# A new Experimental Model-Flow?

## Double Cascade Turbulence and Richardson Dispersion due to Faraday Waves

by

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Florian Huhn (Phd)

# Content

- **Motivation:**
  - Reaction-Diffusion-Advection Systems
  - Scales
  - Active media, chemical wave
- **Experiment:**
  - The T-Rex Experiment
  - Faraday Waves
  - Set up
- **Results:**
  - Faraday Flow
  - Double Cascade Turbulence
  - Dispersion – Mixing Statistics
- **Summary + Outlook**

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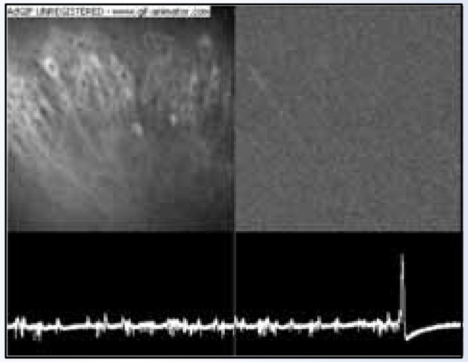
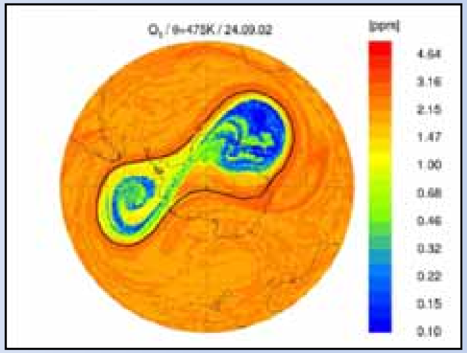
# Reaction – Diffusion – Advection

- **RDA is a model system for:** Spreading of fire fronts influenced by wind, pollution or ozone in the atmosphere, plankton growth in the ocean, disease spreading, neuronal cluster formation. [Murray (1993), Hernández-García et Neufeld (2010), Hufnagel et al. (2003), Tél et al. (2005)]

$$\frac{\delta \vec{c}}{\delta t} = \vec{f}(\vec{c}) + \nabla \cdot (D \nabla \vec{c}) + (\vec{v} \cdot \nabla)\vec{c}$$





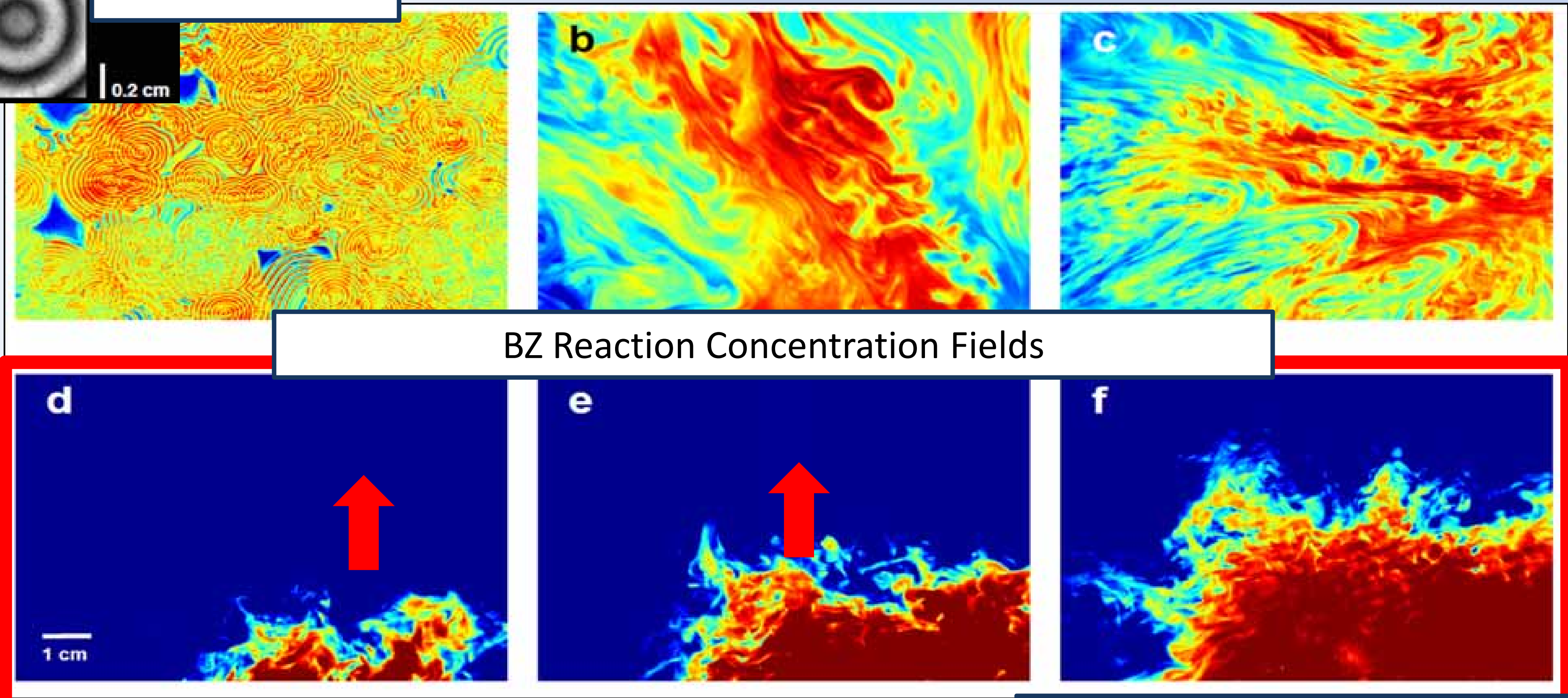

Tél et al. 2005

Table 1
Typical length and velocity scales of important flows, as detailed in the main text

| | Microfluids | Laboratory | Ocean | Atmosphere |
|---|---|---|---|---|
| $L$ (m) | $5 \times 10^{-4}$ | 1 | $10^5$ | $10^6$ |
| $U$ (m/s) | $10^{-2}$ | $10^{-2}$ | $10^{-1}$ | 10 |

# Our System and Scales


No flow




BZ Reaction Concentration Fields

Strong advection

Table 1
Typical length and velocity scales of important flows, as detailed in the main text

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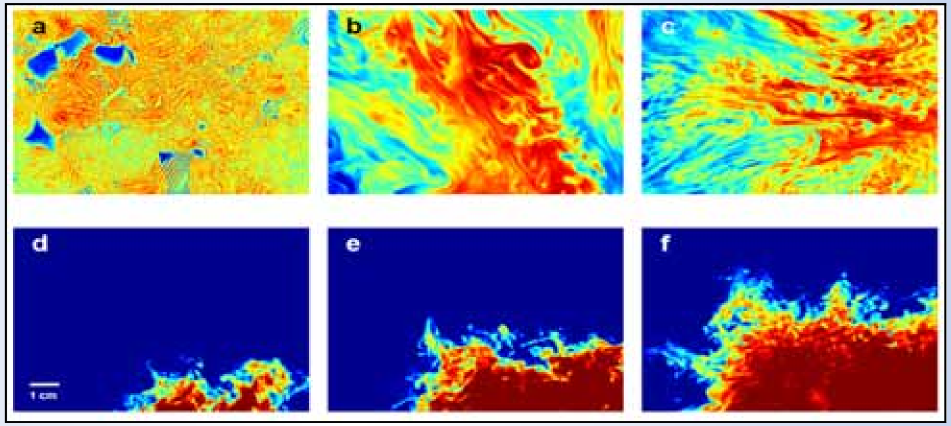
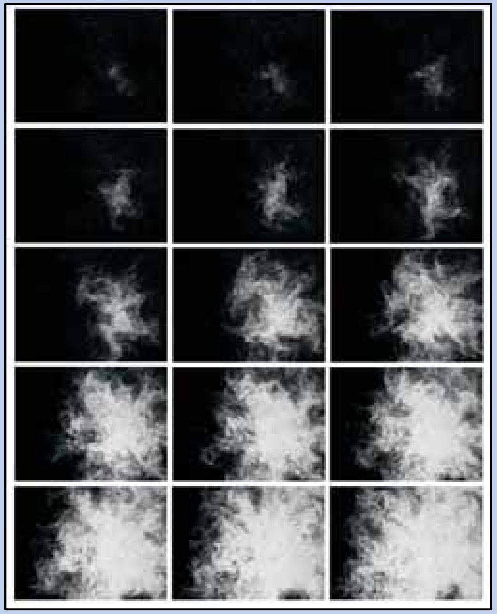
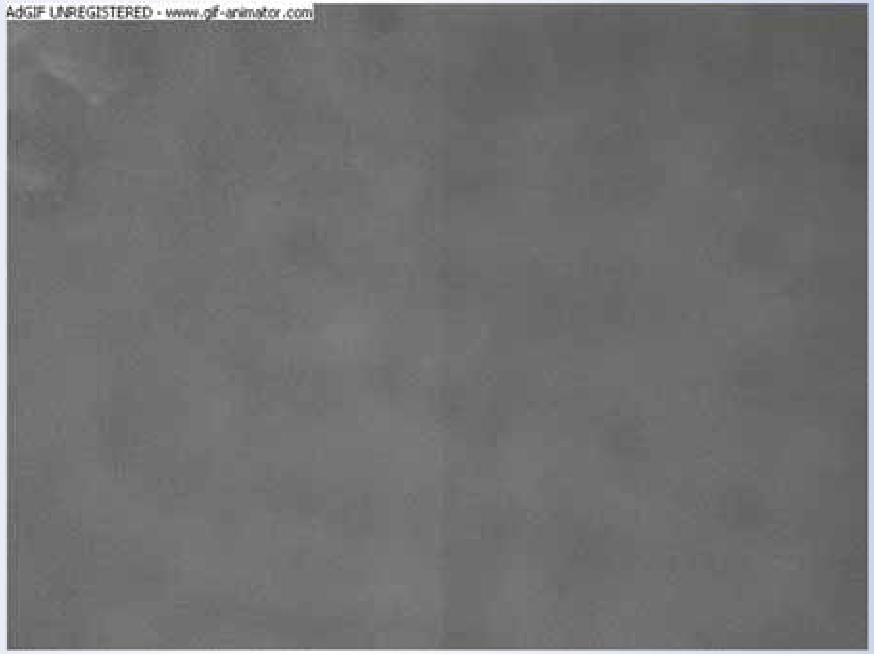
# The Chemical BZ Wave

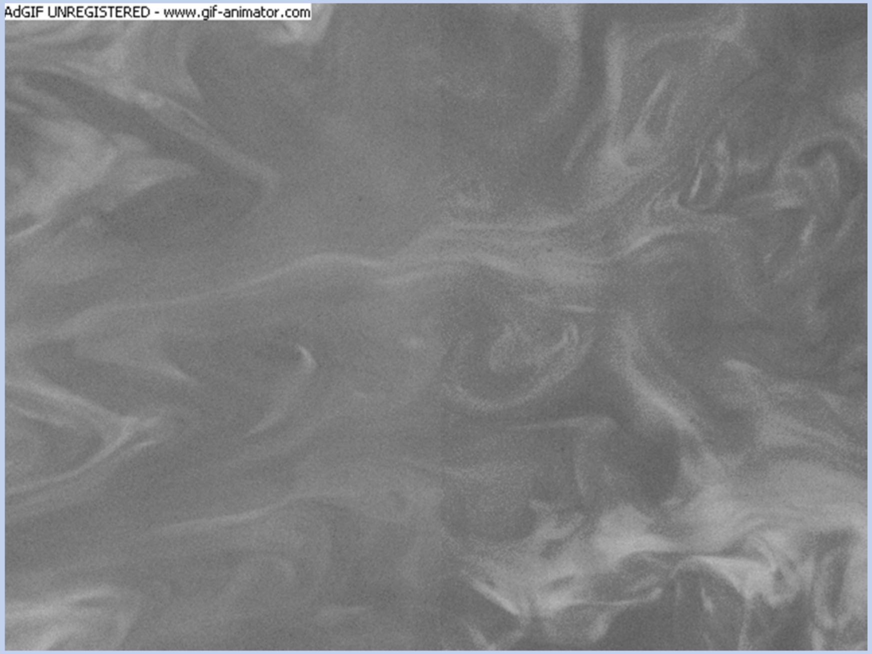






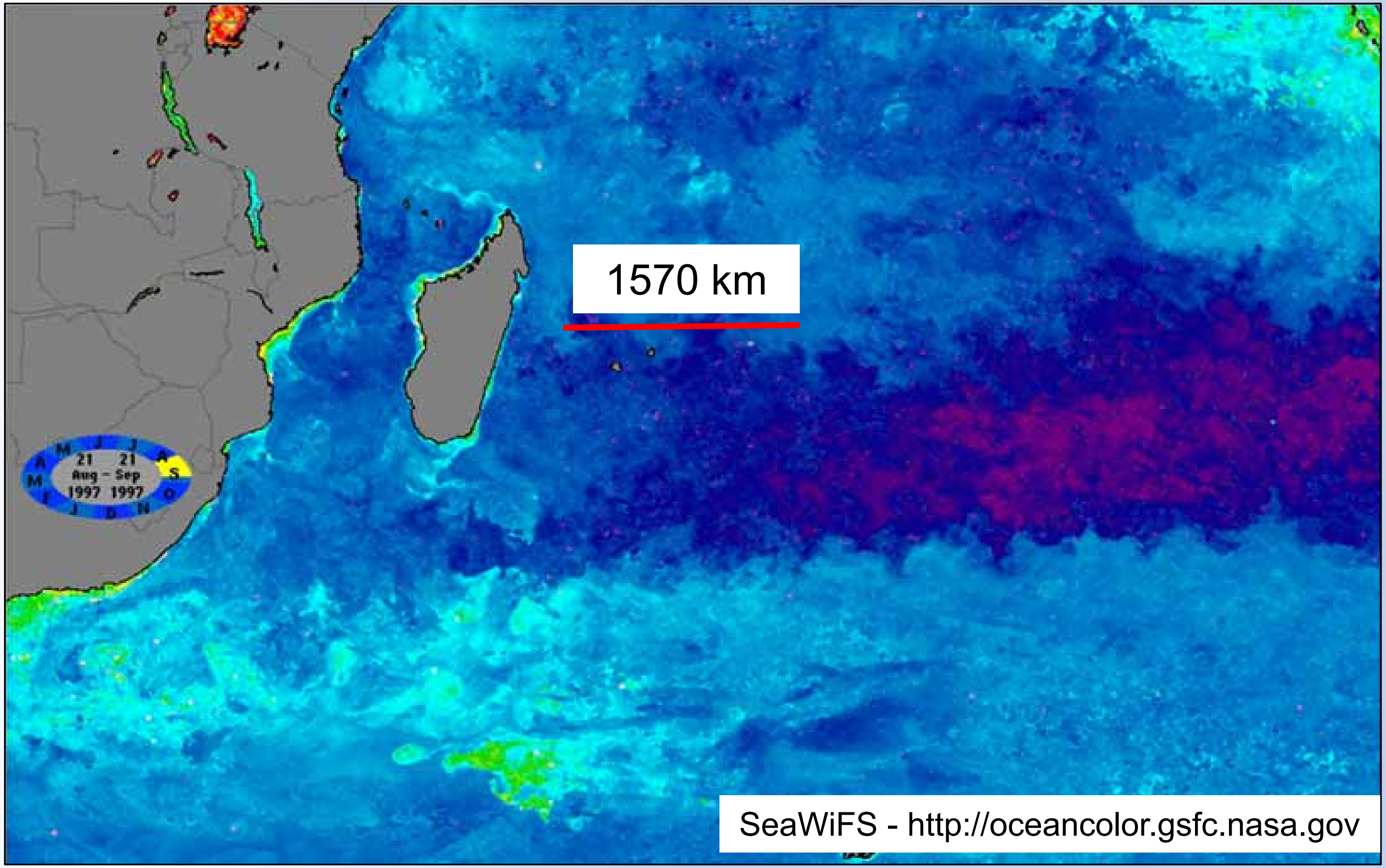



~ 6 cm




~ 6 cm

# Your scales




***Jet-like Lagrangian Coherent Structures in the Madagascar plankton bloom***, tomorrow at 12 am, Florian Huhn

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- **Experiment:**
  - Jurassic Park: The T-Rex Experiment
  - Faraday Waves
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  - Faraday Flow
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- **Summary + Outlook**

# How we induce Fluid Flow?

(Jurassic Park - The T-Rex Experiment)


+


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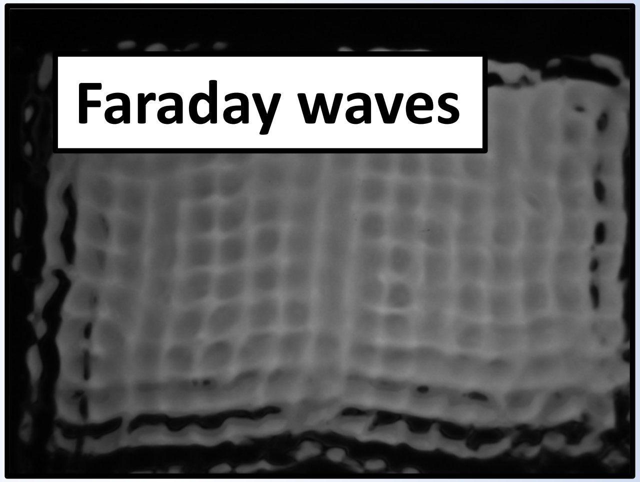
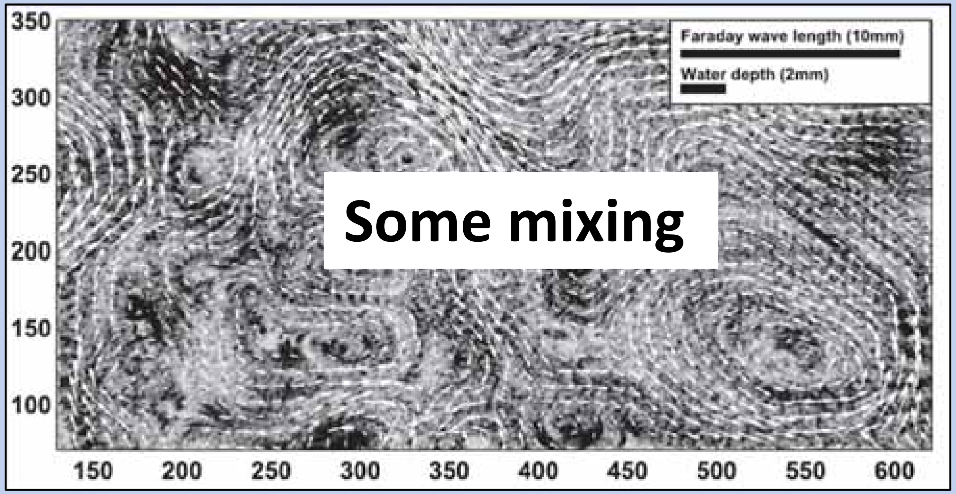


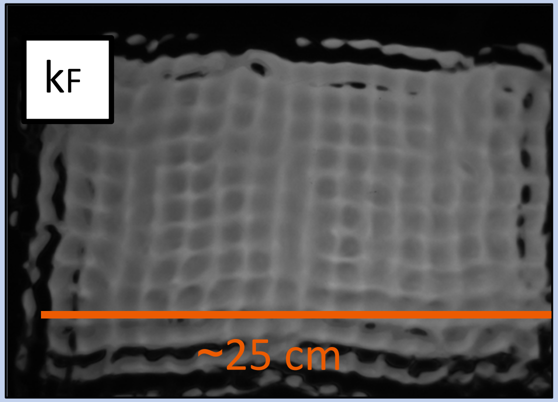



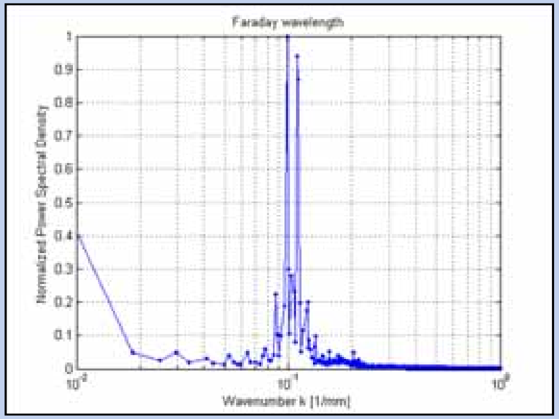
# Faraday Waves

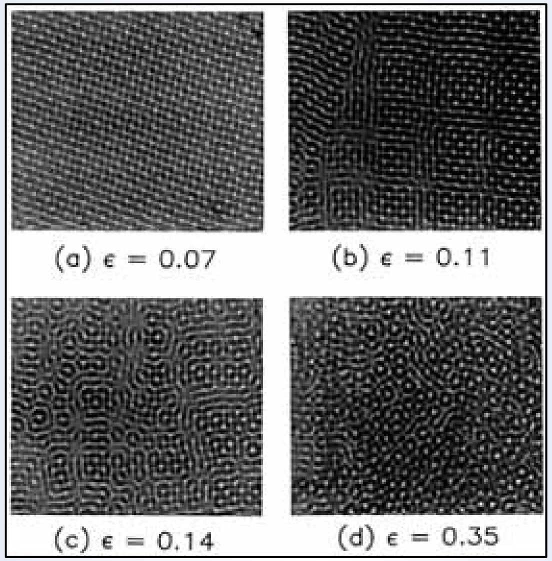
- **Faraday Waves** (1831) gravity **- capillary waves** on free fluid surface (Westra et al. 2003)

- Subharmonic, characterstic wavelength/wavenumber, $\lambda_F$, $k_F$

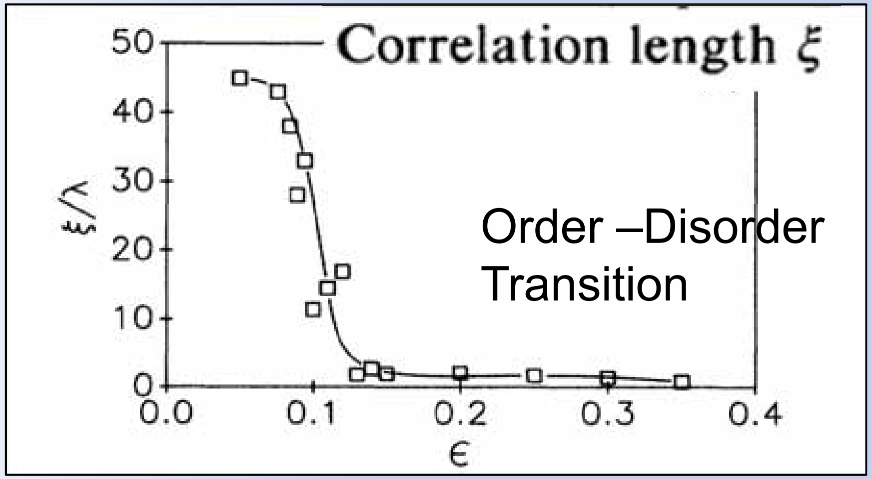
- The **patterns are time dependend.** Flow becomes disordered **-> TRANSITION!!** (Mesquita et al. 1992)



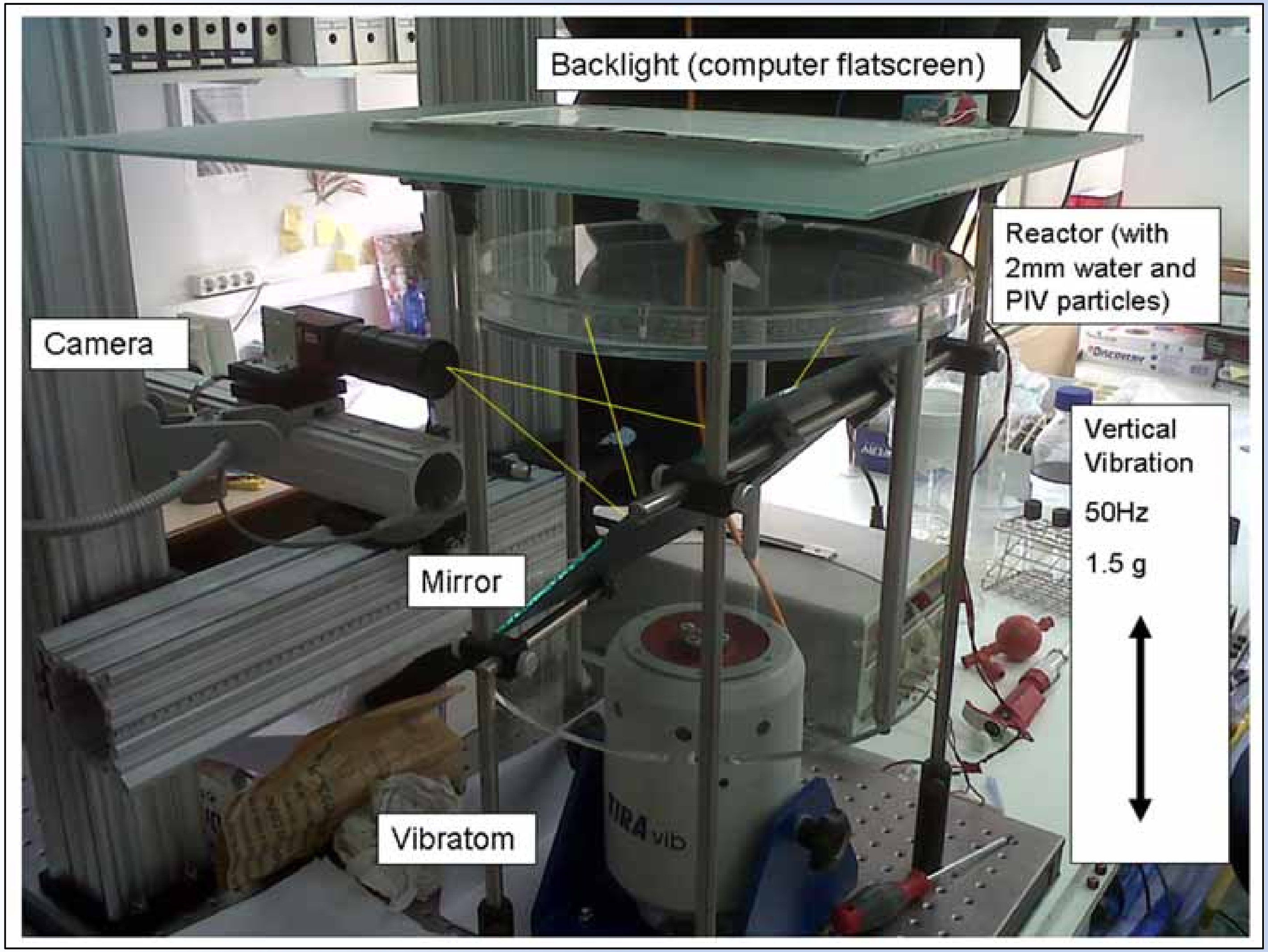




Tufillaro et al., 1989



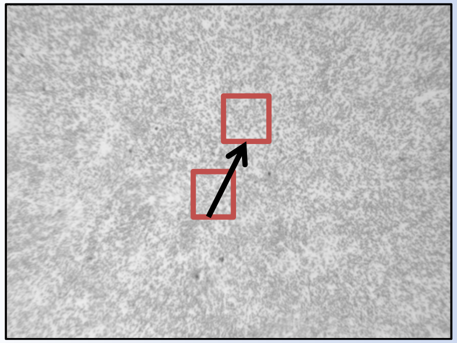
# Set-up

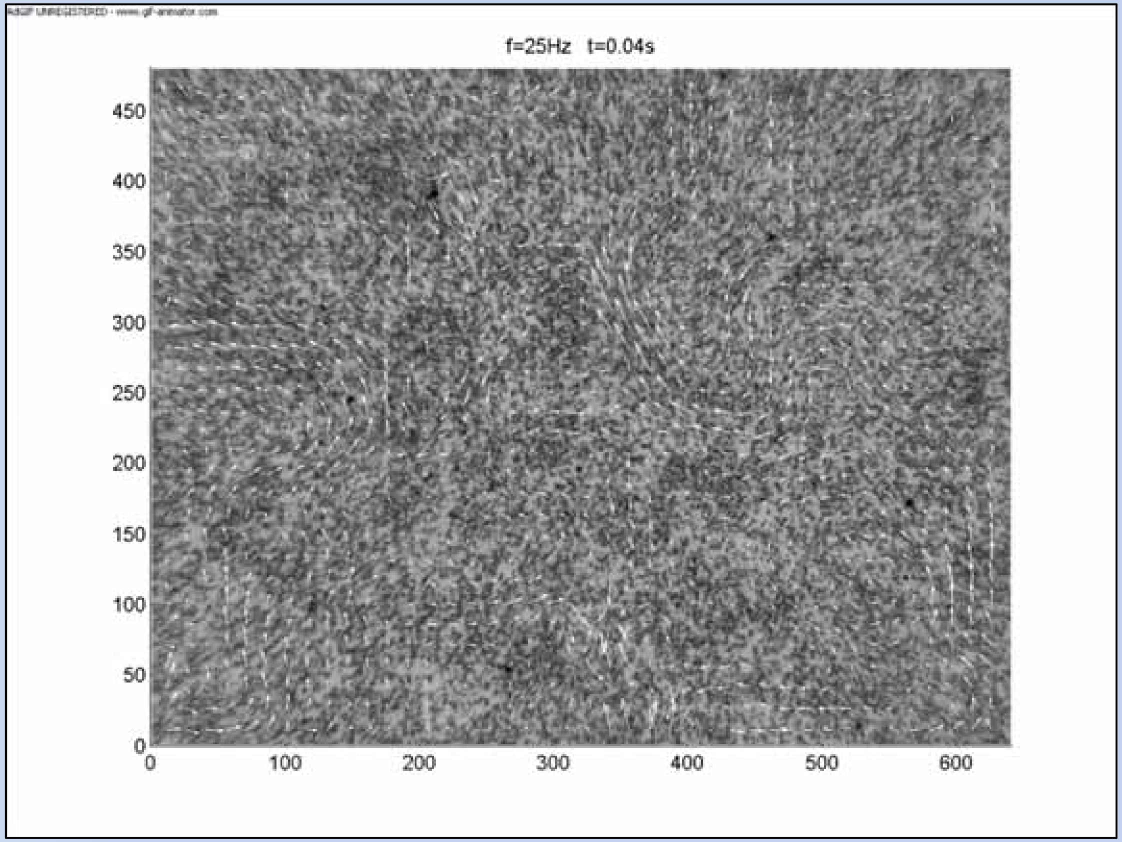



- Quasi 2-dim. - **2mm** liquid height

# Faraday Flow

- **Floating** Particles (hollow glass spheres)
- Perform Image Analysis (mpiv – Matlab based **PIV** algorithm http://www.oceanwave.jp/softwares/mpiv/ )
- **$T_{Eint}$ ~ 0.5 s** (Eulerian integral time scale calculated from velocity fields)
- **$T_{Lint}$ ~ 0.8 s** (Lagrangian integral time scale calculated tracers in vel. field)
- **Re ~ 100**
- **$v_{rms}$ ~ 1.15 cm/s**





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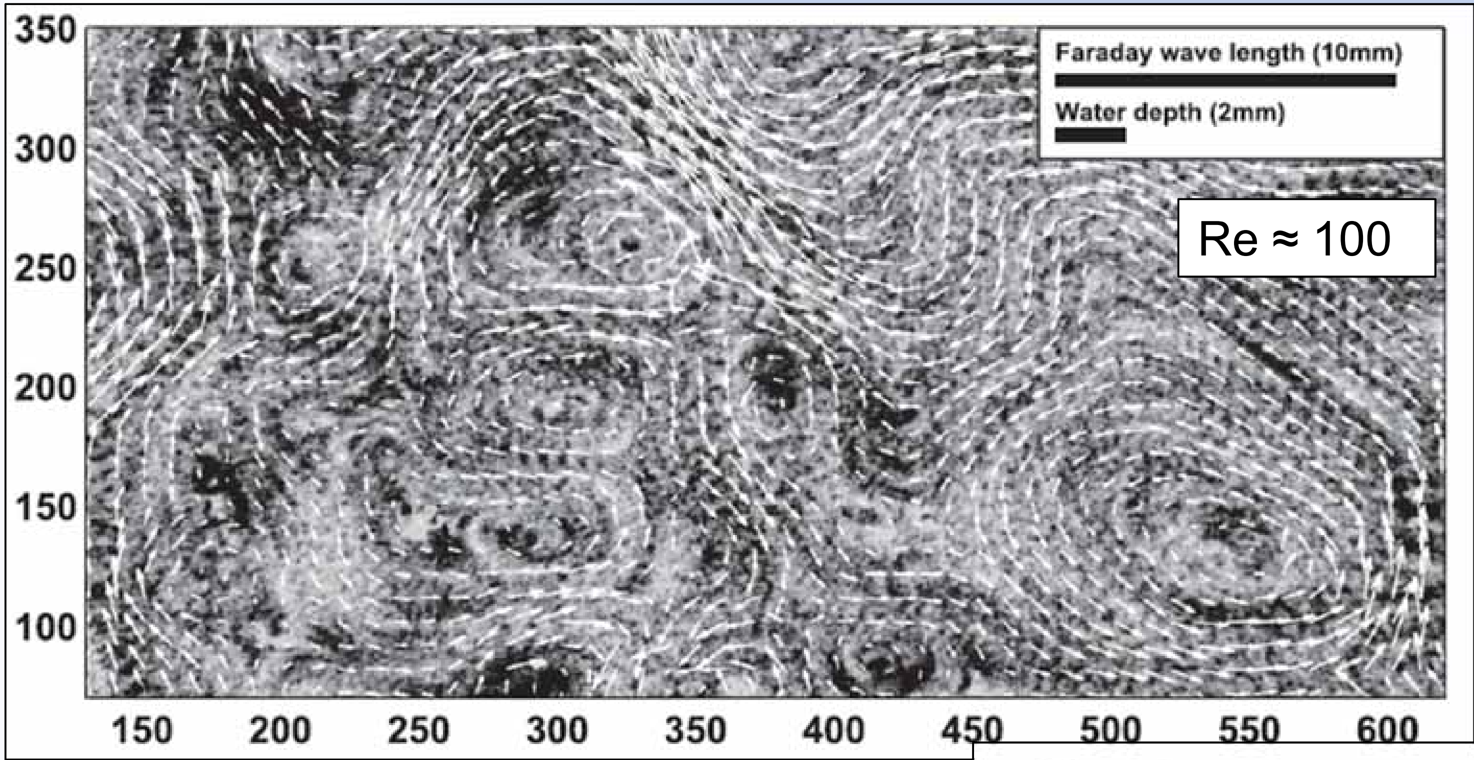
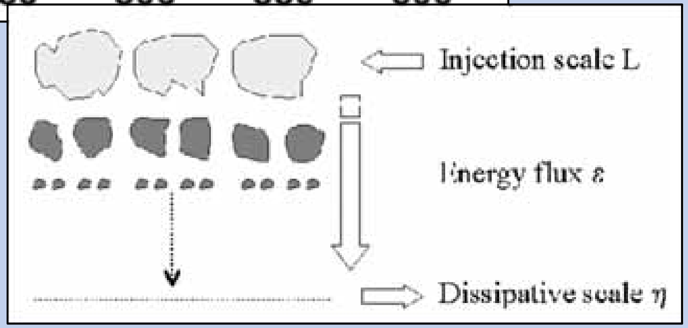
# Faraday Flow has Characteristics of 2D Turbulence




Navier Stokes Equation:

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla \mathbf{p} + \mathbf{f}_{ext} + \nu\Delta\mathbf{u}$$

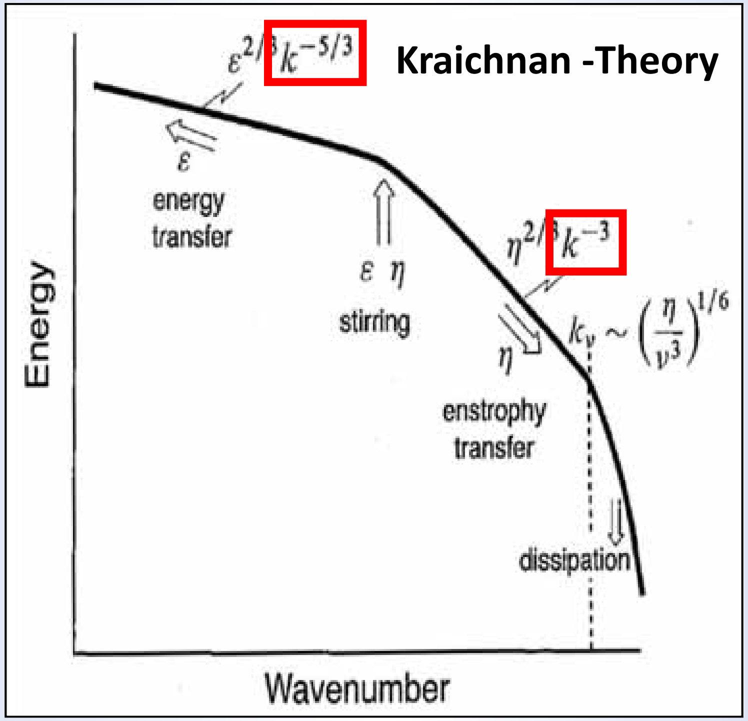
Triad interaction generate new length scales in velocity field



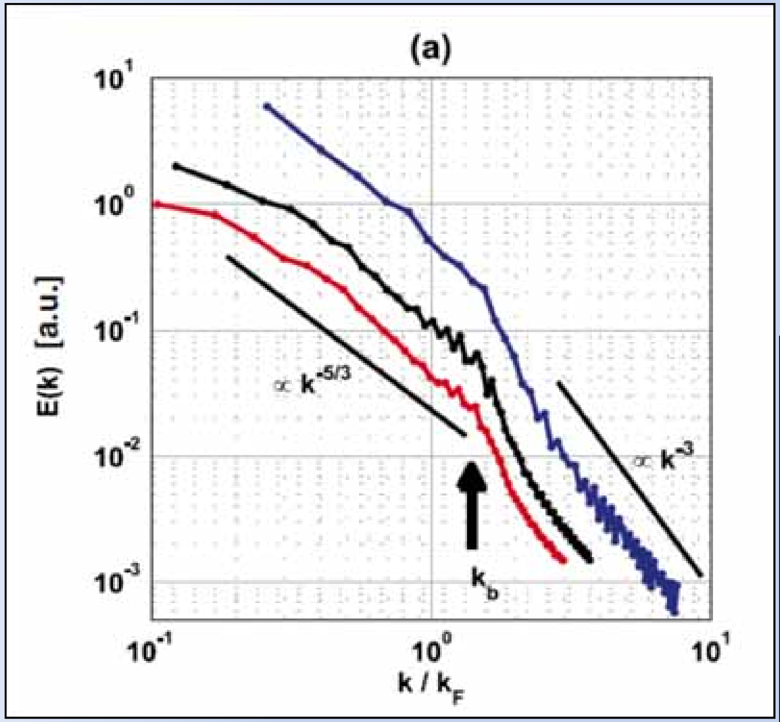

S. Musacchio, 2004

# Characteristics of 2D Turbulence

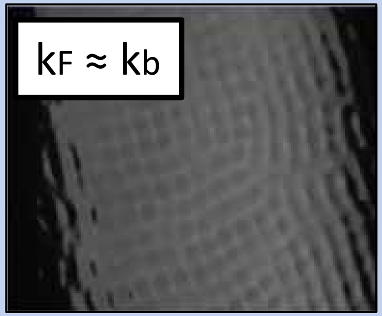
- 2D turbulence spectrum with **double cascade**
- Energy input around **k of Faraday** surface waves
- Narrow **forcing spectrum** caused by Faraday surface waves

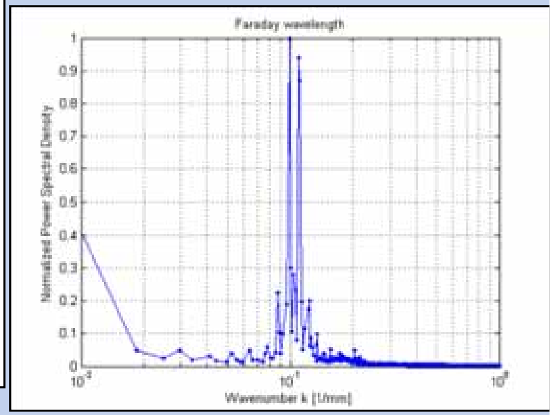



From: Textbook, G. Vallis (2006)



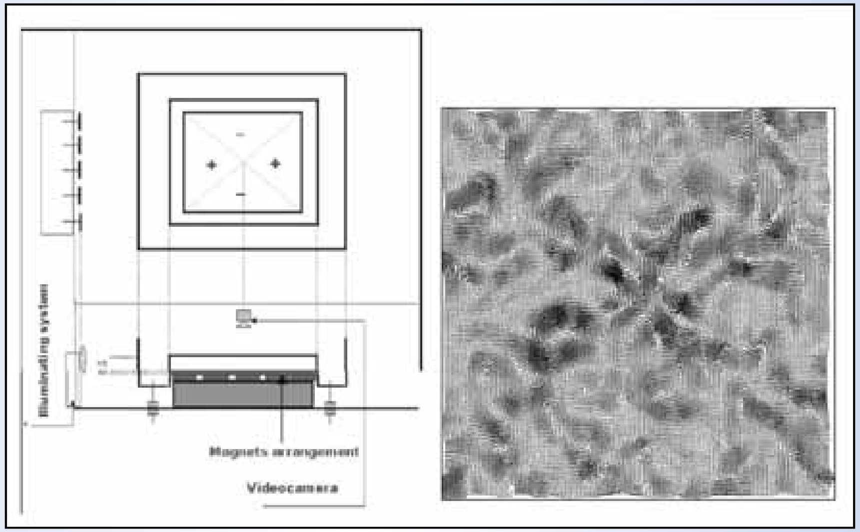

Measured: Faraday wave flow



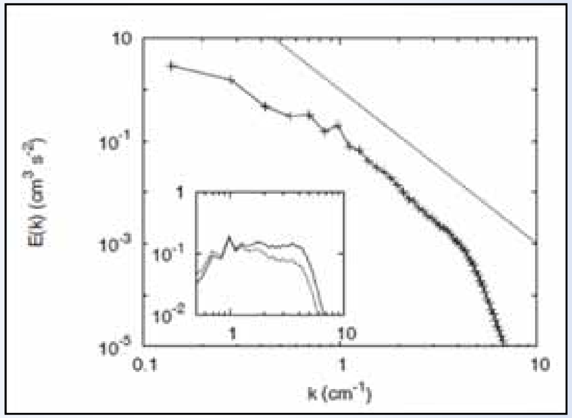



# Characteristics of 2D Turbulence

- Value of γ: $\gamma = -(3+\mu),\ \mu \geq 0$ correction of **Kraichnan scaling** due to **bottom friction** (Boffetta et al. 2005)




**Boffetta et al., 2005**

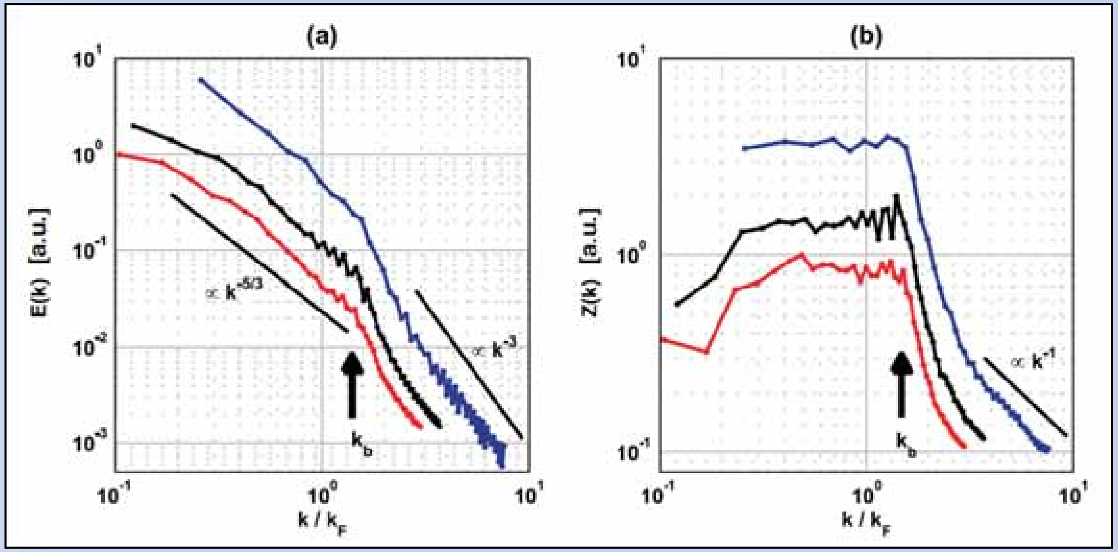



**Energy**

$$E_k \propto k^{\gamma}$$

**Enstrophy**

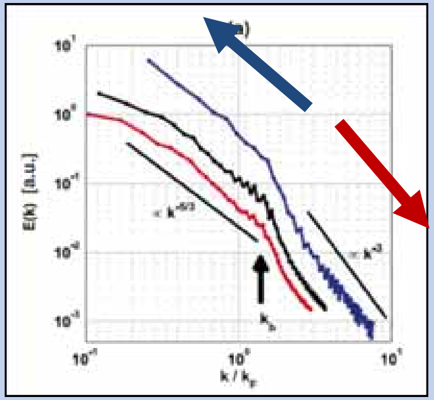
$$Z_k \propto k^{\beta}$$




**Measured: Faraday wave flow**

# Inverse Cascade - Spectral Fluxes

- Spectral flux at scale λ – **time averaged deformation work** that is done by large eddies > λ on smaller eddies < λ
- **Energy transported to larger scales** (inverse cascade)
- **Enstrophy transported to smaller scales** (enstrophy cascade)




$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla \mathbf{p} + \mathbf{f}_{\text{ext}} + \nu \Delta \mathbf{u}$$

Triad interaction generate new length scales in velocity field

**Calculating spectral flux:**

1. **Convolve velocity field with spectral low-pass filter:**

$$u^{(r)} = \int G^{(r)}(x - x')u(x)\mathrm{d}x'$$

2. **Construct filtered strain rate, subgrid stress:**

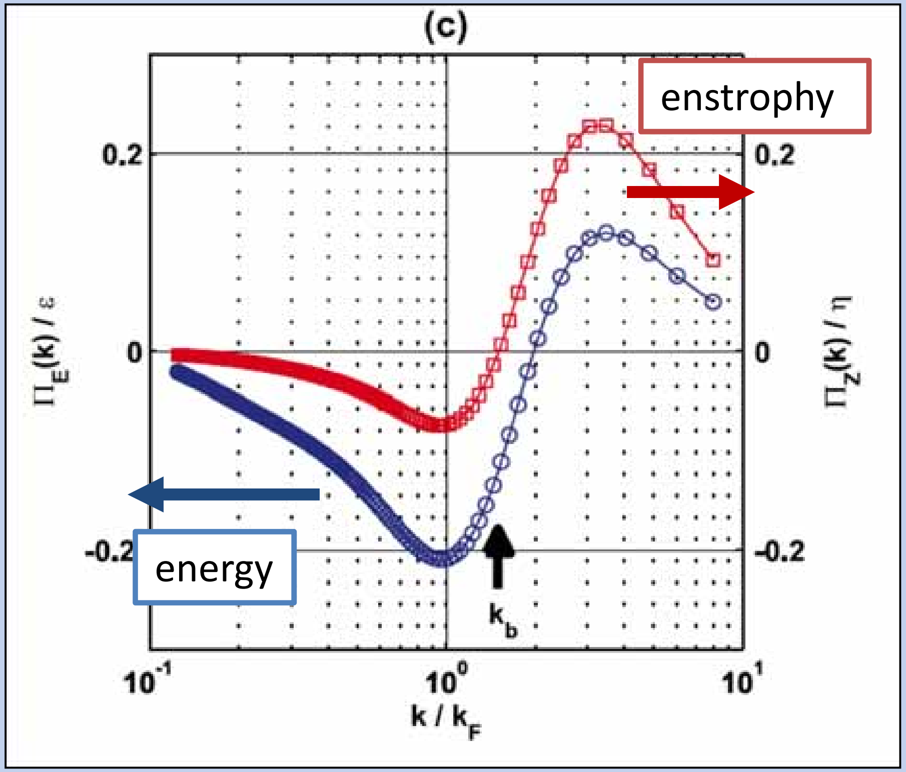
$$s_{ij}^{(r)} = \frac{1}{2}\left(\frac{\partial u_i^{(r)}}{\partial x_j} + \frac{\partial u_j^{(r)}}{\partial x_i}\right) \qquad \tau_{ij}^{(r)} = (u_i u_j)^{(r)} - u_i^{(r)} u_j^{(r)}$$

3. **Compute spectral flux:**

$$\Pi^{(r)} = -\tau_{ij}^{(r)} s_{ij}^{(r)}$$

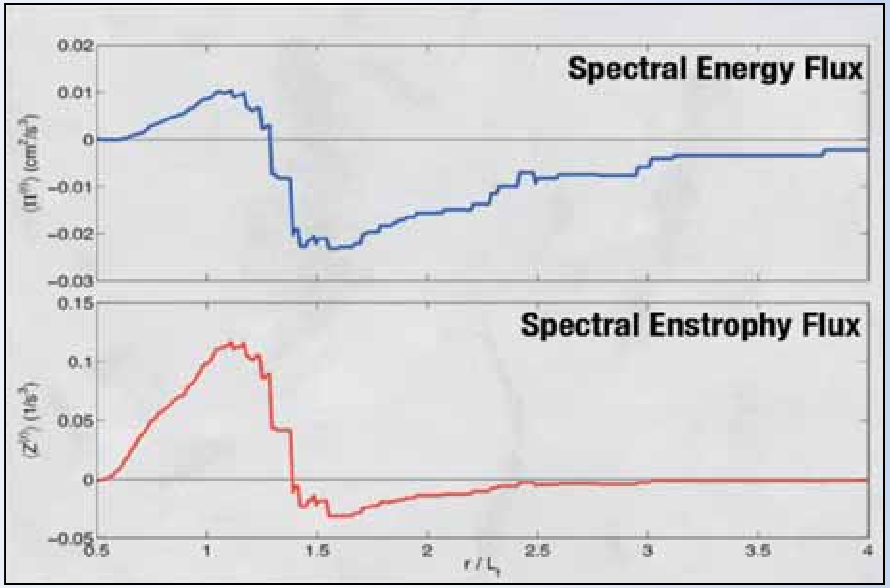
**From: Oullette (2011)**

M.K. Rivera et al., Phys. Rev. Lett. (2003)

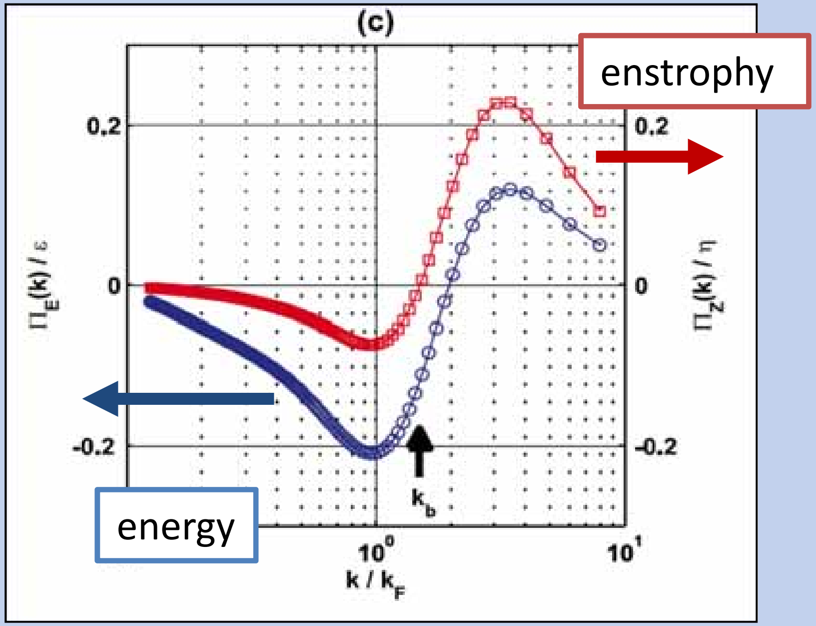



**Measured Mean Energy Flux in Faraday-Flow**

# Inverse Cascade - Spectral Fluxes

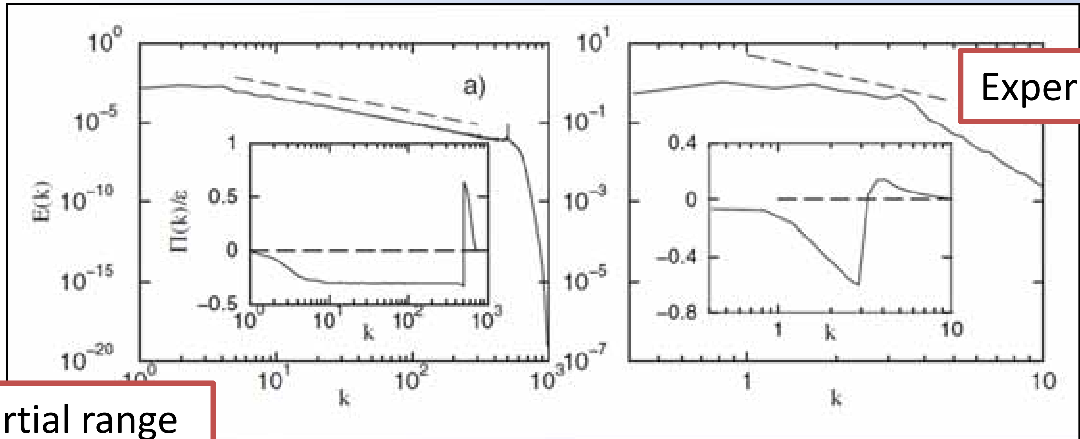



From: Oullette (2011)
Lorentz force driven 2D flow – "state-of-the-art"




OurMeasured Mean Energy Flux in Faraday-Flow

Experimentally: No clear inertial range

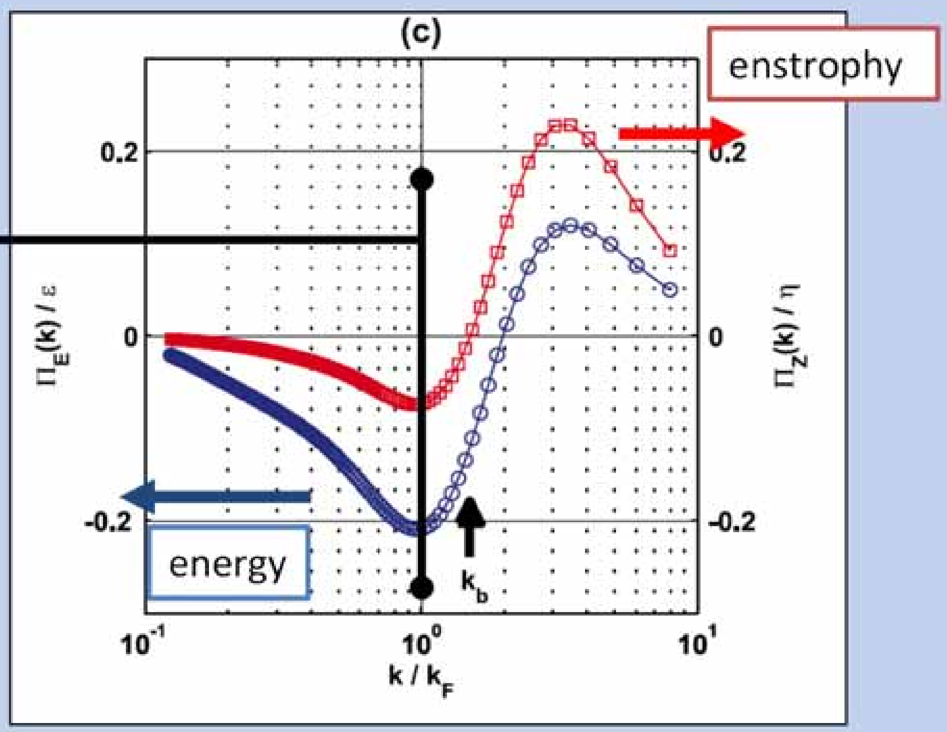



Numerically: Clear inertial range

From: Chen (2006), Left: numerical, Right: Experimental

# Inverse Cascade - Spectral Fluxes

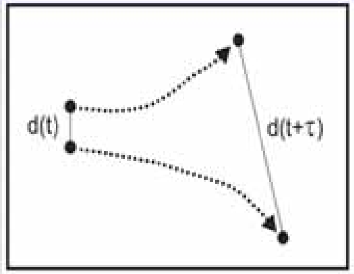
- Local Information about **spectral flux** at Faraday-Wavelength

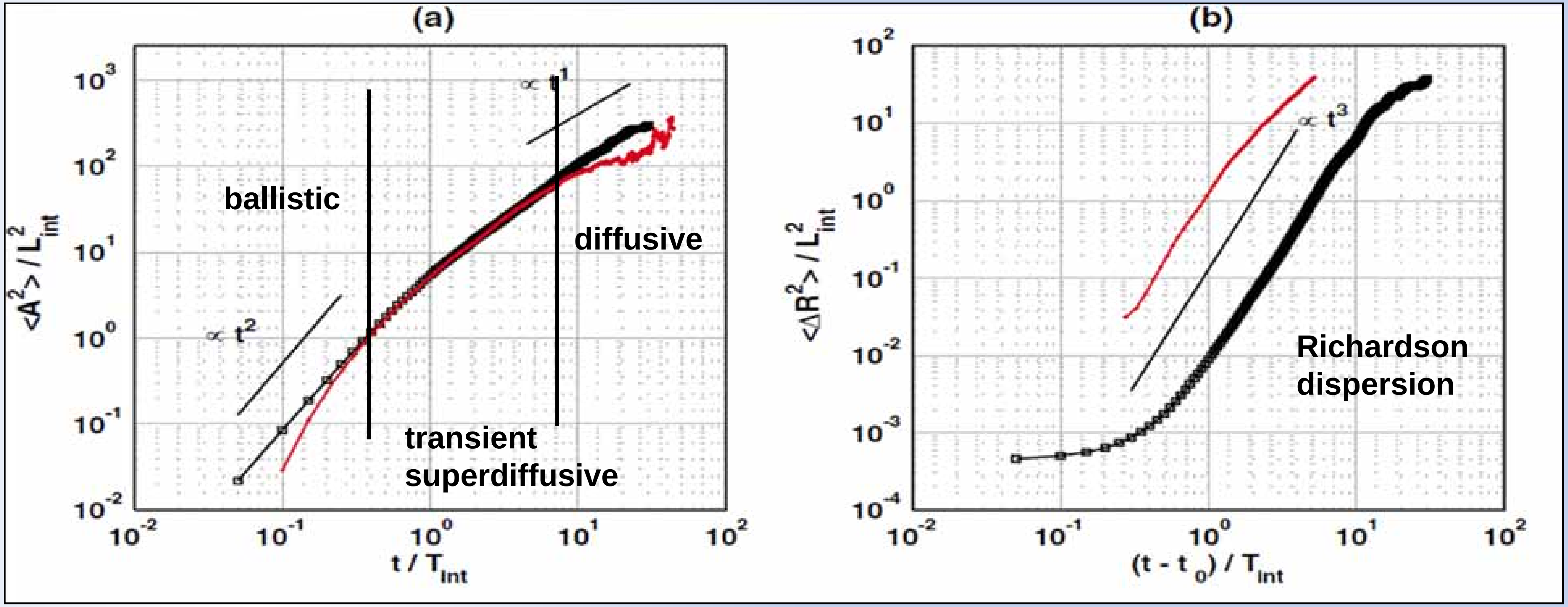



**Measured Mean Energy Flux in Faraday-Flow**

# Dispersion – Mixing Statistics

- PIV – velocity fields – virtual tracers (black)
- Particle tracking of 300μm real particles (red)
- **Richardson dispersion <-> -5/3 energy cascade**




$$\langle A^2 \rangle = \langle |\vec{r}(t) - \vec{r}(t_0)|^2 \rangle \propto t^\nu$$

**Absolute dispersion**

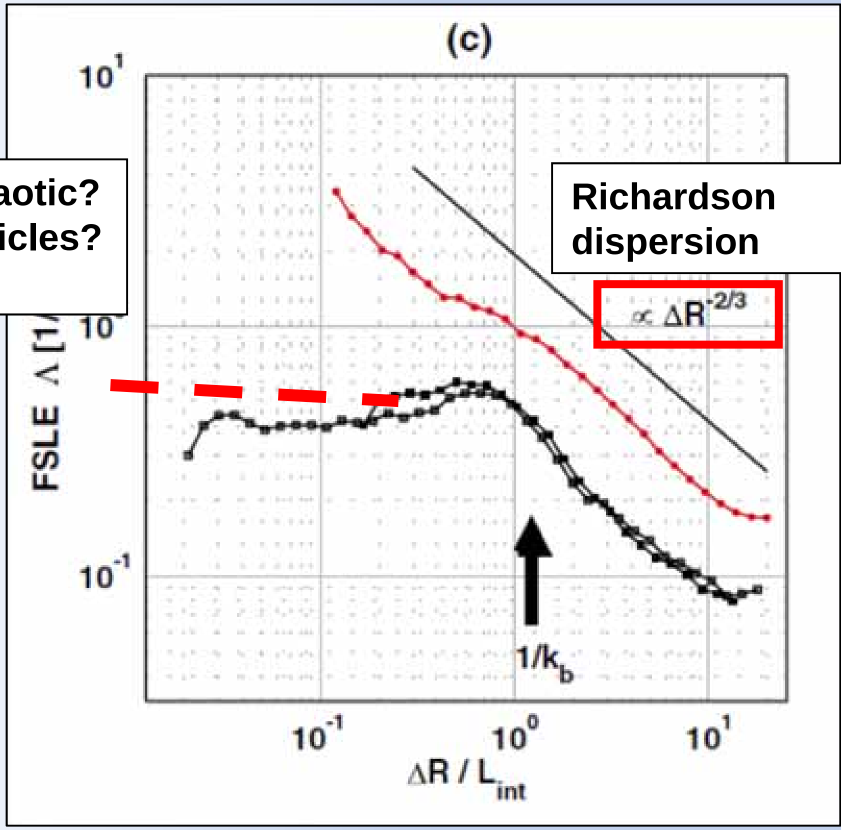
$$\langle \Delta R^2 \rangle = \langle |\vec{r}_i(t) - \vec{r}_j(t)|^2 \rangle$$

**Relative dispersion**

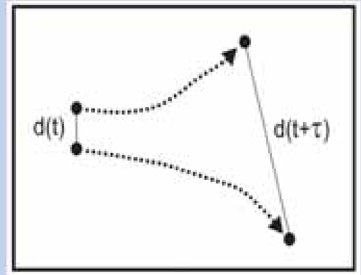
# Dispersion – Mixing Statistics

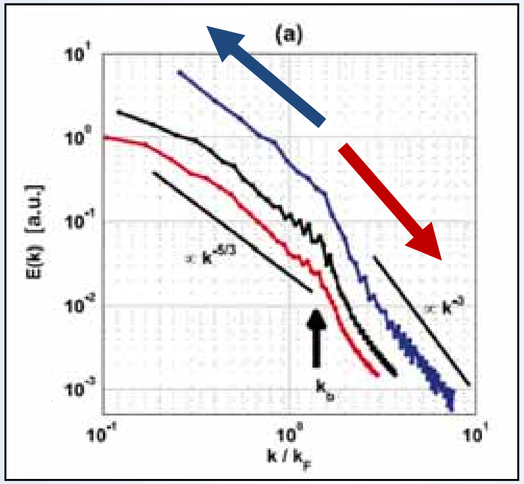
- Scale dependent exponential separation rate – **Finite Size Lyapunov Exponent (FSLE)**

**Enstrophy cascade - chaotic?**
**Inertia effect of real particles?**




**Richardson dispersion**




- Virtual tracers
- Real tracers

enstrophy

energy

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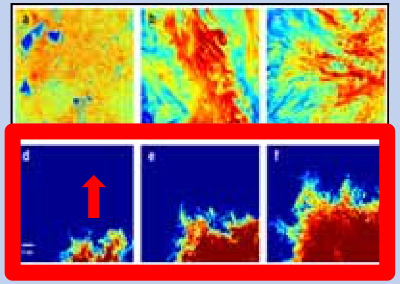
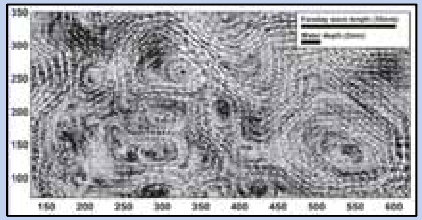
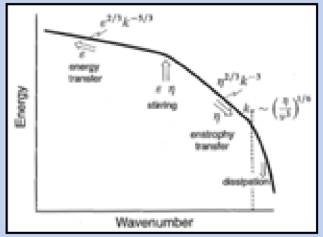
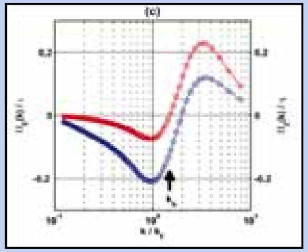
# Summary

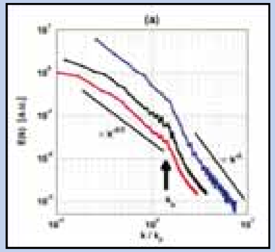




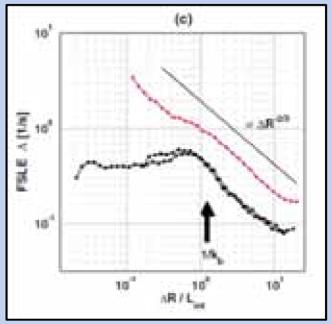
- **Transition in BZ-chemical-reaction** when subject to fluid flow
- Surface **Faraday flow** reveals important **characteristics of 2D turbulence**
- Energy input by waves sufficiently **monochromatic** to see **doble cascade**





- **Inverse cascade (only possible in 2D)**
- Mixing statistics show **Richardson 'superdiffusive' dispersion** and possibly **chaotic mixing**



PHYSICAL REVIEW E 81, 066211 (2010)

**Propagation of a chemical wave front in a quasi-two-dimensional superdiffusive flow**

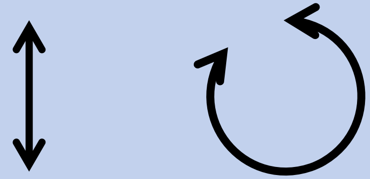
A. von Kameke,* F. Huhn, G. Fernández-García, A. P. Muñuzuri, and V. Pérez-Muñuzuri
*Group of Nonlinear Physics, University of Santiago de Compostela, E-15782 Santiago de Compostela, Spain*
(Received 6 April 2010; published 18 June 2010)

PRL 107, 074502 (2011) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2011

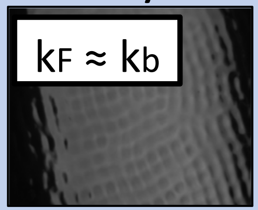
**Double Cascade Turbulence and Richardson Dispersion in a Horizontal Fluid Flow Induced by Faraday Waves**

A. von Kameke,* F. Huhn, G. Fernández-García, A. P. Muñuzuri, and V. Pérez-Muñuzuri
*Group of Nonlinear Physics, University of Santiago de Compostela, E-15782 Santiago de Compostela, Spain*
(Received 16 May 2011; published 12 August 2011)

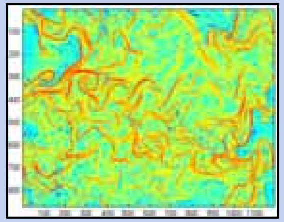
# Open Questions



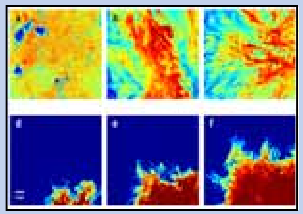
- **Wave – vortex** interaction , Vibration → Stirring, 3D→2D?
- **Alteration of forcing** (liquid height, frequency, amplitude) and effect on Faraday flow

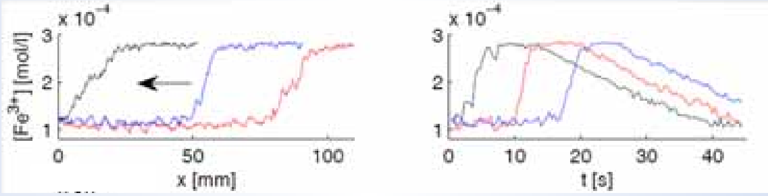



- **Extension of inertial range** → System Size vs. Faraday wavelength ?



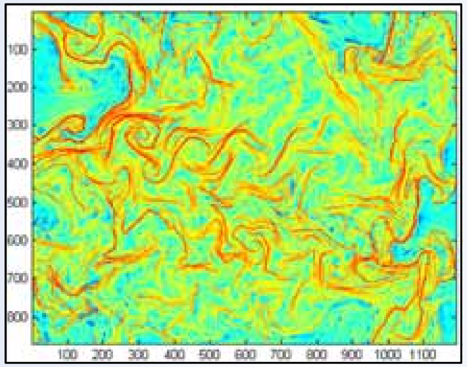
- Role of **Lagrangian Coherent Structures** in the Faraday flow?
- Transition from **Superdiffusivity to Diffusivity**?
- **Resonance of active media and Faraday flow**? Time and length scales adjusted to those of the reaction?

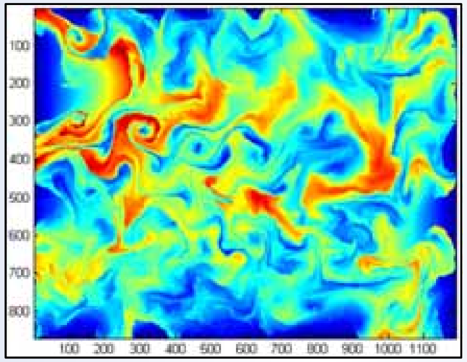


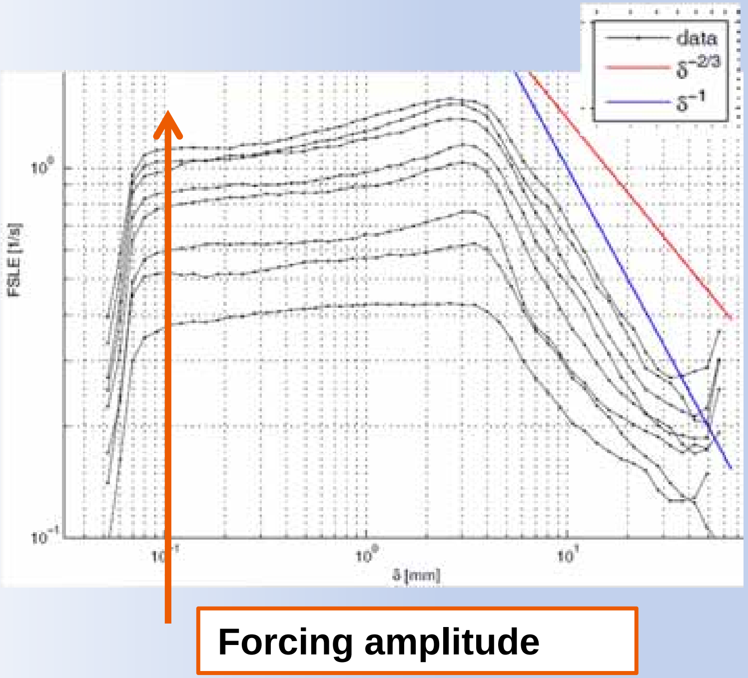


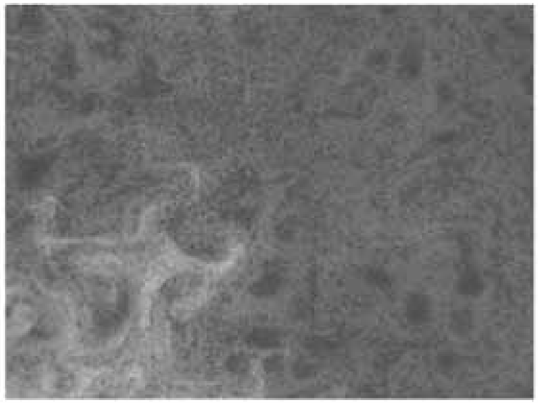

# Outlook

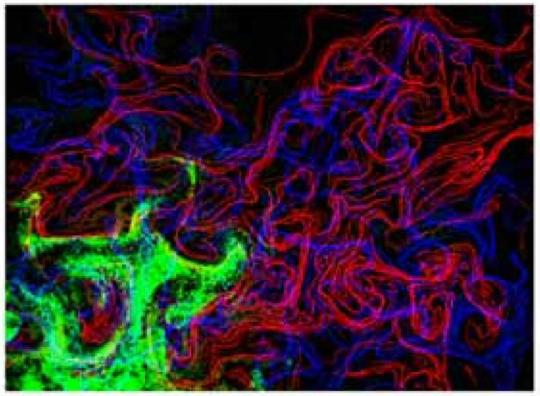
- Calculating **FTLE's**, **M - Function**, etc..
- Measure fluid flow and **reactant concentration simultaneously**
- Variation of **reaction dynamics**
- Variation of **Mixing Intensity**









# Thanks

.... For your **attention!**

Faraday Flow is
cheap and beautiful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