Intermittency, Front motion and Nucleation in a Stochastic Extended System

by

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Keywords:

- Argentina, et al, "Chaotic Nucleation of Metastable Domains", PRE 56,3,R2359, (1997).
- Van den Broeck, et al, "Nonequilibrium phase transitions Induced by Multiplicative Noise", PRE 55, 4,4084, (1997).
- Muñoz, "Nature of different types of absorbing states", PRE, 57, 2, (1998).

Spatiotemporal intermittency in Shells

Figure 1: Cymbiola Vespertilio found in Sulawesi Is., Indonesia

Spatiotemporal intermittency in PDE's

Several systems display spatiotemporal intermittency: spatially extended laser with injected signal, autocalytic reaction-diffusion (Gray-Scott model), catalysis on Pt surface model (Phys. D110, 92, [1997]):

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= -\frac{1}{\epsilon}u(u-1)(u-\frac{b+v}{a}) + \frac{\partial^2 u}{\partial x^2} \\
\frac{\partial v}{\partial t} &= f(u) - v \\
f(u) &= \begin{cases} 0, & 0 \le u < 1/3 \\ 1 - 6.75u(u-1)^2, & 1/3 \le u \le 1 \\ 1, & 1 < u \end{cases}
\end{aligned} \tag{1}
$$

with periodic boundary conditions in $x \in [0, L]$:

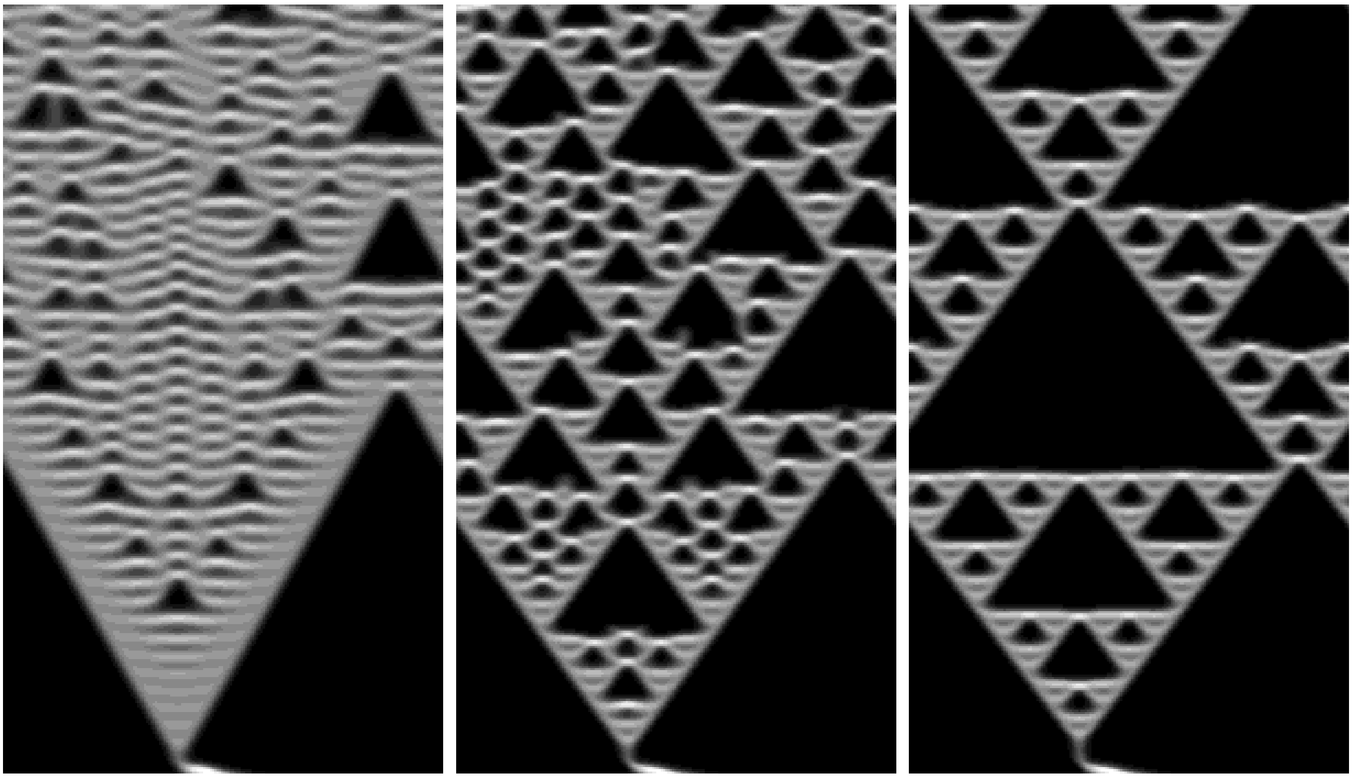

Figure 2: Reaction-diffusion simulations $L = 100$, $N = 200$, $b = 0.19$, (a) $\epsilon = 0.215$, (b) $\epsilon = 0.155$, (c) $\epsilon = 0.147$.

Stochastic extended system

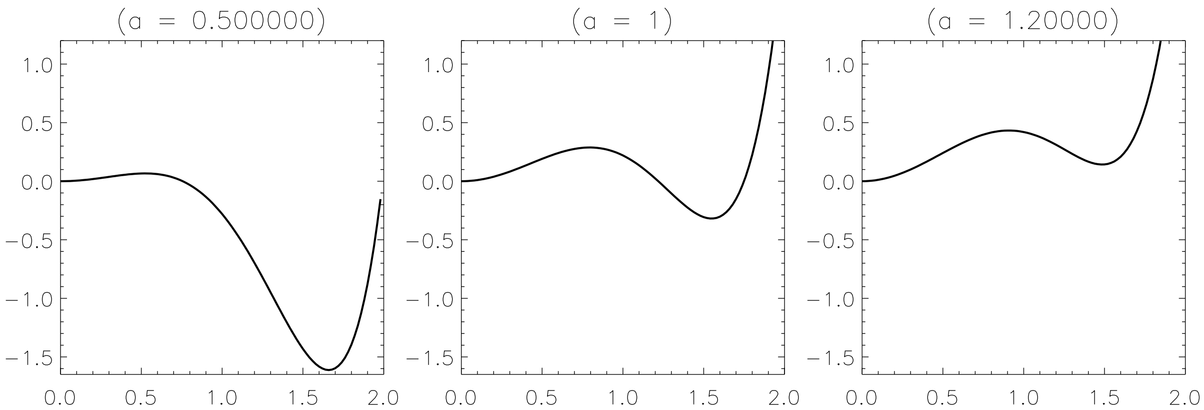

Figure 3: Potential $\phi(u)$ for $a = 0.5$, $a = 1.0$, $a = 1.2$. The Maxwell point occurs at $a_M = 1/4h = 1.1364$.

$$u_t = -\frac{\partial \phi}{\partial u} + D\, u_{xx} + \sqrt{\epsilon}\, g(u)\, \xi \tag{2}$$

with:

$$\phi(u) = a\, u^2 - u^4 + h\, u^6, \quad g(u) = u \tag{3}$$

$$< \xi(x,t)\xi(x',t') > = 2\, \delta(x - x')\, \delta(t - t') \tag{4}$$

- $\epsilon = 0, D \neq 0$: Deterministic fronts anniquilate metastable state.
- $\epsilon \neq 0, D = 0$: No front motion. Eventually after some time all sites will collapse to the absorving state... $\Rightarrow P^{st}(u) = \delta(u)$.

So we expect for $\epsilon \neq 0, D \neq 0$:

a noise-induced transition

Spatiotemporal intermittency in SPDE

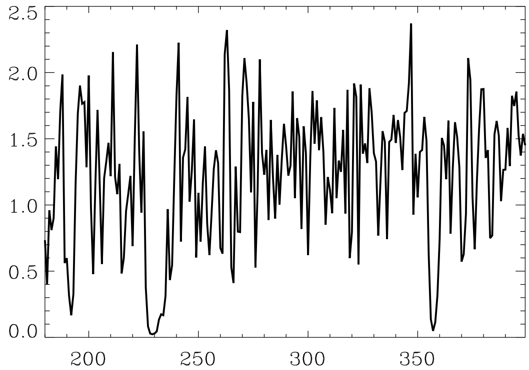

Figure 4: $a = 0.5$. Space profile for $\epsilon = 0.9$.

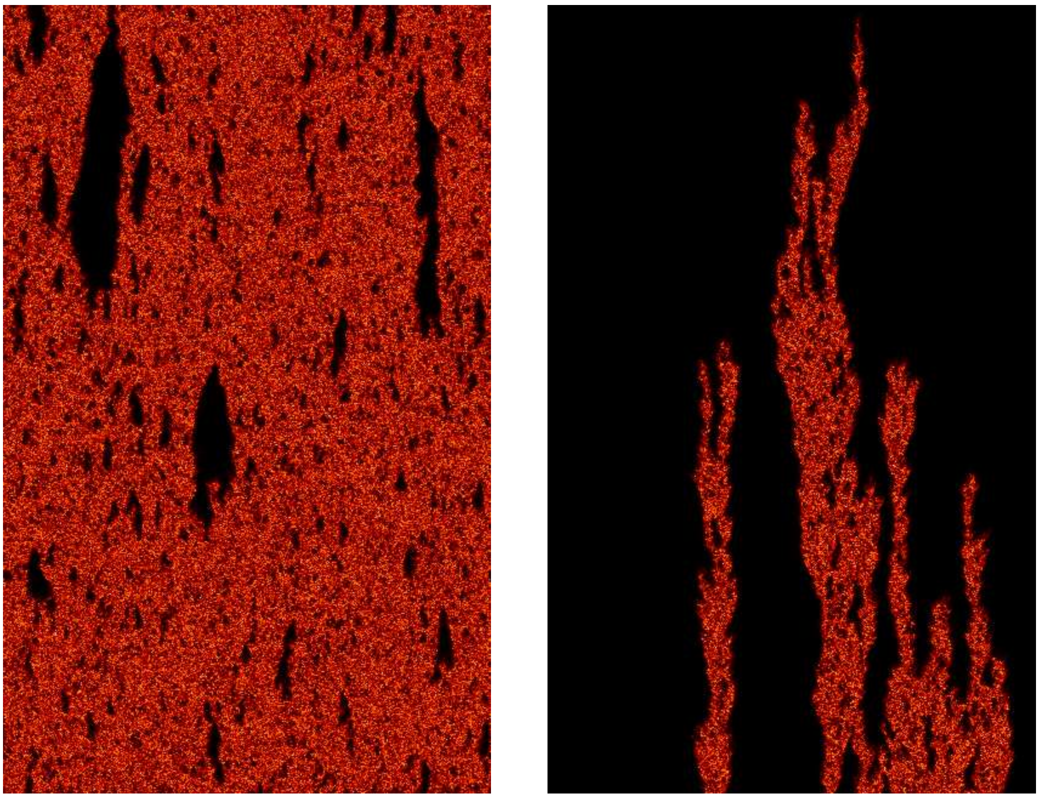

Figure 5: $a = 0.5$. (left) $\epsilon = 0.9$, (right) $\epsilon = 1.0$.

Numeric characterization:

- Front velocity reversal
- Fraction of absorbing phase
- One-site probability distribution
- Phase diagram

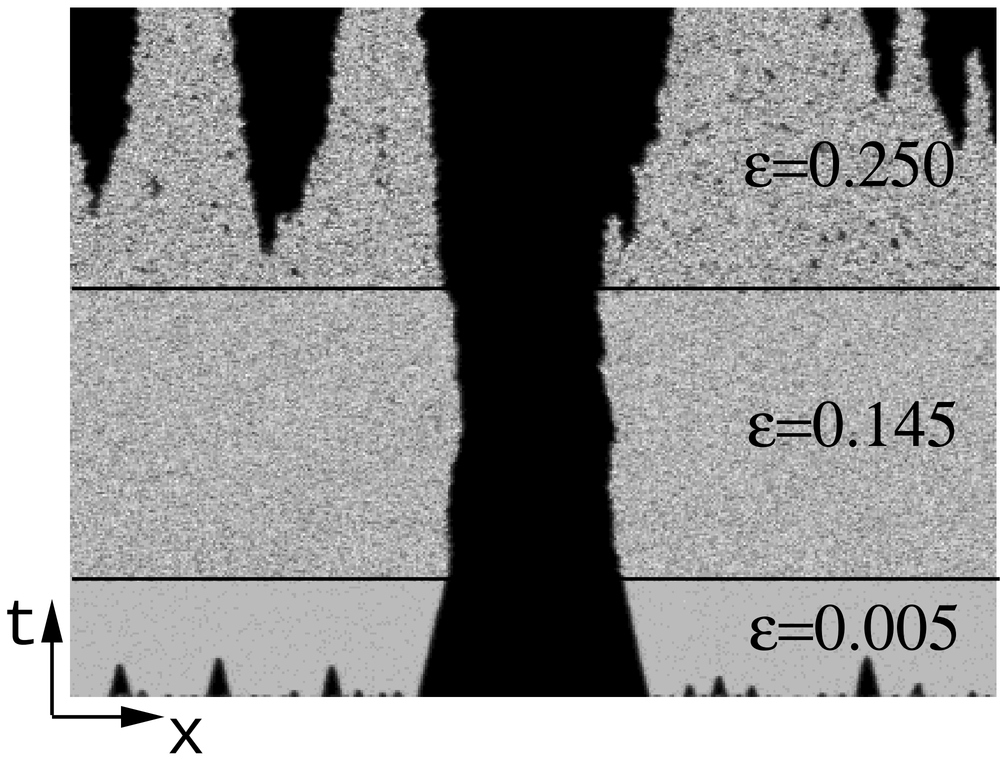

Figure 6: Space-time plot for the field $u(x,t)$ starting from a random initial condition $u_0(x)$ of mean u_+ for $x \in (0,400)$ except in the interval $x \in (150,250)$ where $u_0(x) = 0$. The noise intensity ϵ is changed during the evolution to show the dependence of the front velocity on ϵ: $\epsilon = 0.005$ for $t \in (0,30)$, $\epsilon = 0.145$ for $t \in (30,105)$ and $\epsilon = 0.250$ for $t \in (105,180)$. Other parameters are $D = 2.0$, $a = 1.0$, $h = 0.22$.

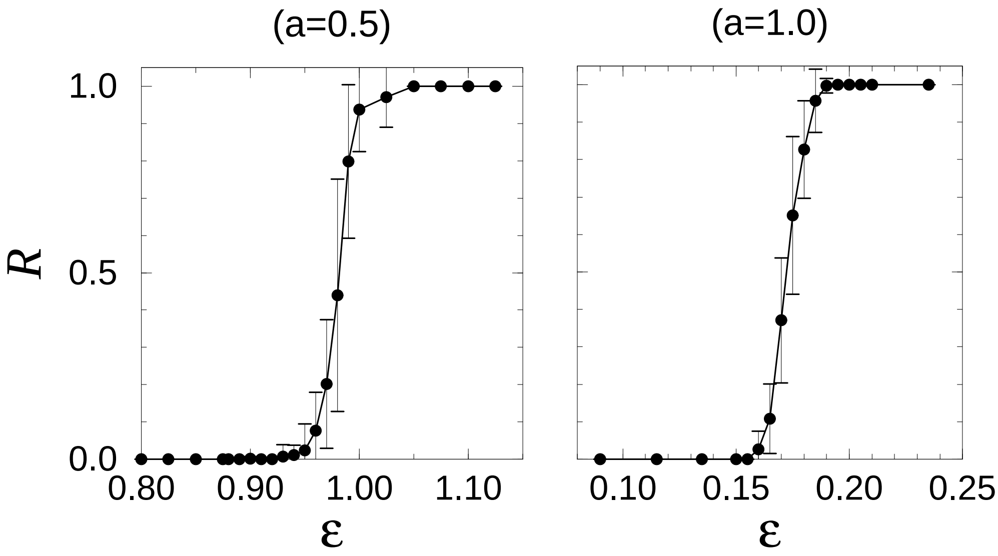

Figure 7: Fraction portion of absorbing phase R as a function of ϵ and two different values of a. The data are the result of averaging over 40 realizations, each one of them integrated the SPDE for a time $t = 900$.

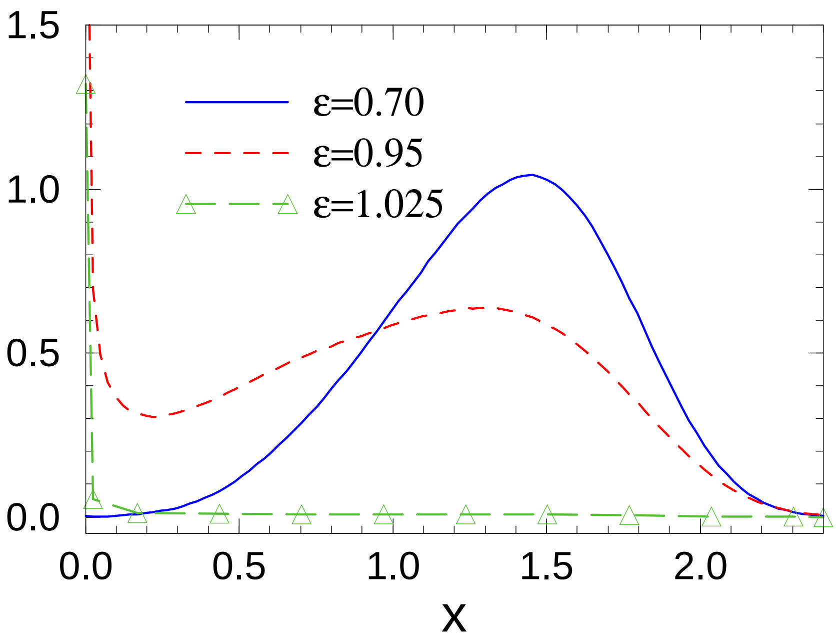

Figure 8: One-site probability distribution $P(u)$ for different values of ϵ. The ordinate axis of the pdf corresponding to $\epsilon = 1.025$ is scaled down 10 times.

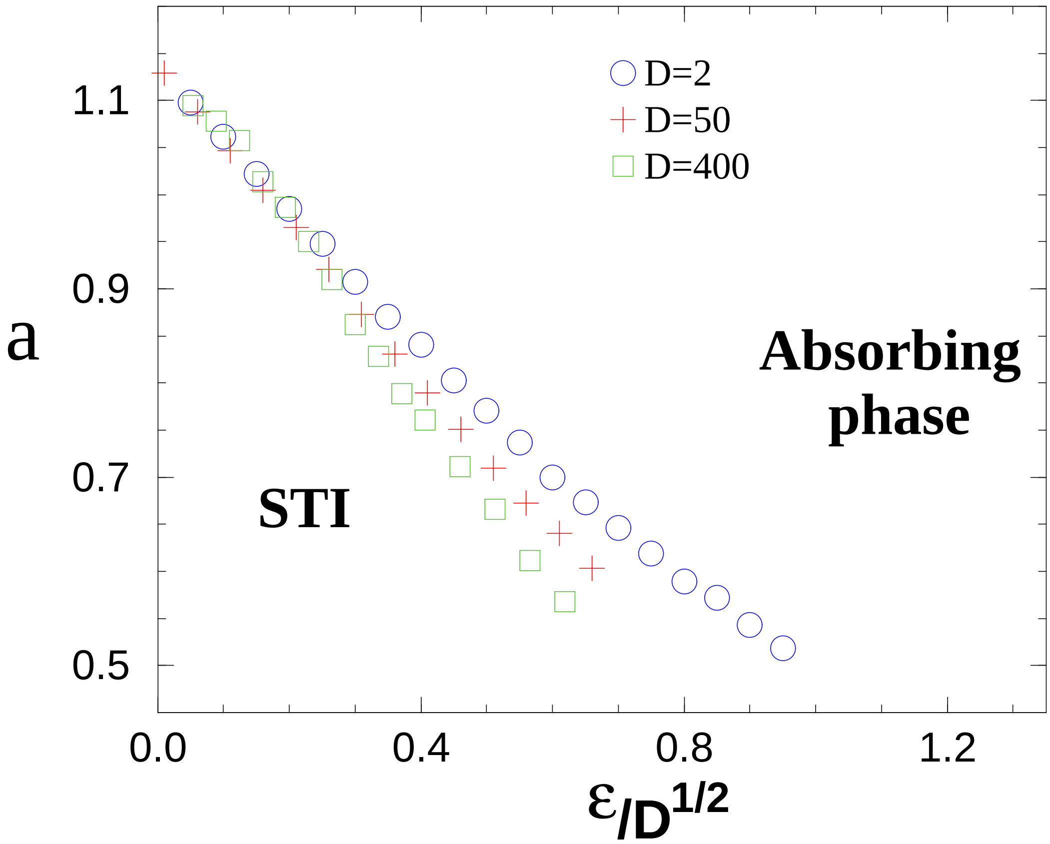

Figure 9: Phase diagram for stochastic system.

Mean field analysis

For this we take Van der Broeck et al (1994,1997) work and take over their calculations under Ito interpretation...

- discretize SDE: $u^i_t = f(u^i) + D_0\,(u^{i+1} + u^{i-1} - 2u^i) + g(u^i)\,\xi^i$

  where $D_0 \equiv D/\epsilon^2$ and $u^i = u(x_i, t)$.

- Write a multivariate Fokker-Planck equations and integrate under some assumptions all except one-site at $u^i = u$:

$$\partial_u\left[(-f(u) + 2D_0\,(u - E(u)))P\right] + \partial_u^2(g^2(u)P) = 0 \tag{5}$$

  where as a first approximation one can take $E(u) \simeq \bar{u}$.

$$P(u;\bar{u}) = \frac{1}{Z\;g(u)^2}\,\exp\int_0^u \frac{f(v) - 2D_0(v-\bar{u})}{g(v)^2}\,dv \tag{6}$$

- $D = 0$, $\epsilon \neq 0$ case is easy: $P^{st}(u,\bar{u}) \sim u^{-2(1+a)} \Rightarrow \delta(u)$.

- $D = \infty$: after a saddle-point expansion: $f(\bar{u}) = 0$, which implies that there is always one solution $\bar{u} = 0$, and for $0 < a < 1/3h$, two non-trivial $\bar{u} > 0$ solutions exists (for all ϵ).

- We solve numerically the consistency equation ,

$$\bar{u} = \int_0^\infty v\,P(v;\bar{u})\,dv \tag{7}$$

  and obtain:

$$P(u,\bar{u}) = \begin{cases} \delta(u), & \text{for all } a, \epsilon \\ P_\pm(u,\bar{u}), & \text{for } \epsilon < \epsilon^*(a), \text{ and } 0 < a < 1/3h \end{cases} \tag{8}$$

- a Maxwell like construction permits us to compute the relative stability between the 3 branches, and we determine the transition point numerically.

Complete phase diagram:

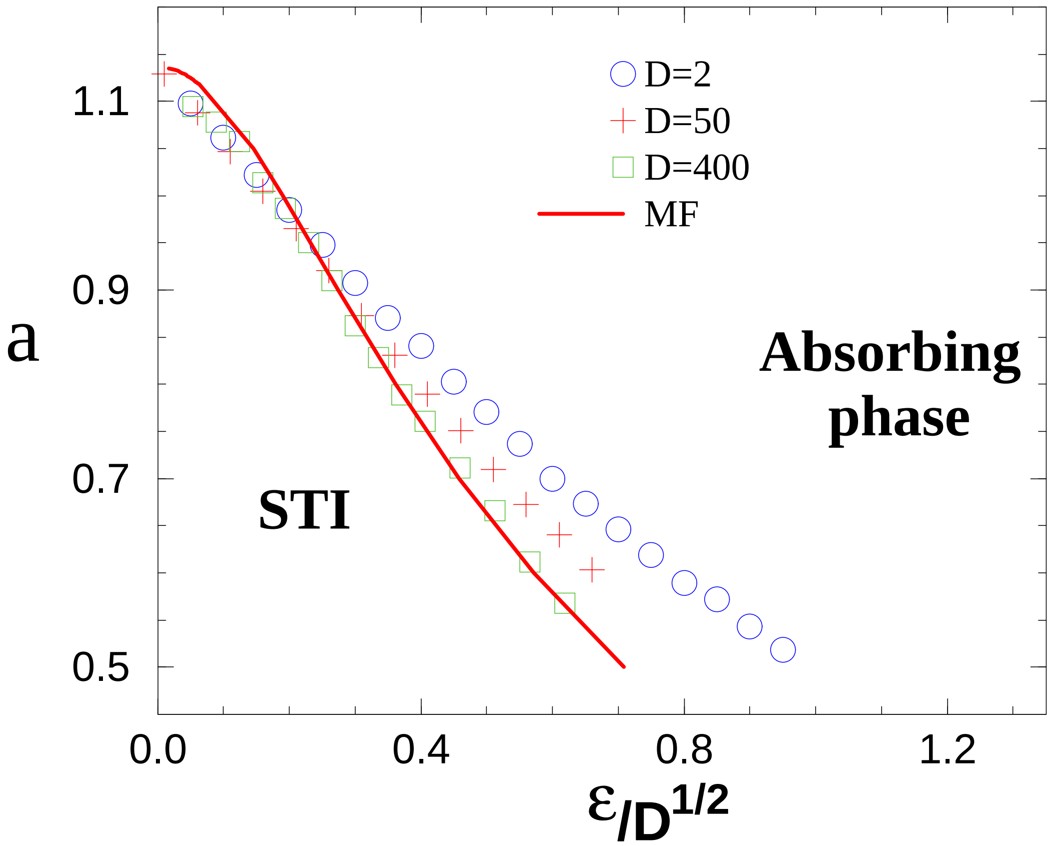

Figure 10: Phase diagram of the noise induced transition from STI to absorbing phase. The symbols correspond to numerical simulations of (1) for different values of D and ϵ. The solid line comes from the solution of the mean-field consistency (7). We have taken $h = 0.22$ and the Maxwell point of the potential $\phi(u)$ is at $a = a_M = 1/4h = 1.136$.

Colored noise

Changed the noise correlations to $\langle \xi(x,t)\xi(x',t')\rangle = \exp(-\frac{|x-x'|}{\gamma})\delta(t-t')$

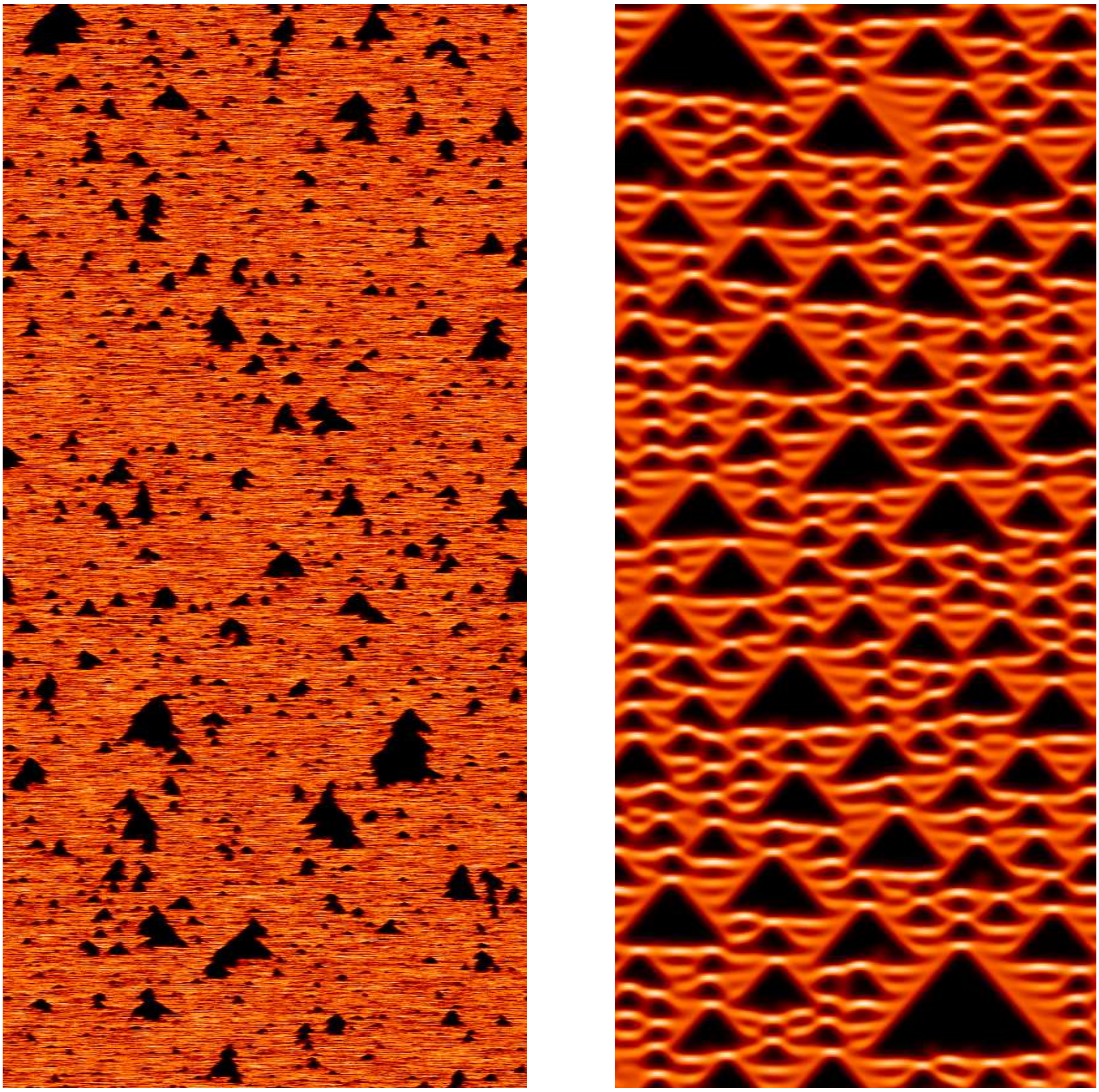

Figure 11: $a = 0.5$. (left) Colored noise, $\epsilon = 0.35$, $\gamma = 8.0$, (right) Reaction-diffusion simulations $L = 100$, $N = 200$, $b = 0.17$, $\epsilon = 0.175$.

Conclusions:

- SPDE model with multiplicative noise, reproduces STI behaviour.
- Velocity of fronts connecting an STI region to an absorbed region, reverses for increasing noise intensity.
- Noise induced transition characterized by mean field approach.