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Coherent Structures in Three Dimensional Flows

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Abstract

Coherent structures are known to drive biological dynamics, from plankton to top predators, thus it is very important to be able to characterize them in realistic three dimensional flows. The Finite-Size Lyapunov Exponent (FSLE) is a measure of particle dispersion in fluid flows and the ridges of this scalar field locate regions of the velocity field where strong exponential separation between particles occur. These regions are referred to as Lagrangian Coherent Structures (LCS). We have identified LCS in two different 3D flows: a canonical turbulent velocity field, that is the turbulent flow between two parallel stationary plates, driven by a pressure gradient in the mean flow direction and a primitive equation model (ROMS) simulation of the oceanic flow in the Benguela region.

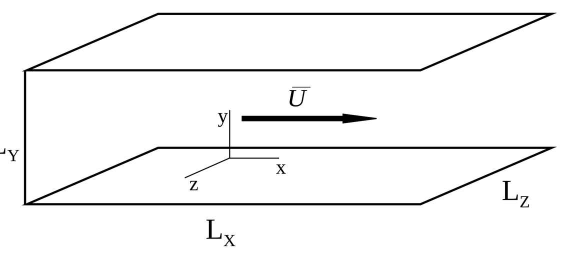
Turbulent channel flow

We used the DNS algorithm of **Channelflow.org** to simulate turbulent channel flow at low Reynolds number. The simulation domain is a periodic (in x and z) box.

The simulation parameters follow Moser et al (1999):

$$(L_x, L_y, L_z) = (4\pi, 2L, 4/3\pi)$$

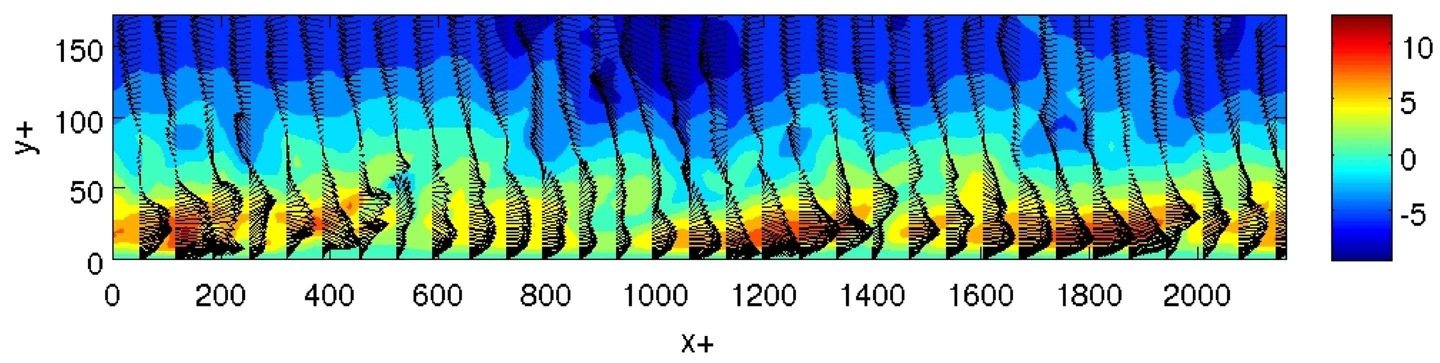
$$N_x \times N_y \times N_z = 128 \times 129 \times 128$$



In terms of wall variables the Reynolds number of the flow is:

$$R_{e\tau} = \frac{L u_\tau}{\nu} = 172$$

with the friction velocity: $u_\tau = \sqrt{\frac{\mu}{\rho}\frac{dU}{dy}\Big|_{y=0}}$



Snapshot of the turbulent velocity field in the bottom half of the channel at z=Lz/2.

FSLE calculation algorithm

The Finite Size Lyapunov Exponent measures the time τ it takes for two fluid particles initially at a distance δ_0 to separate by a distance δ_f

The FSLE fields were calculated using a 3D version of the algorithm used by d'Ovidio et al.(2004).

The velocity field domain is covered by a regular grid with mesh spacing δ_0. At time t_0, a fluid particle **P** is released from every grid node, together with particles that are at a distance δ_0 from **P** in the three coordinate directions.

The algorithm then tracks the particles, checking, at each integration time step t if the distance between **P** and any other particle is greater than δ_f. If so, the FSLE for the initial location of P, **x**(**P**,t_0) is

$$\Lambda(\mathbf{x}, t_0, \delta_0, \delta_f) = \frac{1}{\tau}\log\left(\frac{\delta_f}{\delta_0}\right)$$

If at the end of the tracking interval the distance is lower than δ_f, the FSLE for the spatial location of **P** at t_0 is set to zero.

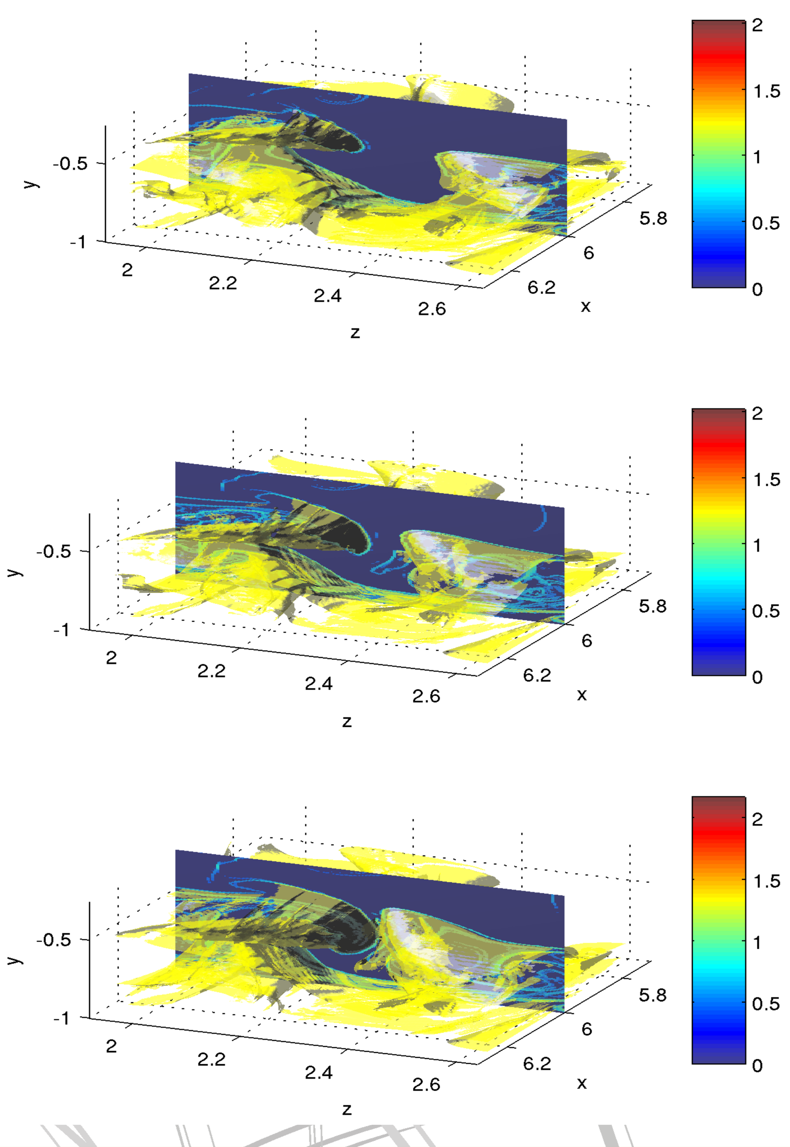
The particles positions do not coincide in general with the spatial locations where the velocity field is specified, so we use linear interpolation in all spatial directions and also time to obtain velocity values at the particle's position at the required integration time step.

3D LCS in the turbulent channel flow

Lagrangian Coherent Structures are defined as ridges in the FSLE field. Intuitively, a ridge can be defined as if a person was standing on a ridge moving perpendicularly to it would lead this person down.

Turbulent channel flow was previously studied with lagrangian techniques (Direct Lyapunov Exponent) by Green et al (2007) who showed that lagrangian measures were able to define more precisely the boundaries of coherent structures when compared to eulerian measures.

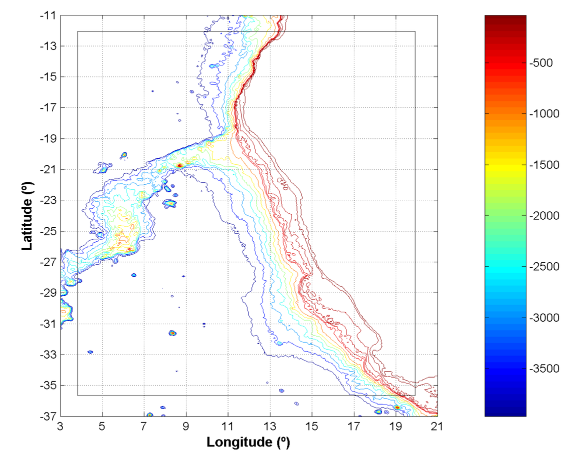
3d LCSs are rendered in the figure to the right (mean flow in the positive x direction), in a sequence of time instants, as they pass through the calculation domain. They have a clearly 3d shape and move with the flow. The LCS seem to create a boundary between the inner turbulent region and the outer region that is practically devoid of FSLE. The highest LCS have L_y-scale heights above the wall, and have a distinct mushroom shape enclosing the regions of the channel closer to the wall, where high FSLE values can be found. Near the wall, the LCS adopt the shape of sheets parallel to it, which reflects the high rates of shear that occur in that region. These sheets form the base of the mushroom-shaped excursions up to the channel center.



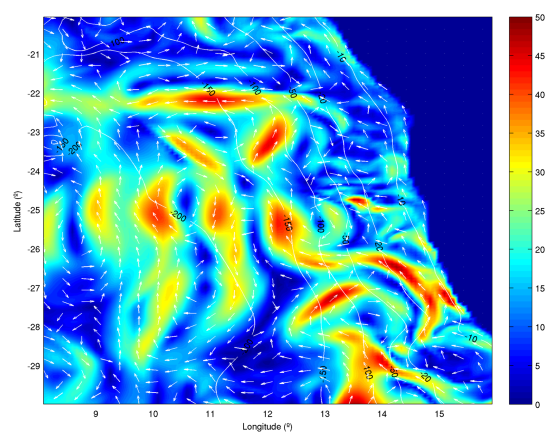
Benguela mesoscale ocean flow

The three dimensional velocity field used in this study was obtained from a ROMS (Regional Ocean Modeling System) simulation of the Benguela ocean region (Le Vu et al, 2012). The horizontal resolution was 1/12º and in the vertical direction 32 terrain following layers were used. The model was run with climatological forcing.

Limits of the domain of the velocity field set used in the FSLE field calculation in the Benguela ocean region. The set contained two years of daily averaged horizontal (u,v) and vertical (w) velocity components. Color bar shows depth values in meters.



Snapshot of the horizontal velocity field (cm/s) at layer 16 (200 m depth at the offshore area). There is a strong mesoscale activity for which the Benguela ocean region is known.



3D LCS ridge extraction

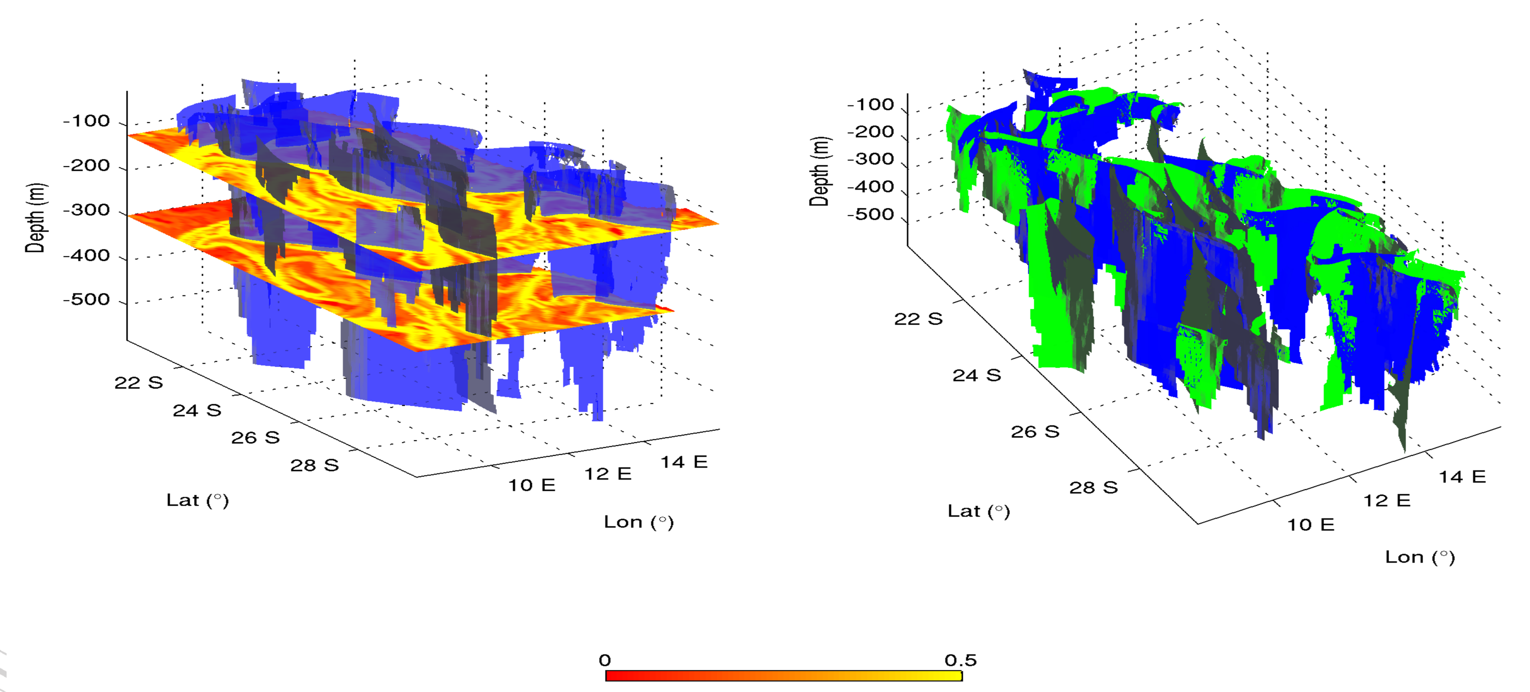
By using a rigorous mathematical definition of ridge (Eberly et al, 1994), the ridge surfaces can be extracted directly from the FSLE fields.With $g_i = \partial_i f$ and $h_{ij} = \partial^2_{ij} f$ the hessian matrix of f with eigenvalues α^k ($\alpha_1 > \ldots > \alpha_n$) and eigenvectors e^k, a d-dimensional ridge point is given by: $\forall\, d < k \leqslant n \quad g_i e_i^k = 0 \wedge \alpha^k < 0$

The extraction method (Schultz et al,2008) marches through the grid checking the ridge condition. Ridge points with α^3 lower than a predefined threshold are excluded. There is no systematic way of finding an "optimum" threshold.

LCS in the Benguela region

The ridge extraction algorithm will search for the points where the ridge criteria are met. The surfaces are defined based on the triangulation of the ridge points.

The left panel shows the attracting (blue) LCS extracted from a snapshot of the 3D FSLE field superposed on 2D slices of the FSLE field showing tha the ridges coincide with the lines of higher FSLE. There is a vigorous mesoscale activity due to the high number of LCS,as can be seen in the right panel, where the attracting (blue) and repelling (green) LCS are shown, with the mutual intersections.



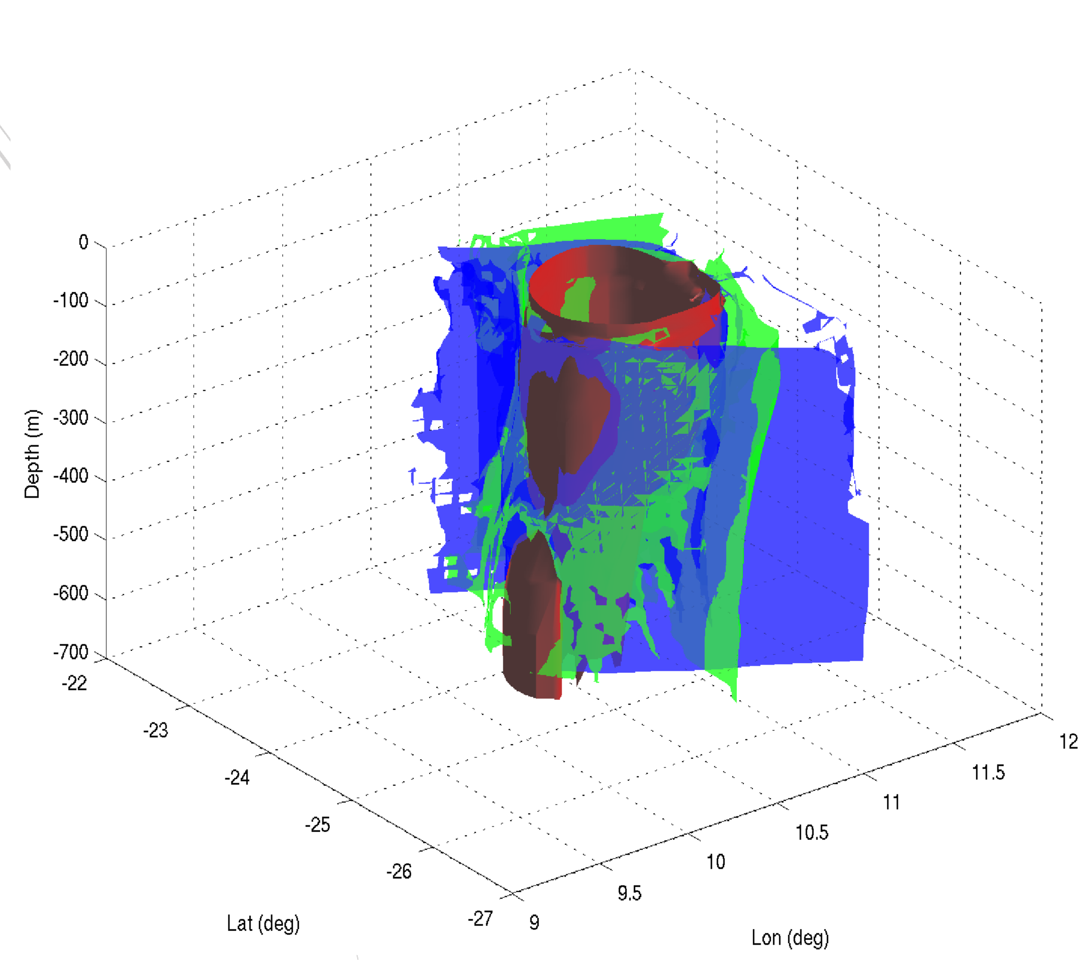
A mesoscale eddy from 3D LCS

Attracing (blue) and repelling (green) LCS on day 1 of the calculation period together with Q-criterion isosurface at $Q = 10^{-10}$ (red), around a mesoscale eddy.

Eulerian and Lagrangian measures limit approximately the same region, but are substantially different.

The Q-criterion is related to local information about fluid flow processes.

The Lagrangian perspective, on the other hand, provides an integration of the temporal evolution of material properties of the flow, e.g. material transport, and thus should give more meaningful information about processes that rely on ocean material transport.



References

•**Mixing Structures in the Mediterranean Sea from Finite Size Lyapunov Exponents**, d'Ovidio, F; Fernandez, V; Hernandez-Garcia, E; Lopez, C, Geophysical Research Letters 31, L17203 (2004)

•**Direct Numerical Simulation of Turbulent Channel Flow up to Ret=590**, Moser, R D; Kim, J; Mansour, N N; Physics of Fluids 11,4 (1999)

•**Detection of Lagrangian coherent structures in three-dimensional turbulence**, Green, M. A.; Rowley, C. W.; Haller, G.; Journal of Fluid Mechanics, 572 (2007)

• **Ridges for Image Analysis**, Eberly, D; Gardner, R; Morse, B; Pizer, S; Scharlach, C; Journal of Mathematical Imaging and Vision, 4 (1994)

•**Crease Surfaces: From Theory to Extraction and Application to Diffusion Tensor MRI**, Schultz, T; Theisel, H; Seidel, H; IEEE Transactions on Visualization and Computer Graphics, 13 (2010)

•**Processes maintaining the OMZ of the Benguela upwelling system using an eddy resolving model**, Le Vu, B., Gutknecht, E., Machu, E., Dadou, I., Veitch, J., Sudre, J., Paulmier, A., Garçon,V., submitted for publication,Journal of Marine Research, 2012

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