Finite size effects in the dynamics of opinion formation

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Does size matter?

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In Statistical Physics we are used to the thermodynamic limit

$$N \to \infty$$

But in real physical systems, we should be happy with

$$N \approx 10^{23}$$

In computer simulations we struggle for larger and larger sizes and always try to extrapolate to infinite size.

Social systems are never that large and new phenomena can appear depending on the size or the number of individuals considered.

1.- Apparent phase transitions

-sine-Gordon model

-Galam's model for minority opinion spreading

-Axelrod's model for culture formation

2.- System size stochastic resonance

-Simple majority model for opinion formation www.imedea.uib.es

References

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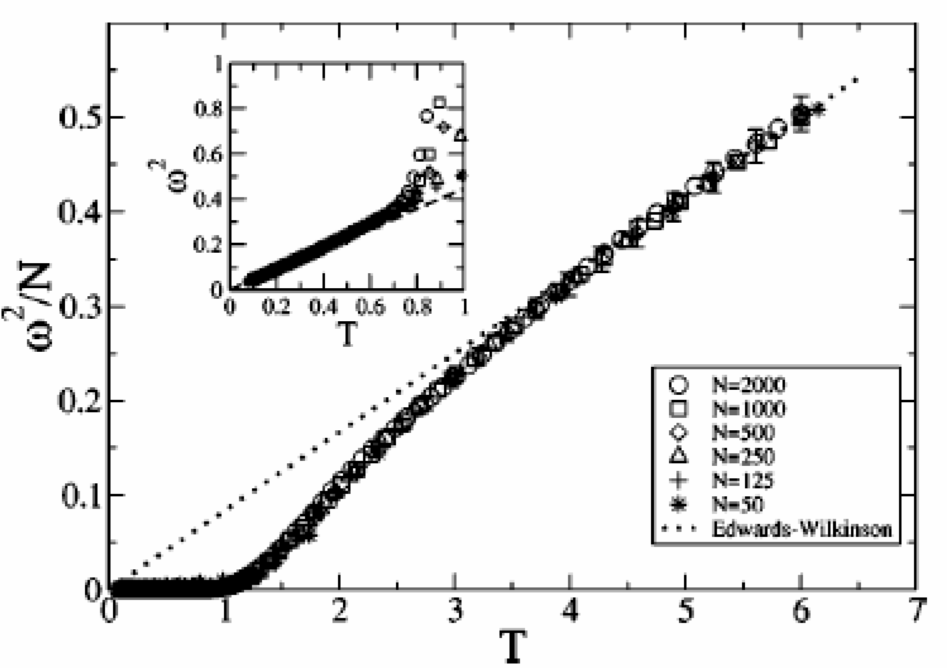
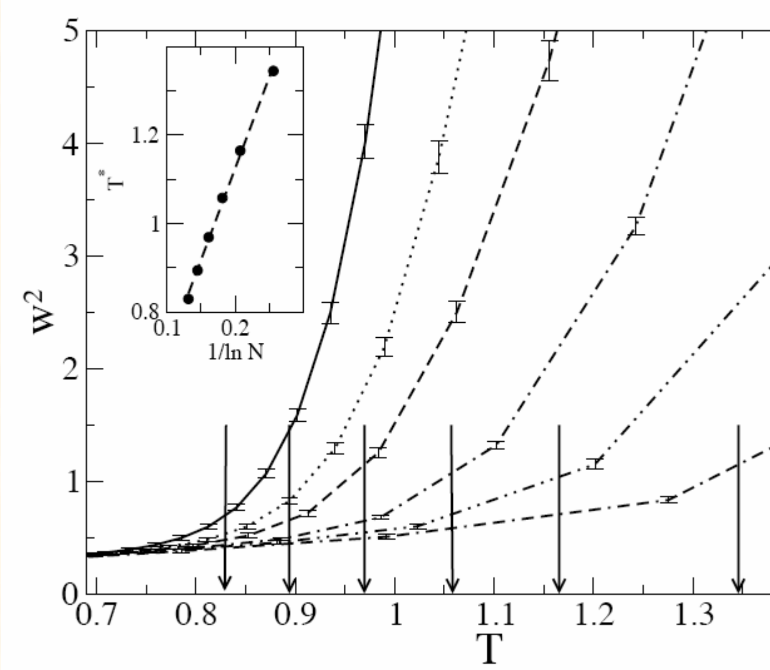
Apparent phase transitions

Things are not always what they appear to be

Roughness in the 1d-sine-Gordon growth model





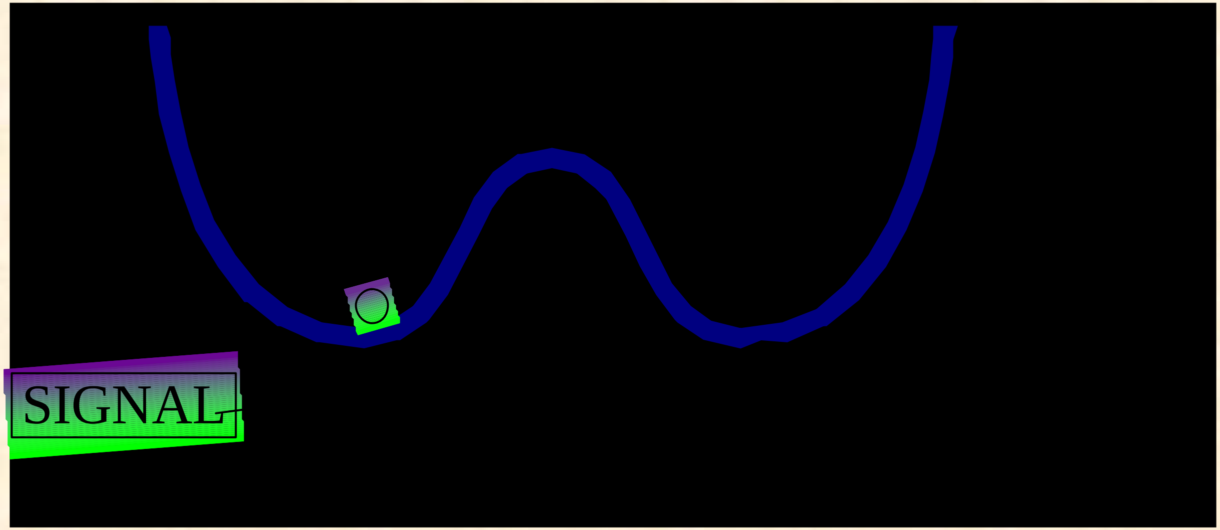
$$T_c(N) \approx 1/\ln N$$

System Size Stochastic Resonance in a majority model for opinion formation

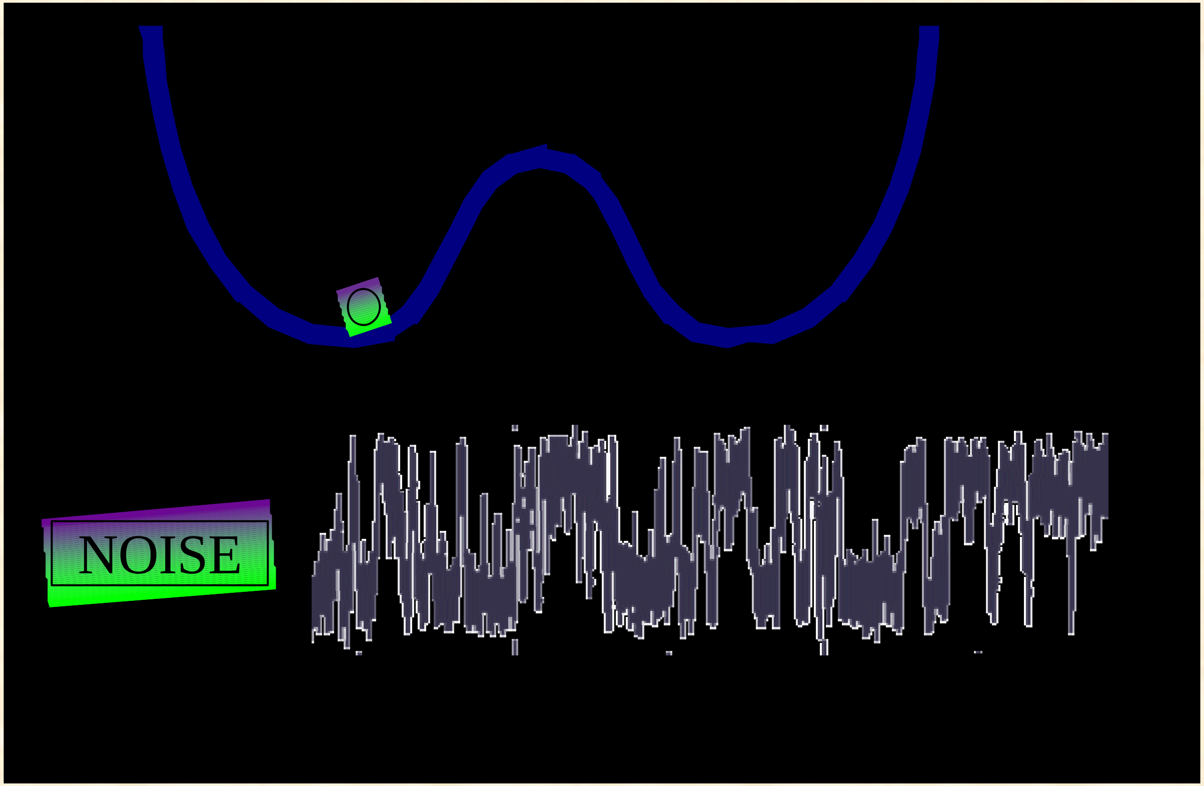
Influence of advertising in a population of interacting people with free will

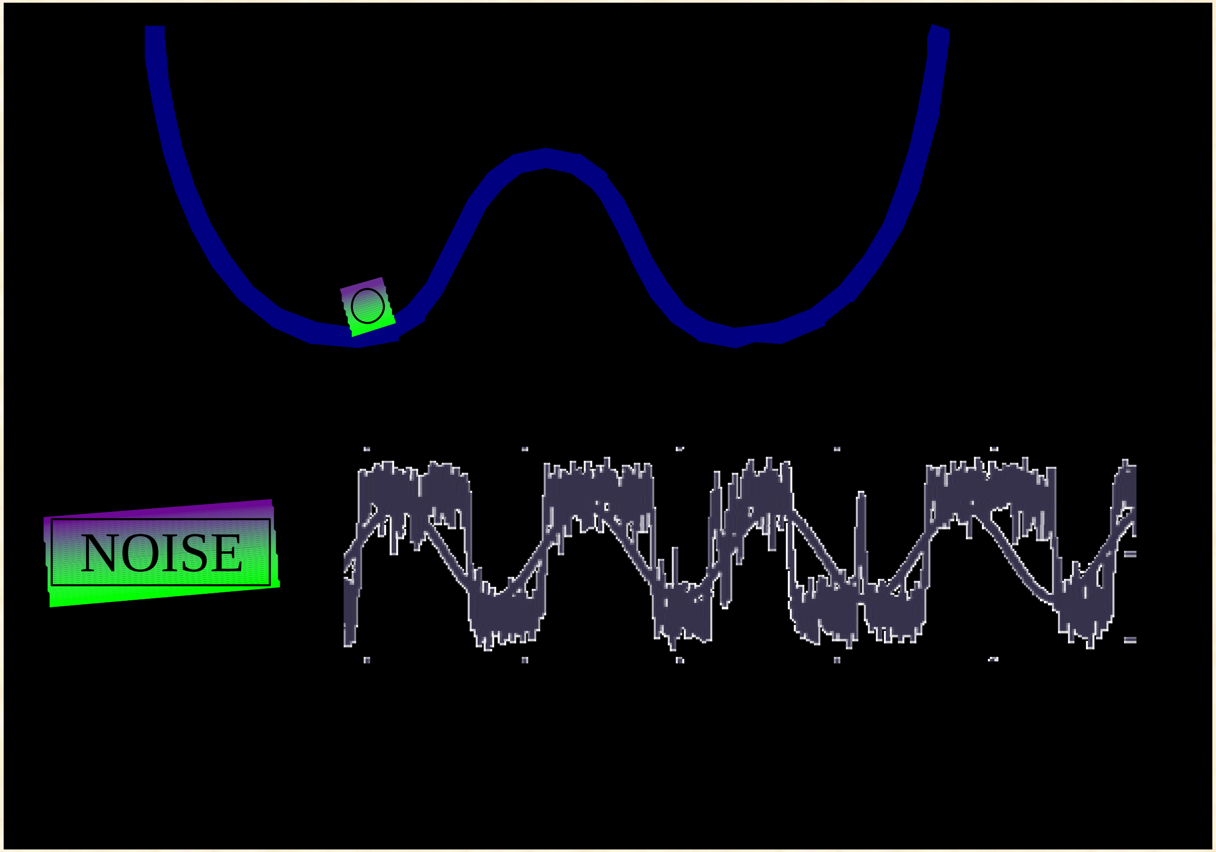
Stochastic Resonance

-Amplification of sub-threshold signals helped by noise

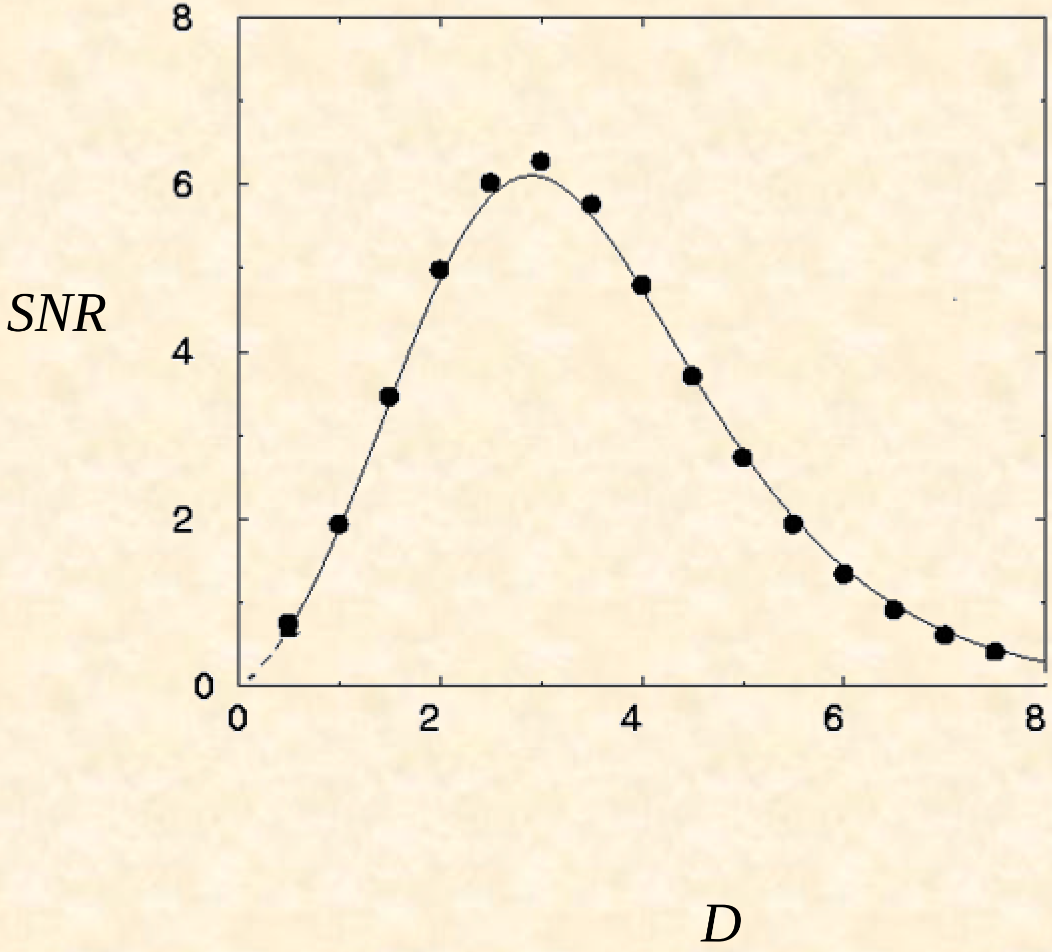


$$\dot{x} = x - x^3 + A\cos(\Omega t)$$


NOISE


NOISE

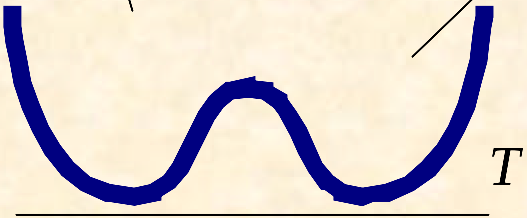
Signal-to-noise ratio



Earth climate

Weak signal: earth eccentricity (~ $3 \cdot 10^4$ y)
Noise: Changes in Sun luminosity
Global temperature changes

Bi-stable climate potential



System Size Stochastic Resonance

[Pikovsky, Zaikin, De la Casa, Phys. Rev. Lett. 88 (2002) 050601]

Consider N coupled bistable systems

$$\dot{x}_i = x_i - x_i^3 + \frac{\varepsilon}{N}\sum_{j=1}^{N}(x_j - x_i) + \sqrt{2D}\,\xi_i(t) + f(t)$$

x_i Gaussian white noise, uncorrelated

$f(t) = A\sin(\Omega t)$ periodic signal

Look at the collective variable X(t):

$$X = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad x_i = X + \delta_i$$

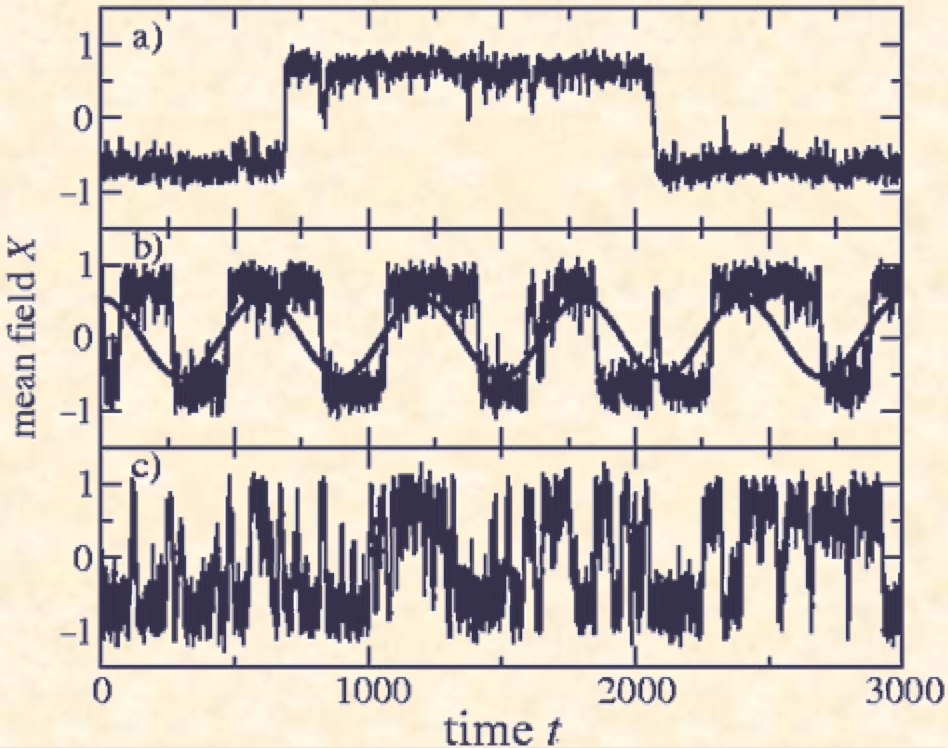
After some approximations

$$\dot{X} = aX - bX^3 + \sqrt{\frac{2D}{N}}\,\eta(t) + f(t)$$

Effective noise intensity can be controlled by varying D and N.

System Size Stochastic Resonance

Dynamical evolution

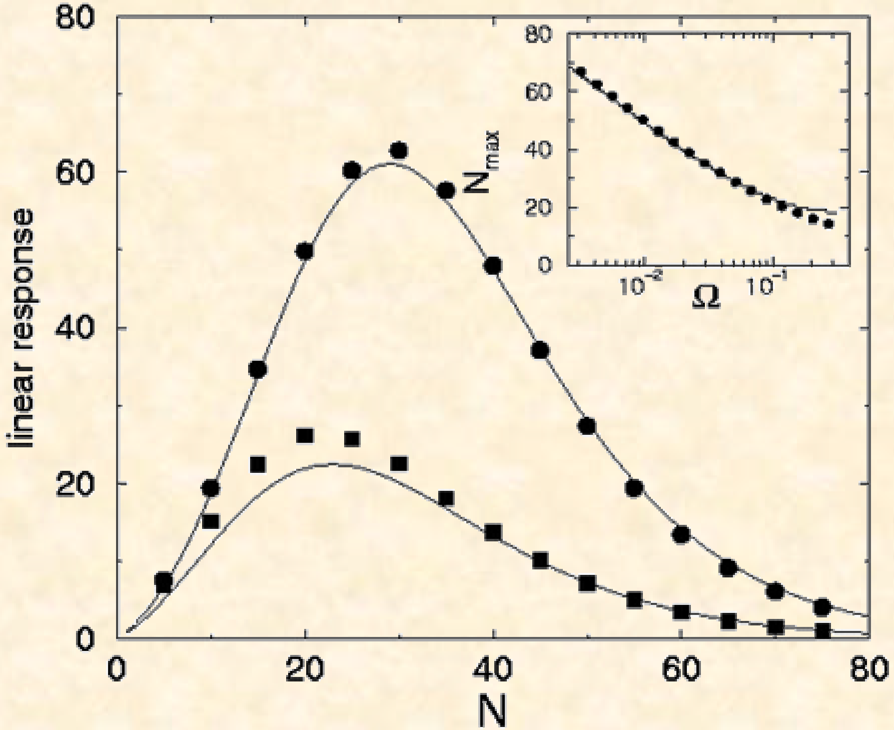


N=80

N=35

N=15

System Size Stochastic Resonance



- System size resonance appears to be generic: local coupling, Ising model, ...

[Pikovsky, Zaikin, De la Casa, Phys. Rev. Lett. 88 (2002) 050601]

- FitzHugh-Nagumo excitable model

[Toral, Mirasso, Gunton, Europhys. Lett. **61**, 162 (2003)]

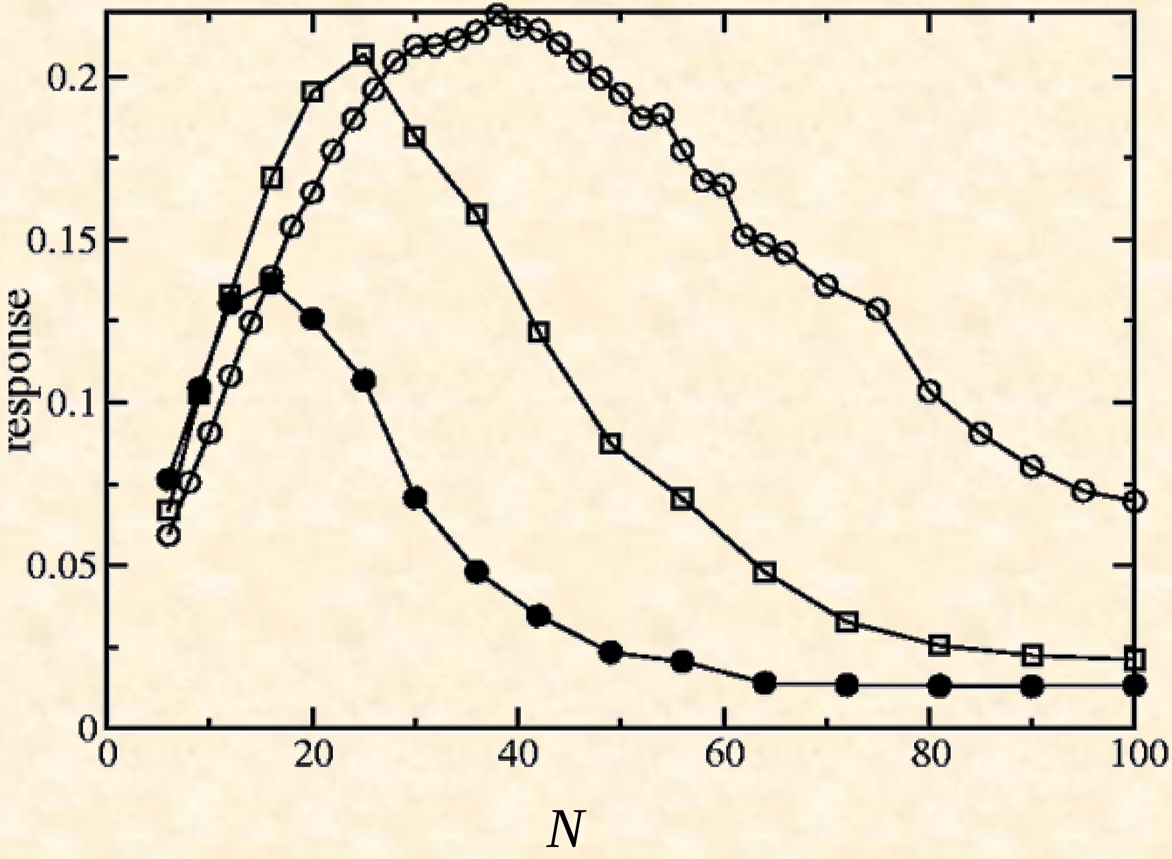
- Other models for ion transport in biological membranes

[Schmid, Goychuk, Hänggi, Europhys. Lett. **56**, 22 (2001)]

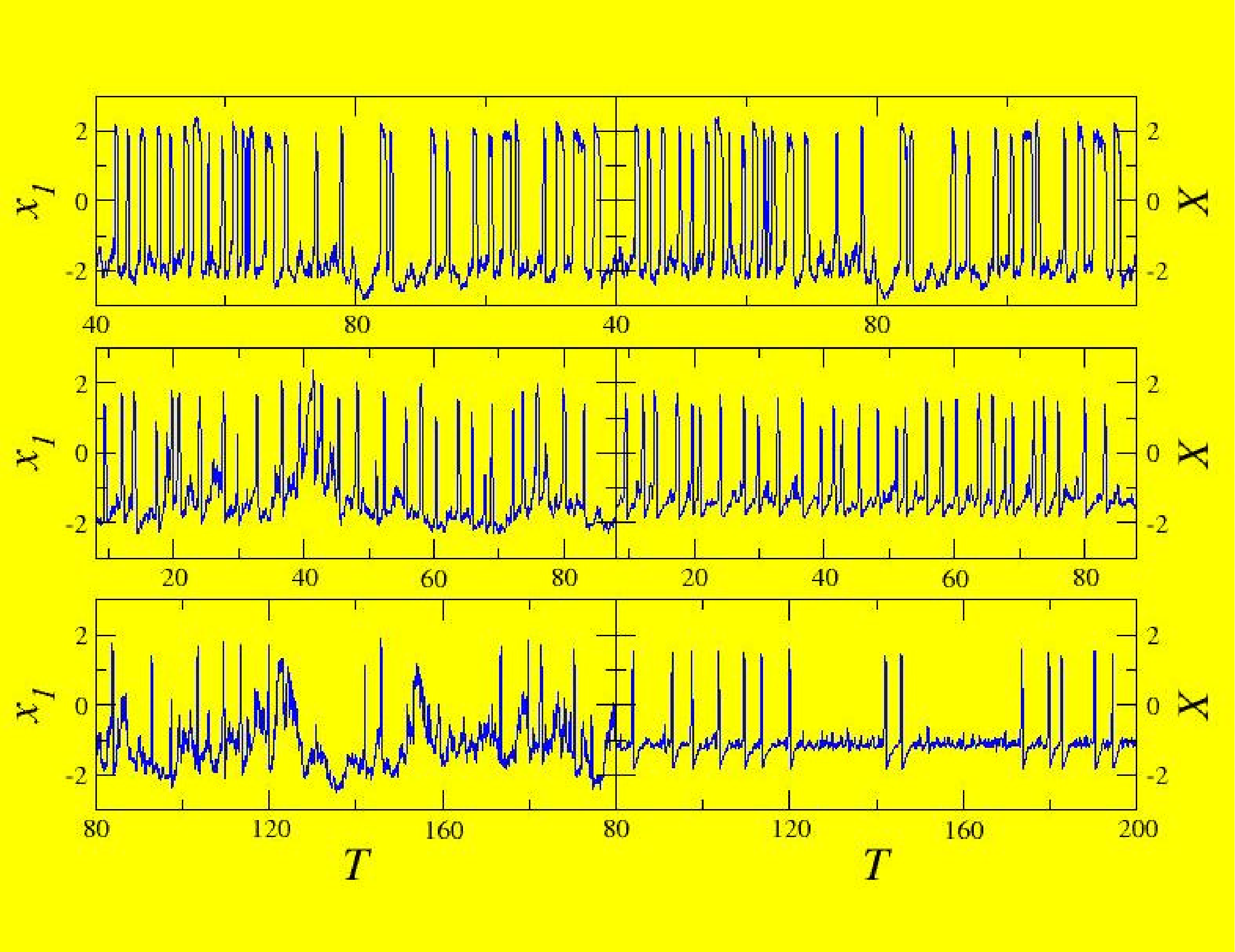
[Jung, Shuai, Europhys. Lett. **56**, 29 (2001)]

System Size Stochastic Resonance

Ising Model



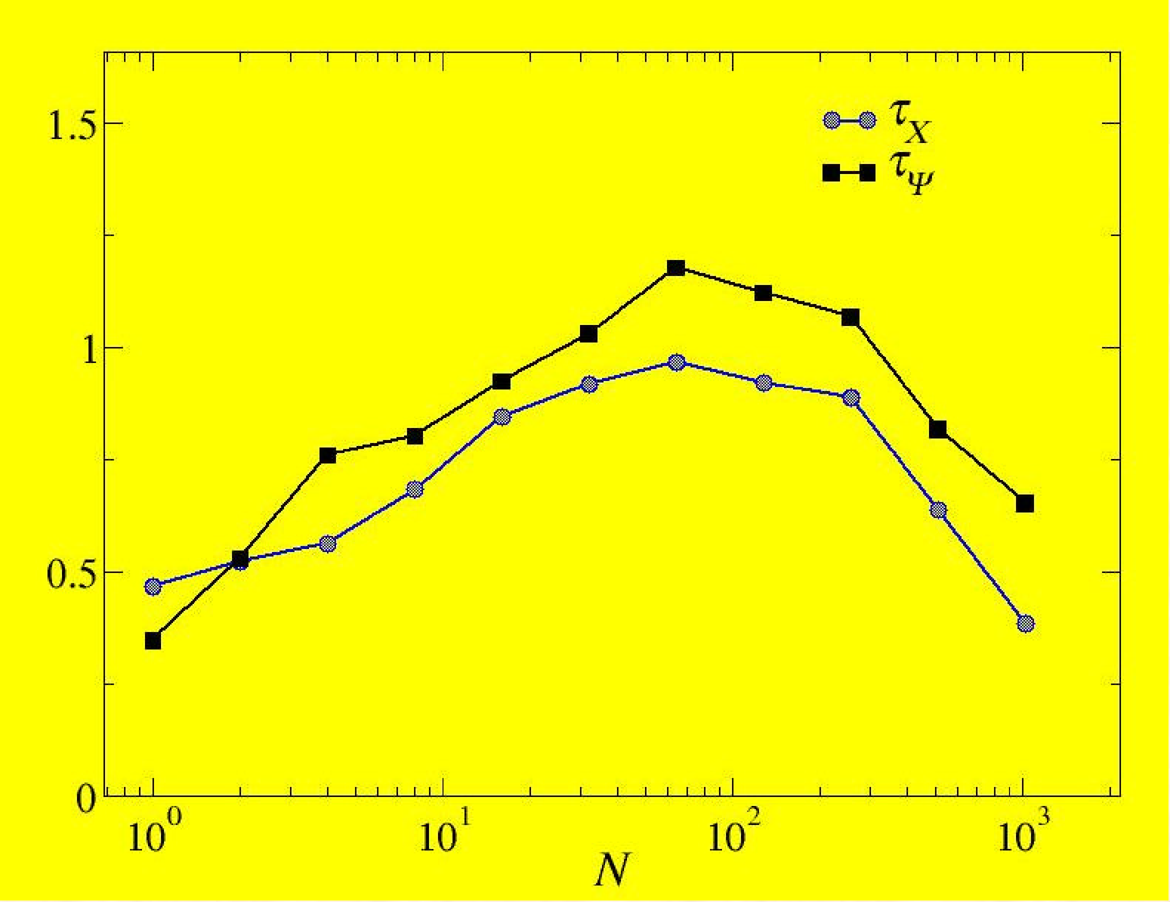
FitzHugh-Nagumo excitable model



N=1

N=128

N=1024


1.5
1
0.5
0
10^0
10^1
10^2
10^3
N
τ_X
τ_Ψ

Model for opinion formation

[M. Kuperman, D. Zanette, *Eur. Phys. Jour. B*, **26** 387 (2002)]

- It is an Ising-like model
- Incorporates social imitation
- Takes into account the structure of the society

Model for opinion formation

Opinion is a binary variable:

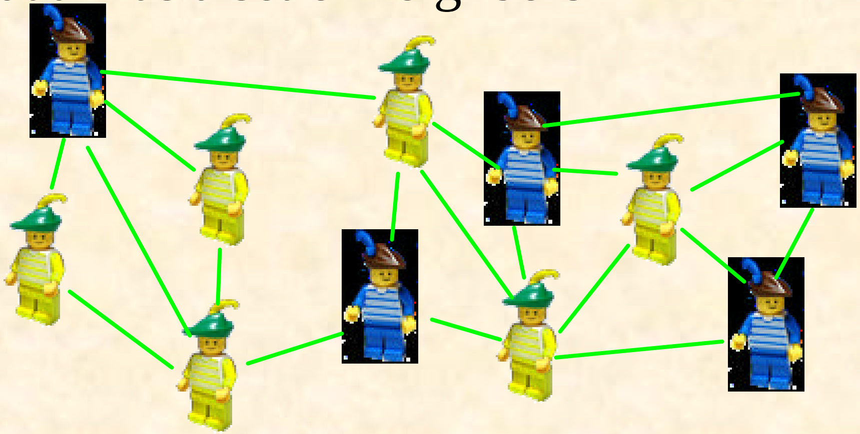
Individuals have an opinion: $m_i=+1,-1$

Opinion changes by 3 effects:

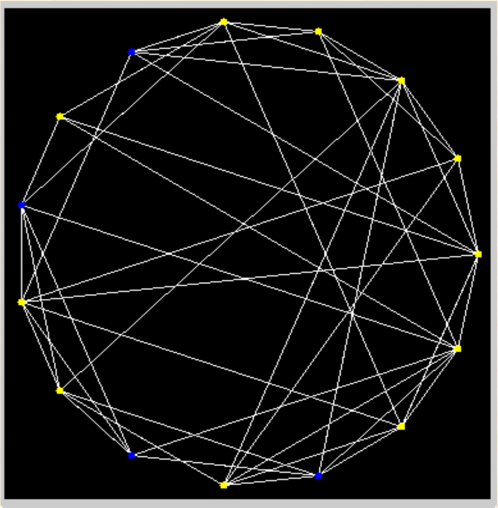
- **Imitation**
- **External influence**
- **Random effects**

?System formed by *N* individuals which have one of two opinions

?Each individual has a set of neighbors



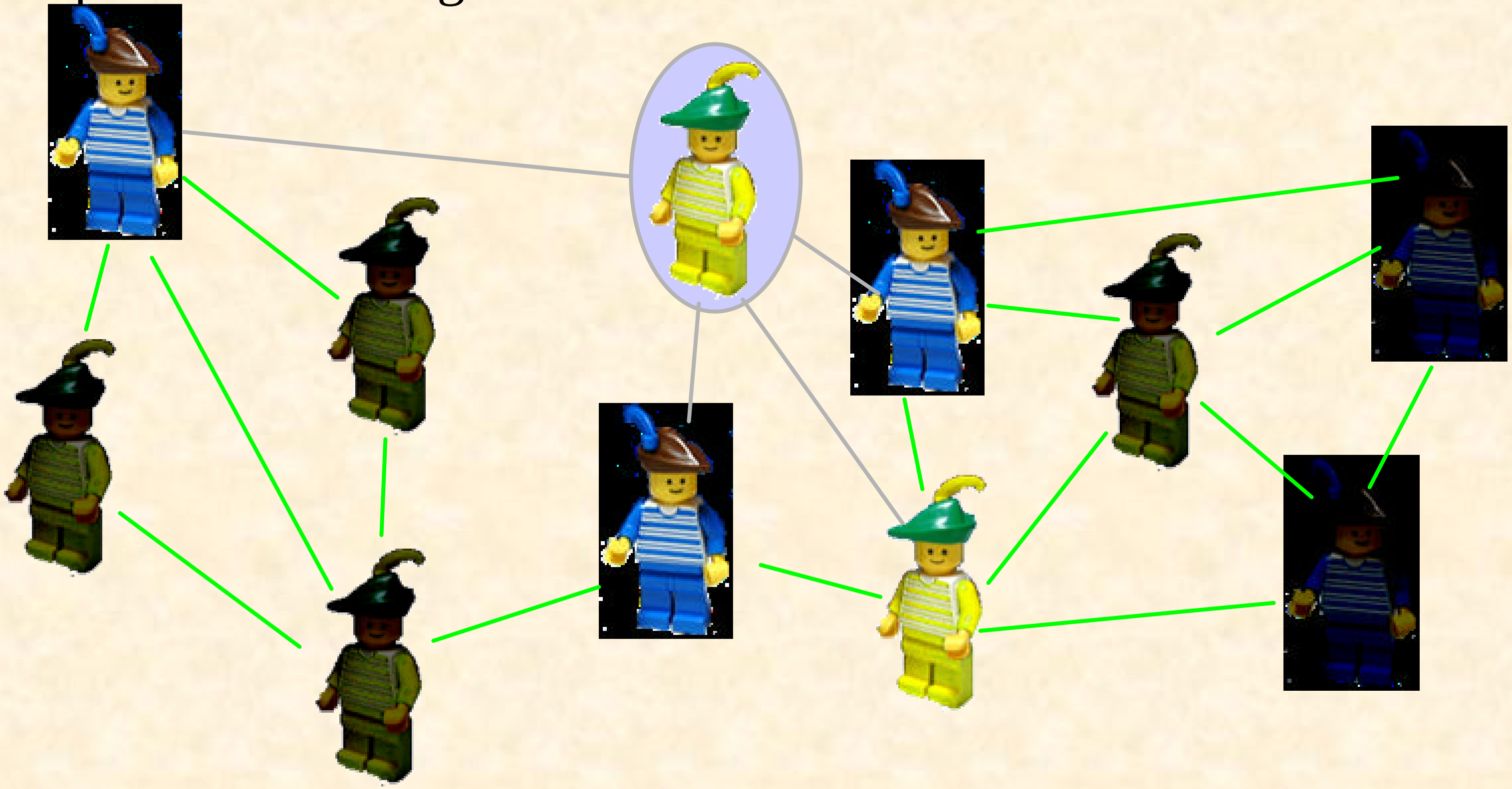
The network of neighbors is a small-world one, constructed in the following way:



- A regular 1D network built up to *k* neighbors
- with probability *p* each link is rewired, i.e. another destination node is selected

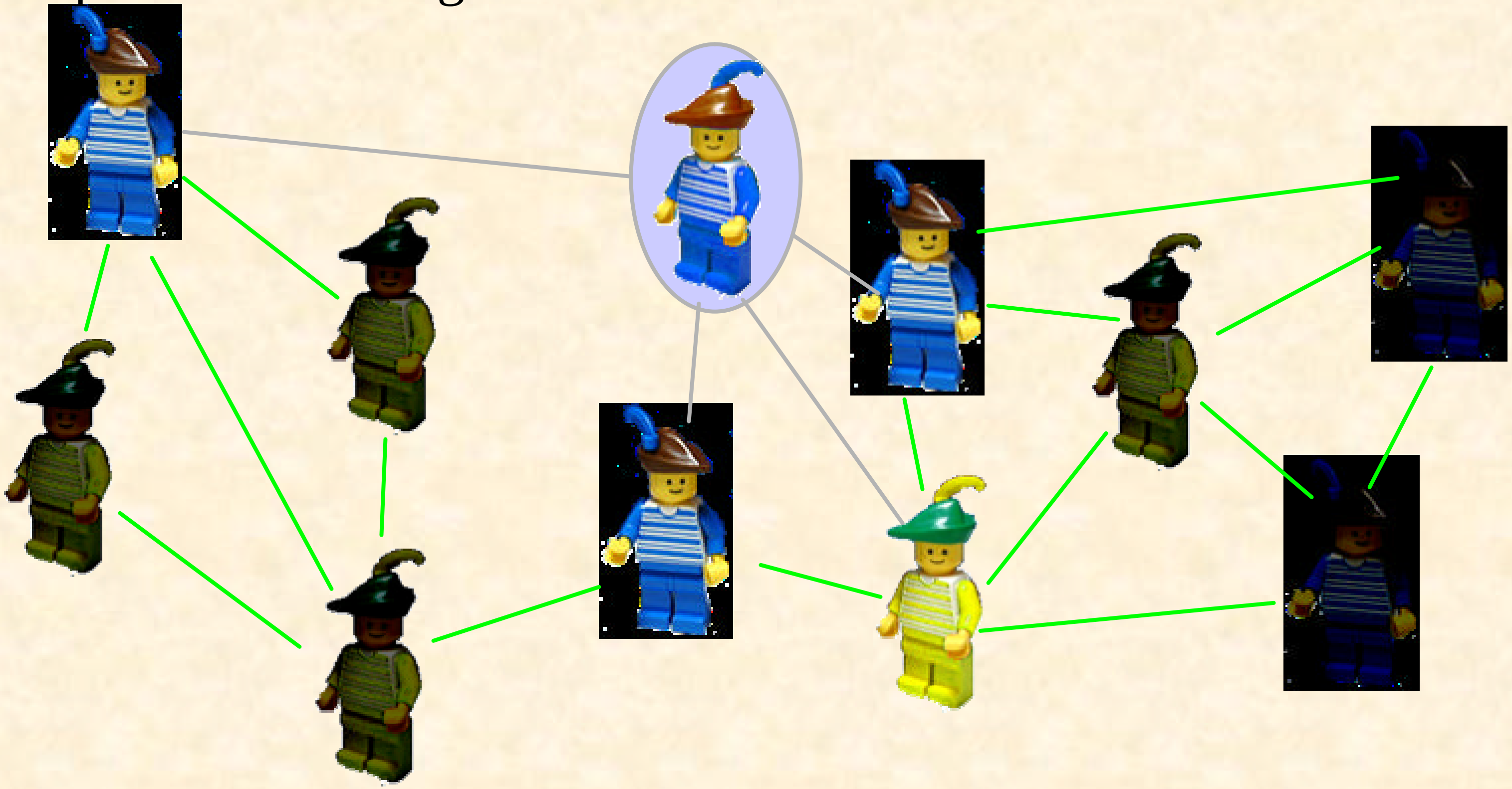
? **Opinion Update: 1 *imitation***

An individual is randomly chosen and takes the majoritary opinion of his neighbors



? **Opinion Update: 1 *imitation***

An individual is randomly chosen and takes the majority opinion of his neighbors



? **Opinion Update: 2** ***External influence***

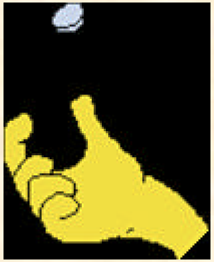
The social preference for one of each opinions is assumed to change periodically in the form

$$\epsilon \cos(\omega t) \begin{cases} < 0 \\ > 0 \end{cases}$$

With probability $p_f(t) = |\epsilon \cos(\omega t)|$
the favored opinion is taken

- **Opinion Update: 3 *random choice***

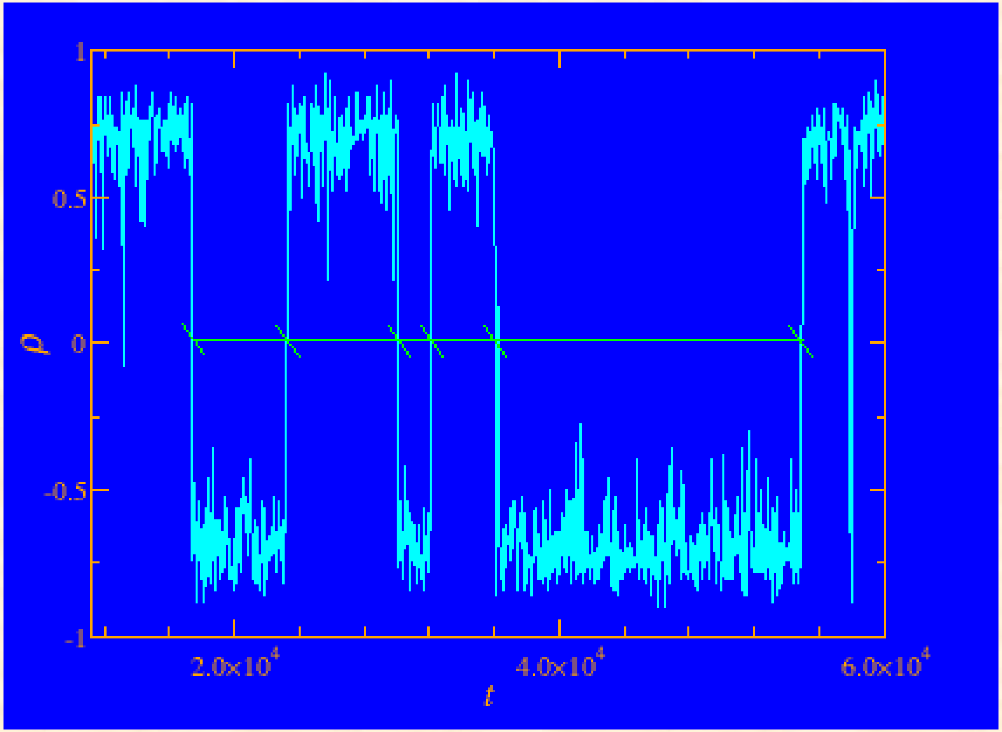
With probability **p** a random opinion is taken



- Steps **1,2,3** are applied ***CONSEQUTIVELY***
- After each repetition, *t* increases by 1/N

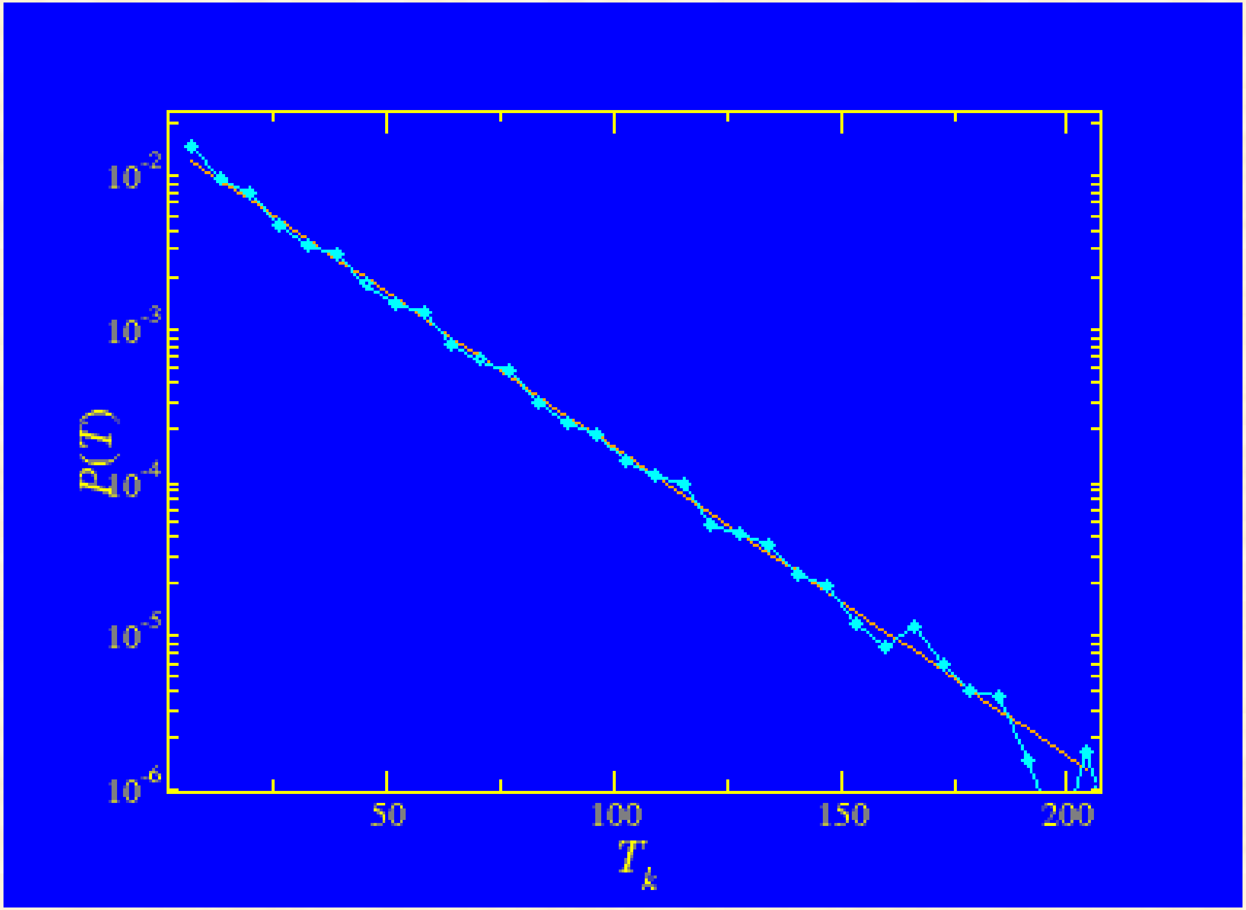
Results: Dynamical Evolution

i) In absence of external forcing, behaves as a bistable system

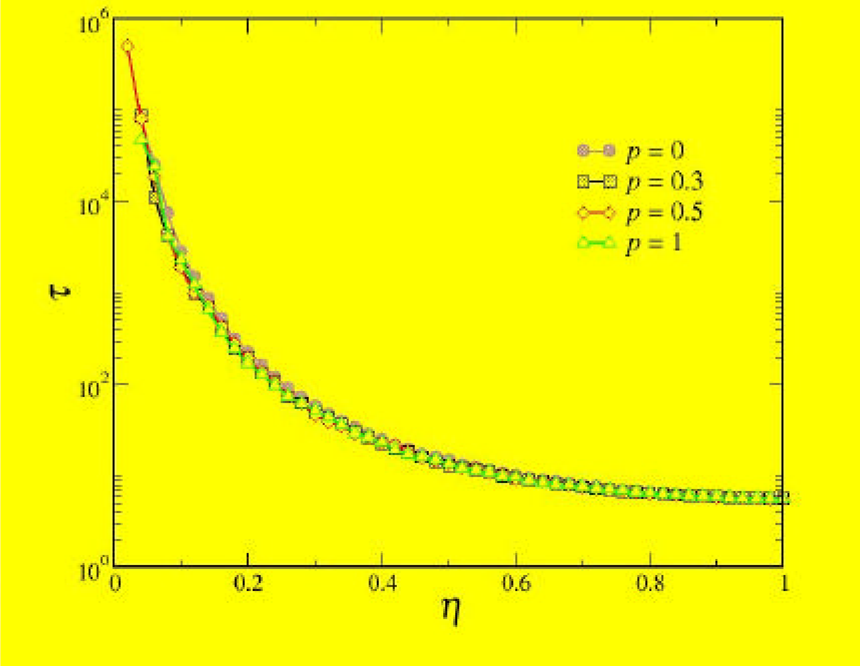


ii) the residence time distribution follows Kramer's law

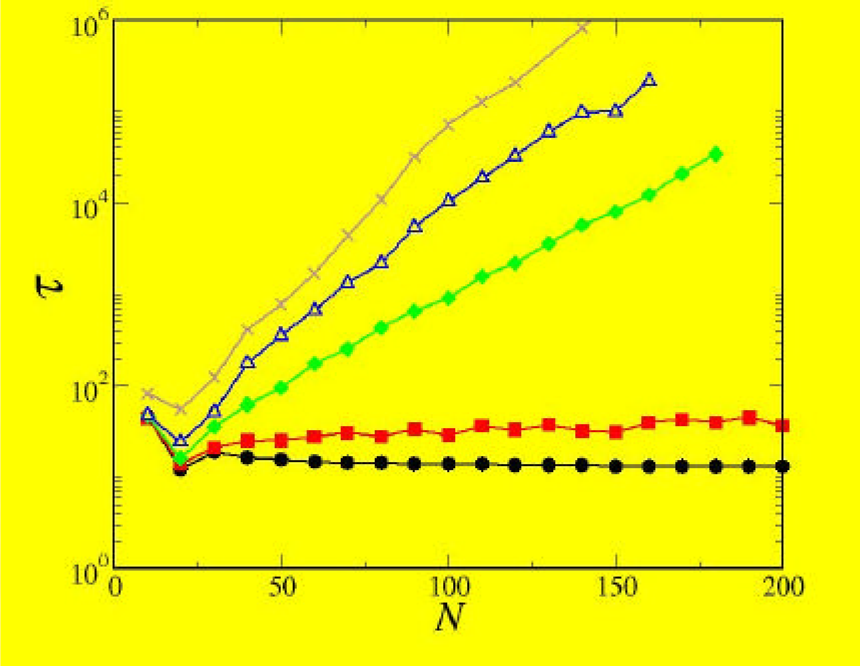
$$p(T) = \tau \exp(-T/\tau)$$



t decreases with noise rate



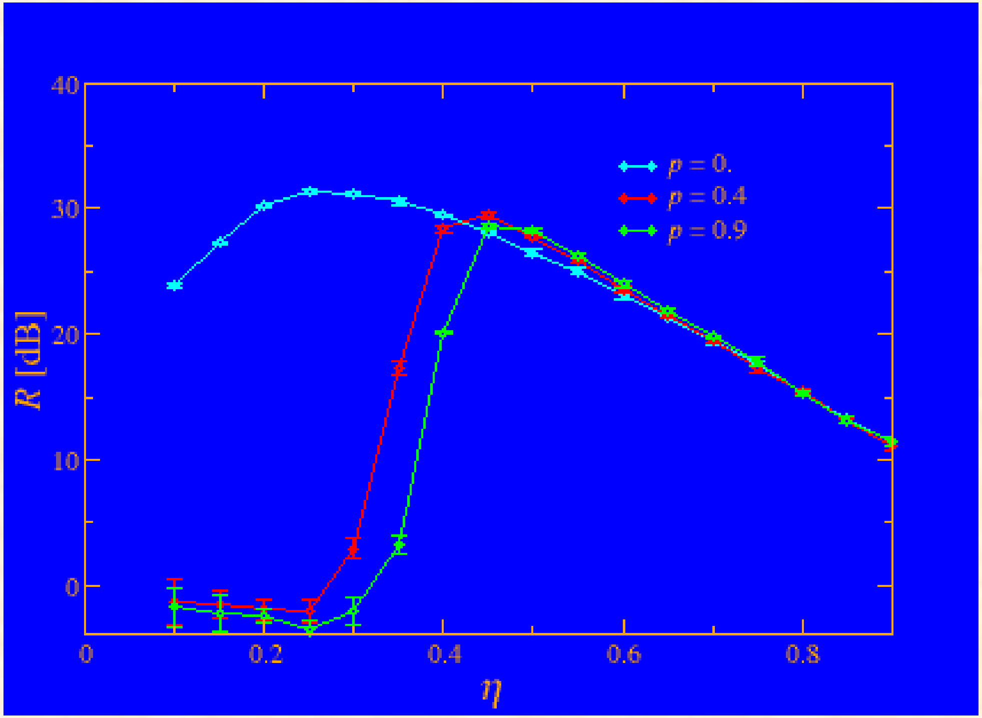
τ increases with systems size



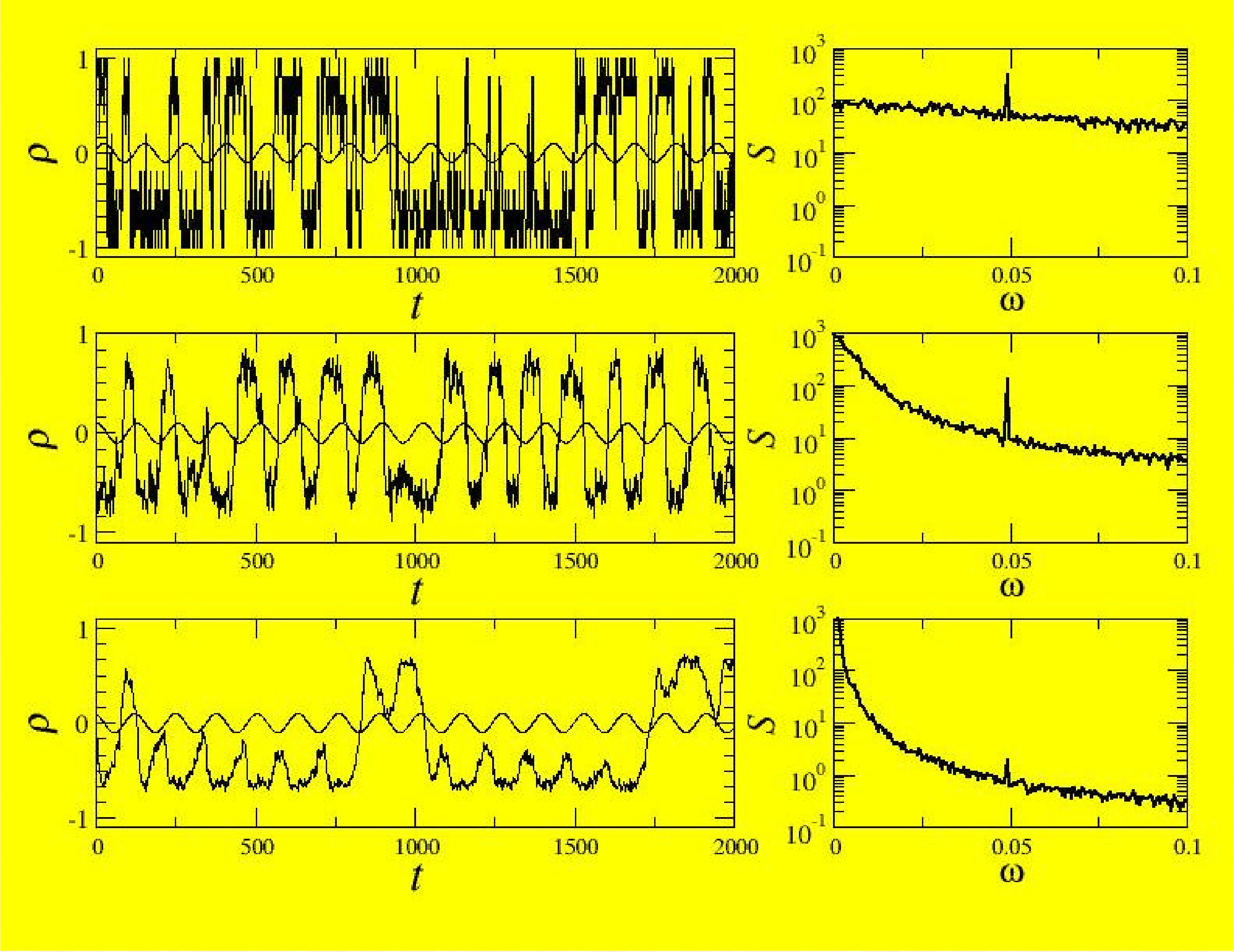
-We have all the ingredients for stochastic resonance:

- -Bistable system
- -Coupling
- -Noise
- -External forcing

$N = 1000$



We have all the ingredients for ***system size*** stochastic resonance

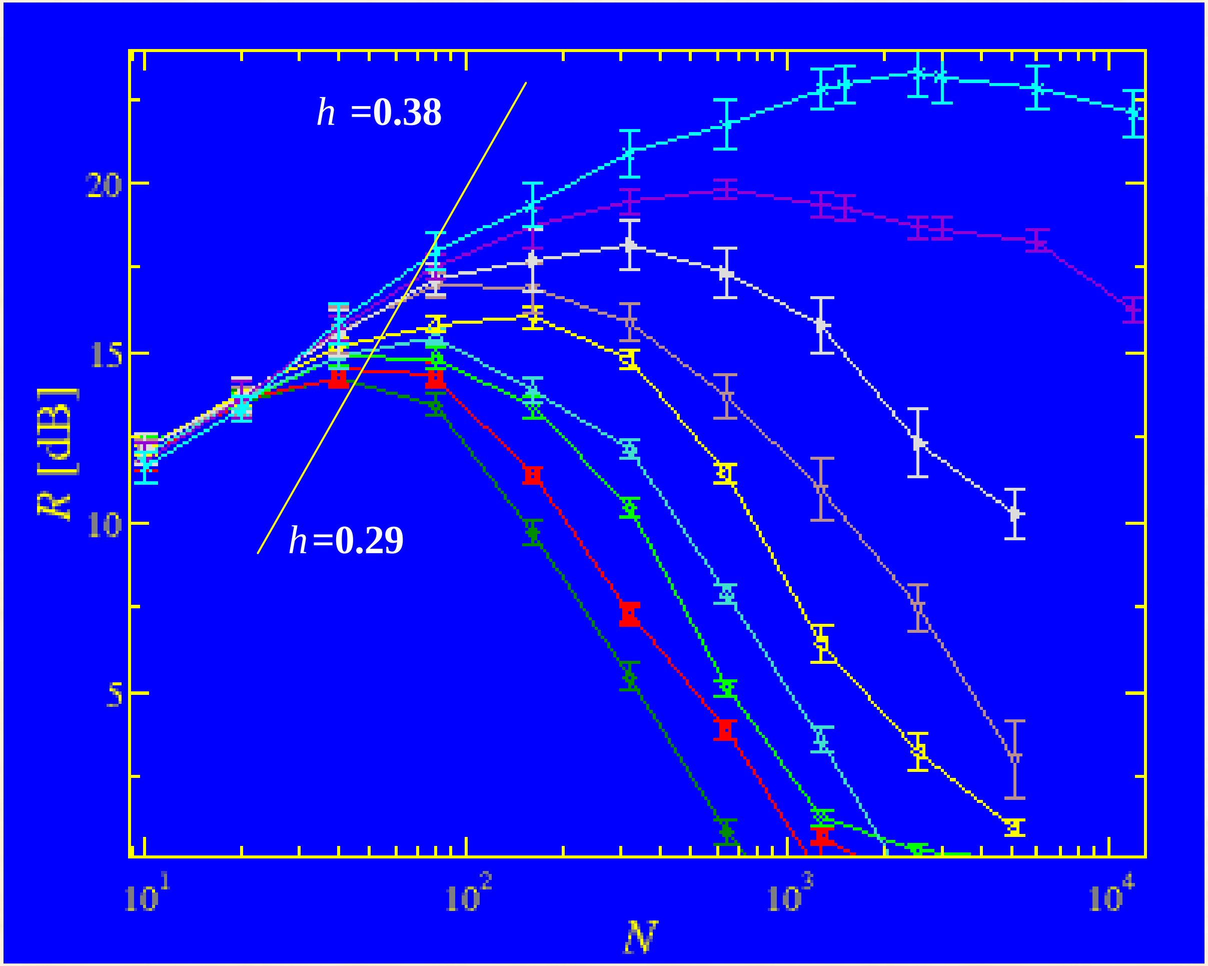


N=10

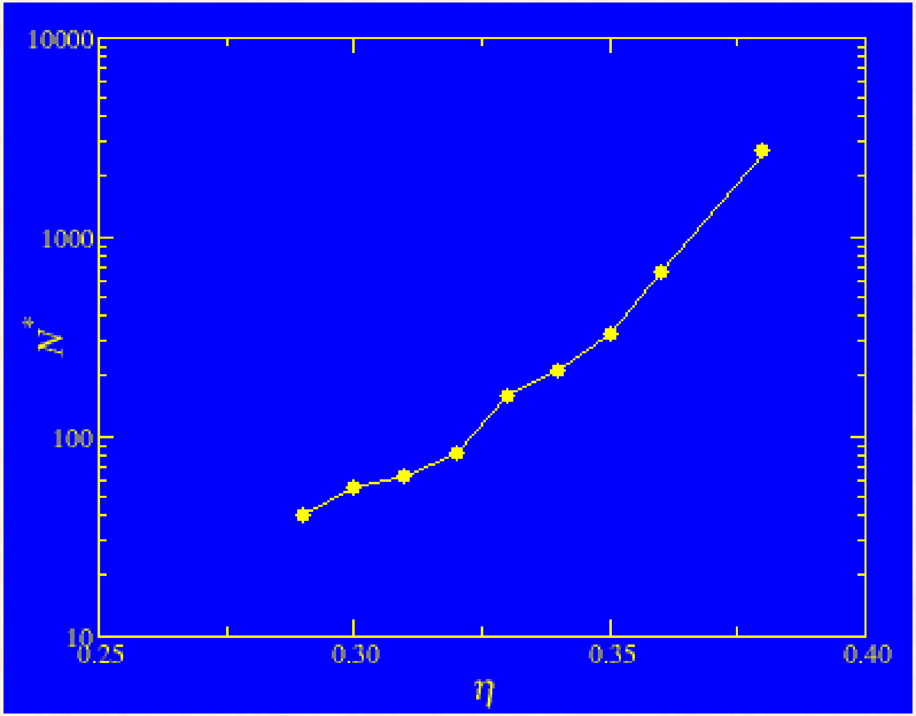
N=100

N=1000

Signal-to-noise ratio shows an optimum system size



Optimum system size increases with noise rate



Conclusions

-System size can have a non-trivial role in some phase transitions in models of social interest.

-Apparent phase transitions appear in models of social interest (biased opinion and culture formation).

-In noise driven systems, the “quality” of the output (synchronization with an external forcing or its regularity) depends on the system size.

- In a majority opinion formation model, an external influence works optimally in a society of the proper system size.

-This work stresses the non-trivial role that the system size has in the dynamics of social systems

-The thermodynamic limit should not be taken routinely in those models.

Does size matter?

Yes, in social relations size matters

In contrast to Statistical Physics, larger is not always better