Minimizing the environmental impact of wastewater discharges with SOS

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ABSTRACT

Elimination of waste is one of the most important environmental problems. For the particular case of wastewater in any urban area, sewage domestic and trade waste are collected from the different districts and transported to purifying plants via pipes and pumping stations. These plants treat sewage by different methods and finally, treated effluents are discharged through a submarine outfall into an aquatic media (a lake, a river, an estuary...). Sewage treatment is not only a necessary task but also a very expensive one, and determining the intensity of the treatment becomes a very difficult problem involving environmental and economical aspects. The problem can be even more complicated if there are several plants discharging wastewater in the same area. In this case, different (cooperative and non-cooperative) multi-objective problems can be formulated.

In this work we use numerical simulation, and combine optimal control theory of partial differential equations with multi-objective optimization techniques to formulate and solve this type of problems. Specifically, the main objective of this paper is to detail the mathematical models and numerical techniques that we have used, and which is the kernel of the free software SOS recently developed by the authors (see [3]) to asses in decision making related to the sewage treatment intensities in each one of the purifying plants.

Moreover, we assume the existence of several sensitive areas (representing beaches, fisheries or marine recreation areas) where the water quality should be guaranteed with pollution levels lower than allowed thresholds (usually fixed by administrative directives). The environmental problem consists of determining the optimal intensities of treatment in each plant along an arbitrary period of time $(0, T)$. This optimal strategy should satisfy two main objectives: it should be *inexpensive* (low economic cost), and it should be *green* (low environmental impact).

We begin presenting the environmental problem and proposing a mathematical model (a 2D uncoupled hyperbolic-parabolic system) for hydrodynamics and faecal coliform concentration simulation in a shallow water domain [1]. Within this model, the problem of determining the intensity of the treatment can be formulated in different scenarios. The resultant multi-objective optimal control problems

are studied from a cooperative and/or a non-cooperative viewpoint [2, 4] and, in order to solve them in both cases, a detailed algorithm is proposed. Particularly, a useful method based on *sensibilities* to evaluate objective functions and their gradients is presented.

Finally, we briefly show how these techniques have been organized to develop a specific software (SOS) which potentialities are made clear by solving a realistic problem posed in the estuary of Vigo, one of the most populous and industrialized cities in the NW of Spain. In this estuary two sensitive areas and two purifying plants discharging wastewater are considered. An example of numerical result provided by SOS is shown in Fig. 1.

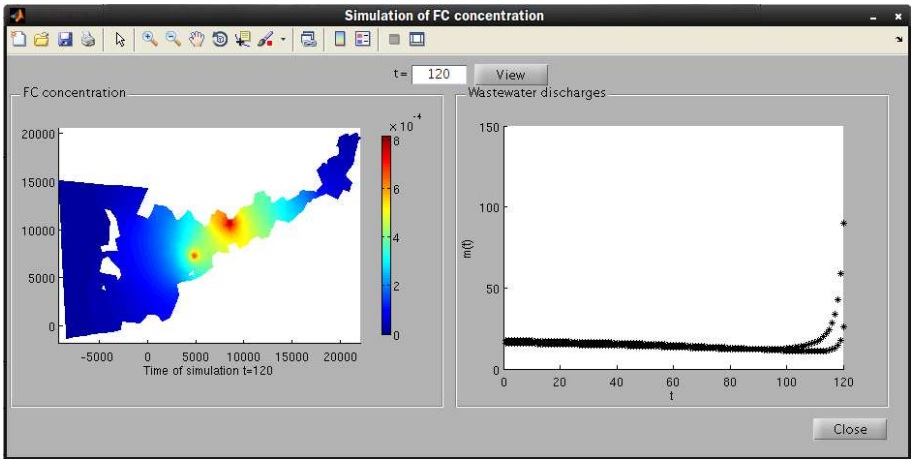

Figure 1: Pareto-optimal discharges and faecal coliform concentration corresponding to an optimal strategy.

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