A multi-objective approach to an atmospheric pollution problem

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Keywords: multi-objective optimization, air pollution control, Pareto solutions, industrial plant location.

ABSTRACT

This work deals with the problem of determining the optimal location of a new industrial plant, considering the framework of numerical simulation and multi-objective optimal control of partial differential equations (PDE). We take into account both ecological and economic objectives, and we look not only for the optimal location of the plant but also for the optimal management of its emissions to atmosphere. With these purposes in mind, we propose a mathematical model (a system of parabolic PDE) to simulate air pollution and, based on this model, we formulate the problem in the field of multi-objective optimal control. This problem is studied here from a cooperative point of view, looking for Pareto-optimal solutions. A numerical algorithm (*via* a characteristics-Galerkin discretization of the adjoint model) is proposed, and preliminary numerical results for a hypothetical situation in the region of Galicia (NW Spain) are also presented.

Let $\Omega \subset R^2$ be a bounded domain, where several industrial plants located at points $p_i \in \Omega$, $i = 0, \ldots, N$ send contaminant emissions to air. Let us assume that we need to control the concentrations of N_S pollutants and, for $j = 1, \ldots, N_S$, let $\phi_j(x,t)$ be the concentration of pollutant j at point $x \in \Omega$ and at time $t > 0$. Under these hypotheses hipótesis, for the time interval $(0,T)$ the concentrations of each pollutant satisfies the following system:

$$\frac{\partial \phi_j}{\partial t} + \vec{u} \cdot \nabla \phi_j - \nabla \cdot (\mu_j \nabla \phi_j) + k_j \phi_j = \sum_{i=0}^{N} Q_i^j(t)\, \delta(x - p_i) \qquad \text{in } \Omega \times (0,T) \tag{1}$$

$$\phi_j(x,0) = \phi_j^0(x) \qquad \text{in } \Omega \tag{2}$$

$$\mu_j \frac{\partial \phi_j}{\partial \vec{n}} - \phi_j \vec{u}.\vec{n} = 0 \qquad \text{on } S^-, \tag{3}$$

$$\mu_j \frac{\partial \phi_j}{\partial \vec{n}} = 0 \qquad \text{on } S^+ \tag{4}$$

where S^- and S^+ denote, respectively, the inflow and outflow boundaries for the wind velocity $\vec{u}$.

Assuming the existence of N industrial plants already built (points $p_1, \ldots, p_N \in \Omega$ and functions $Q_1^j(t), \ldots, Q_N^j(t)$, for $j = 1, \ldots, N_S$ are known), we consider the problem related to the location and management of a new plant, located at a point $p_0 \in \Omega$ to be determined, and producing pollutant emissions $\vec{Q}_0(t) = (Q_0^1(t), \ldots, Q_0^{N_S}(t))$, also to be determined. We want the plant to be as economically profitable as possible, but also to be environmentally harmless.

From an economic viewpoint we consider an objective function to be minimized

$$J_E(p_0, \vec{Q}_0) = -\int_0^T F\left(\vec{Q}_0(t)\right) dt + \int_0^T s(t)||p_0 - p_0^I||^2 dt + G(p_0) \tag{5}$$

involving the maximization of profitability (F) and the minimization of construction (G) and energy (s) costs. From an environmental viewpoint, we assume the existence of N_Z protected zones $A_k \subset \Omega$, for $k = 1, \ldots, N_Z$, where the average pollutant concentrations

$$J_k^j(p_0, \vec{Q}_0) = \frac{1}{|A_k|T} \int_{A_k \times (0,T)} \phi_j(x,t) dx dt \tag{6}$$

must be minimized.

Previous works in the literature deal only with a unique objective function (see, for instance, [2]). In this paper, we present a multi-objective approach, in the spirit of related works of the authors [1]: We try to characterize the Pareto-optimal solutions in order to assess decision maker. For determining the Pareto front corresponding to all above objective functions, we propose an adjoint method - combined with a characteristics-finite element algorithm - for evaluating the cost functions, and a weighting method for the computation of the Pareto-optimal solutions. A first numerical test for the region of Galicia (NW Spain) is solved, and graphical results (Fig. 1) are presented.

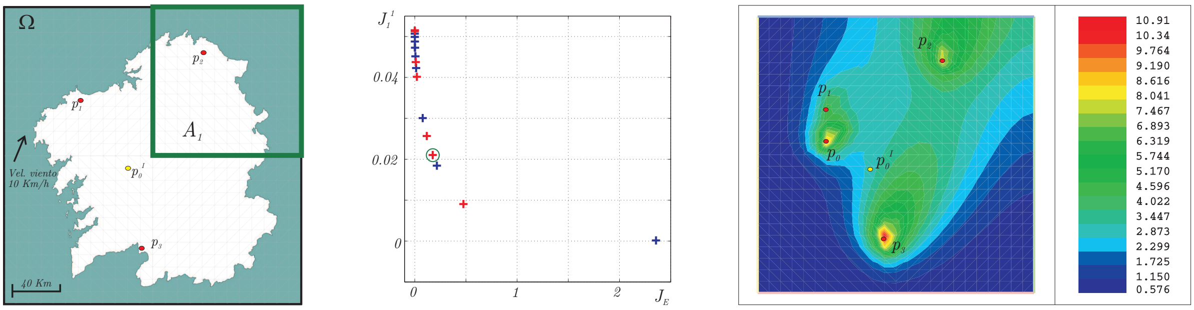

Figure 1: Domain Ω for the numerical example corresponding to Galicia showing problem data (left), Pareto front (center), and final pollutant concentration for the circled Pareto-optimal solution (right).

Work partially supported by Project MTM2009-07749 of Ministerio de Ciencia e Innovación (Spain).

References

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