

Neighborhood Models of Minority Opinion Spreading



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Models of Consensus vs. Polarization, or Segregation:

Voter model, Minority opinion models, Axelrod, Sznajd, Schelling, Bounded confidence and Relative agreement, Threshold models, Social impact theory, ...

Two general questions:

 Effects of "Who interacts with whom?": Comparison with average, Fully connected population, Random pairing, Neighboring interaction in regular lattices, Complex networks.

Here: Neighborhood models

2) Mechanisms for reaching or stopping consensus with convergent interactions: Barriers, topology, size, noise, updating rules,...

Here: Mechanisms for consensus on initial minority

NEIGHBORHOOD MODELS?

T. Schelling (Micromotives and Macrobehavior, 1978):

1) Spatial Proximity Model:

- -Interactions weighted by spatial location
- -Agents interact with neighbors in lattices

2) Bounded Neighborhood Model:

- -Common definition of neighborhood (system) and its boundaries
- -Interactions in a completely connected population (mean field)
- -Recurrence relations for the fraction of population with one option
- -Threshold models of collective action (M. Granovetter, Am. J. Soc. (1978))

PROPOSAL OF A THIRD WAY: Neighborhood Models

- -Consider locally defined neighborhood cells within a system
- -Complete connectedness within the neighborhood cell
- -Neighborhood cells change shape and size during evolution

Mechanisms for consensus on initial minority?

Question: How an initially minority opinion can become majority?

Serge Galam's model discusses a mechanism of social inertia resulting in democratic rejection of social reforms initially favored by a majority:

A TIE IN A CELL VOTING MEANS NO FOR THE REFORM

(Eur. Phys. J. B 25, 403(2002); Physica A 320, 571 (2003))

- a) Irish vote on the EU Nice treaty
- b) September 11 French rumor: no plane crashed the Pentagon

Madrid March 11 terrorist attack:

ETA or AL-QAEDA

Galam's Model

Eur. Phys. J. B 25, 403(2002) Physica A 320, 571 (2003)

- Models how an initially minority opinion can become majority
- •Examples:
 - •The September 11 no-plane Pentagon hoax the spread of such rumor in France and UK
 - Minoritary opinion against an structural change in society finally becomes a majority

Maastritch related Ireland & France voting

Authorship of recent terrorists attacks in Madrid
 Eta versus Al-Qaeda

Minority opinion blocks social reforms

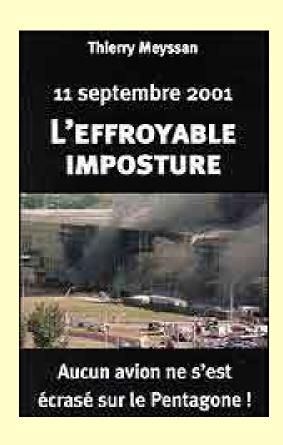


EU Nice treaty: Ireland referendum yielded results against reforms, although initially a majority supported it. Result came as a surprise to the Irish people themselves.

Social inertia: Conservative response to the risk of a change

Maintain social status quo

Minority Opinion Spreading: September 11th no-plane Pentagon hoax









Social inertia: Feelings of French society about US politics

Minority Opinion Spreading: Madrid, March 11



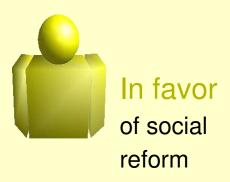
Government representatives swiftly put the blame for bombs on ETA yesterday morning. However, throughout the day doubts circulated regarding whether or not the basque group was in fact responsible

Minority Opinion Spreading Model

Serge Galam, Eur. Phys. J. B 25, 403 (2002)

Binary opinion:

Each agent has one opinion: blue (+) or yellow (-)

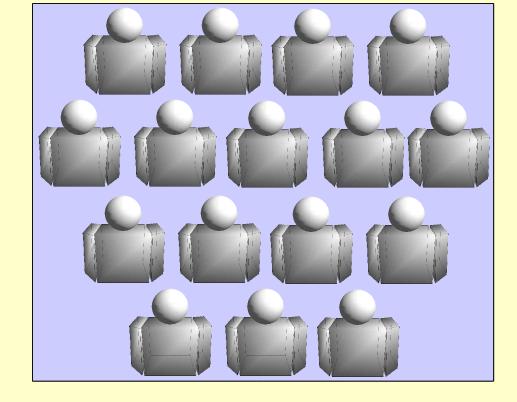




Initially there is a (blue) minority against social reform

Social life: Agents gather and discuss in *meeting cells* (offices, houses, bars, clubs, etc.)

Cells are defined only by their size k



k=16

M= Maximum size of a cell

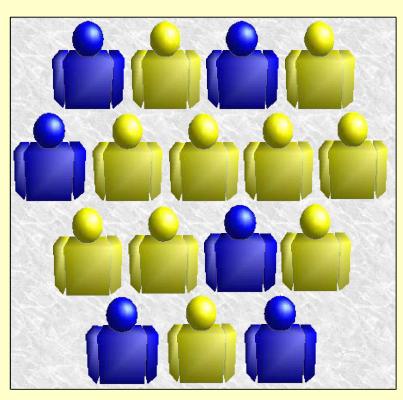
Interaction rules in meeting cells:

Pre-campaign political party meeting

Majority of *yellow* opinion

6

10



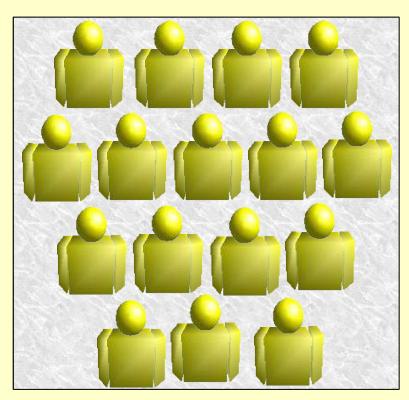
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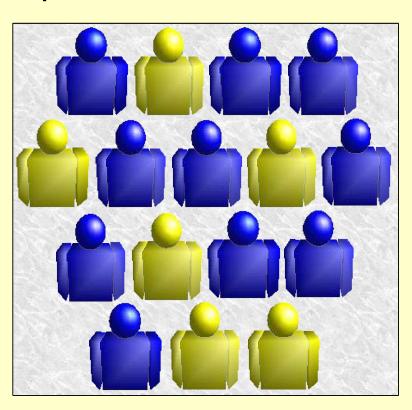
0

16



ALL the agents in the cell adopt the yellow opinion

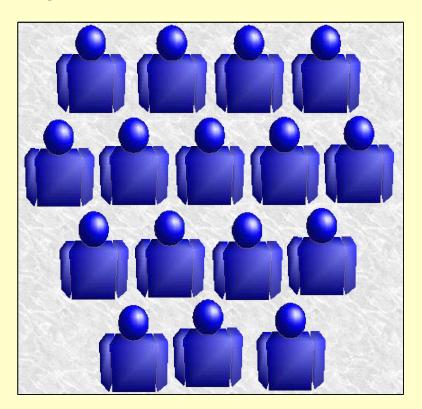
Majority of blue opinion



Majority of blue opinion

16

0



ALL the agents in the cell adopt the blue opinion

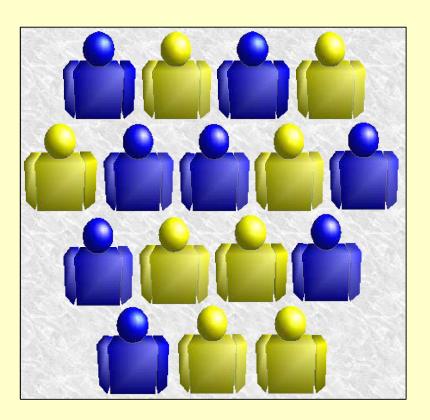
Individuals leave the cell as supporters of the majority

Social inertia:

A tie in the voting is a NO for social reform

8

8



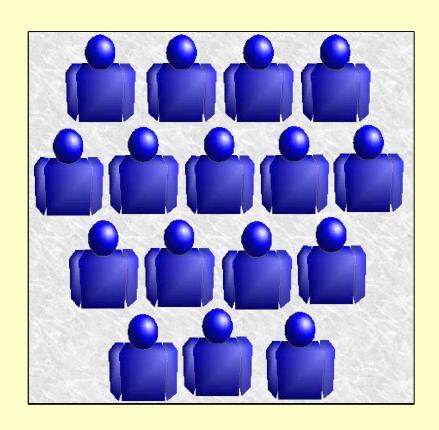
There is a bias toward one opinion in the society

Social inertia:

A tie in the voting is a NO for social reform

8

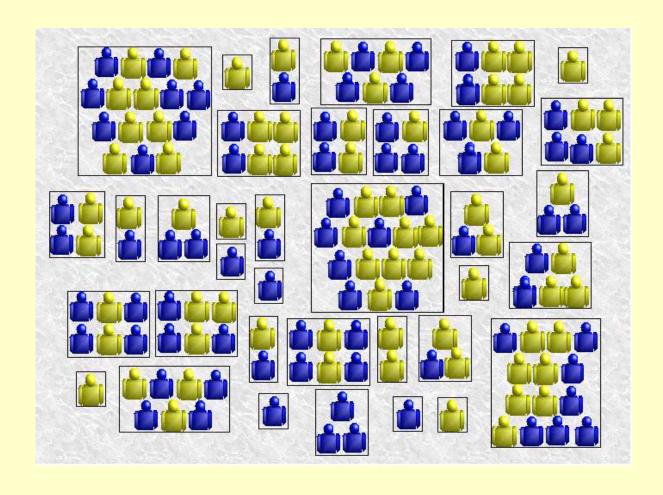
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ALL the agents in the cell adopt blue opinion minority against reform is favored

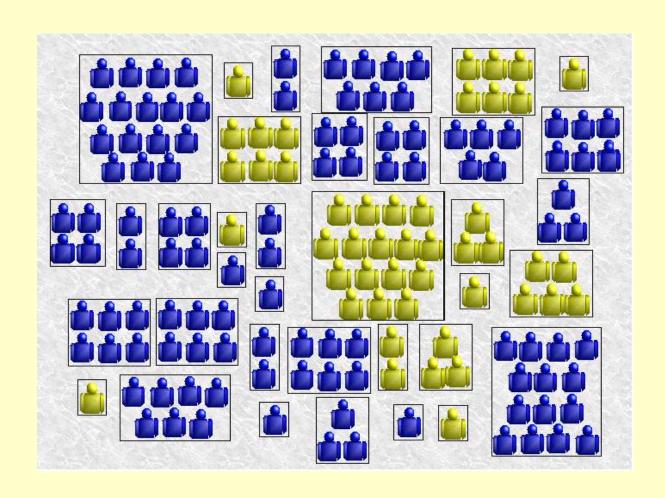
Dynamical evolution

Agents join a meeting cell randomly selected



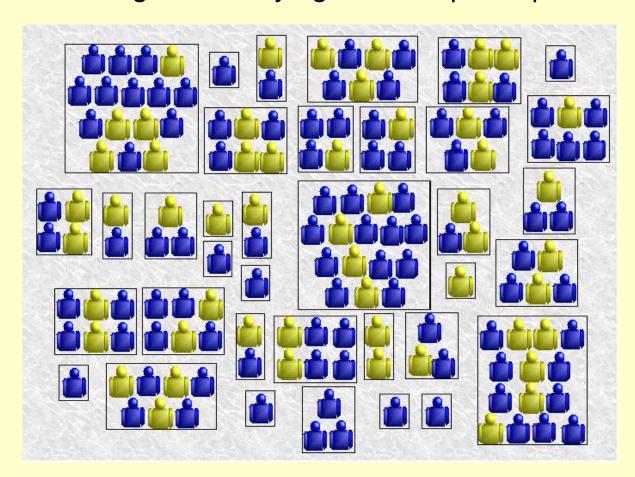
Dynamical evolution

Decision rule is applied in all the cells



Dynamical evolution

Following cycle: Agents randomly redistributed in the meeting cells carrying their adopted opinion



Mean-field analysis

 $P_{\perp}(t)$: density of agents with blue (+) opinion

P(t): density of agents with yellow (-) opinion

$$P_{-}(t) + P_{+}(t) = 1$$

M: maximum cell size

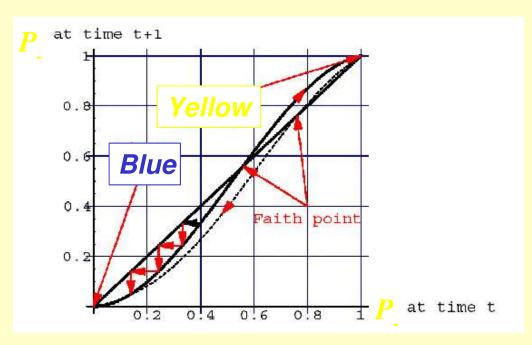
 a_k : probability for an agent to be in a cell of size k

Recursion relations:

$$P_{+}(t+1) = \sum_{k=1}^{M} a_{k} \sum_{j=\left[\frac{M}{2}+1\right]}^{M} {k \choose j} P_{+}(t)^{j} \left[1 - P_{+}(t)\right]^{k-j}$$

$$P_{-}(t+1) = \sum_{k=1}^{M} a_k \sum_{j=\left[\frac{M+1}{2}\right]}^{M} {k \choose j} P_{-}(t)^j \left[1 - P_{-}(t)\right]^{k-j}$$

Asymmetric unstable fixed point: *FAITH POINT*



There is a threshold value of initial minority supporters $p_c(M) < 1/2$ such that for $P_+(0) > p_c$ the minority opinion finally becomes majority:

$$P_{+} \longrightarrow 1$$
 $P_{-} \longrightarrow 0$

S. Galam, Eur. Phys. J. **B 25**, 406 (2002)

For
$$P+(0) > p_c$$
:
Social reform is rejected and status quo maintained

<u>Dynamics</u>: Time to reach consensus is fast and <u>system</u>—size independent

Beyond mean-field recursion relations:

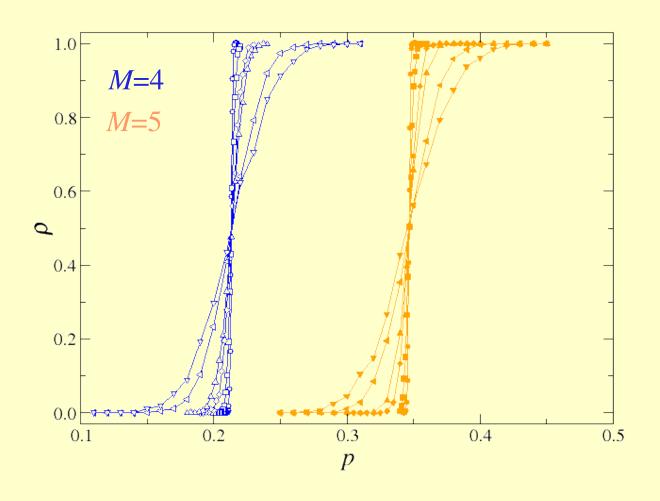
- •Numerical simulations of the model with N agents
- •We consider decision cells whose size is uniformly distributed in the interval [1,M]
- •p<1/2: Initial proportion of agents with the minority (blue) opinion
- Consensus (unique opinion) is always reached for all values of M and p
- •Order parameter: ρ

Mean field prediction:

probability (over realizations) that the consensus coincides with the initial minority blue opinion (rejection of social reform).

Finite Size Fluctuations:

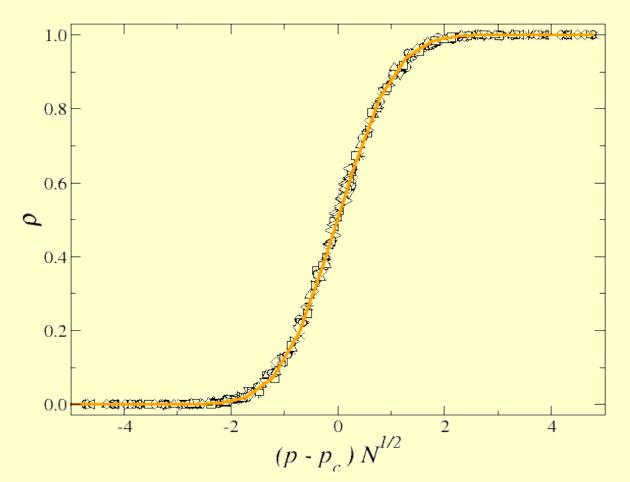
Smoothed 1st-order transition



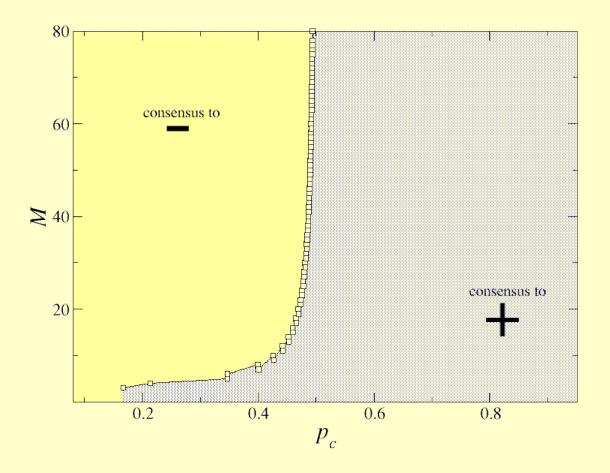
Finite Size Fluctuations:

Smoothed 1st-order transition

There is a region of width N^{-1/2} in which the outcome of a run is not well known



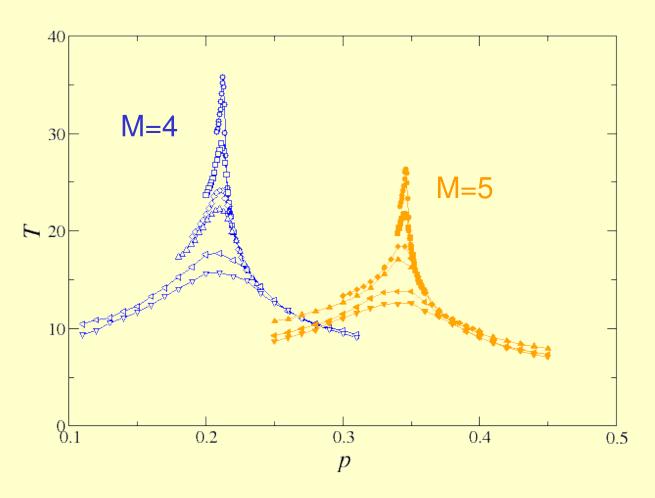
Consensus phase diagram



Small meeting cells required for minority opinion spreading

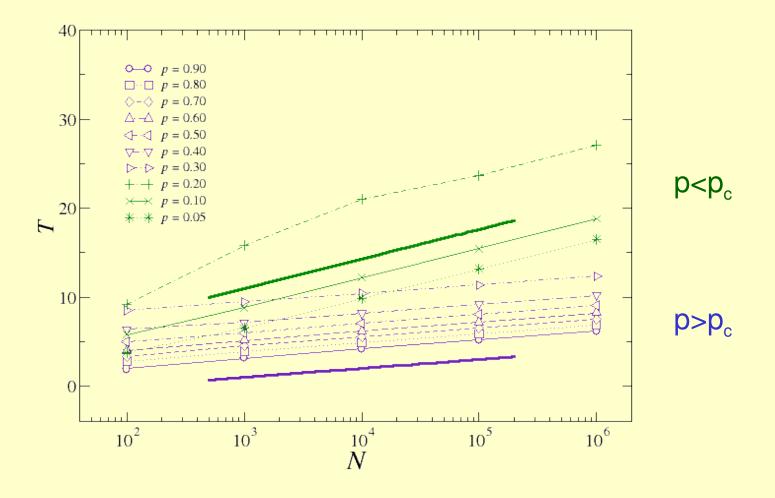
Dynamics:

Time to reach consensus, T, for different sizes N



T diverges with N for p=p_c

Time to reach consensus can be shown to scale as $T \sim \ln N$



Also, D. Stauffer, Int. J. Mod. Phys C 13, 975 (2002)

Neighborhood Models (NMs)

Meeting cells defined by their size do not incorporate local effects

"Primitive" society: Individuals interact predominantly with neighbors

NMs: Individuals are fixed at the sites of a regular lattice

Meeting cells locally defined by a tessellation of the lattice

Individuals in a meeting cell interact with original rules

Meeting cells redefined at each time step of dynamics

Consider 1D and 2D square lattices with **synchronous** and **asynchronous** updating schemes

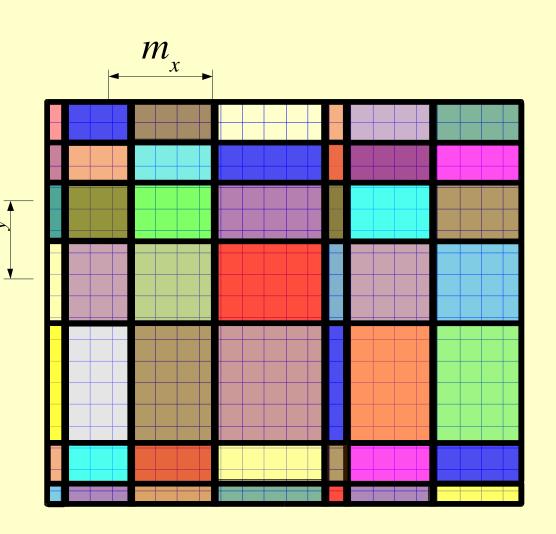
2D Neighborhood Model: Synchronous update

Rectangular tessellation

$$1 \le m_x \le M$$

$$1 \le m_y \le M$$

- Interaction rules simultaneous in each cell.
- Time step defined by each iteration



2D Neighborhood Model:

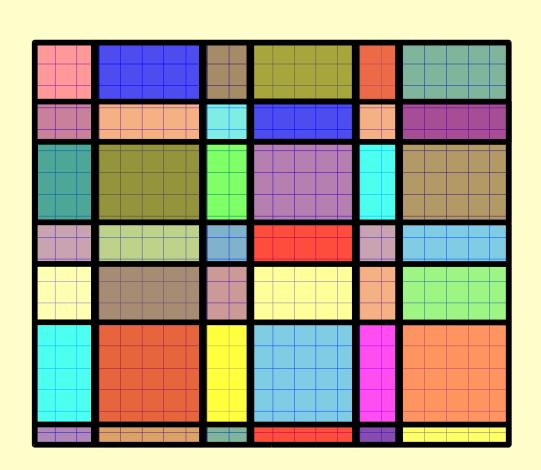
Synchronous update

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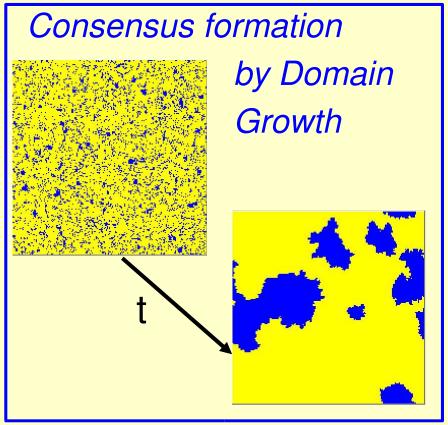


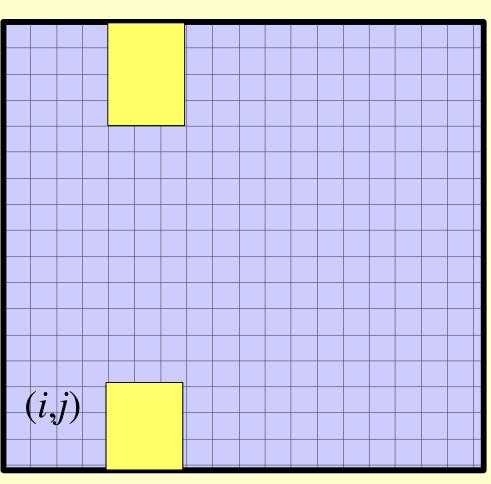
2D Neighborhood Model:

Asynchronous update

$$1 \le m_x \le M$$
$$1 \le m_y \le M$$

For each update, time increases as $t \rightarrow t + m_x \cdot m_y / N$





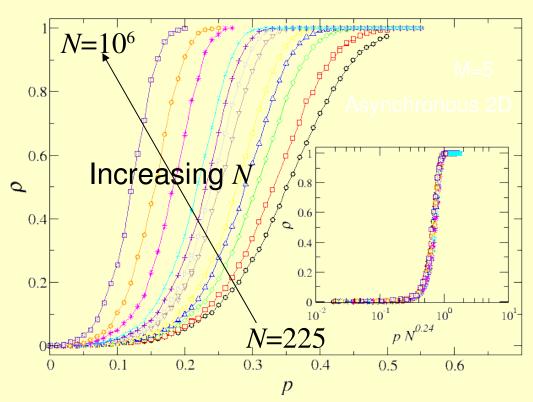
Neighborhood Models: Steady State

Consensus always reached in finite systems in a finite number of steps.

For
$$N \rightarrow p_c \rightarrow 0$$

$$p_c \sim N^{-\alpha}; \alpha(\mathsf{D}, \mathsf{M})$$

$$\rho(p,N)=f(pN^{\alpha})$$



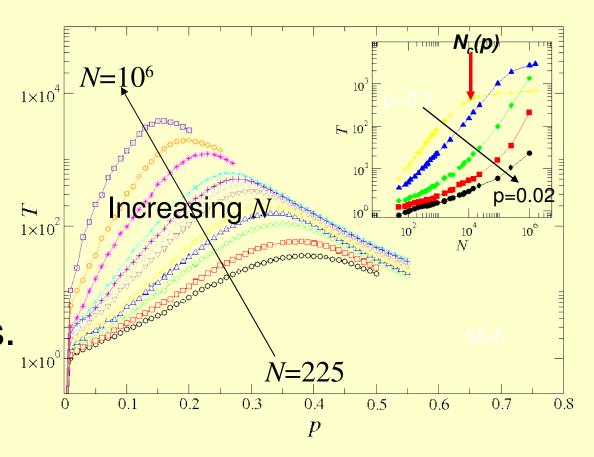
In an infinite system, the blue initial minority opinion wins regardless the amount of initial supporters:

Social reform always rejected in a large system

Neighborhood Models: Time to reach consensus

T has a maximum for $p_c(N)$.

T much larger than for nonlocal models.

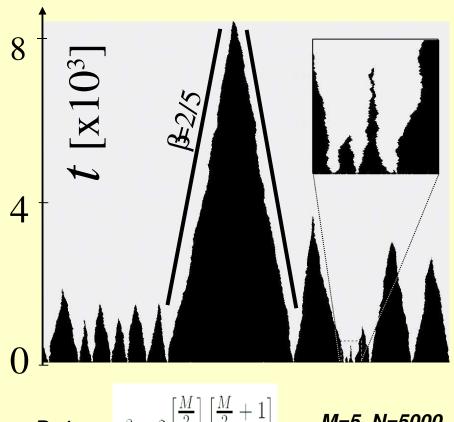


The time to reach consensus, scales as two power laws of N, below and above the transition

Neighborhood Models: Domain growth

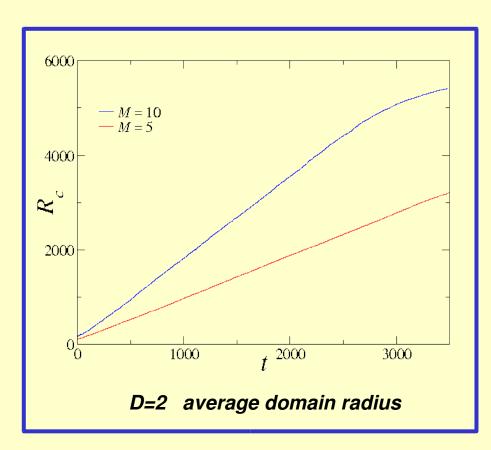
Domains of a given opinion shrink or grow.

Characteristic size scales linearly with time: $R(t) = \beta t$



 $\beta = 2 \frac{\left\lfloor \frac{M}{2} \right\rfloor \left\lfloor \frac{M}{2} + 1 \right\rfloor}{M(M+1)}$

M=5, N=5000

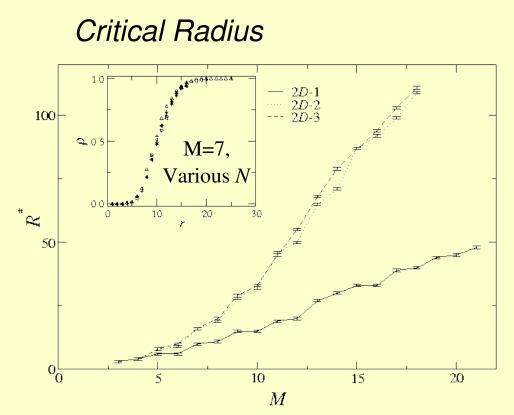


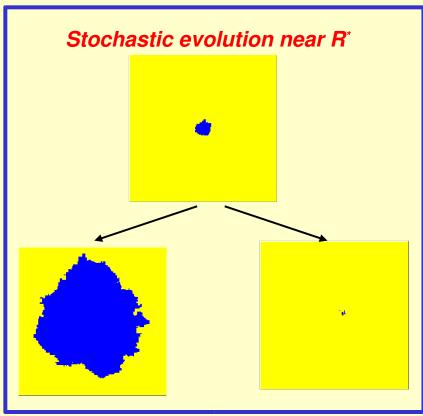
Why minority blue opinion always wins in Neighborhood Models for large systems?

A critical size for an initial local domain of minority supporters exists.

Domains of larger size grow and occupy the system.

A domain of overcritical size always exists in a large enough population





Critical Size in NMs and Nucleation theory

Nucleation:

- -R*: competition between surface tension and different bulk energies
- -R* meaningless in D=1 (no surface tension)
- -Overcritical droplet created by a dynamical fluctuation, and grows deterministically

NMs:

- -Overcritical droplet appears in random initial condition. Stochastic dynamics of growth.
- -Critical size (probabilistic concept) appears in D=1
- -R*: competition between favoured opinion and stochastic interface dynamics

Conclusions

General interest of Neighborhood Models as a third way between strictly local interactions and completely connected populations

Importance of the population size in consensus models:

- -Analytic recursion relations neglect finite size effects
- -NM local effects: Social reform is always rejected by a large population in the model of Minority Opinion spreading

Neigborhood models of minority opinion spreading:

- -Treshold value of initial concentration of minority suporters for minority overcome: $p_c \sim N^{-\alpha}$ Time to reach consensus grows as a power law of N.
- Mechanism: Critical size for an initial local domain of of minority supporters
- -Neighborhood models describe a more efficient spreading of minority opinion, but spreading takes a much longer time.