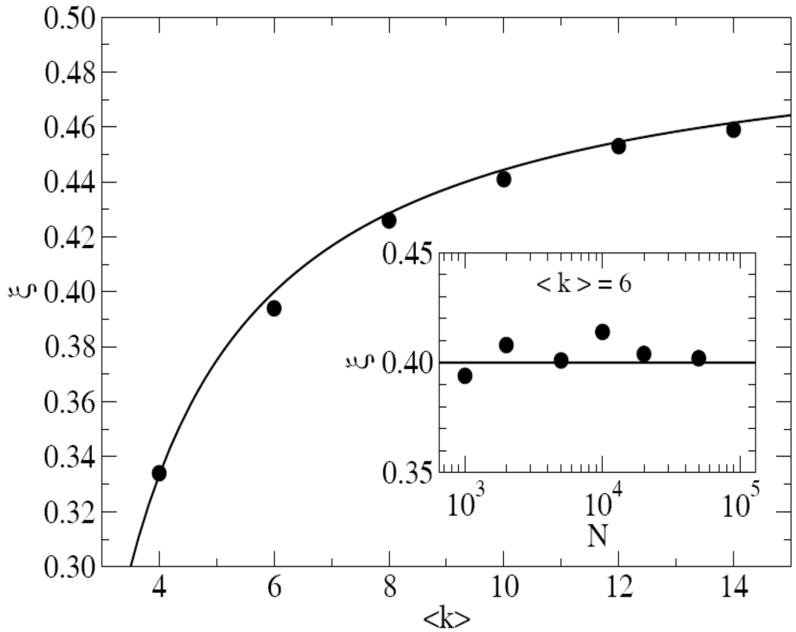
Single parameter theory

$$\rho^s = \xi = \frac{<k> - 2}{2(<k> - 1)}$$

Network topology independence



Barabasi-Albert Scale Free Networks

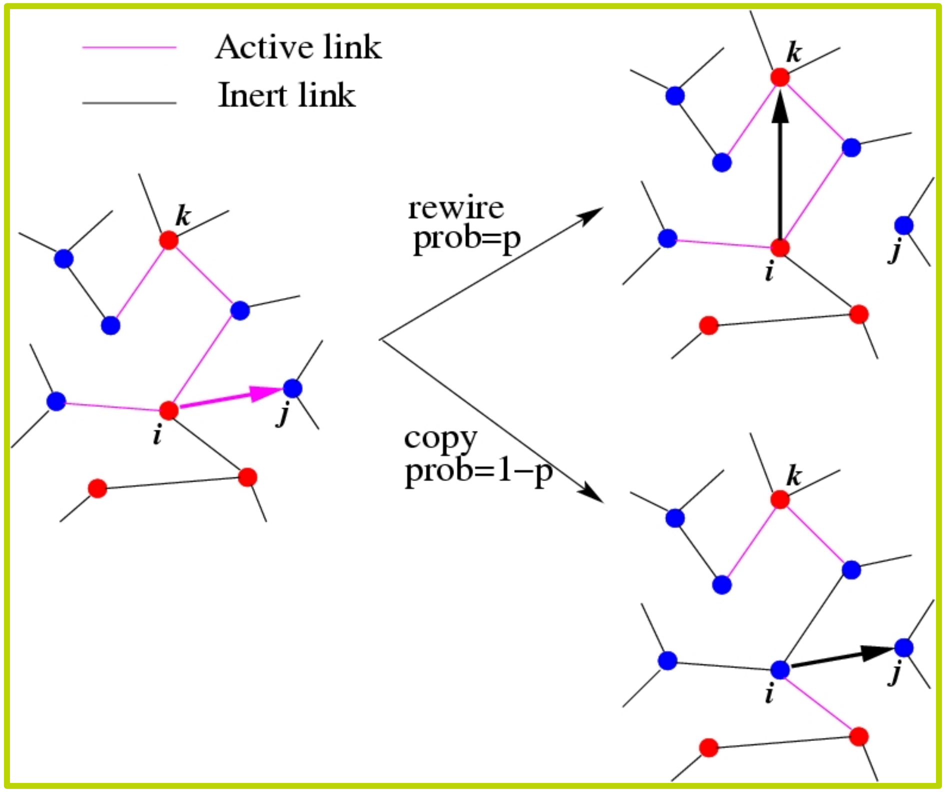


Coevolution Voter Model

F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. ***100****, 108702 (2008)*

Initial: Degree-regular random graph with μ neighbors.
Nodes take state $S = -1$ *or* $S = +1$ with the same probability 1/2.

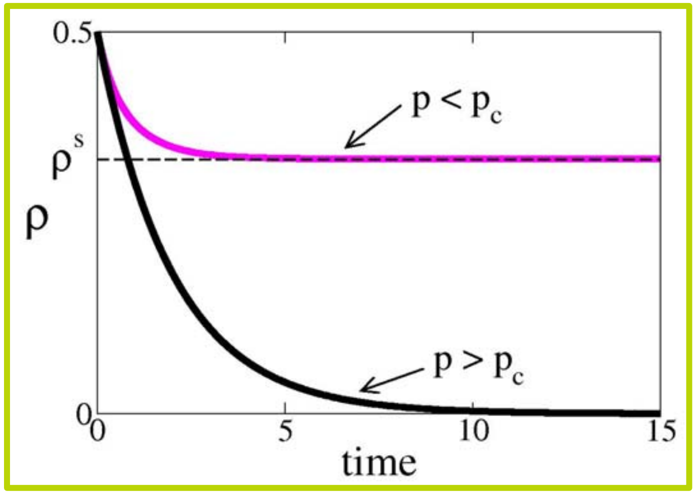
1. Pick a node i and a neighbor j at random.
2. If $S_i = S_j$ nothing happens.
3. If $S_i \neq S_j$ then:
   - Network dynamics: *rewire*
     with probability *p* delete link $i - j$
     and create link $i - k$ $(S_i = S_k)$.
   - State dynamics: *copy*
     with probability *1-p* set $S_i = S_j$.
4. Repeat ad infinitum.



- **Agents select interacting partner according to their state**
- **p gives a ratio of time scales of evolution of state of nodes and network**

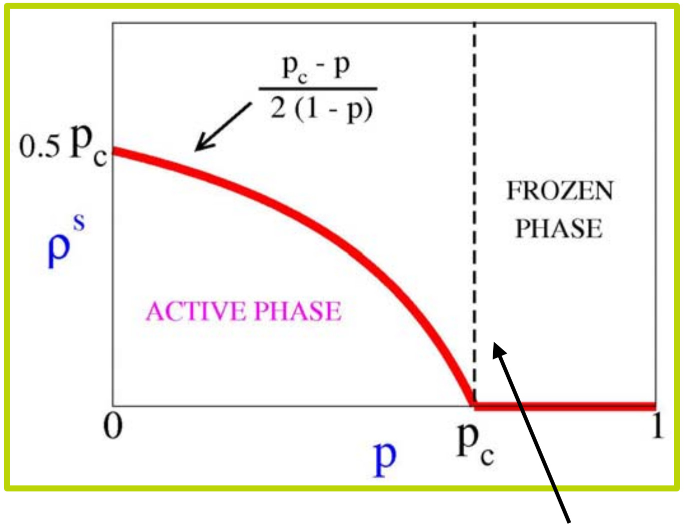
Master equation for the density of active links in the N→∞ limit:

$$\frac{d\rho}{dt} = \frac{2\rho}{\mu}\left[(1-p)(\mu-1)(1-2\rho)-1\right]$$



Active - Frozen Transition at

$$p_c = \frac{\mu-2}{\mu-1}$$



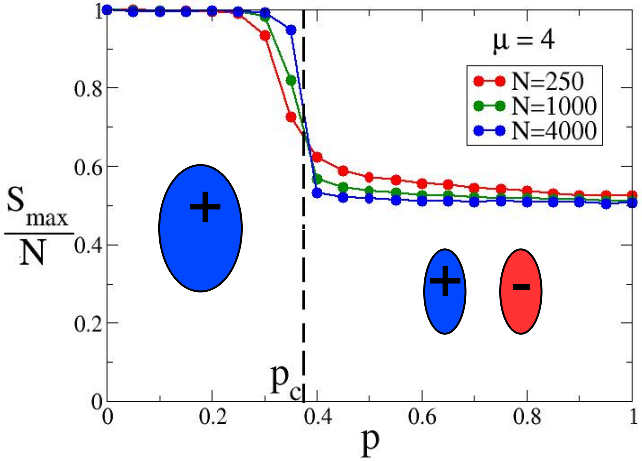
$\rho_{++} = \rho_{--} = 1/2$?

- **Active phase:** Links continuosly being rewired and nodes flipping states
- **Frozen phase:** Fixed network where connected nodes have the same state

Fragmentation transition in a FINITE coevolving network

Fragmentation Transition

Size of largest network component.

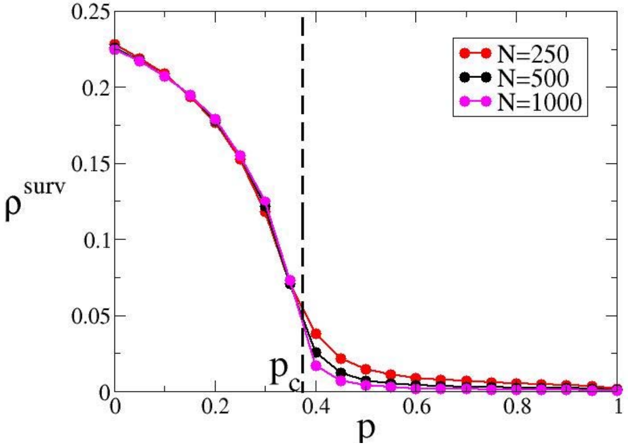


Active phase→Connected network ($S_{max}/N = 1$)
($N = \infty$)

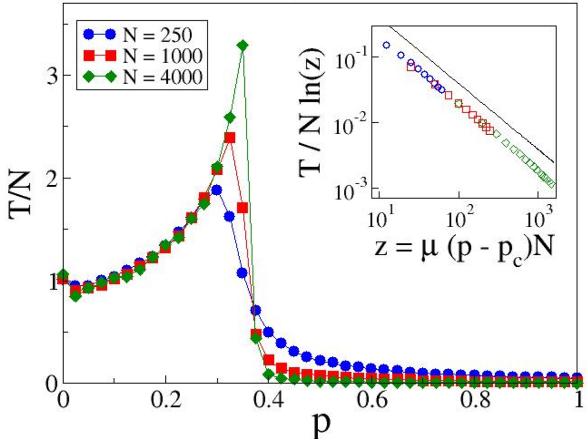
Frozen phase→Fragmented network ($S_{max}/N \approx 0.5$)

- $p<p_c$: slow rewiring keeps network connected until system fully orders and freezes in a single component.
- $p>p_c$: fast rewiring leads to fragmentation of network into two components before system reaches full order.

Active links in surviving runs.

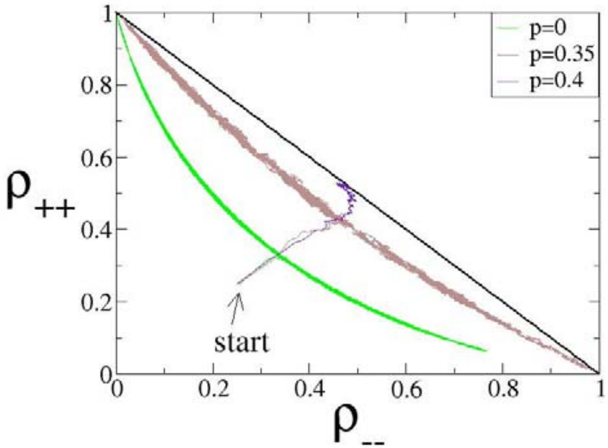


Convergence times

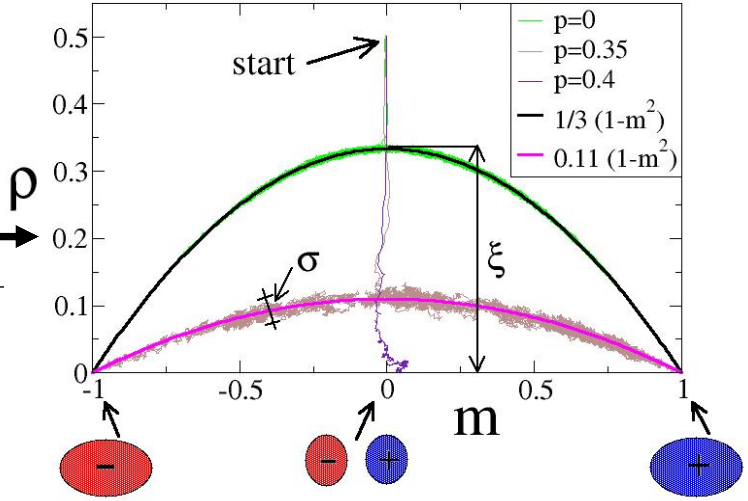


$$T \sim \begin{cases} N\,(p_c - p)^{-1} & \text{for } p \lesssim p_c \\ (p-p_c)^{-1}\,\ln\left[\mu(p-p_c)N\right] & \text{for } p \gtrsim p_c \end{cases}$$

Absorbing phase transition in a coevolving networks



$$m=\rho_{++}-\rho_{--} \xrightarrow[\rho=1-\rho_{--}-\rho_{++}]{}$$



absorbing points

$$p < p_c : \quad \rho \sim \xi(1-m^2) \quad \rightarrow \quad m=-1,1$$

(one component)

$$p > p_c : \quad \rho \sim e^{-t/\tau} \quad \rightarrow \quad m=0$$

(two components)

$p<p_c$: slow rewiring keeps network connected until system fully orders and freezes in a single component.

$p>p_c$: fast rewiring leads to fragmentation of network into two components before system reaches full order.

Axelrod's model of cultural dynamics

J. Conflict Res. 41, 203 (1997))

Proposal: Model to explore mechanisms of competition between *globalization* and persistence of *cultural diversity ("polarization")*

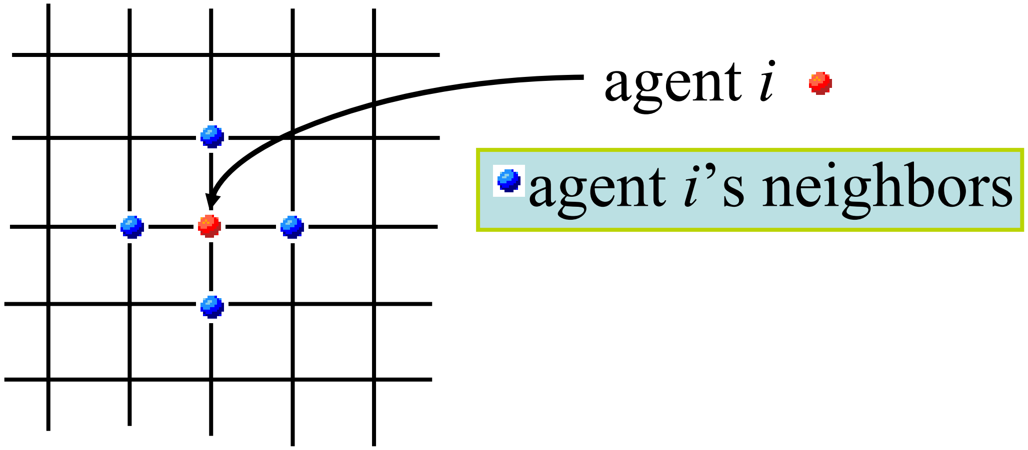
Definition of culture: Set of individual attributes subject to social influence

Principle of Homophily: Promotes interaction between similar.
"*like attracts like*"

Principle of Social Influence: Promotes cultural similarity. *The more two interact the more similar they become.*

Axelrod´s conclusion: Combination of homophily and social influence produces and sustains polarization (cultural diversity)

Axelrod's agents based model: interaction



agent i ●

● agent i's neighbors

$$\begin{pmatrix}\sigma_{i1}\\ \sigma_{i2}\\ \vdots\\ \sigma_{iF}\end{pmatrix}$$

F = # **Features**

q = # **Traits** per feature

$\sigma_{if} \in \{0, \dots, q\text{-}1\}$

F=3; q=10 q^F (10^3) *equivalent* cultural options.

Mechanism of local convergence:

$$\begin{pmatrix}0\\0\\\boxed{7}\end{pmatrix}\begin{pmatrix}5\\9\\\boxed{7}\end{pmatrix} \xrightarrow[\frac{\text{Common features}}{F}=\frac{1}{3}]{\text{Prob to interact}=} \begin{pmatrix}\boxed{5}\\0\\\boxed{7}\end{pmatrix}\begin{pmatrix}\boxed{5}\\9\\\boxed{7}\end{pmatrix}$$

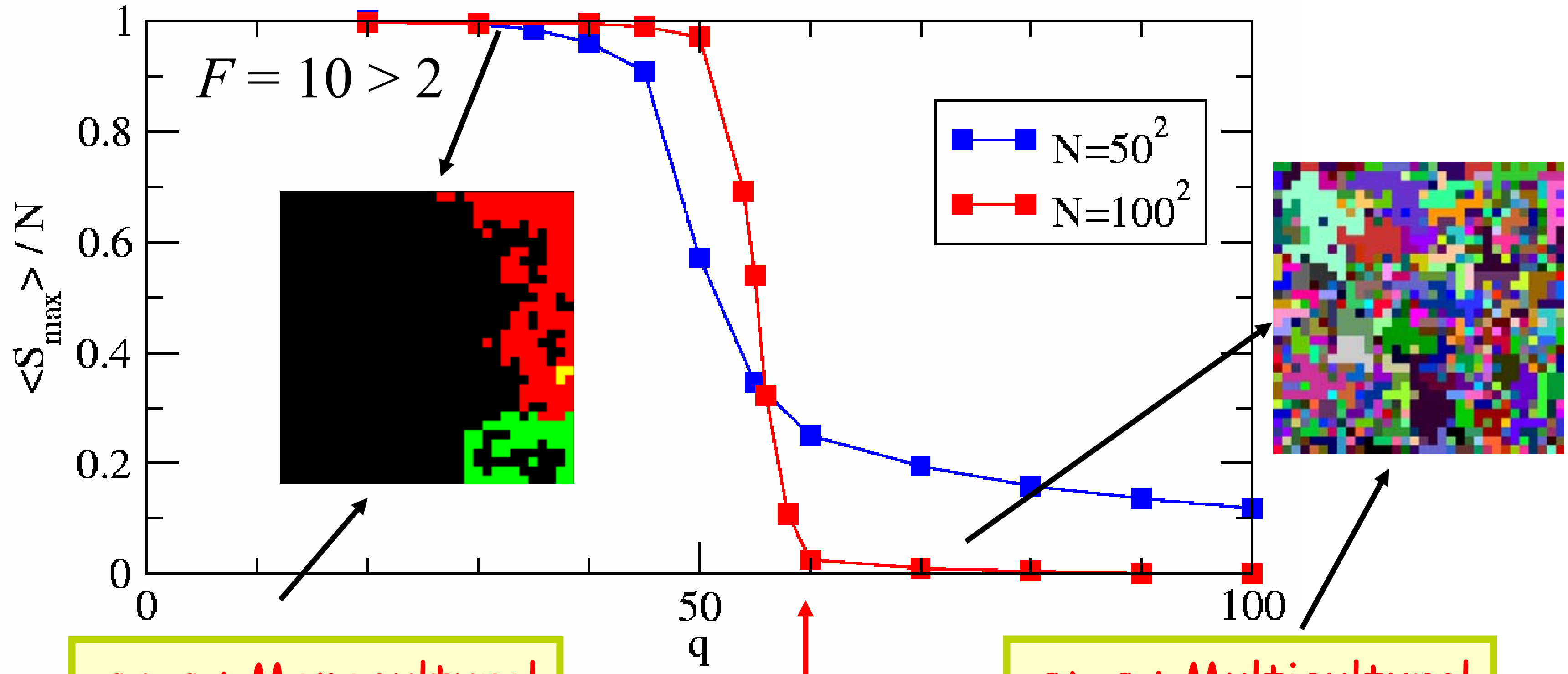
Globalization-Polarization transition

Castellano et al, Phys. Rev. Lett. ***85****, 3536 (2000)*

- **Order parameter:** S_{max} size of the largest homogeneous domain

Lewenstein et al (1992)

- **Control parameter:** q measures initial degree of disorder.



$q < q_c$: Monocultural

Global culture

q_c

$q > q_c$: Multicultural

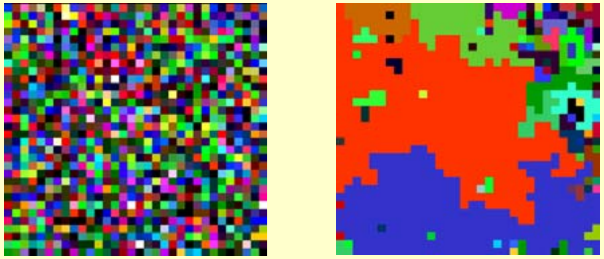
Cultural diversity

Global polarization

http://ifisc.uib.es/eng/lines/APPLET_Axelrod/Culture.html

Robustness: Cultural Drift and Coevolution

Illustration of how **local convergence** can generate **global polarization.**



$t = 0 \longrightarrow$ System freezes in an absorbing multicultural state

➡ **Frozen polarized states stable?**

➡ **Robustness of globalization-polarization transition?**

- **Cultural drift:** *"Perhaps the most interesting extension and at the same time, the most difficult one to analyze is cultural drift (modeled as spontaneous change in a trait)."*

R. Axelrod, J. Conflict Res. (1997)

⇨ Polarized states are not stable and cultural diversity is destroyed

Klemm et al., Phys Rev. E ***67****, 045101R (2003); J. Economic Dynamics and Control* ***29****, 321 (2005)*

- **Coevolution:**

⇨ New specification of homophily

⇨ Transition robust. Culturally polarized states robust vs cultural drift

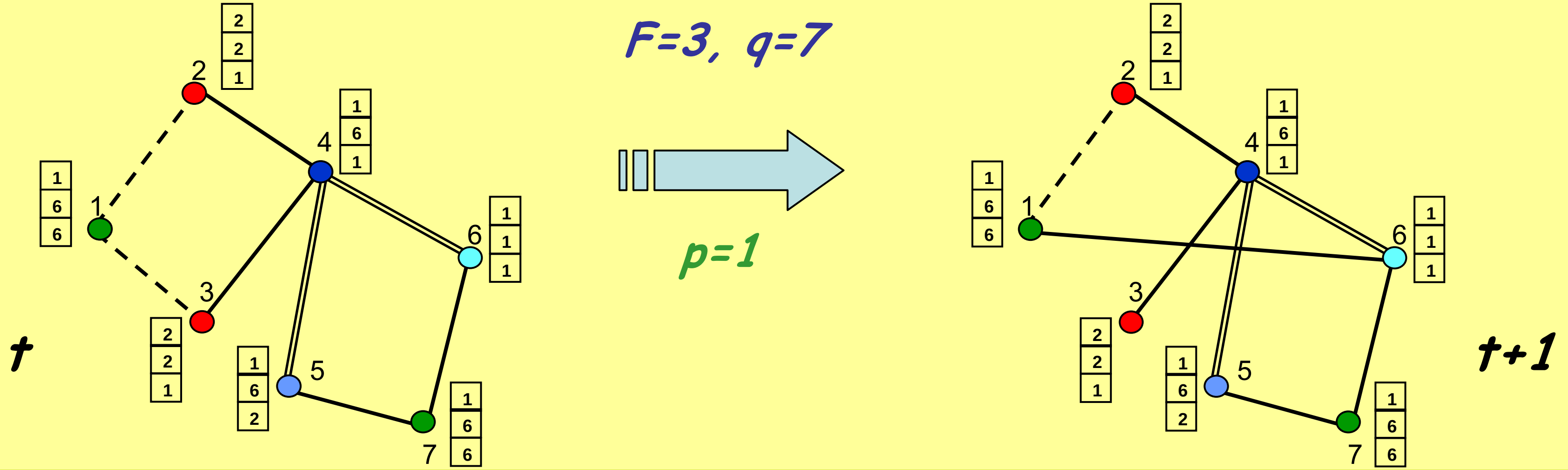
Axelrod´s model in a Co-evolving Network

Step 1: Choose randomly a link connecting two agents and calculate the overlap (number of shared features). Probability of interaction is proportional to the overlap (if overlap is not maximum)

Step 2: **Social influence dynamics:** interaction results in one more common trait

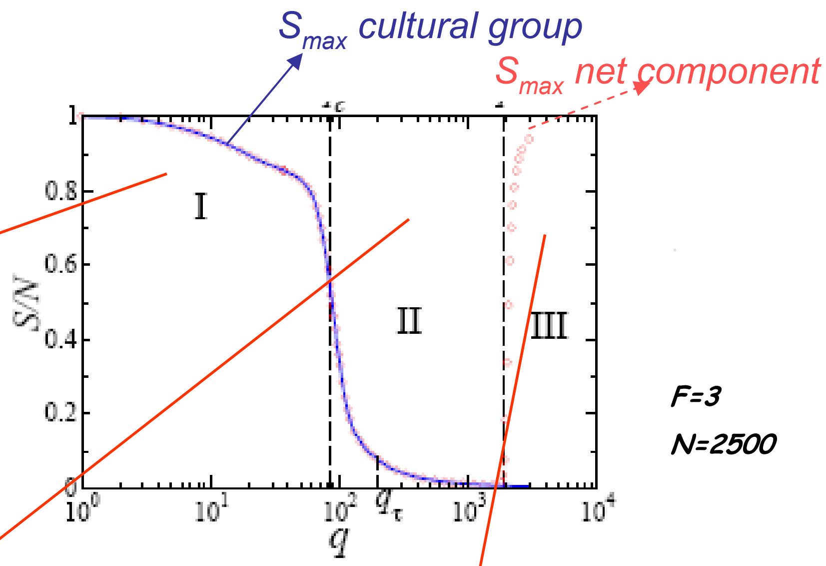
Step 3: ***NETWORK DYNAMICS:*** New homophily specification

A link with zero overlap (cleavage-link) is dropped + new link established



Network fragmentation and recombination

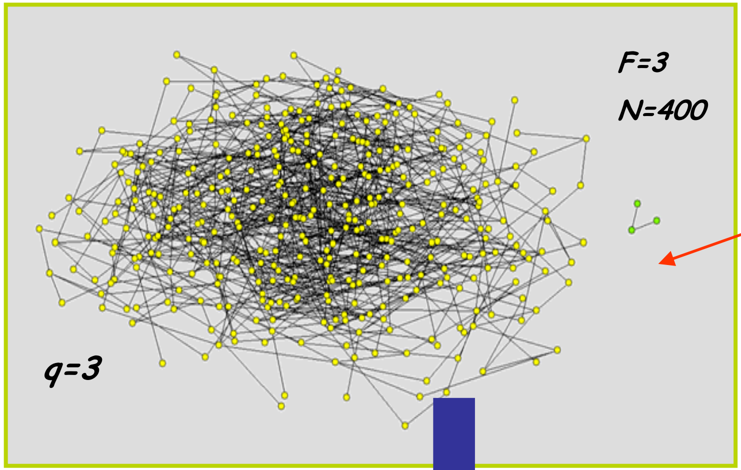
*F. Vázquez et al. Phys. Rev. E **76**, 046120(2007)*



F=3

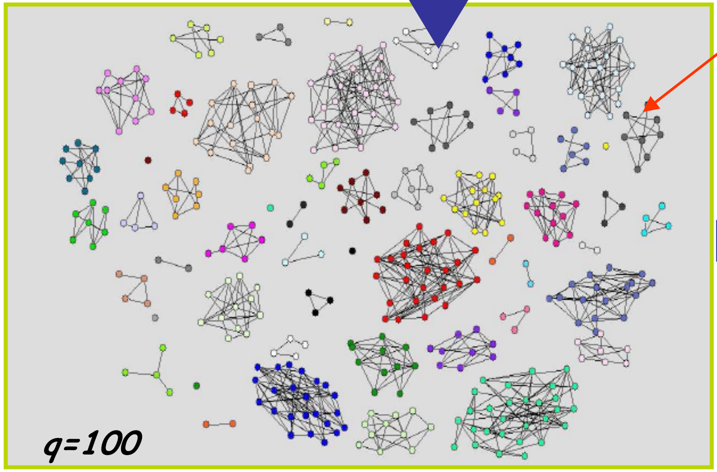
N=2500

Region I (frozen configuration)



Fragmentation

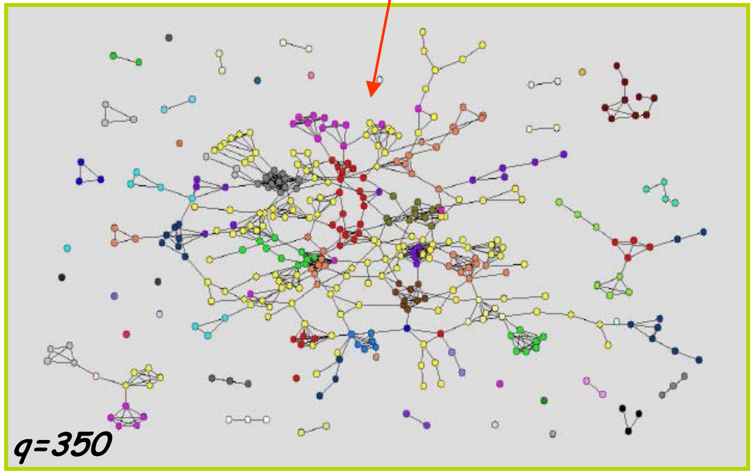
Region II (frozen)



Recombination

$$q^* \cong \frac{NF}{<k>}$$

Region III (dynamic frustrated configuration)



Network fragmentation transition

*F. Vázquez et al. Phys. Rev. E **76**, 046120(2007)*

Region I

giant network component

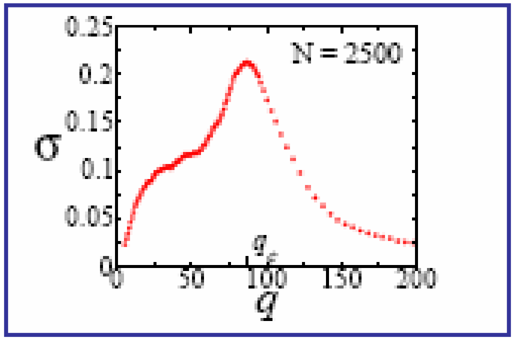
q_c'

Region II

many small network components

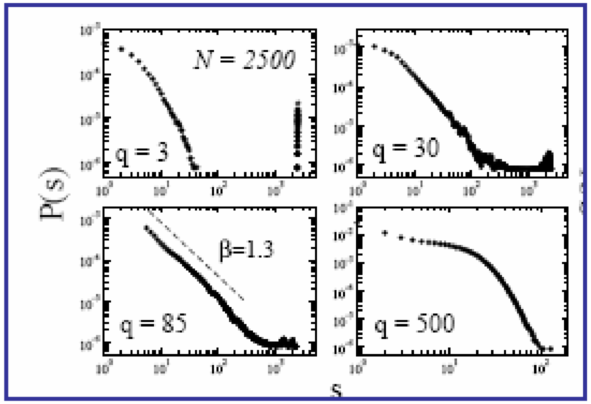
Maximum of fluctuation in S

F=3



$q_c' = 85$

Power law distribution for size components



$P(s) \sim s^{-\beta}$

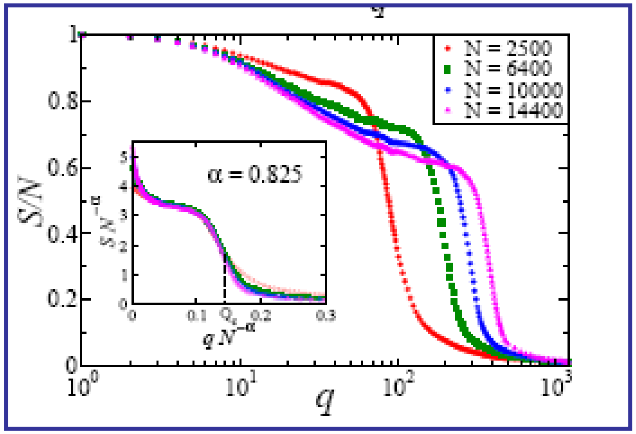
$\beta = 1.3$

Finite size scaling

$S = N^\alpha f(N^{-\alpha} q)$ for $q < q_c$.

$q_c \sim N^\alpha \to \infty$

$S/N \sim N^{\alpha-1} \to 0$ as $N \to \infty$



Transition becomes continuous and diasappears in the limit $N \to \infty$

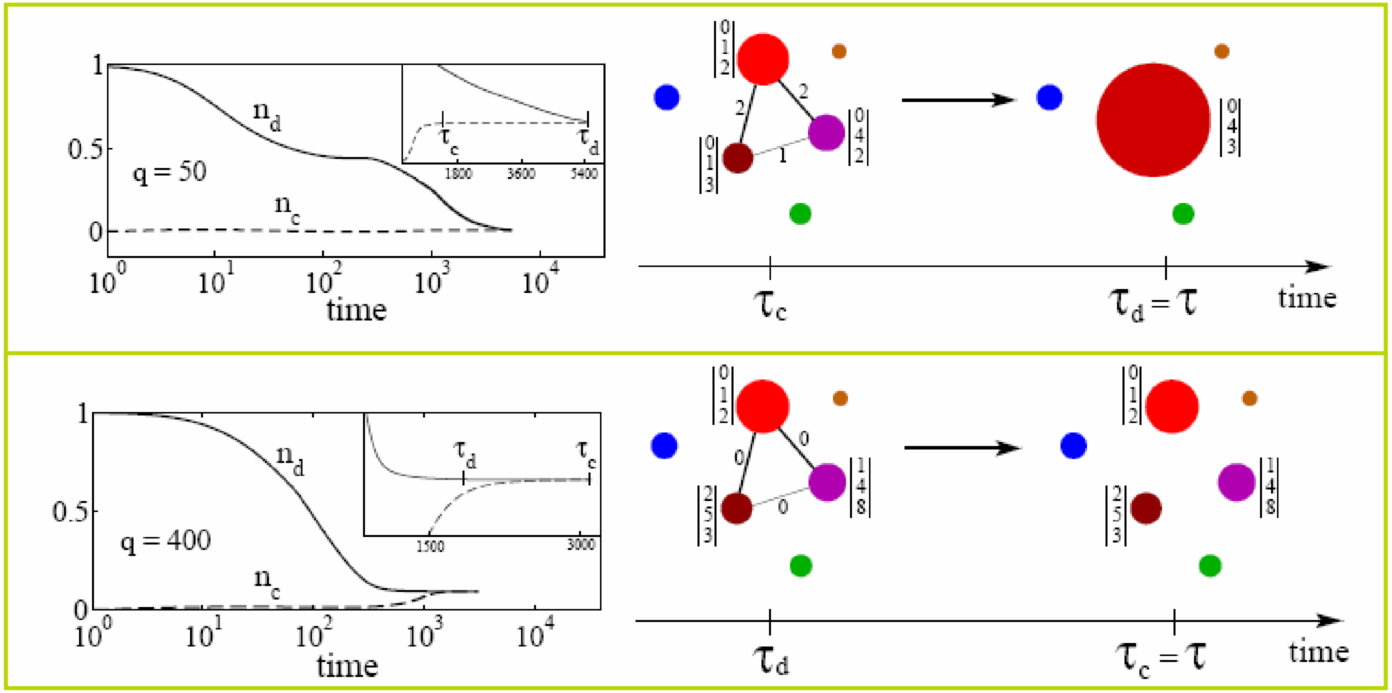
Dynamics of Network Fragmentation

F. Vázquez et al. Phys. Rev. E ***76****, 046120(2007)*

Two internal time scales (τ_c and τ_d) spontaneously emerge from a model in which states and network are updated at the same rate

n_c (t) = # of network components /N

n_d (t) = # of cultural domains /N



$\tau_d > \tau_c$

τ =
freezing time
Max (τ_d , τ_c)

$\tau_d < \tau_c$

n_d reaches stationary value at τ_d → Controls states (domains) formation

n_c reaches stationary value at τ_c → Controls network (component) formation

Fragmentation transition occurs for $\tau_c \approx \tau_d$

Cultural Drift and Co-evolution

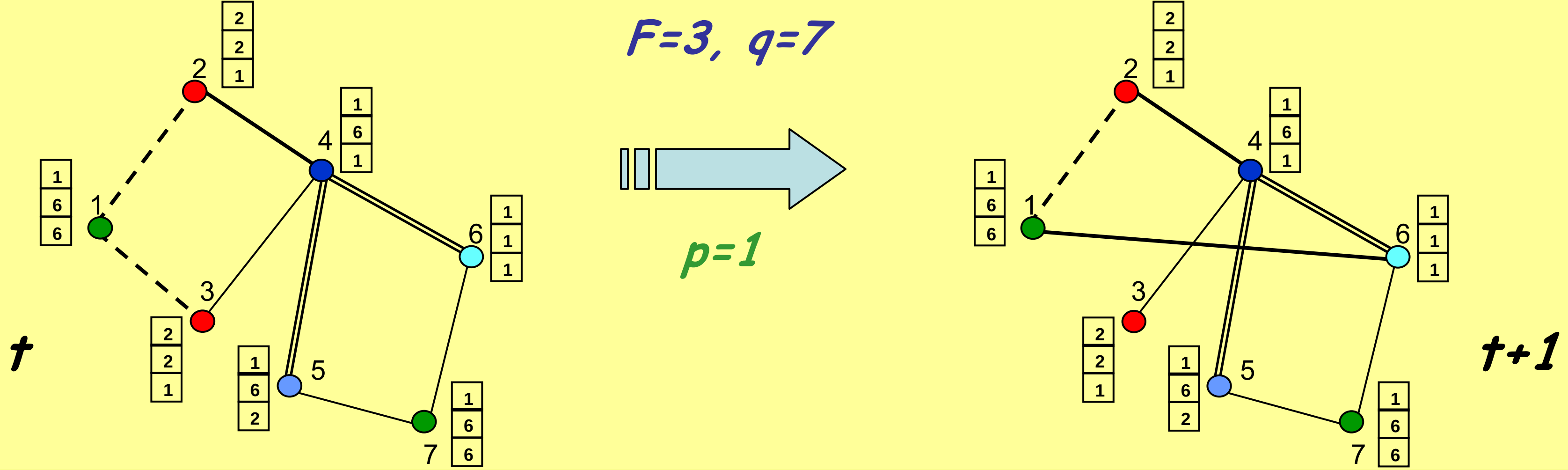
D. Centola et al. J. of Conflict Resolution ***51****, 905 (2007)*

Step 1: Choose randomly a link connecting two agents and calculate the overlap (number of shared features). Probability of interaction is proportional to the overlap (if overlap is not maximum)

Step 2: **Social influence dynamics:** interaction results in one more common trait

Step 3: ***NETWORK DYNAMICS:*** New homophily specification

A link with zero overlap (cleavage-link) is dropped + new link established



$F=3$, $q=7$

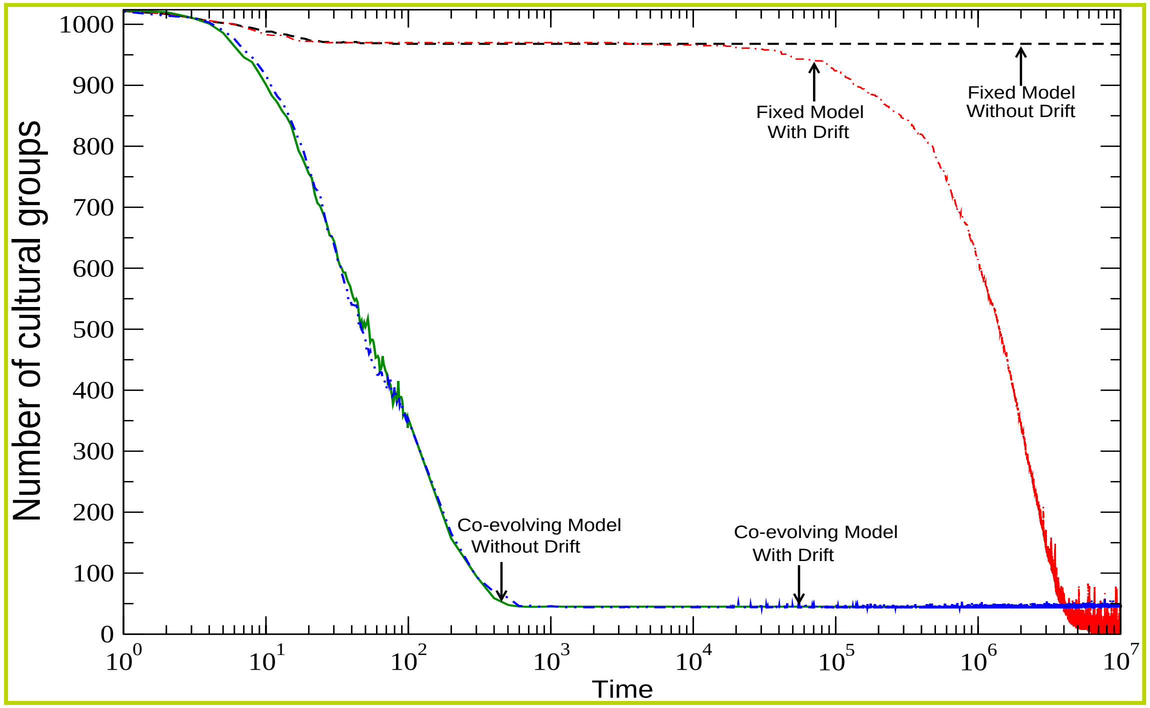
$p=1$

Step 4: **Cultural drift:**

Single feature perturbation with probability r

Cultural drift in a Co-evolving Network

*D. Centola et al. J. of Conflict Resolution **51**, 905 (2007)*



Region II

F=3, q=100

N=1024

$r=10^{-5}$

- Dynamical network maintains polarization in spite of cultural drift of slow rate: Insensitive to noise

- Noise is not efficient to produce globalization in a co-evolvig network during large time scales

Summary of Axelrod´s cultural dynamics

•**Basics:** Interaction of several cultural features based on homophily and social influence produces a transition between global culture and polarization.

•**Fixed networks:** Long range links and degree heterogeneity favor globalization. High clustering restores polarization in scale free networks with large number of nodes. *Klemm et al., Phys. Rev. E* ***67****, 026120 (2003)*

•**Cultural drift in fixed networks:** Essential → Qualitative changes. q-independent, N-dependent noise induced transition between metastable global culture and noise dominated polarized state.

Klemm et al., Phys. Rev. E ***67****, 045101 (2003); J. Econ. Dyn. Control* ***29,*** *321(2005)*

Co-evolution (Dynamic networks):

- Network Fragmentation and recombination transitions

F. Vázquez et al., Phys. Rev. E ***76****, 046120(2007)*

- Stable cultural polarization: Cultural drift of slow rate becomes inefficient. *D. Centola et al. J. of Conflict Resolution* ***51****, 905 (2007)*

Thanks!

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