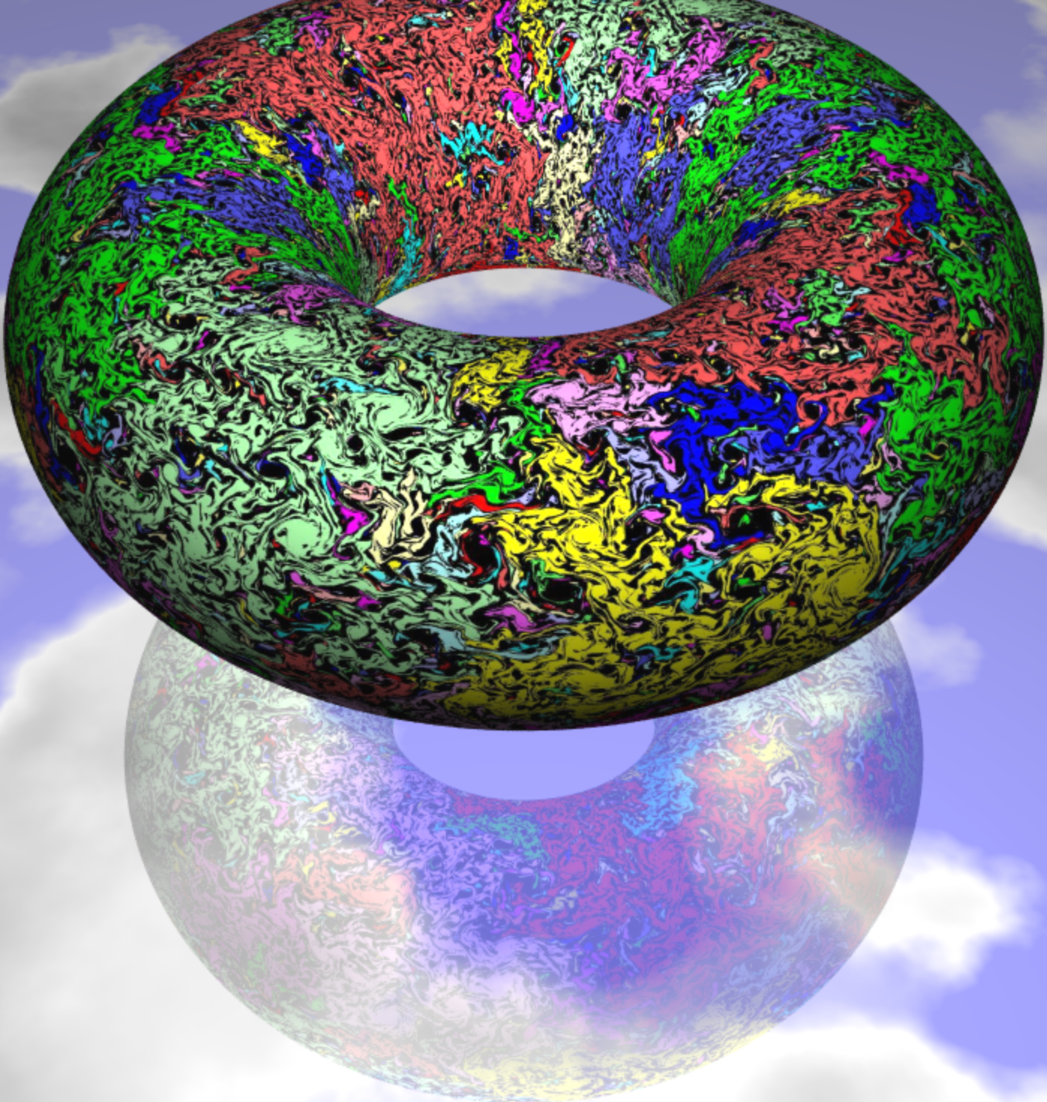
Every Thing You Always Wanted to Know About Two-Dimensional Turbulence



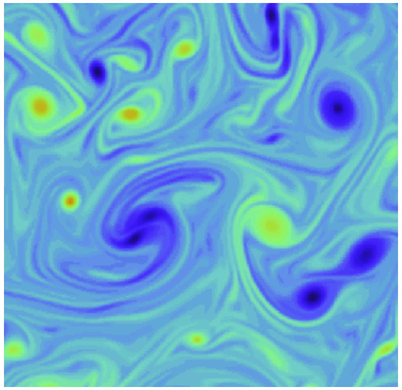
Guido Boffetta
University of Torino

A physical motivation for two-dimensional turbulence

2D Navier-Stokes equations are a simple model for large scale motion of atmosphere and oceans: thin layers of fluid in which stratification and rotation supress vertical motions.

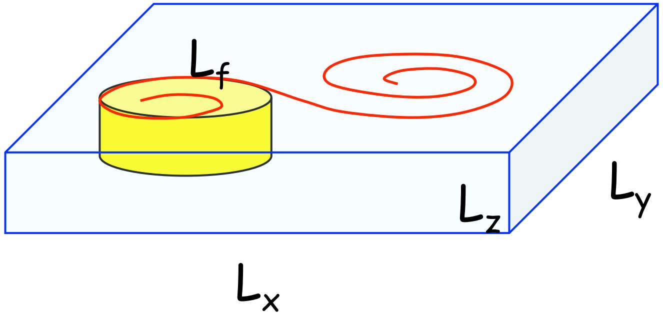
“More easily simulated on digital computers than 3d flows [...] a valuable testing ground for dynamical theories” RHK, 1967

Outline



* Zero thickness: 2D Navier-Stokes

- double cascade theory by Kraichnan
- experimental and numerical studies
- statistics of the inverse cascade



* Finite thickness: 2D or not 2D ?

- split energy cascade
- 2D phenomenology in 3D flows

G. Boffetta, R. Ecke, *Ann. Rev. Fluid Mech.* **44**, 427 (2012)

2D Navier-Stokes

Prehistory: 2D Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

Introducing the stream function $\mathbf{u} = \hat{\mathbf{z}} \times \nabla \psi$

$$\boxed{\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega + f - \alpha \omega}$$

f: external forcing
α: friction coefficient (from boundary conditions on the bottom)

In the inviscid, unforced limit $\nu=\alpha=f=0$, ω is a Lagrangian invariant $\frac{d\omega}{dt} = 0$
two inviscid quadratic invariants:

$$E = \frac{1}{2} \int u^2 d^2 x$$

energy

$$Z = \frac{1}{2} \int \omega^2 d^2 x$$

enstrophy

Pre-Kraichnan studies

$$\lim_{\nu\to 0}\frac{dE}{dt} = -\lim_{\nu\to 0} 2\nu Z = 0$$

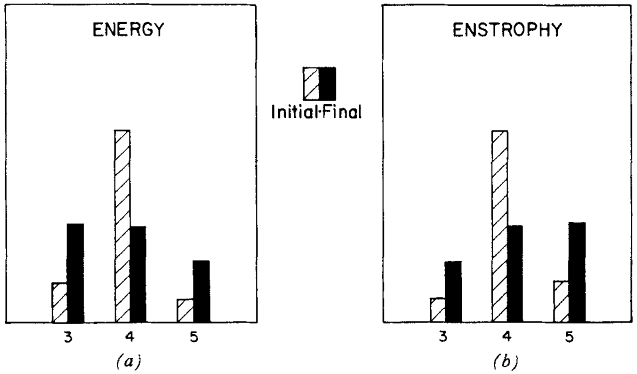
* TD Lee (1951): enstrophy conservation is incompatible
with direct energy cascade (i.e. the energy flux is inverse)

* Fjørtoft (1953)

$$E_3 + E_4 + E_5 = E'_3 + E'_4 + E'_5$$

$$k_3^2 E_3 + k_4^2 E_4 + k_5^2 E_5 = k_3^2 E'_3 + k_4^2 E'_4 + k_5^2 E'_5$$

energy center of mass moves to larger scales
enstrophy center of mass moves to smaller scales



* Neuman (1967)

Define characteristic wavenumbers:

$$\begin{cases} k_E^2 = \int k^2 E(k)\,dk \,/ \int E(k)\,dk = Z/E \\ k_Z^2 = \int k^4 E(k)\,dk \,/ \int k^2 E(k)\,dk = P/Z \end{cases}$$

from energy/enstrophy balance:

$$\frac{dE}{dt} = -2\nu Z \qquad \frac{dZ}{dt} = -2\nu P$$

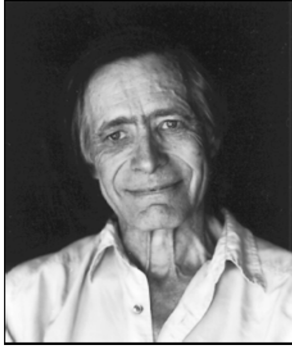
using Schwarz inequality :

$$\dot{k}_E^2 = 2\nu k_E^2 (k_E^2 - k_Z^2) \le 0$$

i.e. energy characteristic scale increases.

A simple argument for cascade directions

(Kraichnan, 1967; Eyink 1996)



$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega + f - \alpha \omega$$

$$\omega = \nabla \times \mathbf{u}$$

Characteristic scales in the cascades:

$$\ell_\nu = \left(\varepsilon_\nu / \eta_\nu\right)^{1/2}$$

$$\ell_f = \left(\varepsilon_f / \eta_f\right)^{1/2}$$

$$\ell_\alpha = \left(\varepsilon_\alpha / \eta_\alpha\right)^{1/2}$$

$$\ell_\nu \qquad \ell_f \qquad \ell_\alpha \longrightarrow$$

From energy/enstrophy balance in stationary conditions:

$$\varepsilon_f = \varepsilon_\alpha + \varepsilon_\nu$$

$$\eta_f = \eta_\alpha + \eta_\nu$$

we have

$$\frac{\varepsilon_\nu}{\varepsilon_\alpha} = \left(\frac{\ell_\nu}{\ell_f}\right)^2 \left(\frac{\ell_f}{\ell_\alpha}\right)^2 \frac{(\ell_\alpha / \ell_f)^2 - 1}{1 - (\ell_\nu / \ell_f)^2}$$

$$\frac{\eta_\nu}{\eta_\alpha} = \frac{(\ell_\alpha / \ell_f)^2 - 1}{1 - (\ell_\nu / \ell_f)^2}$$

developed direct cascade: $\ell_\nu \ll \ell_f$

$\dfrac{\varepsilon_\nu}{\varepsilon_\alpha} \to 0$ energy goes to large scales

developed inverse cascade: $\ell_f \ll \ell_\alpha$

$\dfrac{\eta_\alpha}{\eta_\nu} \to 0$ enstrophy goes to small scales

This argument determines the directions of the cascades but not how the characteristic scales depend on the parameter α and ν

Kraichnan [Phys. Fluids **10** (1967), J. Fluid Mech. **47** (1971)]

Non-linear transfer of energy and enstrophy at wavenumber k:

$$\left.\frac{dE(k)}{dt}\right|_{NL} = T(k) \qquad \left.\frac{dZ(k)}{dt}\right|_{NL} = k^2 T(k) \qquad \text{with} \qquad T(k) = (1/2)\int_0^\infty dp \int_0^\infty dq\, T(k,p,q)$$

and

$$T(k,p,q)+T(p,q,k)+T(q,k,p)=0$$
$$k^2T(k,p,q)+p^2T(p,q,k)+q^2T(q,k,p)=0$$

(energy/enstrophy conservation)

Energy transfer across a given scale

$$\Pi_E(k) = \int_k^\infty T(k')\,dk' \sim \lambda_k k E(k)$$

characteristic deformation frequency: $\lambda_k^2 \sim \int_{k_{min}}^{k} E(p)p^2\,dp$

(smaller scales are incoherent)

Looking for a scale-independent energy flux $\Pi_E(k)=\varepsilon$ one obtains $E(k) = C\varepsilon^{2/3}k^{-5/3}$

Transfer is local: λ_k is dominated by $p \sim k$ and $\lambda_k \sim \varepsilon^{1/3}k^{2/3}$

Friction IR cutoff: $k_\alpha \simeq \varepsilon^{-1/2}\alpha^{3/2}$

Enstrophy transfer across a given scale

$$\Pi_Z(k) = \int_k^\infty k'^2 T(k') dk' \sim \lambda_k k^3 E(k)$$

Looking for a scale-independent enstrophy flux $\Pi_Z(k)=\eta$ one obtains

$$E(k) = C' \eta^{2/3} k^{-3}$$

with viscous UV cutoff: $k_\nu \simeq \eta^{1/6} \nu^{-1/2}$

This argument is not fully consistent as it gives $\lambda_k \sim \log(k/k_{min})$ and therefore a log-k dependent enstrophy flux

Kraichnan (1971): constant flux using log-corrected spectrum:

$$E(k) = C' \eta^{2/3} k^{-3} \left[\ln(k / k_{min}) \right]^{-1/3}$$

Problem: transfer is not local:
λ_k is dominated by wavenumbers p«k
(similar to Batchelor regime for passive scalar)

$$\lambda_k^2 \simeq \eta^{2/3} \int_{k_{min}}^{k} p^{-1} \left[\ln(p / k_{min}) \right]^{-1/3} dp$$

The analogy with passive scalar is even stronger when considering the effects of friction

The double cascade scenario

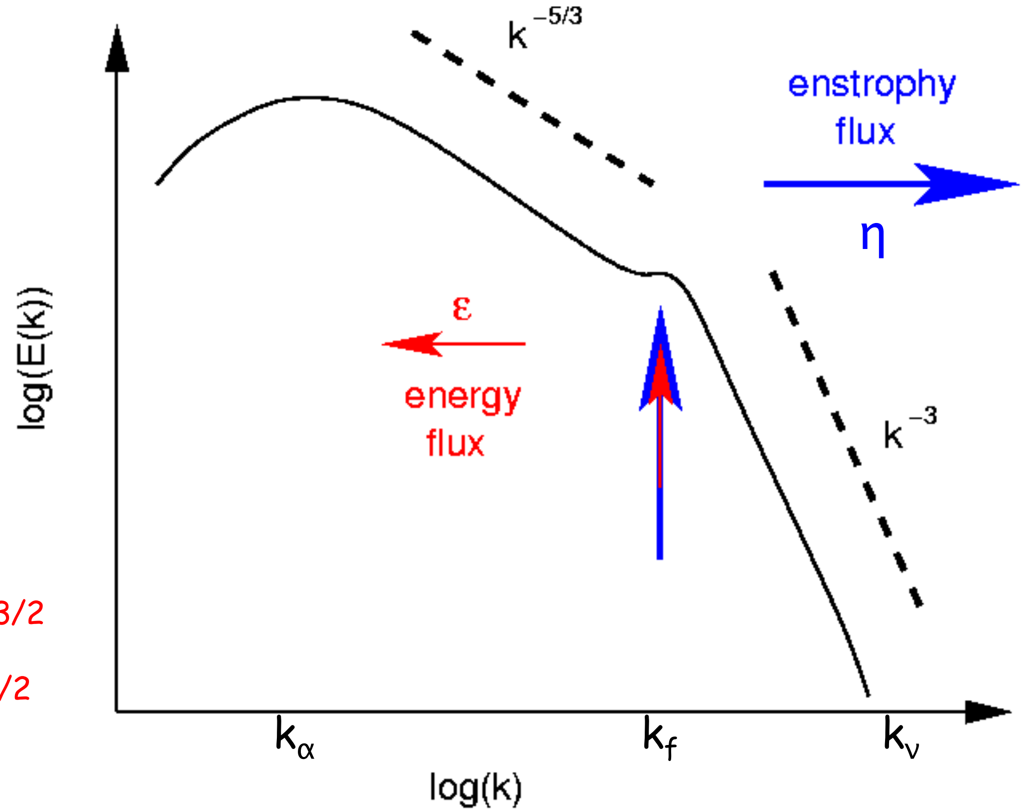
A two dimensional fluid forced at intermediate scales is expected to develop the two cascades (inverse energy to large scales and direct enstrophy to small scales)

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

$$E(k) = C'\eta^{2/3}k^{-3}\left[\ln(k/k_{min})\right]^{-1/3}$$

The dimensionless constants can be estimated by closures.
TFM (Kraichnan, 1971):
C ~ 6.7
C'~ 2.6

$$k_\alpha \simeq \varepsilon^{-1/2}\alpha^{3/2}$$
$$k_\nu \simeq \eta^{1/6}\nu^{-1/2}$$



The double cascade scenario is typical of 2d flows, e.g. plasmas and other geophysical models.

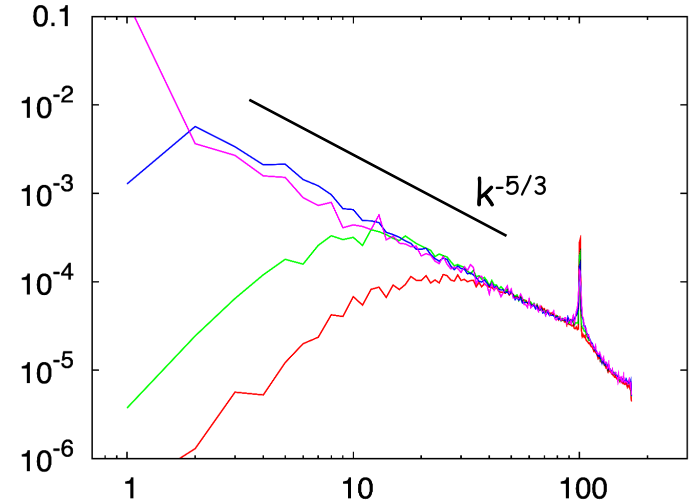
The fate of large scales (Kraichnan, 1967)

In the absence of friction (or with small friction) the inverse cascade is quasi-steady because largest scale grows in time as (from energy balance)

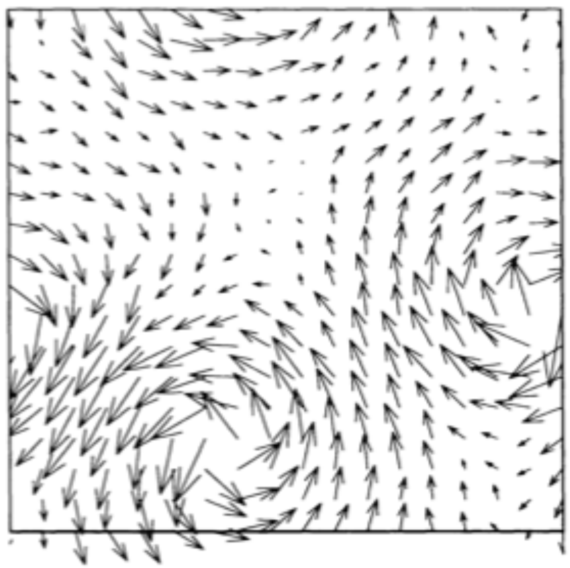
$$L(t) \simeq \varepsilon^{1/2} t^{3/2}$$

When L(t) reaches the box size L_B , energy piles up at scale this scale generating the condensate: a large scale structure (a dipole in the case of periodic BC).

The condensate interacts non-locally with the cascade and modifies the $k^{-5/3}$ spectrum



Evolution of energy spetrum in DNS



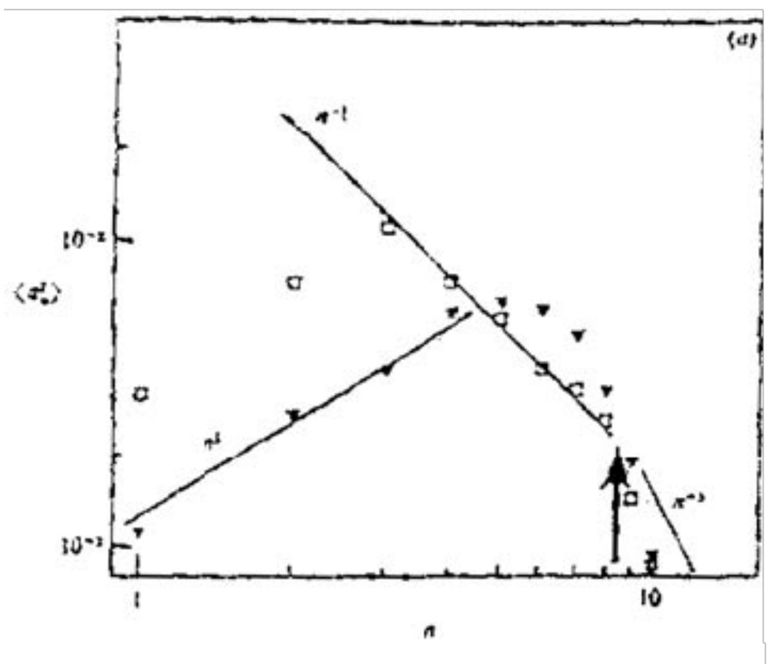
velocity field in presence of the condensate
numerical simulations by L.Smith and V.Yakhot, PRL (1993)

Results on 2D turbulence

Early laboratory experiments

Thin layer of mercury with electrical forcing in a uniform magnetic field suppressing vertical motions (linear friction due to Hartmann layer).
J.Sommeria, JFM **170**, 139 (1986)

Energy spectrum



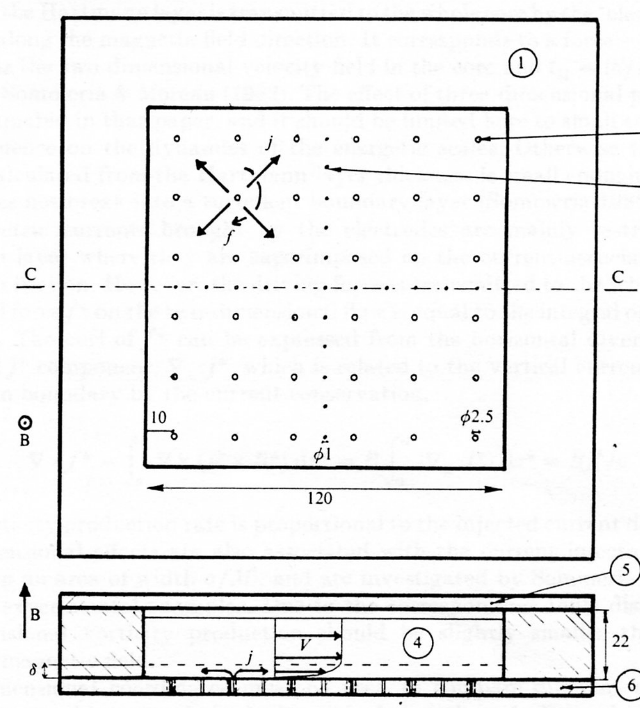


FIGURE 1. The apparatus; the current distribution near one electrode and the velocity profile are schematized. The Hartmann-layer depth is denoted by δ. (1) Copper frame. (2) Electrodes for current injection and electric potential measurements. (3) Electrodes for electric potential measurements only. (4) Mercury. (5) Glass cover. (6) Electrically insulating bottom plate in which electrodes are embedded.

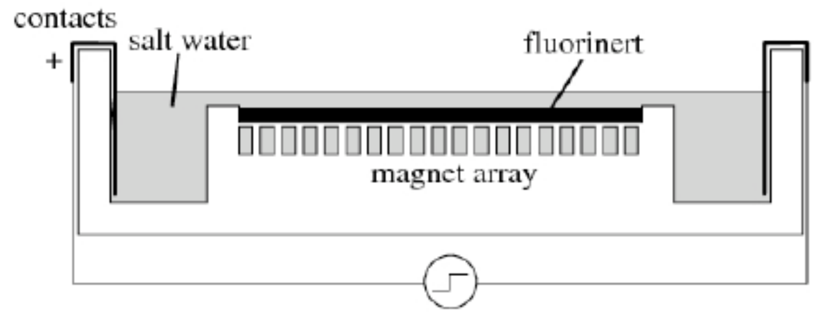
Observation of the inverse cascade
very small inertial range

Electrolyte cell

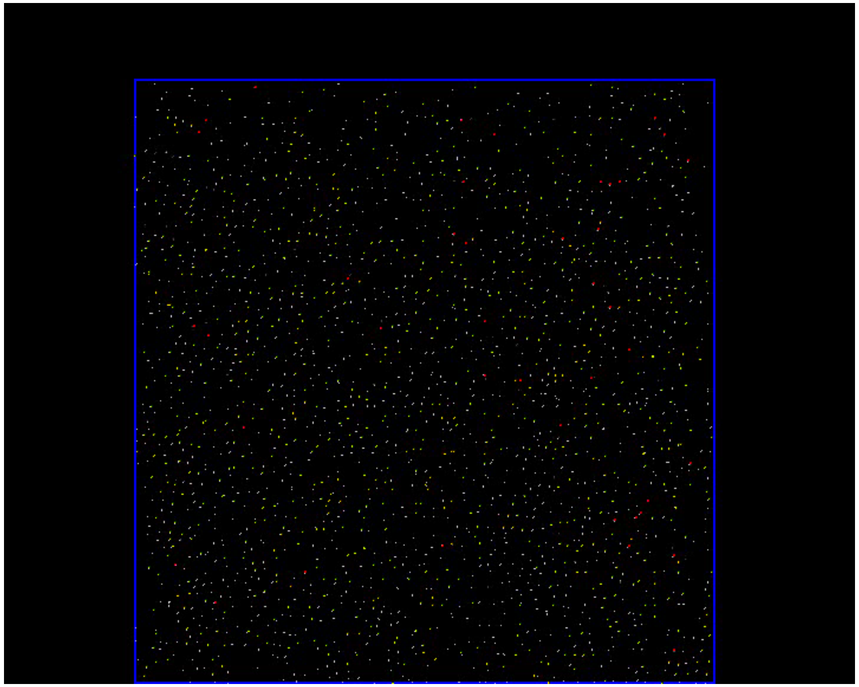
(P. Tabeling, M. Rivera, B. Ecke, J. Gollub, G. van Heijst)

Two layers of different densities
the lower driven by Lorentz force
(array of magnets).

Different implementations:
  * salty water/fresh water
  * immiscible layers (fluorinert/salt water)
PIV data acquisition

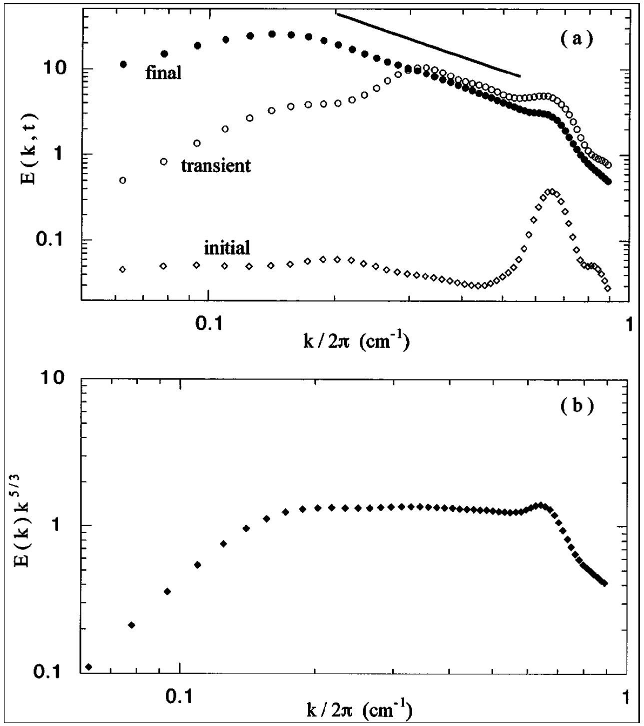


courtesy of B. Ecke



courtesy of S.Espa

Energy spectrum in inverse cascade

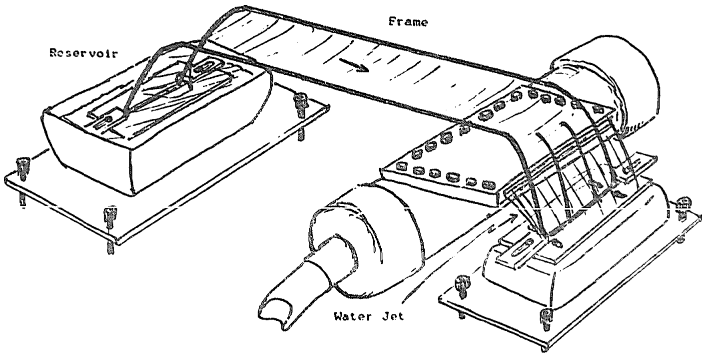


J. Paret, P.Tabeling, PRL **79** 4162 (1997)

Soap films

(Y. Couder, W. Goldburg, H. Kellay, M.A. Rutgers, M. Rivera, R.E. Ecke)

Soap films are very interesting because of the large aspect ratio (~10^4)

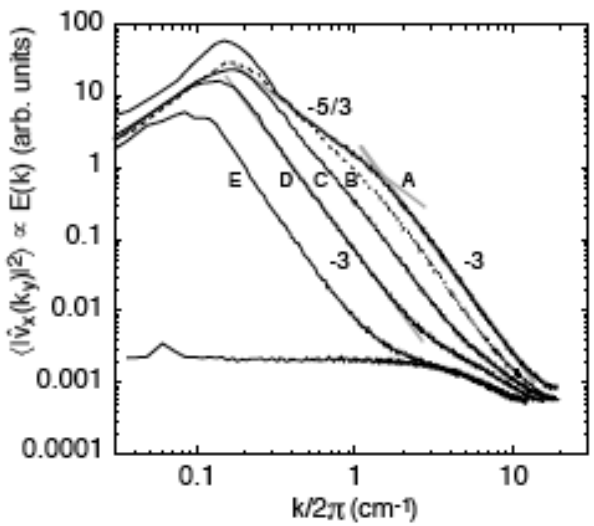


M.Gharib, P.Derango, Phys. D **37** (1989)

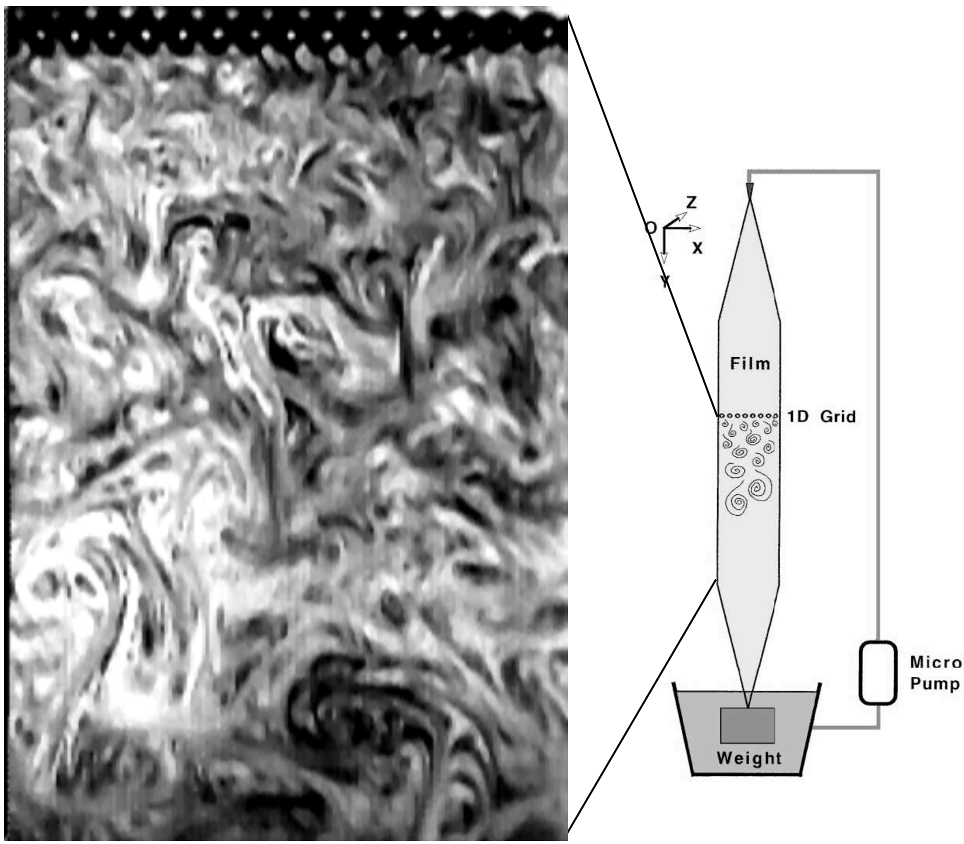
Evaporation limits the stability of the film: continuously running soap film channels with different geometries

Velocity acquisition by LDV and PIV

Turbulent spectrum in soap film channel



M.A. Rutgers, PRL **81**, 2244 (1998)

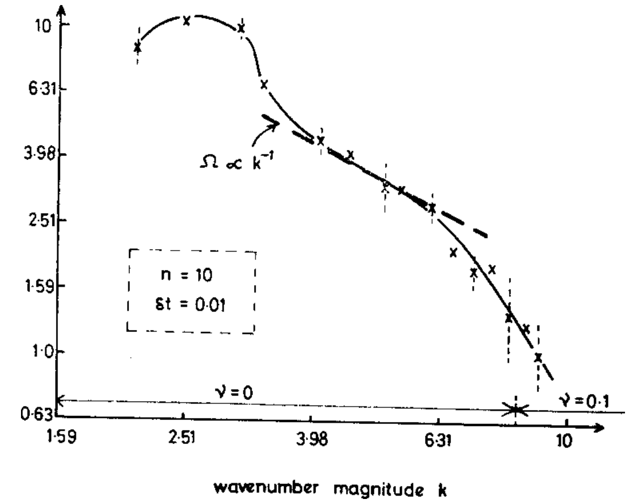


Direct numerical simulation: the beginning

2D turbulent is in principle very convenient for numerical simulations (e.g. a 1024^3 simulation corresponds to about 56000^2 for memory occupation).

Not so convenient for integration time (which is proportional to N^{D+1}, i.e. 1024^3 corresponds to 15000^2).

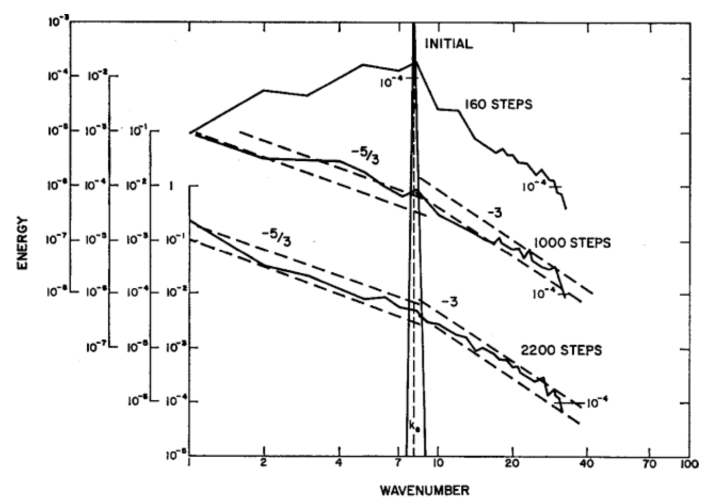
Even less convenient if we want to simulate both cascades (e.g. resolution of 100^2 for each cascade requires $>10000^2$ resolution)



G.K.Batchelor, POF **12** (1969)

First DNS (R.W.Bray, 1961):
pure spectral code with $1 \leq k \leq 10$
Study of the direct cascade

Lilly, 1969
finite difference scheme at 64^2
"Observation" of the double cascade
Kolmogorov constant for inverse C=4.2-6.2



D.K.Lilly, POF **12** (1969)

Pseudospectral simulations

In 1971 Orszag [JAS **28**, 1074] shows how to remove aliasing efficiently (2/3 rule, after Phillips, 1959) paving the way to the use of (pseudo)-spectral codes both in 2D and 3D.

Pseudospectral codes are very efficient: not changed in 35 years of DNS

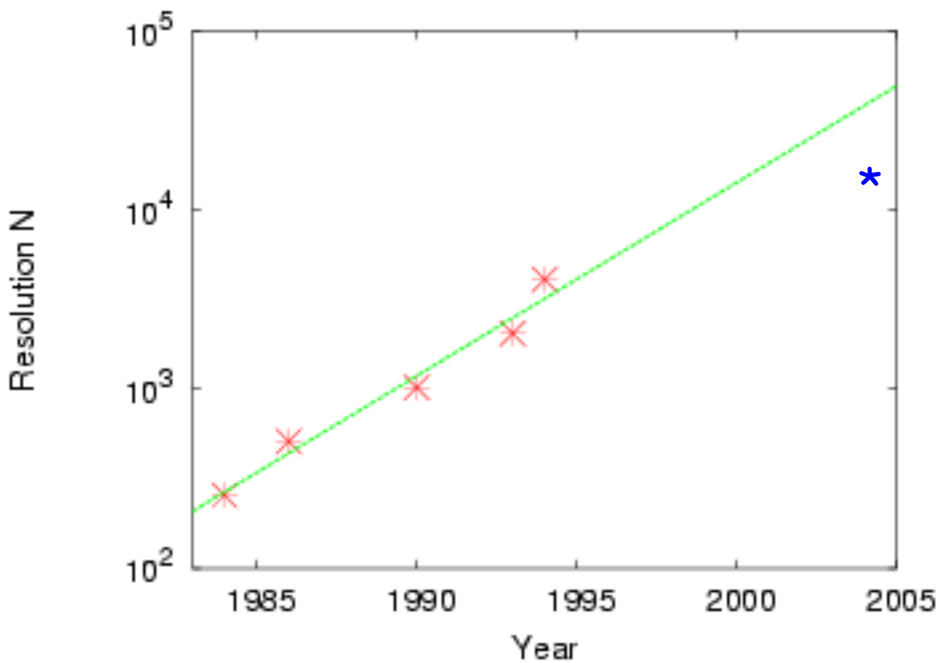
From Frisch & Sulem (1984) at 256^2 to Borue (1993) at 4096^2 the resolution growth is fitted by

$$N = 267 * 2^{(t-1984)/2.8}$$

Doubling time close to Moore's law:

$t=3x1.07=3.2$

Evolution of resolution in 2D simulations



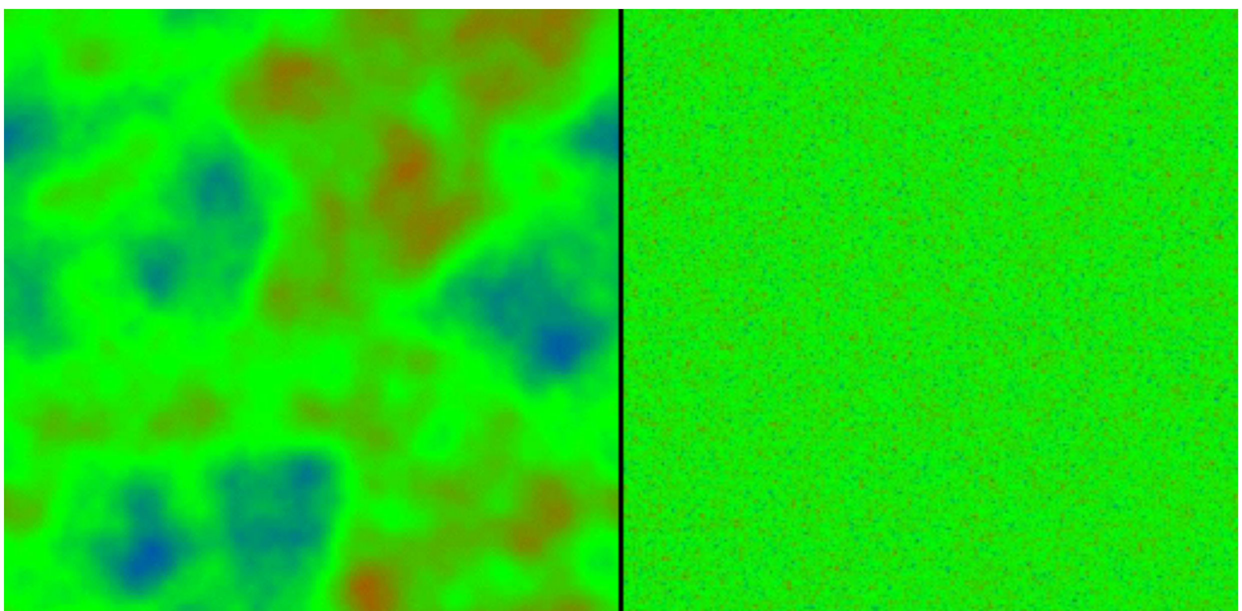
Direct numerical simulations of 2d turbulence

G. Boffetta, JFM (2007)
G.Boffetta and S.Musacchio, PRE (2010)

Set of simulations at high resolutions with a parallel pseudo spectral code.

N	ν	α	L/r_f	r_f/r_ν	$\varepsilon_\alpha/\varepsilon_f$	η_ν/η_f
2048	2×10^{-5}	0.015	100	13	0.54	0.96
4096	5×10^{-6}	0.024	100	26	0.83	0.92
8192	2×10^{-6}	0.025	100	40	0.92	0.90
16384	1×10^{-6}	0.03	100	57	0.95	0.88
32768	2.5×10^{-7}	0.0	100	116	0.98	0.98

ψ ω



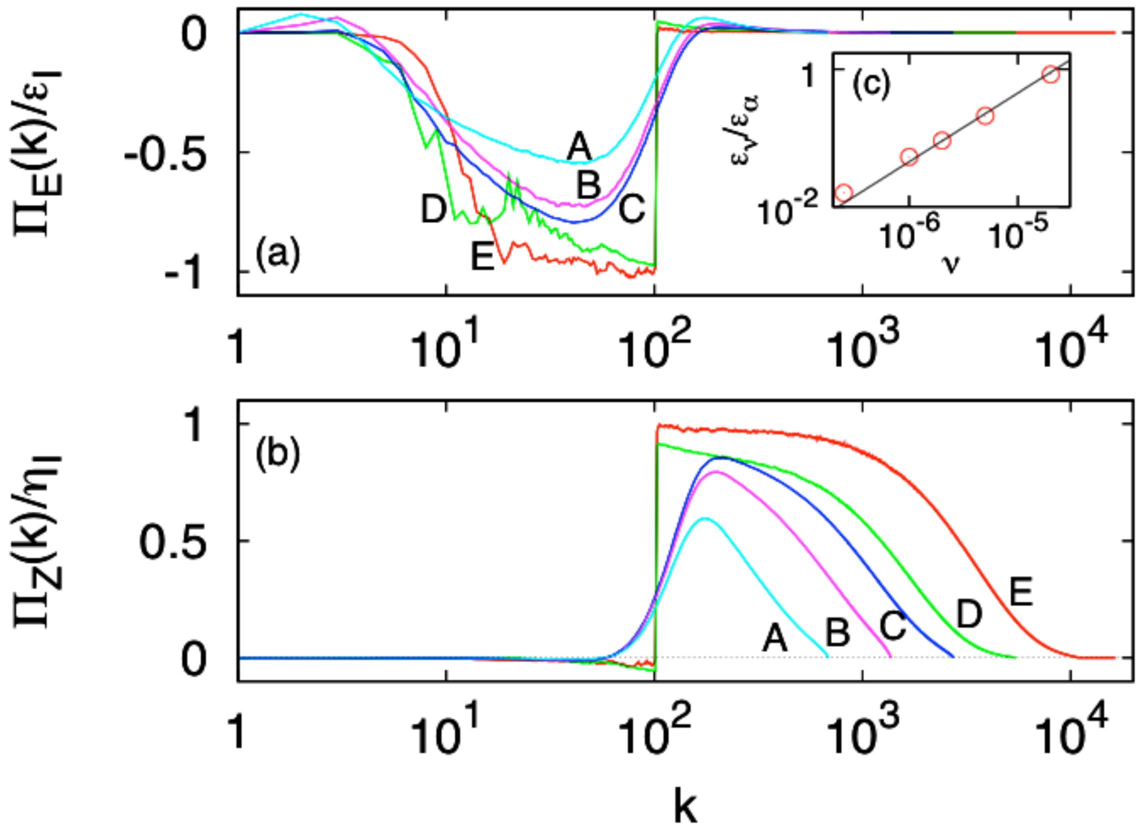
$$\frac{\varepsilon_\nu}{\varepsilon_\alpha} = \left(\frac{\ell_\nu}{\ell_f}\right)^2 \left(\frac{\ell_f}{\ell_\alpha}\right)^2 \frac{(\ell_\alpha/\ell_f)^2 - 1}{1 - (\ell_\nu/\ell_l)^2}$$

$$\frac{\eta_\nu}{\eta_\alpha} = \frac{(\ell_\alpha/\ell_f)^2 - 1}{1 - (\ell_\nu/\ell_f)^2}$$

Energy and enstrophy fluxes

Inverse cascade:
constant flux of energy

Direct cascade:
constant flux of enstrophy



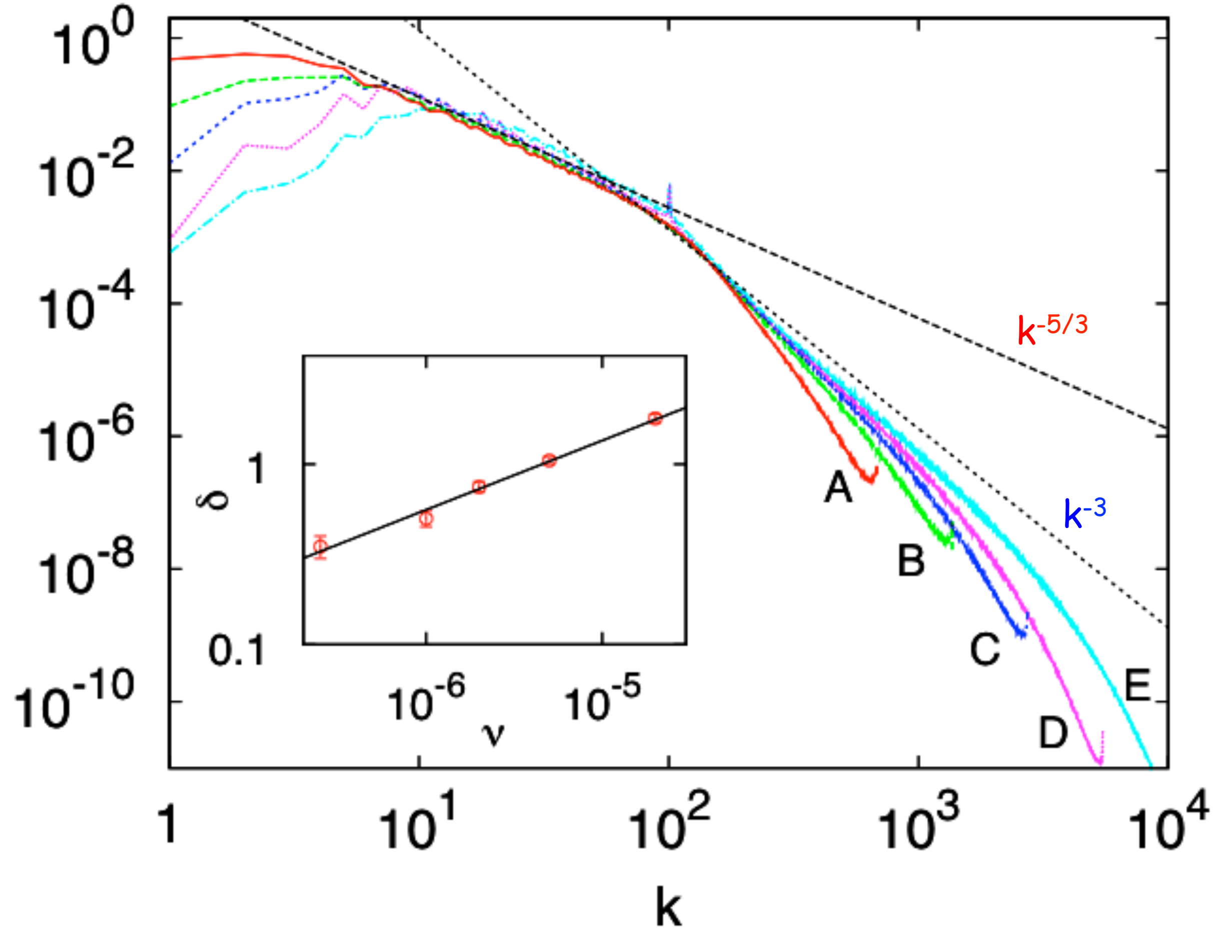
Prediction for fluxes ratio

$$\frac{\varepsilon_\nu}{\varepsilon_\alpha} = \left(\frac{\ell_\nu}{\ell_f}\right)^2 \left(\frac{\ell_f}{\ell_\alpha}\right)^2 \frac{(\ell_\alpha / \ell_f)^2 - 1}{1 - (\ell_\nu / \ell_f)^2} \propto \ell_\nu^2 \simeq \nu$$

enstrophy dissipative scale $\ell_\nu = \nu^{1/2} \eta_\nu^{-1/6}$

Energy spectrum

Simultaneous observation of direct and inverse cascades



* Inverse cascade:
$k^{-5/3}$ spectrum

* Direct cascade:
$k^{-(3+\delta)}$ spectrum with correction δ
which vanishes as $\nu \to 0$

Third order structure function of 2D turbulence

$$S_p(r) \equiv \left\langle \left(\delta u(r) \right)^p \right\rangle$$

$$\delta u(r) \equiv u(x+r) - u(x)$$

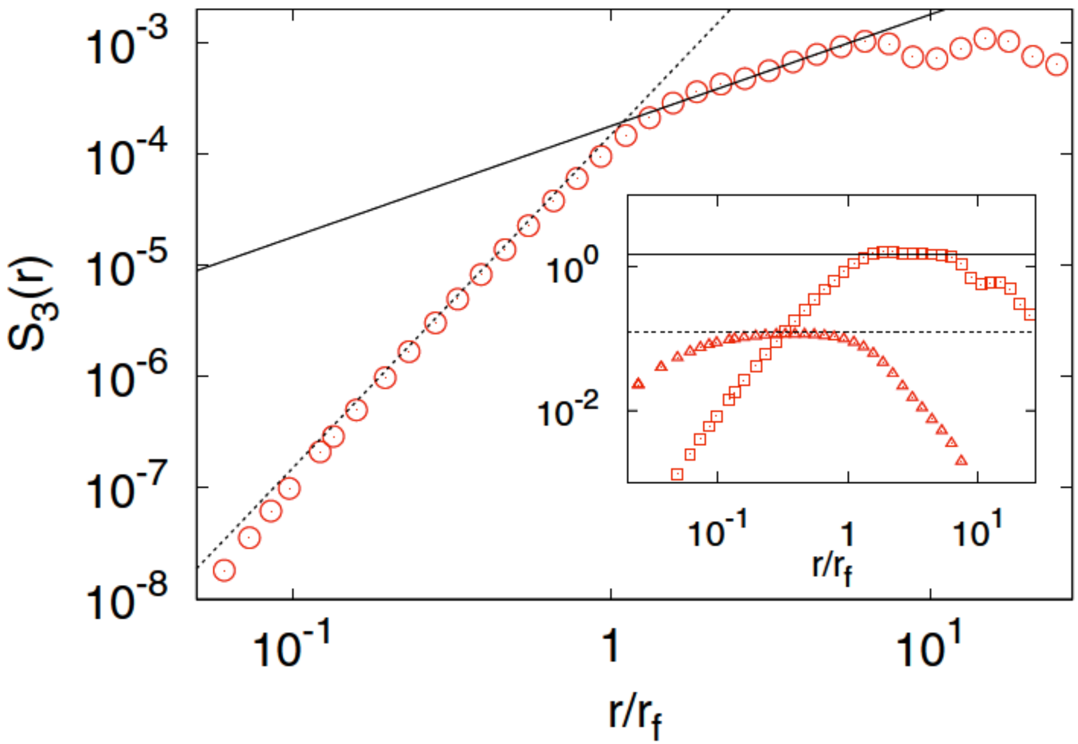
Fluxes of energy and enstrophy in physical space

$$S_3(r) = \frac{1}{8}\eta_\nu r^3 \quad \text{for} \quad r \ll \ell_f$$

Direct enstrophy cascade

$$S_3(r) = \frac{3}{2}\varepsilon_\alpha r \quad \text{for} \quad r \gg \ell_f$$

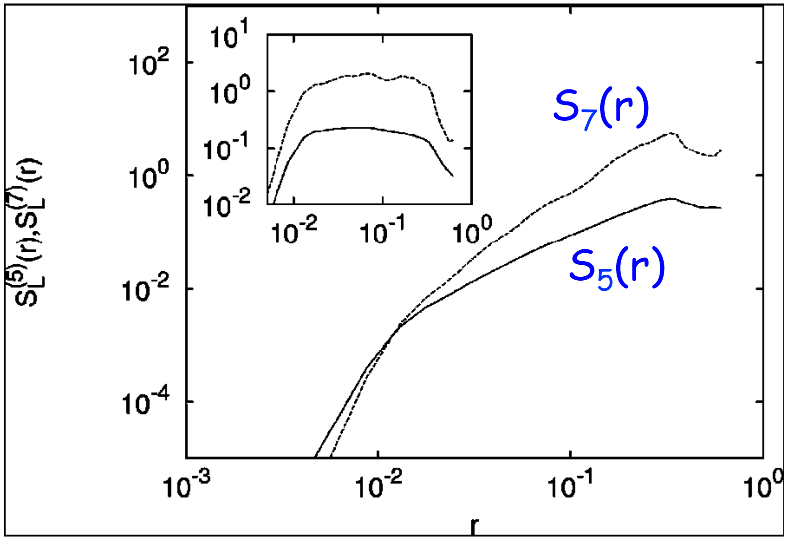
Inverse energy cascade



Higher order structure functions

$$S_p(r) = \left\langle \left(\delta u_{//}(r)\right)^p \right\rangle = C_p \left(\varepsilon r\right)^{p/3}$$

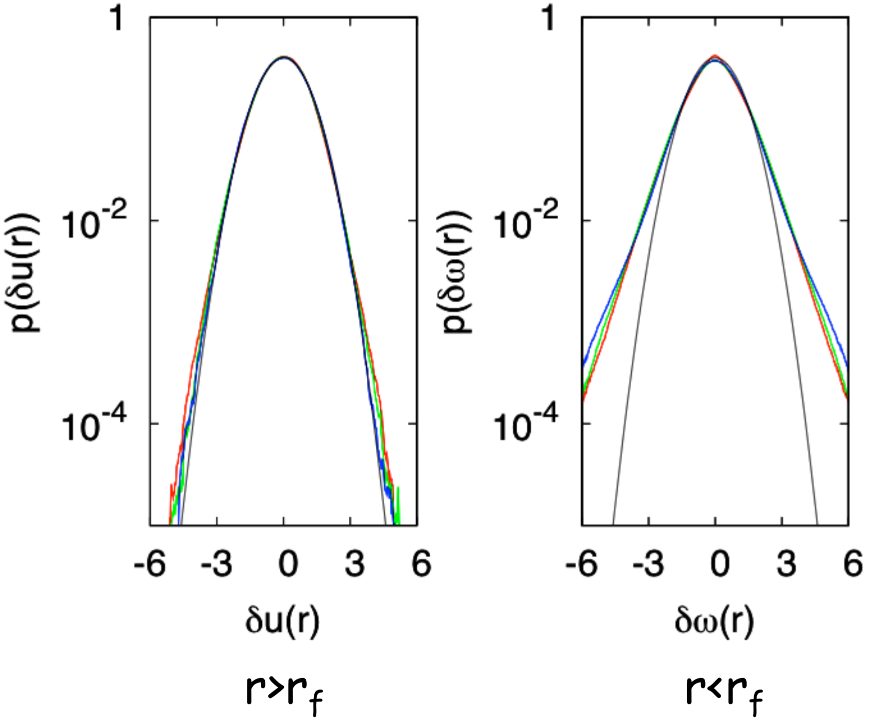
compatible with Kolmogorov scaling
no intermittency



Probability density functions of

velocity: close to Gaussian in the inverse cascade (left)

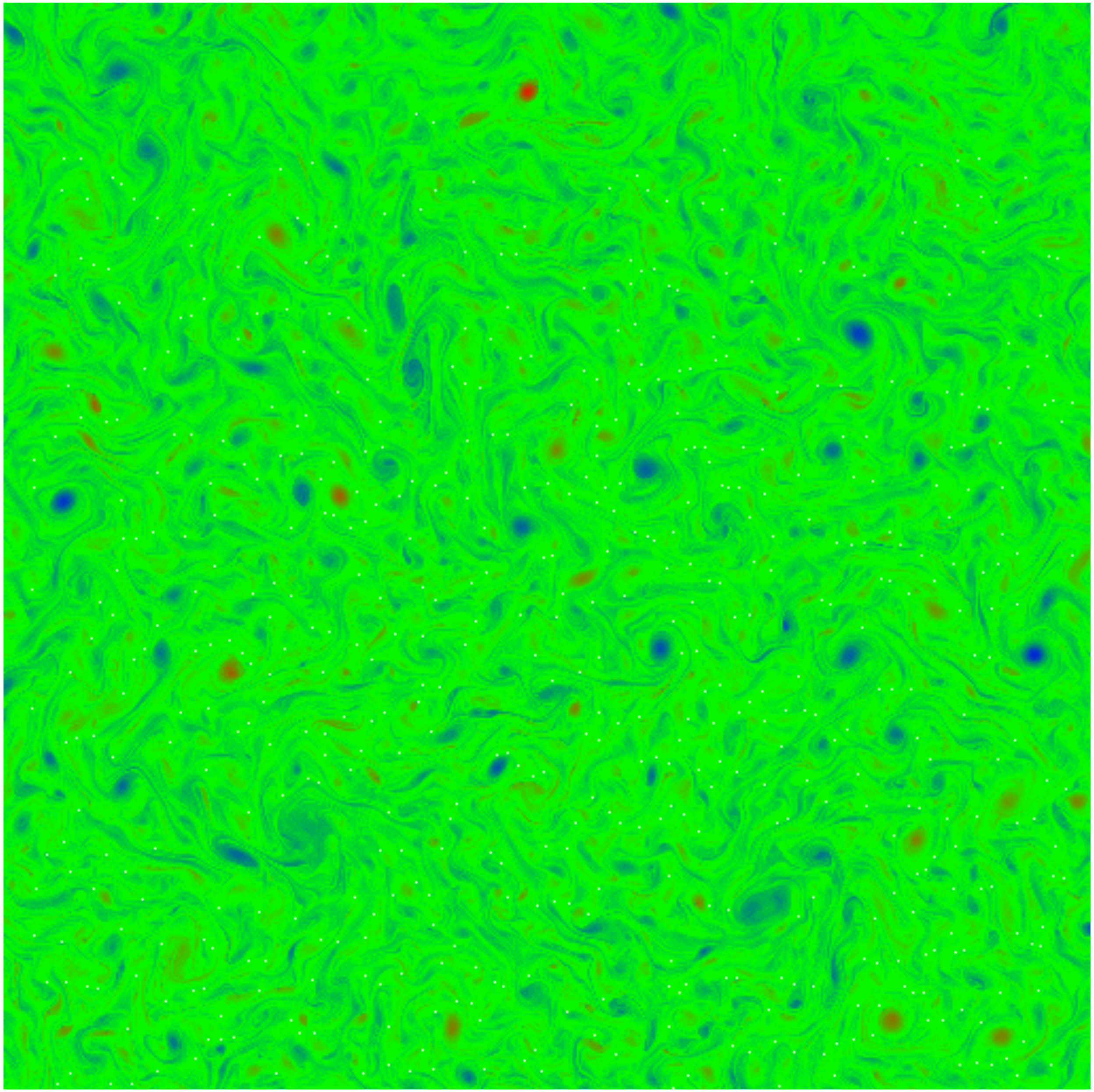
vorticity: self-similar in the direct cascade (right)



Conformal invariance in the inverse cascade

D.Bernard, G.Boffetta, A.Celani and G.Falkovich

Nature Phys. **2**, 124 (2006)
Phys. Rev. Lett. **98**, 024501 (2007).



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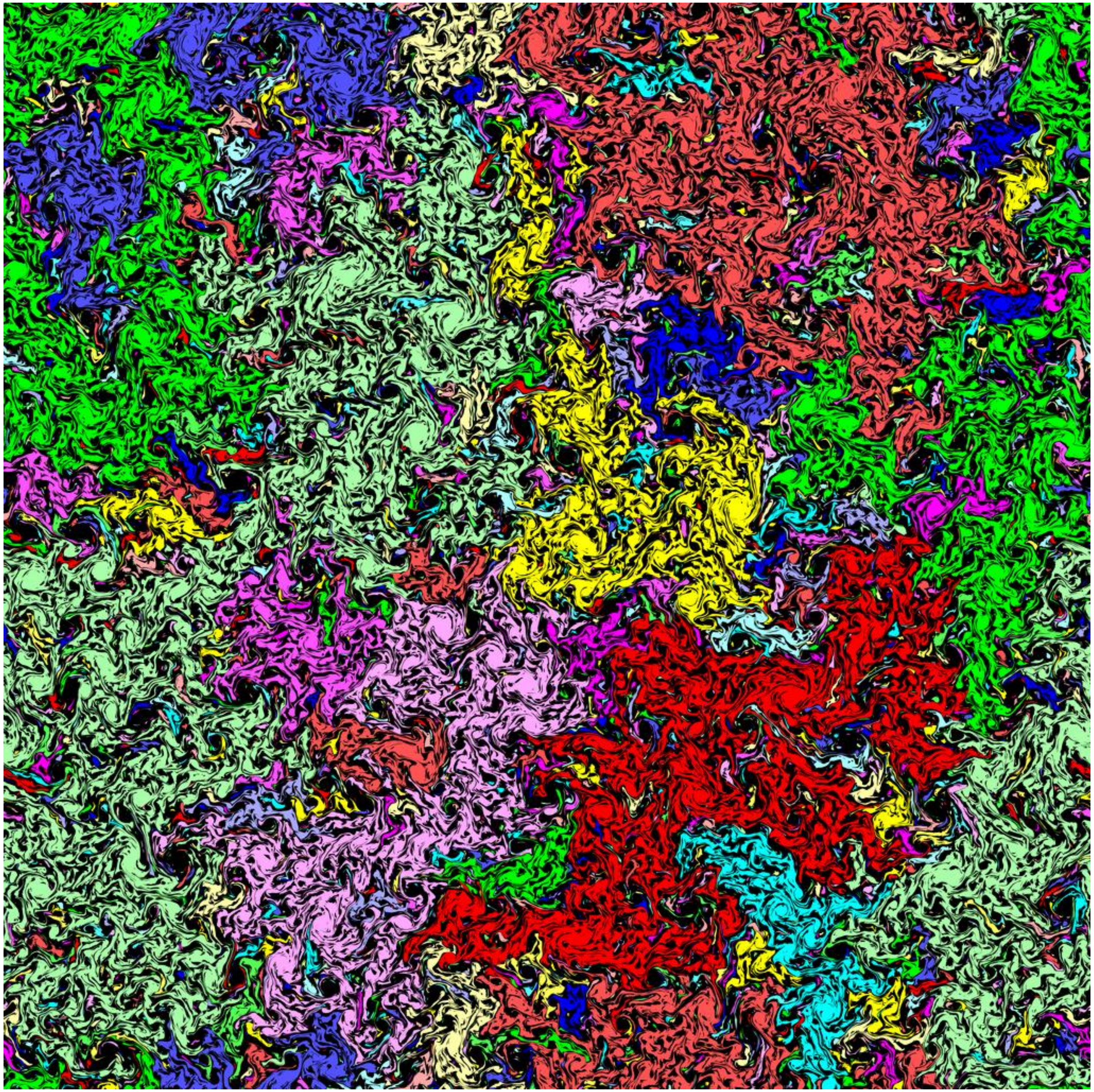
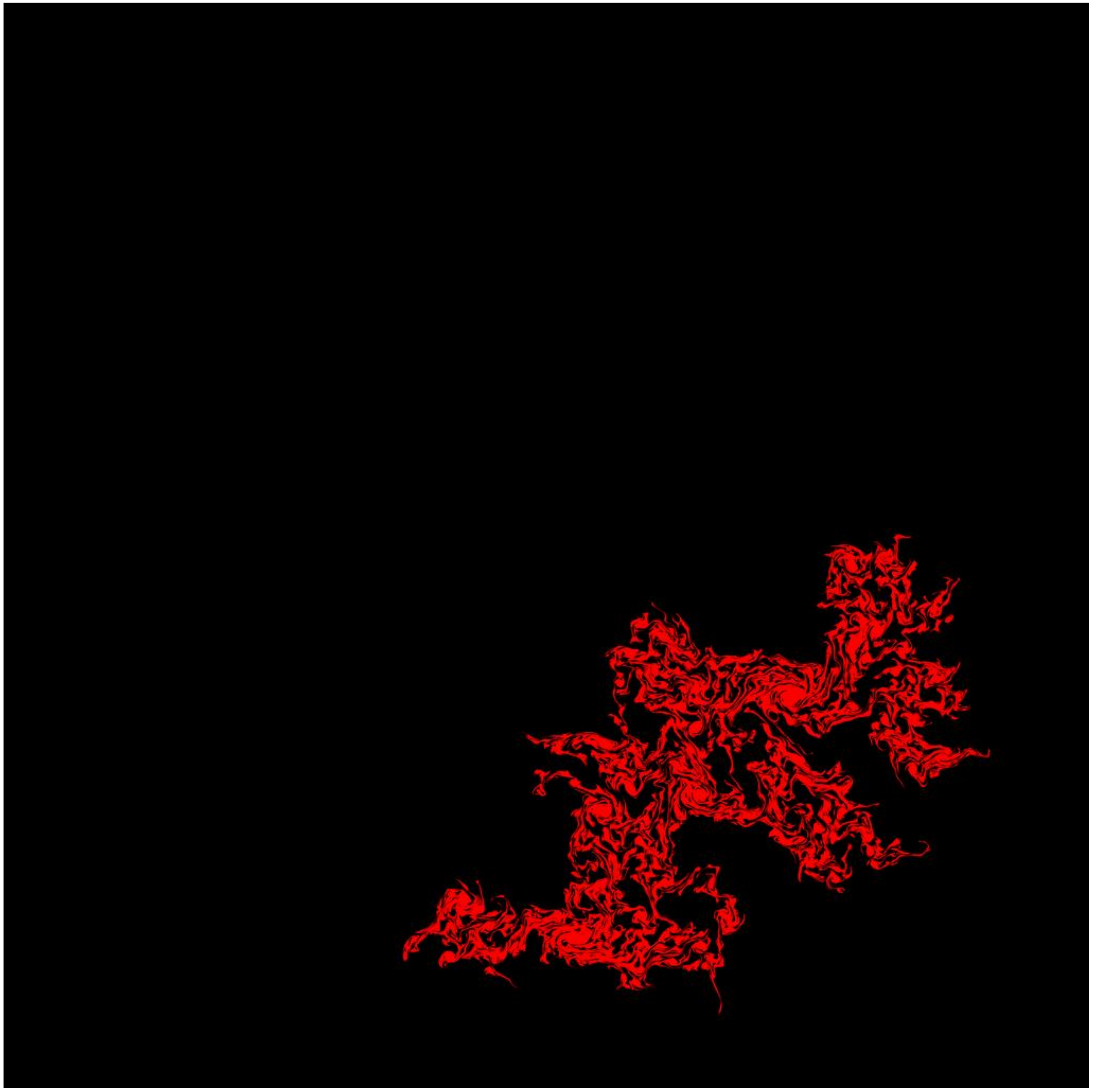
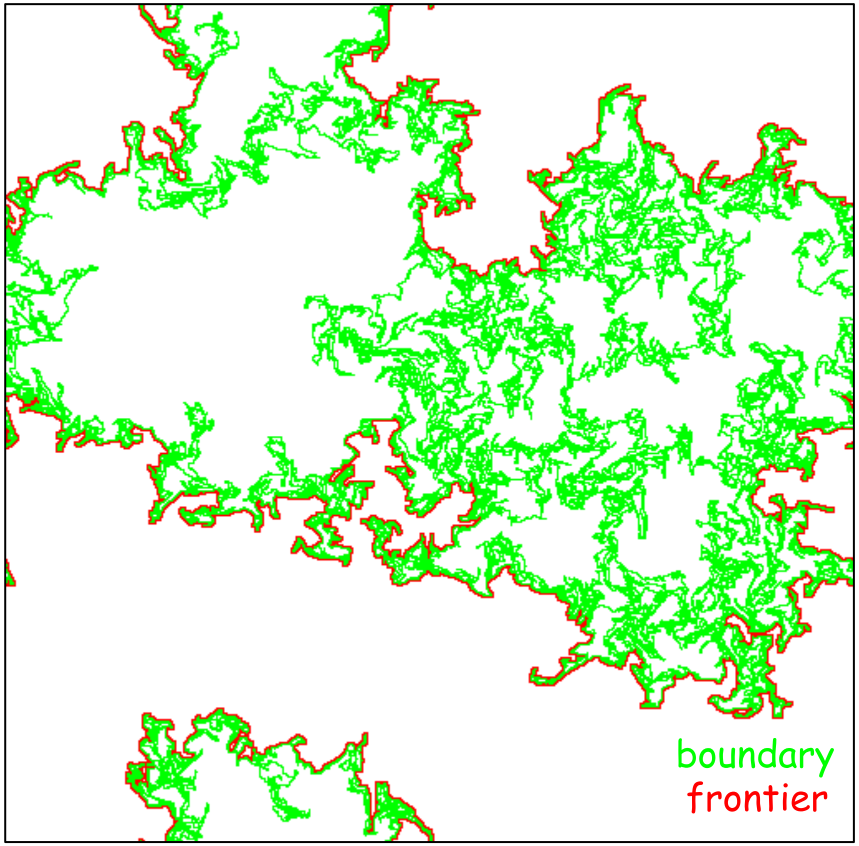


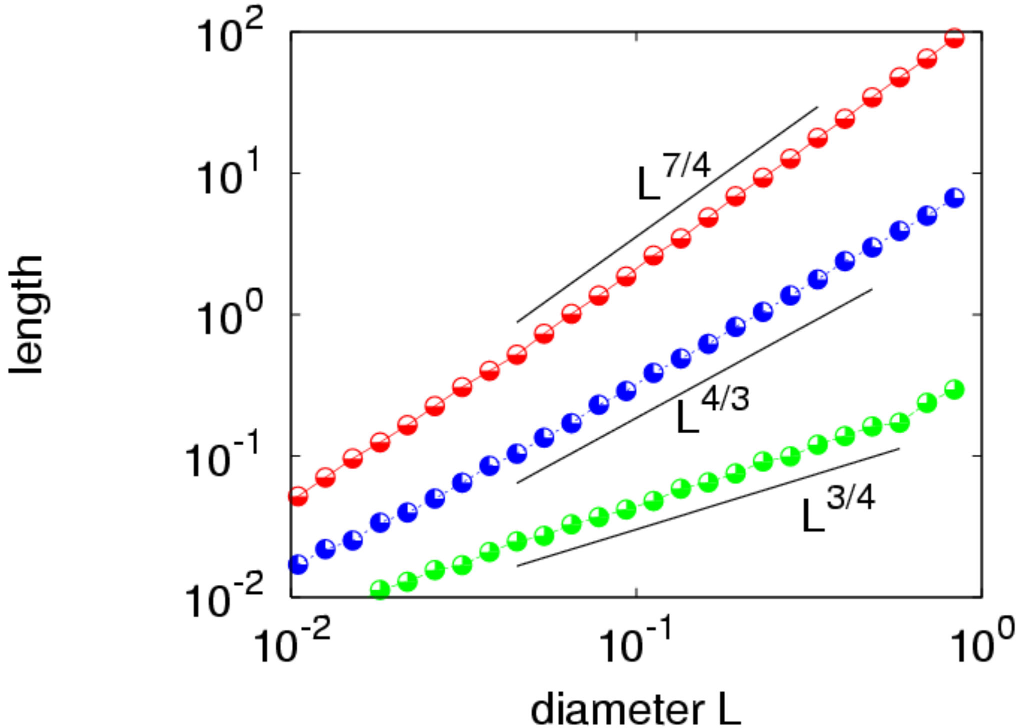
Positive vorticity clusters in the inverse cascade of 2d turbulence



Positive vorticity cluster in the inverse cascade of 2d turbulence

Fractal dimensions of a single vorticity cluster



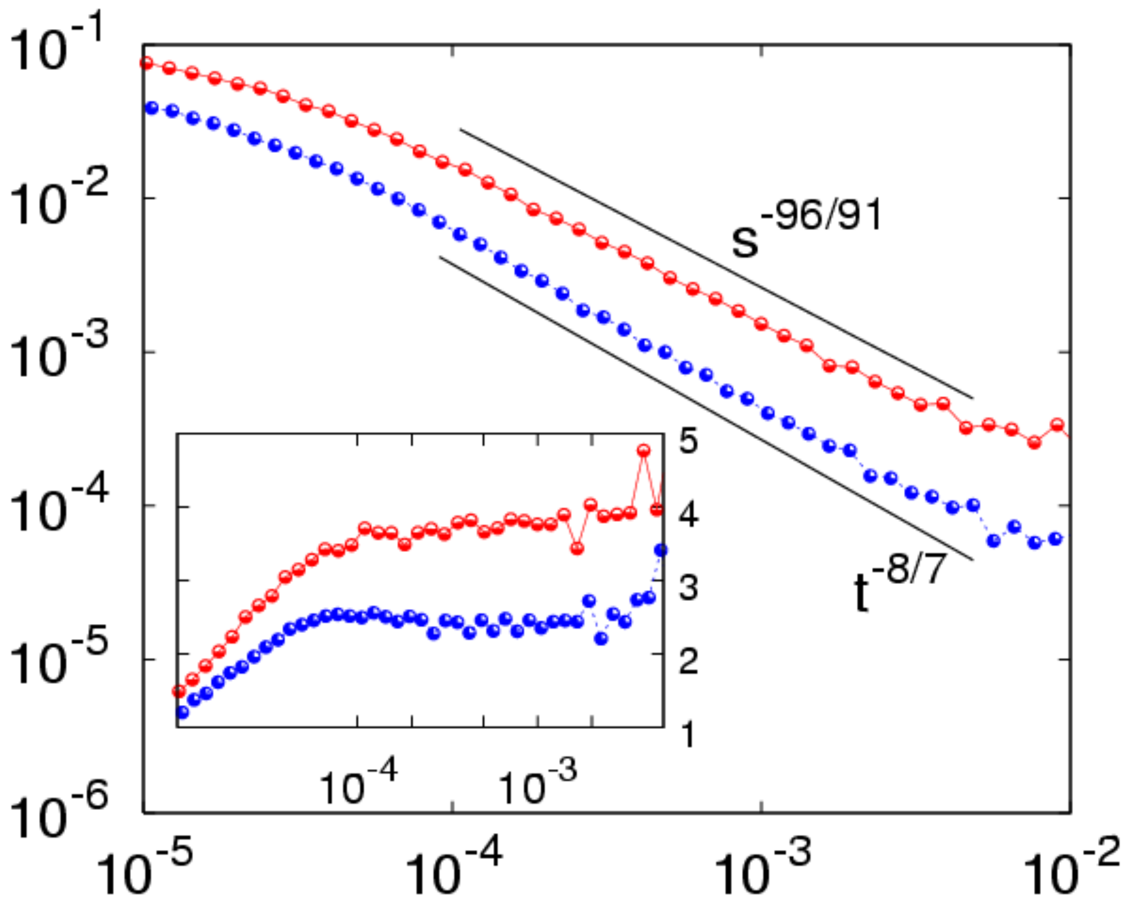


As in critical percolation

- ☆ Boundary
- ☆ Frontier
- ☆ Cut points

L=side of square covering the cluster

Probability distribution of vorticity clusters



☆ Size
☆ Boundary
__ critical percolation

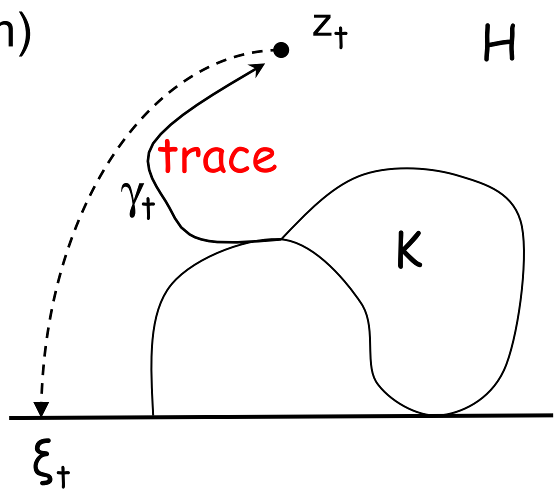
see Cardy and Ziff, J.Stat. Phys. **110**, 1 (2003)

Is the inverse cascade equivalent (geometrically) to critical percolation ?

size s= # connected sites of same sign
boundary t= # connected sites adjacent to opposite sign

Conformal mapping for growth processes: Loewner equation (1923)

A curve γ_t growing in H from the origin (t parameterizes the curve).
The evolution of γ_t can be mapped on the evolution of the map $g_t(z)$ which map the complement of γ_t (or H\K) on H (Riemann theorem) (while γ is mapped on **R**):

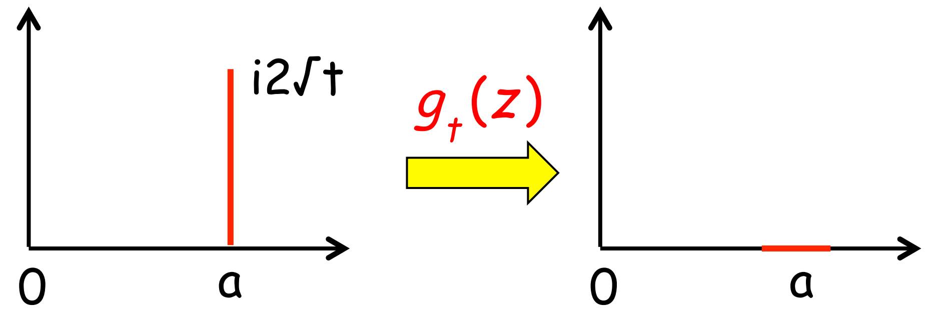


$$\boxed{\frac{dg_t(z)}{dt} = \frac{2}{g_t(z) - \xi_t}}$$

with $g_0(z) = z$
and $g(z) \sim z + O(1/z)$ as $z \to \infty$
driving: $\xi_t \in \mathbf{R}$

Example solution to LE with $\xi_t = a = \text{const}$

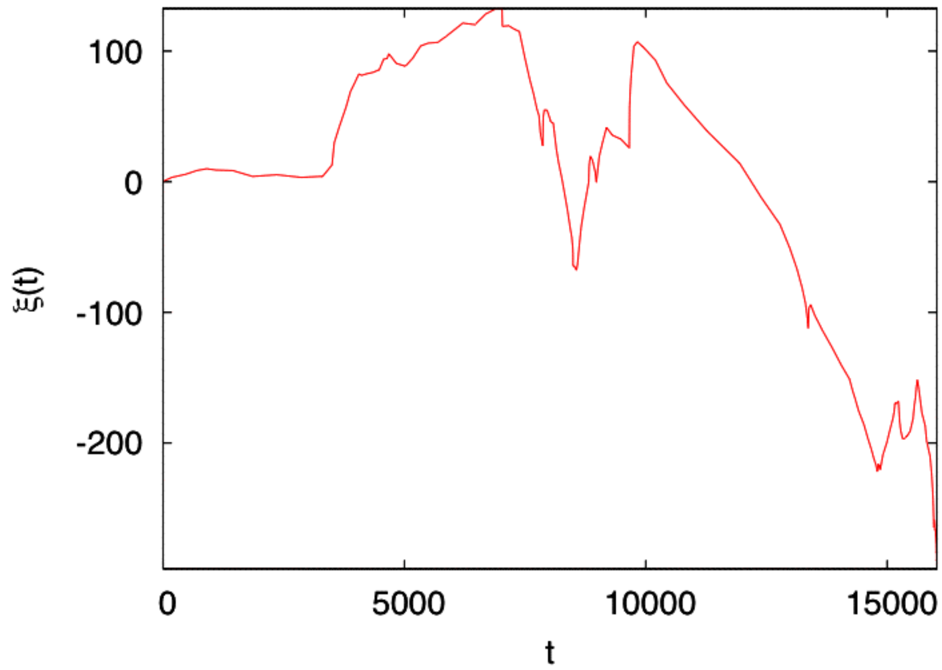
$$g_t(z) = a + \sqrt{(z-a)^2 + 4t}$$



i.e. a vertical segment of length $2\sqrt{t}$

An example
of Loewner evolution
(from driving to trace)

$$\frac{dg_t(z)}{dt} = \frac{2}{g_t(z) - \xi_t}$$



driving

trace

Stochastic Loewner Equation

O.Schramm (2000)
G.Lawler, O.Schramm, W.Werner (2001)
see J.Cardy (2005)

Loewner equation(1923) for conformal mapping with driving ξ_t : **R->R**

$$\frac{dg_t(z)}{dt} = \frac{2}{g_t(z) - \xi_t}$$

LE describes a conformally invariant curve when the driving is proportional to a random walk

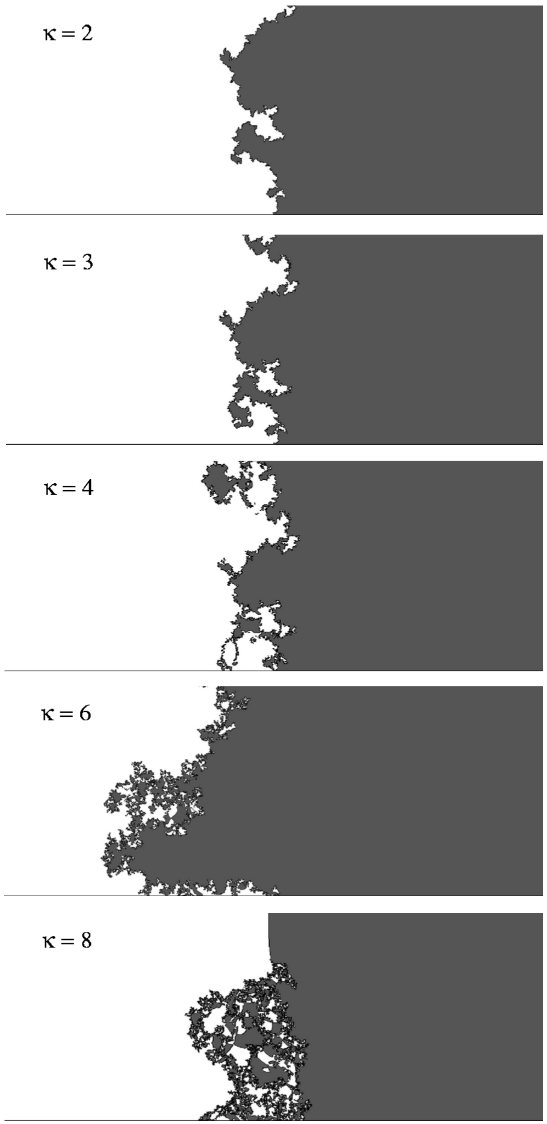
$$\xi_t = \sqrt{\kappa} B_t$$

The "diffusion coefficient" κ controls the fractality of the generated trace (Rohde & Schramm, 2001)

* 0 < k < 4 simple curve
* 4 < k < 8 non-simple curve (∞ intersections)
* k > 8 space filling

Fractal dimension of SLE traces (Beffara, 2002)

$$D_F = 1 + \frac{\kappa}{8}$$



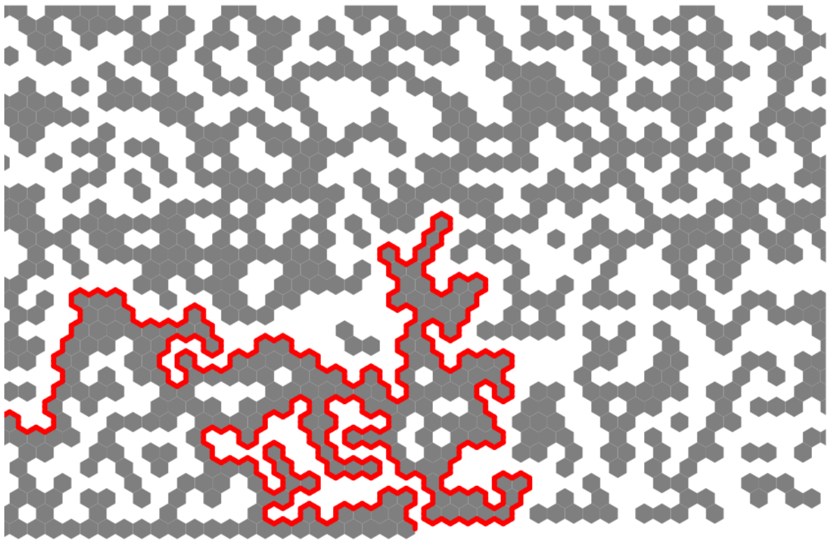
Some applications of SLE

Old conjecture by Mandelbrot (1982):
the frontier of BM is a SAW with D=4/3

Lawler, Schramm & Werner, 2000 (via SLE):

- pioneer points: D=7/4 (SLE_6)
- frontier: D=4/3 ($SLE_{8/3}$)
- cut points D=3/4

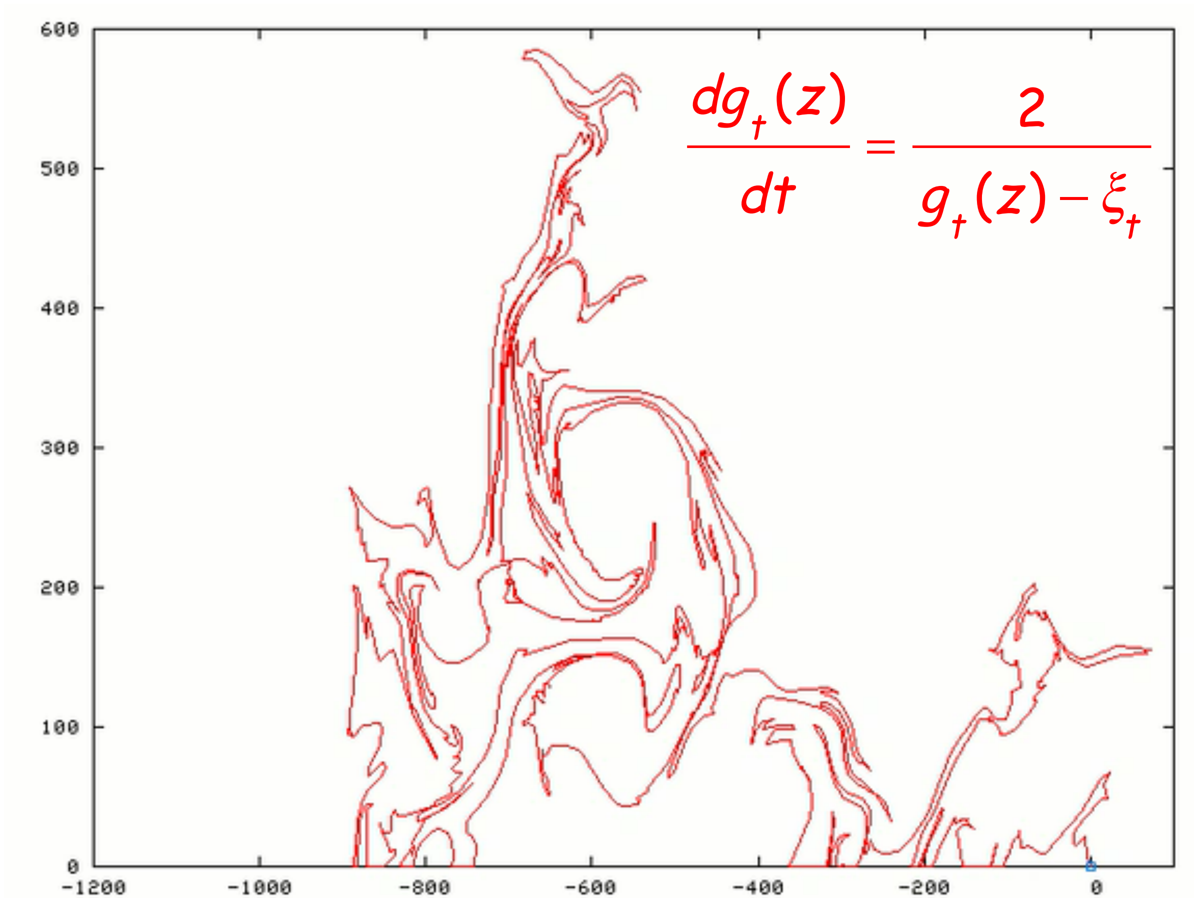
SLE_k and critical systems



- k=2 loop-erased random walk
- k=8/3 self avoiding random walk
- k=3 cluster boundaries in Ising
- k=4 isolines in O(2) model
- k=6 cluster boundaries in percolation
- k=8 uniform spanning trees

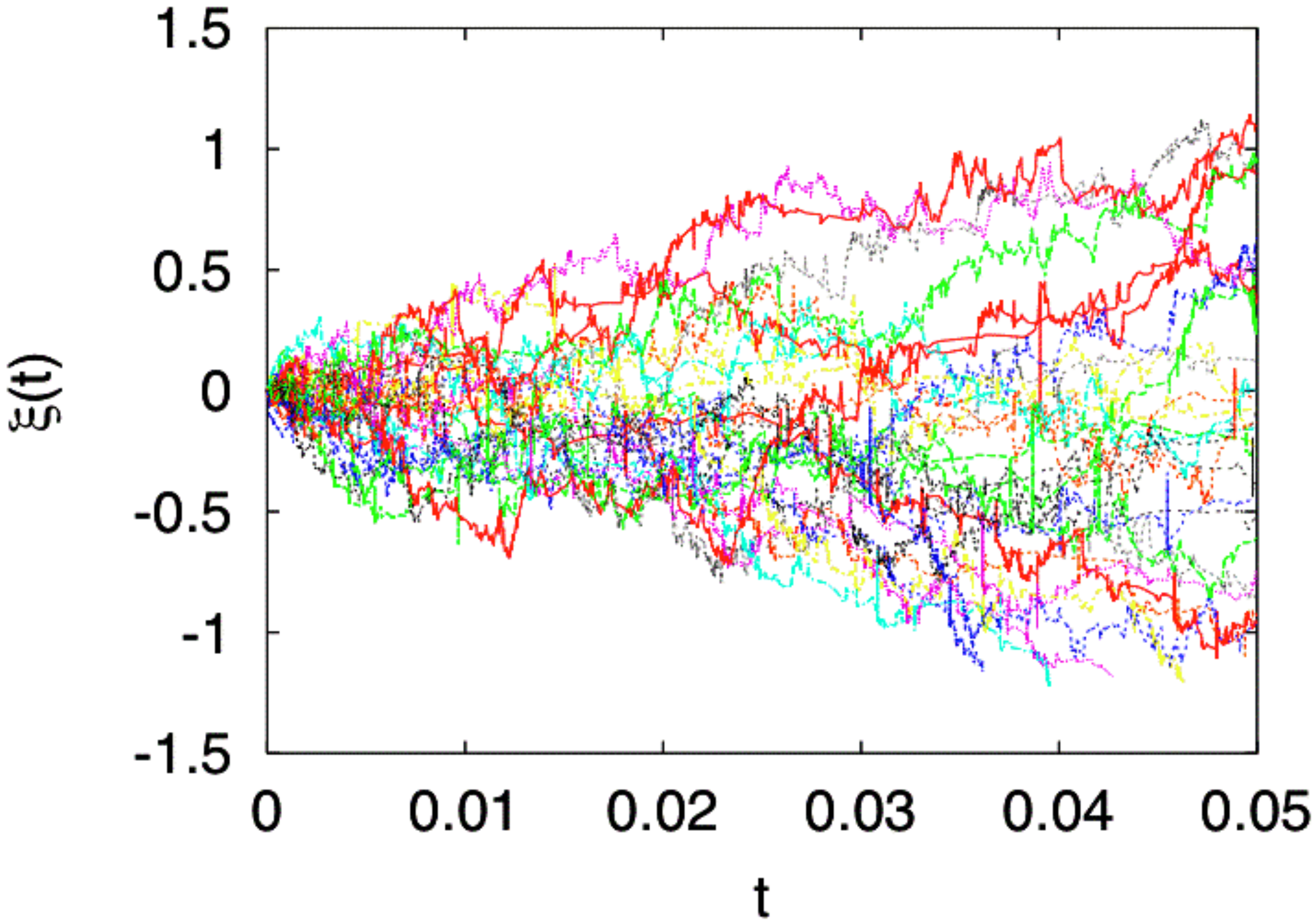
Checking SLE in vorticity clusters

By inverting SLE one computes driving function

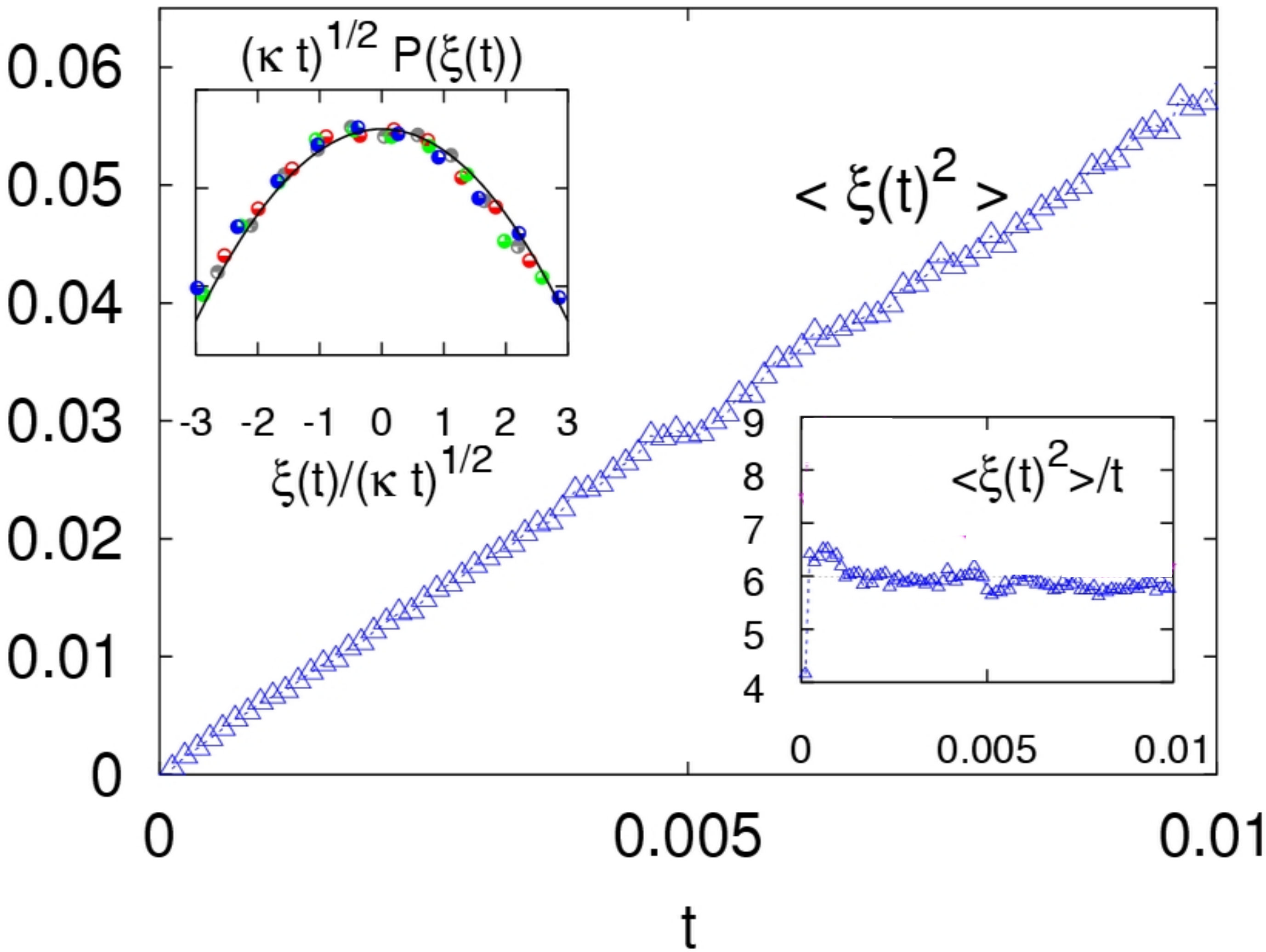


$$\frac{dg_t(z)}{dt} = \frac{2}{g_t(z) - \xi_t}$$

Driving functions



Driving $\xi(t)$ is Brownian motion ➔ zero-vorticity lines are SLE_k



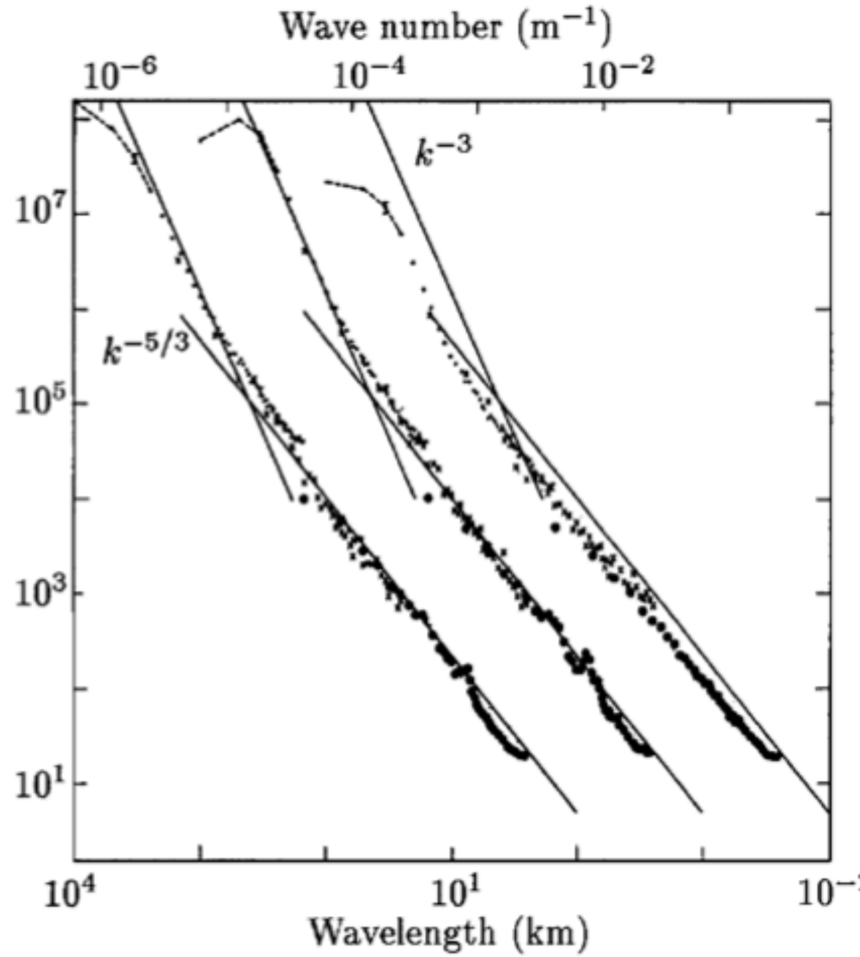
$\kappa = 5.9 \pm 0.3$ vorticity clusters are equivalent to critical percolation

Atmospheric data

Mesoscale wind variability
(radar and balloon): $k^{-5/3}$
K.S. Gage, J.Atmos.Sciences **36** (1979)

Global Atmospheric Sampling Program
(6900 Boing 747 flights):
$k^{-5/3}$ for wavelenghts 10-500 km
k^{-3} for wavelenghts 500-2000 km

see also Mozaic (on Airbus)



Kinetic energy (zonal and meridional wind) and potential energy spectra
Nastrom, Gage, Jasperson, Nature **310** (1984)
Gage, Nastrom, JAS **43** (1986)

Two-dimensional turbulence in the stratosphere ?

Interpretations of Gage-Nastrom spectrum

* Stratified turbulence (Gage, Lilly)

* Gravity wave cascade (Dewan)

* 2-level quasi-geostrophic model (Tung, Orlando)

* Rotation + stratification (Lindborg)

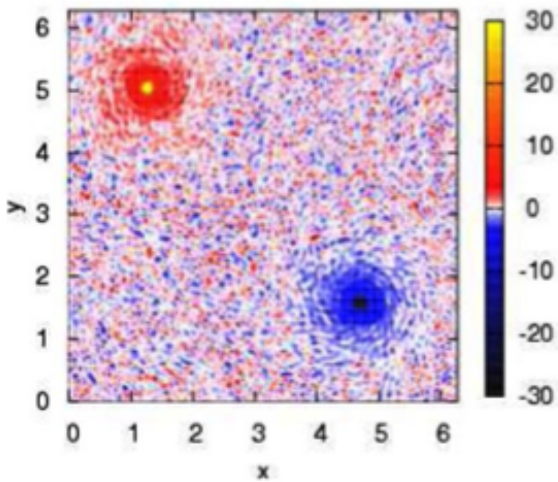
* Two-dimensional turbulence with condensation (Falkovich, Shats)

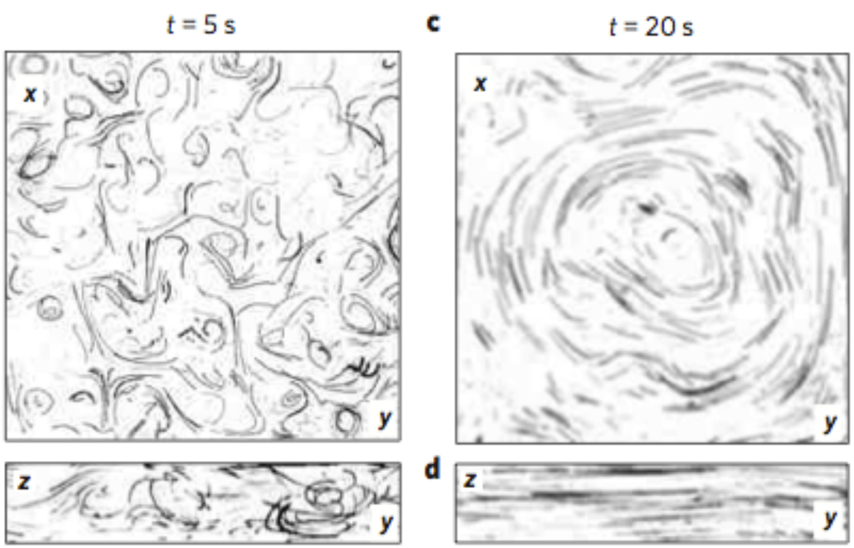
large scale spectrum steeper, close to k^{-3}

energy flux changes sign (nonlocal interactions)

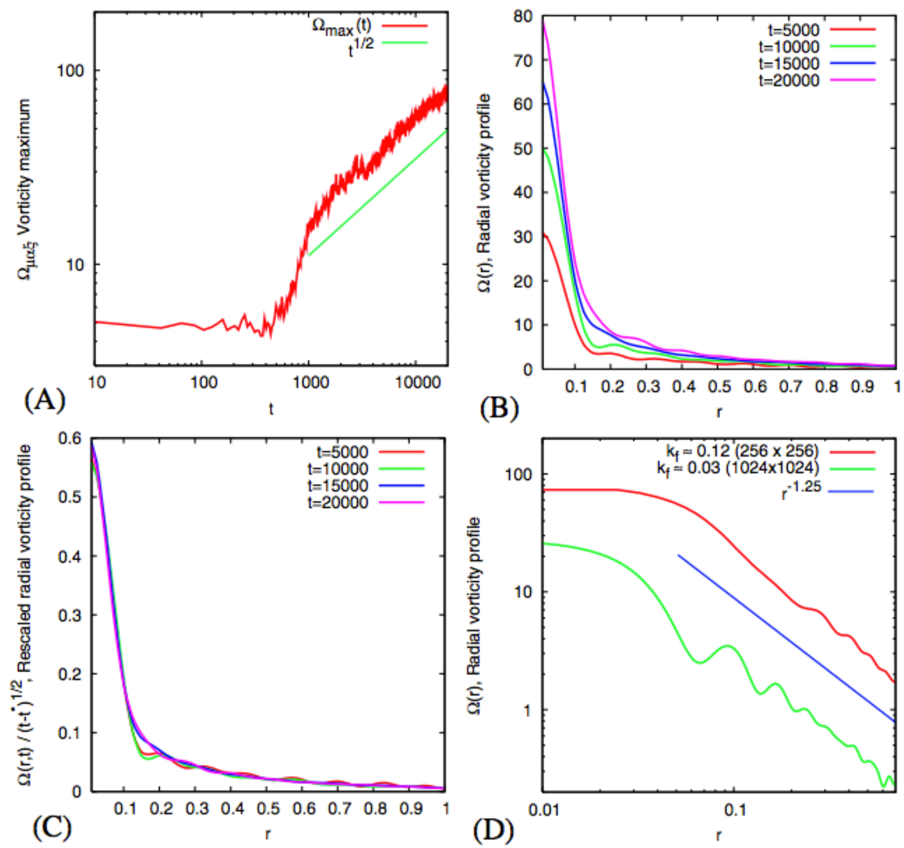
Statistics of the condensate

Experimental/numerical studies on the evolution and the statistics of the condensate



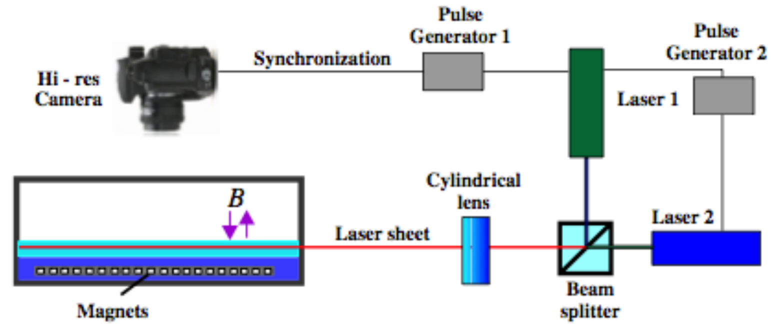


experiments by Xia et al., Nat. Phys. **7** (2011)



self-similar evolution of the condensate
Chertkov et al. Phys. Rev. Lett. **99** (2007)

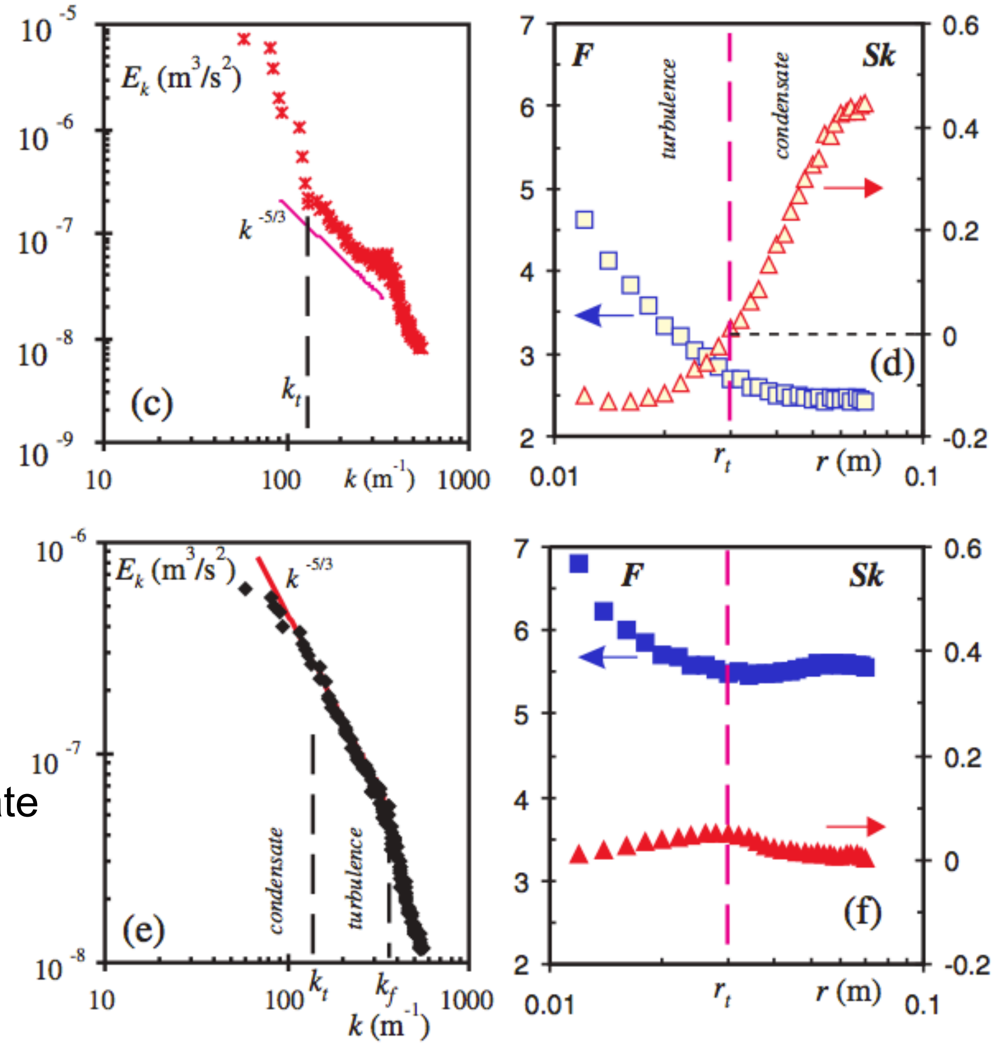
Experimental studies by Shats group
[Xia, Punzmann, Falkovich, Shats, PRL **101** (2008)]



show that the condensate
* makes the spectrum steeper at large scales
* changes sign in the 3rd order velocity SF

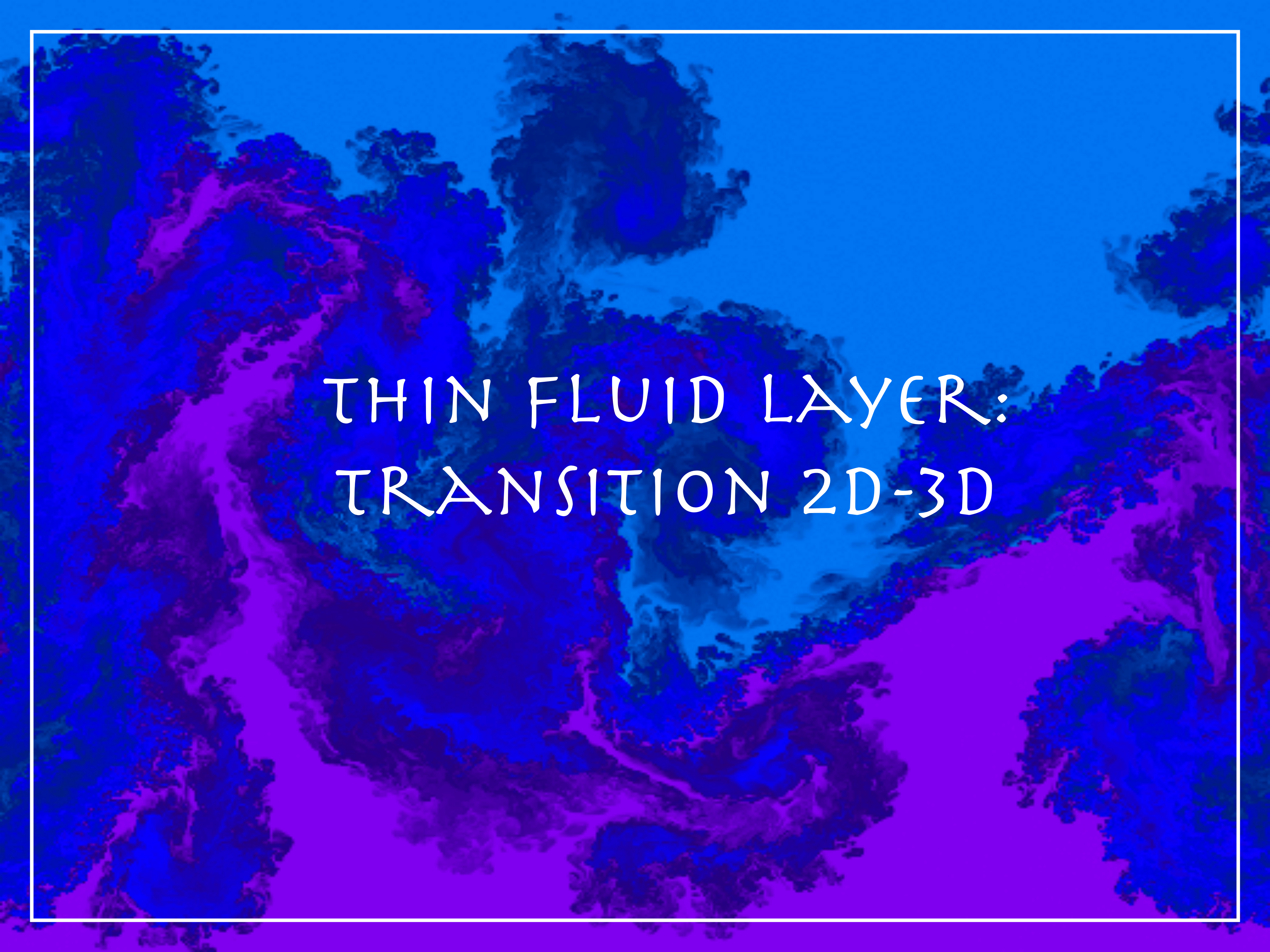
full field

without condensate

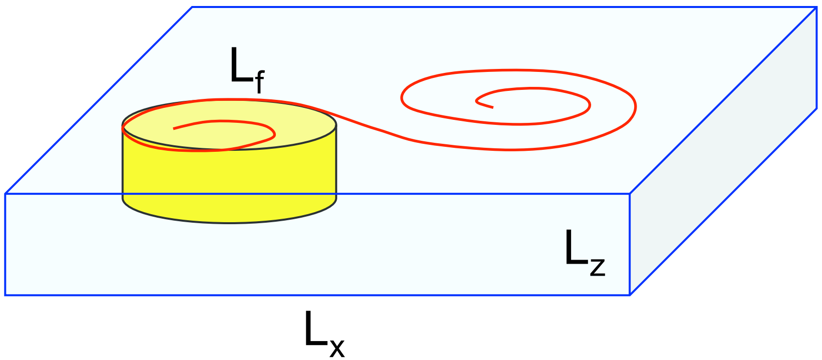


Gage-Nastrom spectrum:
2d turbulence + condensate ?

THIN FLUID LAYER: TRANSITION 2D-3D

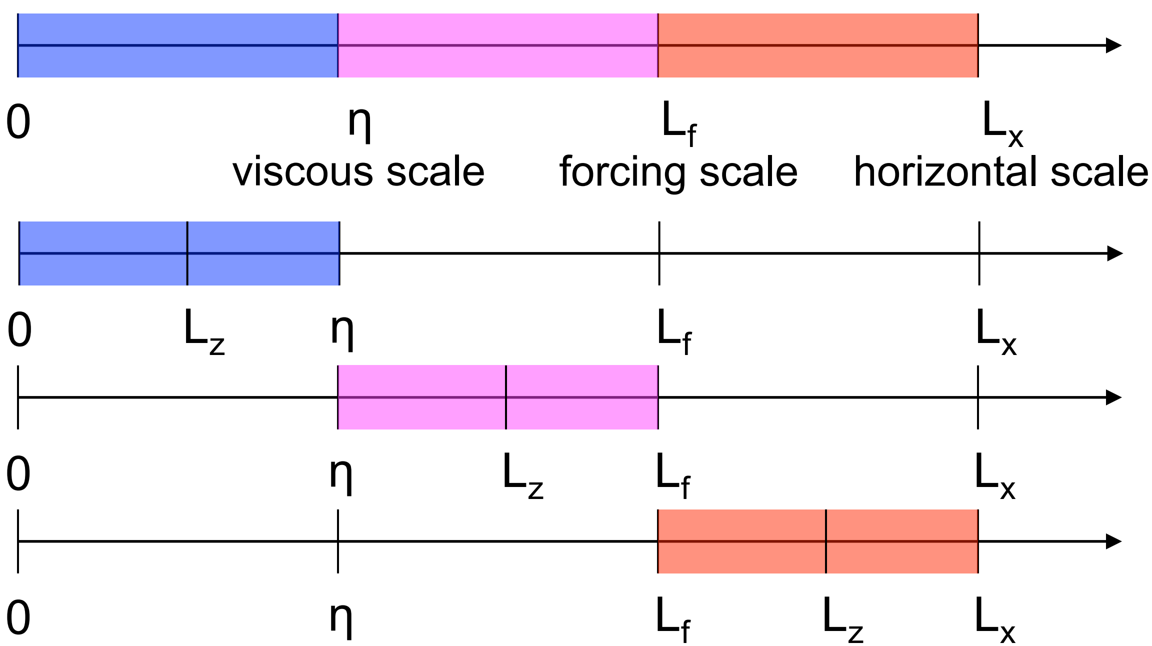


Dimensional transition in thin fluid layers



$L_x = L_y$ = horizontal scale
L_f = forcing correlation scale
L_z = vertical scale (thickness)
η = viscous scale

Transition from 2D to 3D turbulence as the thickness L_z increases



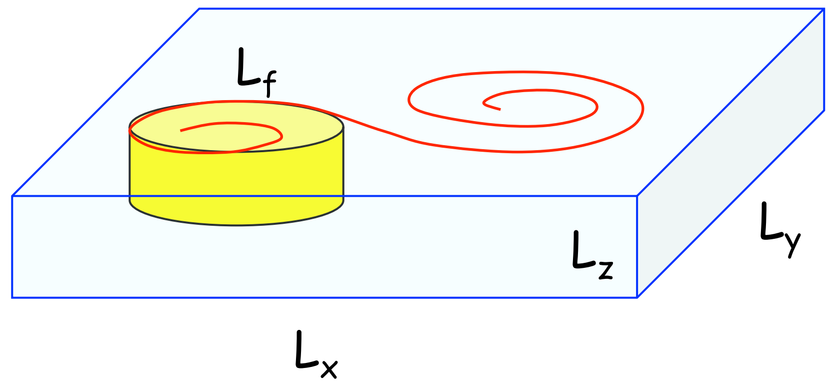
2D inverse cascade

?

3D direct cascade

Numerical simulations of thin fluid layers

3D Navier-Stokes equation for a thin layer of incompressible fluid.



$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

Two-dimensional random force $\mathbf{f}$
No friction

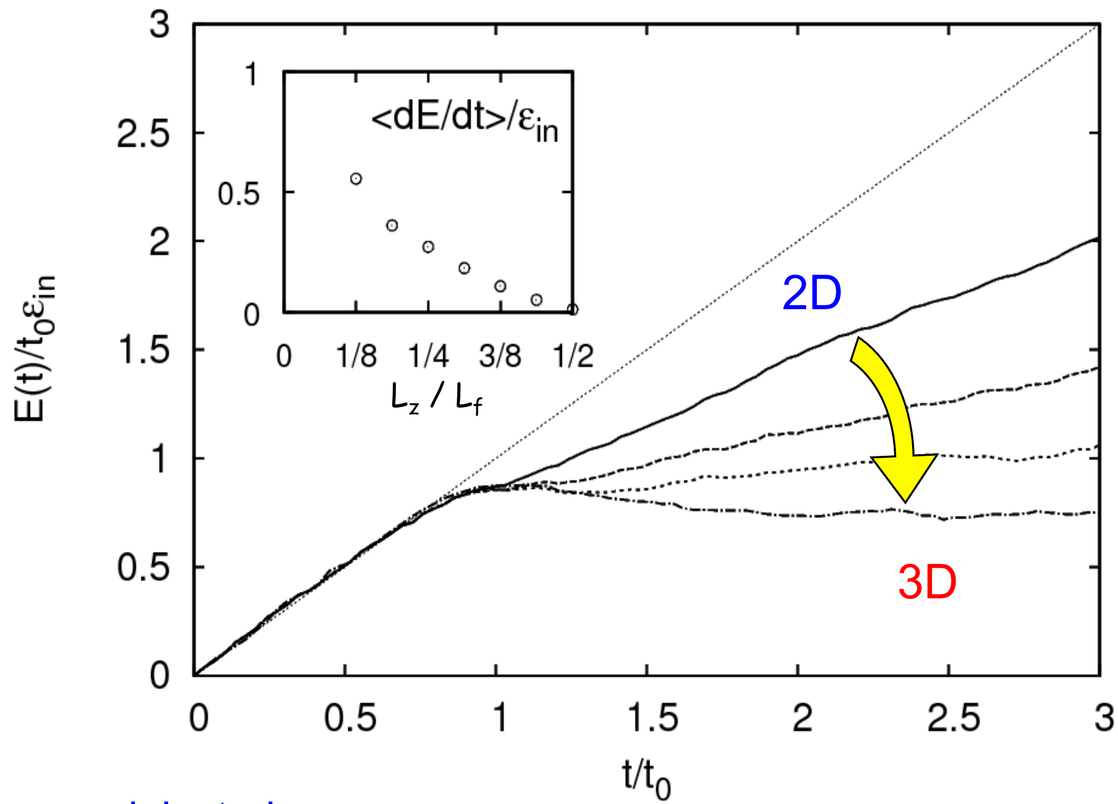
Aspect ratios $L_y / L_x = 1$ $L_x / L_f = 16$ $L_x / L_z = [\,32 - 128\,]$ $L_f / L_z = [\,2 - 8\,]$
Periodic b.c.: no material walls
$N_x \times N_y \times N_z = 4096 \times 4096 \times N_z$ grid points

Celani *et al.* (2010) PRL 104, 184506

Kinetic energy growth

Energy grows linearly in time
(in the absence of friction)

The growth rate decreases
as the thickness is increased



$L_z < \eta$ energy growth rate = power injected
$L_z > \frac{1}{2} L_f$ energy growth rate = 0

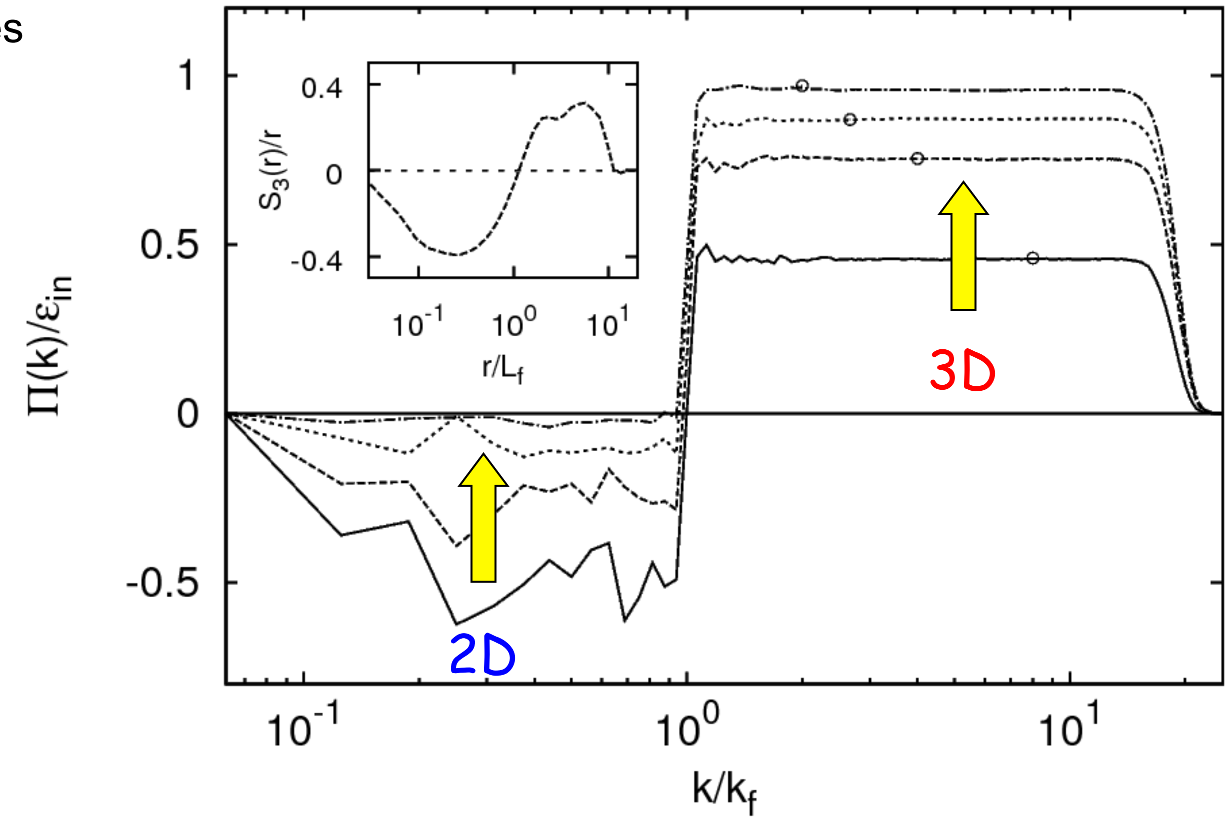
Celani *et al.* (2010) PRL 104, 184506

Smith *et al.* (1996) PRL 77, 2467

Split energy cascade

Part of the energy is transferred toward large scale in an inverse cascade (as in 2D). The remnant energy gives rise to a direct cascade (as in 3D)

As the thickness increases the energy flux in the inverse cascade reduces, and the flux in the direct cascade grows.



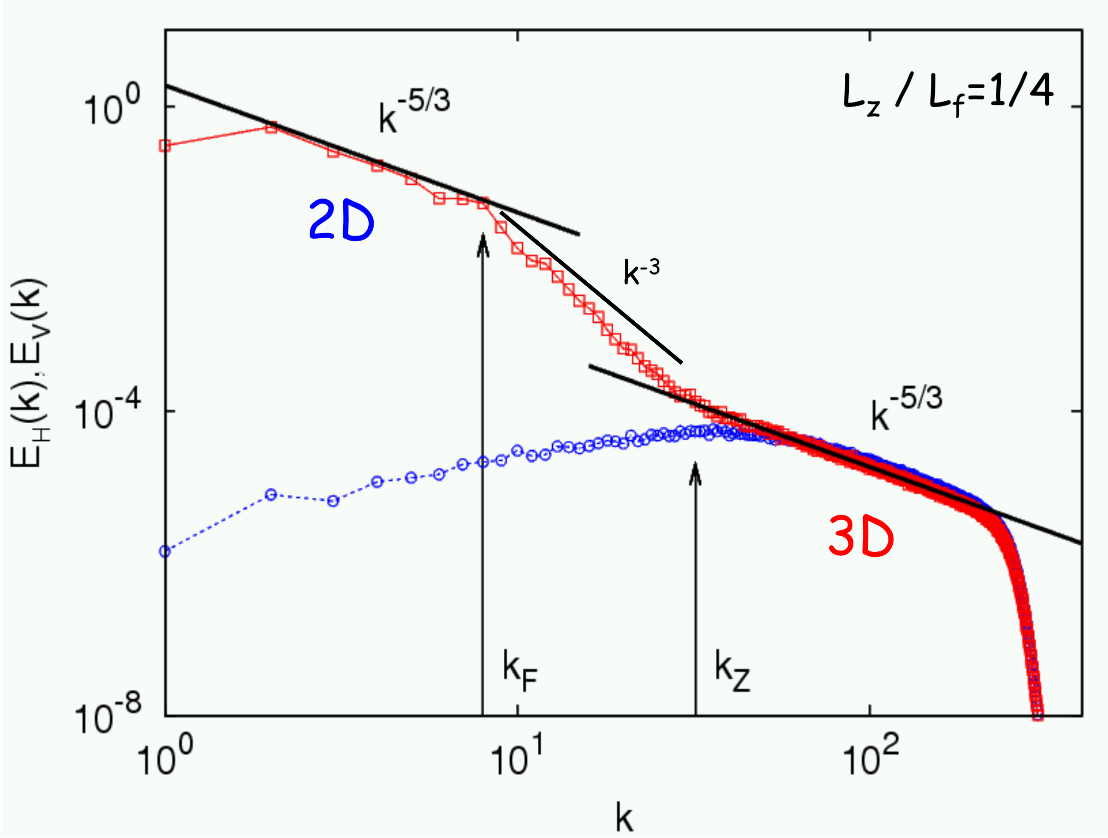
Energy spectrum

When the thickness of the layer is larger than viscous scale and smaller than forcing scale there is coexistence of

2D inverse cascade at large scales and

3D direct cascade at small scales

connected by an intermediate enstrophy cascade.

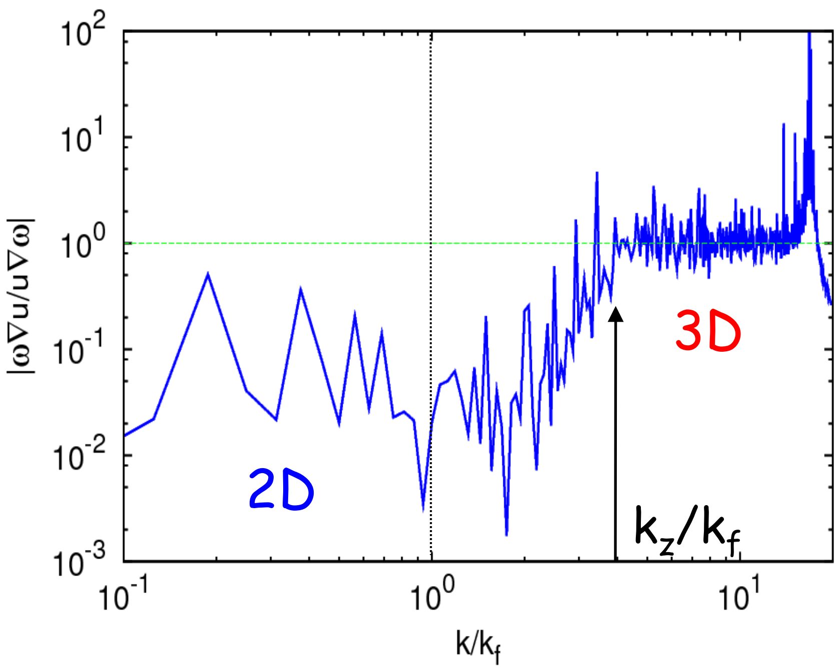


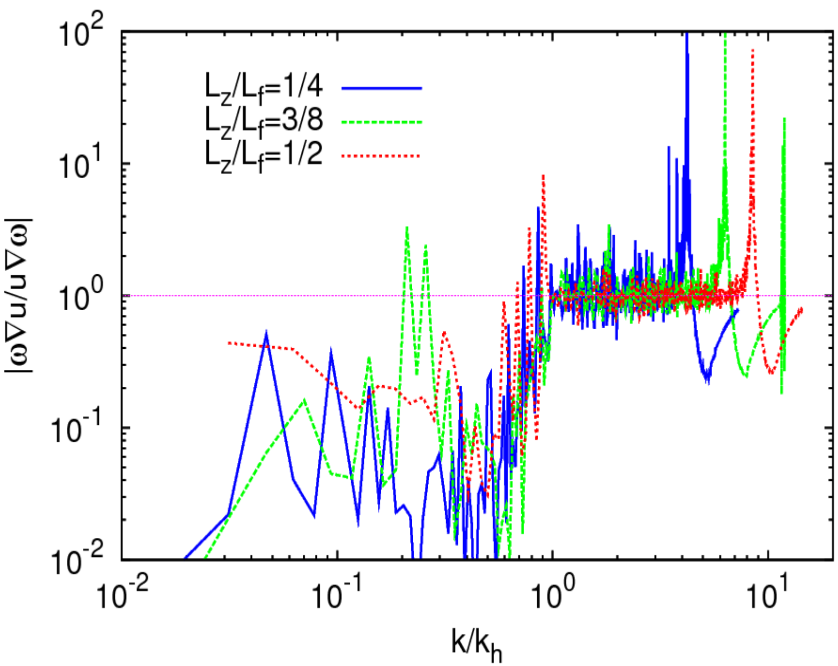
Vortex stretching analysis

3D-NS equation for vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$

$$\partial_t \boldsymbol{\omega} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \Delta \boldsymbol{\omega} + \mathbf{f}_\omega$$

$$\frac{\text{Vortex stretching}}{\text{Enstrophy flux}} = \frac{\boldsymbol{\omega} \cdot \nabla \mathbf{u}}{\mathbf{u} \cdot \nabla \boldsymbol{\omega}}$$





At large scales thin layers are efffectively two-dimensional

Collaborators

S. Berti, M.Shats, R. Ecke, I. Kolokolov, S. Musacchio
G. Falkovich, M. Cencini, P. Muratore-Ginanneschi, A. Mazzino, A. Celani, A. Vulpiani