The extra invariant for Rossby waves and the emergence of zonal jets

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The purpose of this talk is to show that the emergence of *zonal jets* (in atmospheres and oceans of rotating planets) follows from the presence of the extra conservation (like the inverse cascade follows from the conservation of the energy and enstrophy). We show that

- the Rotating Shallow Water dynamics has an adiabatic-type invariant

$$I = \frac{1}{2} \int X_{\mathbf{k}} \, |\mathcal{Q}_{\mathbf{k}}(t)|^2 \, dp \, dq,$$

  which is conserved approximately over long time; $\mathbf{k} = (p, q)$ is the wave vector, $\mathcal{Q}_{\mathbf{k}}(t)$ is the Fourier transform of the perturbational potential vorticity, and X is the following function

$$X_{\mathbf{k}} = \frac{1}{p} \left[\arctan \frac{q + p\sqrt{3}}{k^2} \; - \; \arctan \frac{q - p\sqrt{3}}{k^2} \right]$$

  (the Rossby radius of deformation is normalized to 1);

- the presence of this extra conservation implies that the inverse cascade transfers energy from small scale fluctuations towards large scale zonal jets (unless the forcinng wave-length significantly exceeds the Rossby radius).