Neutral excitations produced on-demand in the Fermi sea

(Injection from a driven Andreev's level)

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Single-particle emission on demand was realized,...

one quantum level - one particle, one voltage pulse - one particle

Dynamic QD	Static QD	Voltage pulse

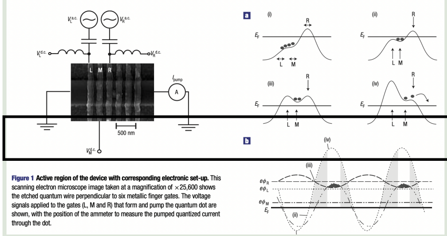
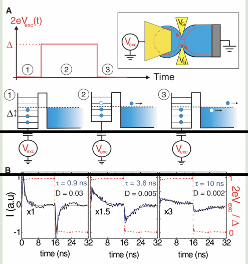
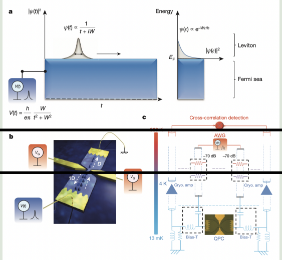


Figure 1 Active region of the device with corresponding electronic set-up. This scanning electron microscope image taken at a magnification of ×25,600 shows the etched quantum wire perpendicular to six metallic finger gates. The voltage signals applied to the gates (L, M and R) that form and pump the quantum dot are shown, with the position of the ammeter to measure the pumped quantized current through the dot.

M. D. Blumenthal, et al. Nat. Physics **3**, 343 (2007)



G. Fève, et al. Science **316**, 1169 (2007)



J. Dubois, et al., Nature **502**, 659 (2013)

but unwanted e-h excitations can also be created

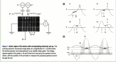
... verified, ...

Fermi sea hosts two types of particles (electrons and holes):

Current is sensitive to the charge sign, $\langle I \rangle \sim \; eN_e + (-e)N_h = e\Delta N$ – It does not see e-h pairs

While the shot noise is not, $\mathcal{P}^{sh} \sim e^2 N_e + (-e)^2 N_h = e^2 N$ – It should be minimal

Minimal low-freq. noise



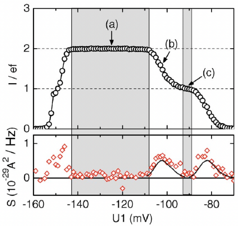
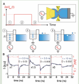
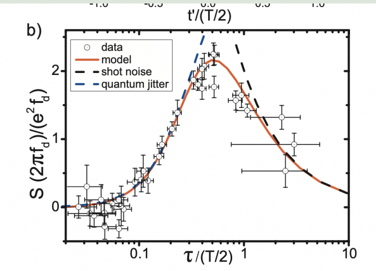


FIG. 3. (Color online) Top: Pumped current of the sample driven at 400 MHz with a power of 1 dBm. Arrows (a), (b), and (c) indicate the voltage positions of the spectra shown in Fig. 2. Bottom: Corresponding noise power (symbols) averaged over the range of 5–15 kHz (10–15 kHz for $-152 \leqslant U1 \leqslant -146$). The solid curves display theoretical estimates given by Eq. (3) (see text).

N. Maire, et al., APL **92**, 082112 (2008)

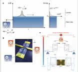
Quantum jitter noise

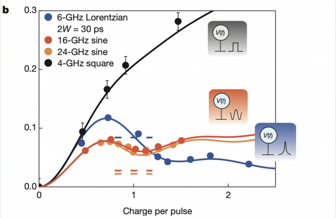




A. Mahé, et al. PRB **82**, 201309(R) (2010)

Minimal shot noise





Lorentzian to observe the manifestation of the DOC[30]. Because non-integer charges require many hole excitations, the larger noise is observed around $q = 0.6$ for both pulse types. The suppression of the DOC at integer charge is signalled by a noise minimum at $q \approx 1$ for the sinusoidal pulses. For the Lorentzian pulse, thermal averaging shifts this minimum to higher value, $q \approx 1.4$. However, it is actually for $q = 1$ that the number of excitations vanishes: indeed, the finite noise observed is entirely due to thermal excitations

J. Dubois, et al., Nature **502**, 659 (2013)

Dynamic QD (na)	Static QD (na/a)	Voltage pulse (a)

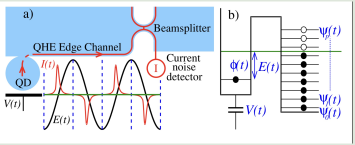
Dynamic QD (na)

$$\rho(E,t) = \frac{\exp\left(-\frac{(E-t\beta_e)^2}{2\sigma_E^2} - \frac{t^2}{2\sigma_t^2}\right)}{2\pi\sigma_E\sigma_t}$$

M. Kataoka, et al., PSS B **254**, No. 3, 1600547 (2017)

Static QD (na/a)

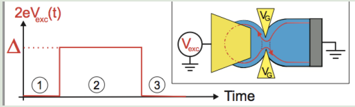
$U(t) = ct$:



$$\Psi_{CS}(t) = \sqrt{\frac{1}{\pi\Gamma_\tau}} \int_0^\infty d\epsilon\, e^{-\epsilon} e^{-i\epsilon \frac{t-\epsilon\tau_D}{\Gamma_\tau}}$$

J. Keeling, et al., PRL **101**, 196404 (2008)

a step potential:



$$\Psi_{Tr}(t) = \frac{e^{-it\frac{\Delta}{2\hbar}}}{\sqrt{\tau_D}}\theta(t)e^{-\frac{t}{2\tau_D}}$$

G. Haack, et al., PRB **87**, 201302 (2013)

Voltage pulse (a)

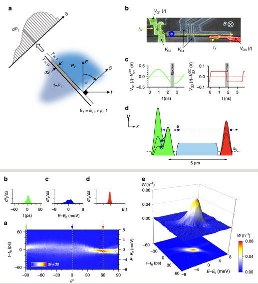
$$\frac{e}{\hbar}U(t) = \frac{2/\Gamma_\tau}{(t/\Gamma_\tau)^2+1},$$

$$\Psi_L(t) = \sqrt{\frac{1}{\pi\Gamma_\tau}}\frac{1}{t/\Gamma_\tau - i}$$

J. Keeling,et al.,PRL **97**,116403 (2006)

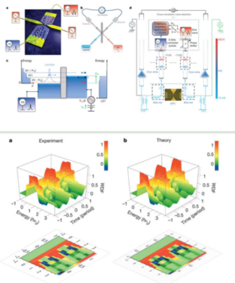
… and single-electron state was characterized

Tomography of a solitary electron

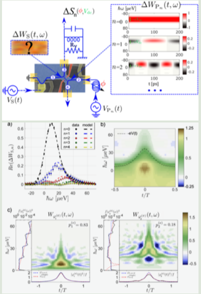


J.D. Fletcher,et al., Nat.Comm.**10**,5298 (2019)

Tomography of a voltage pulse



T. Jullien, et al., Nature **514**, 603 (2014)



R. Bisognin, et al., Nat.Comm.**10**, 071101 (2020)

$$\rho_{exp}(E,t,r) = \frac{\exp\left(-\frac{\left[\frac{E}{\sigma_E}\right]^2 - 2r\frac{E}{\sigma_E}\frac{t}{\sigma_t} + \left[\frac{t}{\sigma_t}\right]^2}{2[1-r^2]}\right)}{2\pi\sigma_E\sigma_t\sqrt{1-r^2}} \sim \rho(E,t), \qquad \Psi_{exp} \sim \Psi_L,$$

a correlation coefficient $r \sim 0.75$

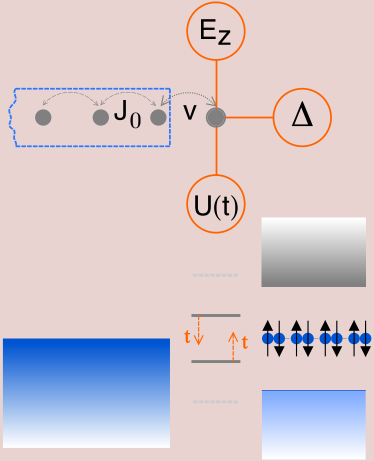
$E-\mu \sim 100meV, \quad \Delta E \sim 1meV, \quad purity < 0.1$

$E-\mu \sim \Delta E \sim 0.01meV, \quad purity \sim 1$

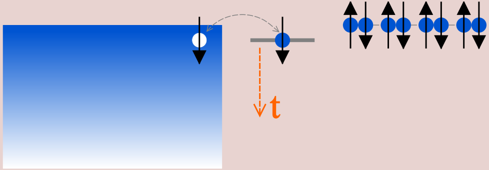
Problem statement and results

What if the quantum level is more complex? Then what is emitted is also more complex

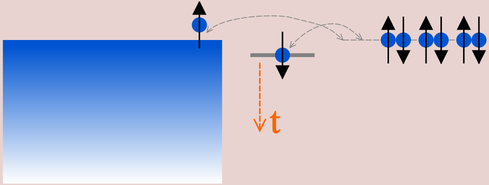
Andreev's QD (Nambu view)	$\|0\rangle \to \|1\rangle \Rightarrow$ e-h pair (real particle view)



an electron absorption



Cooper pair splitting



Problem Solving Algorithm

$$\boxed{\hat{H}_{QD}} \xrightarrow{QD-N} \boxed{\left[\hat{G}^r_{QD}(E)\right]^{-1} = (E-\mu+i0)\,\hat{I} - \hat{H}_{QD} - \hat{\Sigma}_N(E)}$$

$$\downarrow$$

$$\boxed{\hat{S}(E) = \hat{I} - 2\pi i\,\hat{W}_N^\dagger(E)\,\hat{G}^r_{QD}(E)\,\hat{W}_N(E)} \Leftrightarrow \boxed{\hat{S}\hat{S}^\dagger = \hat{I}}$$

(adiabatic drive)

$$\downarrow \hat{H}_{QD}[U(t)]$$

$$\boxed{\hat{G}^{(1,ex)}(t_1;t_2) = \frac{1}{2\pi i}\frac{\hat{S}^*(\mu,t_1)\hat{S}^T(\mu,t_2)-\hat{I}}{t_1-t_2}}$$

$$\downarrow U(t)-?$$

$$\boxed{\hat{G}^{(1,ex)}(t_1;t_2) = \sum_{j=1}^n \eta_j \hat{\Psi}_j^\dagger(t_1)\,\hat{\Psi}_j(t_2)}$$

($\eta = +$ for electrons, $\eta =$ - for holes)

Outline

1. Formalism
   - Excess correlation function
2. Examples: Quasi-particles excited by the voltage pulses
   - Plain quasi-particles: Leviton and anti-Leviton
   - A compound quasi-particle: A dipole
   - Auxiliary correlation function (to identify a pure state)
3. Andreev's ac pump
   - Excitations with variable charge
   - A charge-neutral spin-dipole quartet
4. Summary

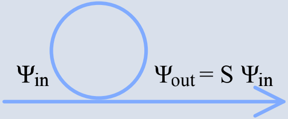
Definitions

$$\mathcal{G}^{(1)}(t_1;t_2) = \left\langle \hat{\Psi}^{\dagger}(t_1)\hat{\Psi}(t_2)\right\rangle ,$$

$$\underbrace{G^{(1,ex)}(t_1;t_2)}_{quasi-particles} = \mathcal{G}^{(1)}_{on}(t_1;t_2) - \underbrace{\mathcal{G}^{(1)}_{off}(t_1;t_2)}_{vacuum},$$

$$\mathcal{G}^{(1)}_{off}(t_1;t_2) = \int\limits_{-\infty}^{\mu}\frac{dE}{h}e^{i\frac{E}{\hbar}(t_1-t_2)} = \frac{e^{i\frac{\mu}{\hbar}\tau}}{2\pi i}\frac{1}{t_1-t_2},$$

$G^{(1)}$ vs S (adiabatic drive)



$$\mathcal{G}^{(1)}_{on}(t_1;t_2) = S^{*}(t_1)\,S(t_2)\,\mathcal{G}^{(1)}_{off}(t_1;t_2),$$

$$G^{(1,ex)}(t_1;t_2) = \frac{e^{i\frac{\mu}{\hbar}(t_1-t_2)}}{2\pi i}\,\frac{S^{*}(t_1)\,S(t_2)-1}{t_1-t_2}$$

$$\stackrel{?}{=} \Psi^{*}(t_1)\,\Psi(t_2) \leftarrow single\ particle$$

below I use $\mu = 0$ and omit ^{ex}

Simple example Plain quasi-particle excited by a Lorentzian pulse

$$S(t) = e^{-i\varphi(t)}, \quad \varphi(t) = \frac{e}{\hbar}\int^t dt' V(t')$$

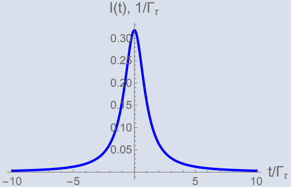
Leviton

$$\frac{e}{\hbar}V_L(t) = \frac{2\Gamma_\tau}{t^2+\Gamma_\tau^2}, \qquad \varphi_L = 2\arctan\frac{t}{\Gamma_\tau},$$

$$S(t) = -\frac{t+i\Gamma_\tau}{t-i\Gamma_\tau}, \qquad G_L^{(1)} = \Psi_L^*(t_1)\Psi_L(t_2),$$

$$\Psi_L(t) = \frac{\sqrt{\Gamma_\tau/\pi}}{t-i\Gamma_\tau},$$

a non-occupied state is added



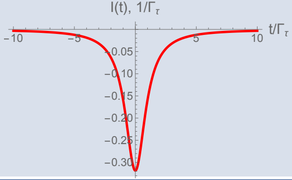
anti-Leviton

$$\frac{e}{\hbar}V_{aL}(t) = -\frac{2\Gamma_\tau}{t^2+\Gamma_\tau^2}, \qquad \varphi_{aL} = -2\arctan\frac{t}{\Gamma_\tau},$$

$$G_{aL}^{(1)}(t_1;t_2) = -\Psi_{aL}^*(t_1)\Psi_{aL}(t_2),$$

$$\Psi_{aL}(t) = \Psi_L^*(t) = \frac{\sqrt{\Gamma_\tau/\pi}}{t+i\Gamma_\tau},$$

an occupied state is removed



$$I(t) = eG^{(1)}(t;t)$$

Simple example A compound quasi-particle: A dipole

(at the threshold, the Andreev's level emits a dipole)

$$\frac{e}{\hbar}V(t) = \frac{4t\Gamma_\tau^2}{t^4+\Gamma_\tau^4}, \quad \varphi = 2\arctan\left(\frac{t}{\Gamma_\tau}\right)^2,$$

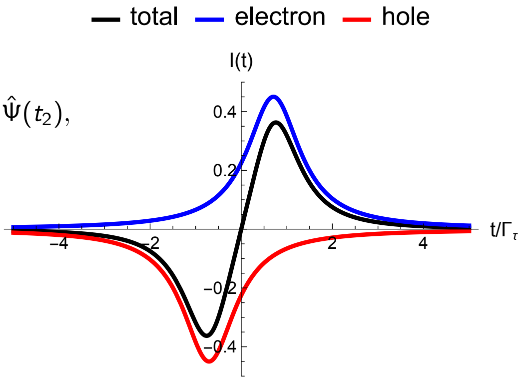
$$G_{dip}^{(1)}(t_1;t_2) = \frac{\Gamma_\tau^2}{\pi}\frac{t_1+t_2}{\left(t_1^2+i\Gamma_\tau^2\right)\left(t_2^2-i\Gamma_\tau^2\right)} = \hat{\Psi}^\dagger(t_1)\hat{G}_{dip}^{(1)}\hat{\Psi}(t_2),$$

Matrix representation:

$$\hat{G}_{dip}^{(1)} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}, \quad \hat{\Psi} = \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix},$$

$$\psi_{e/h}(t) = \frac{\sqrt{\Gamma_\tau/(\sqrt{2}\pi)}}{t \mp \Gamma_\tau e^{i\pi/4}},$$

$$\delta_{ab} = \int dt\psi_a^*(t)\psi_b(t), \quad a,b = e,h,$$



The number of particles: (i) from the quantum state: $N_{dip} = 2\frac{1}{2} = 1$, (ii) from the shot noise: $N_{dip} = Tr\left[\hat{G}_{dip}^{(1)}\right]^2 = 1$

A dipole (a pure state) is a superposition of an e-h pair and a vacuum : $|dip\rangle = \frac{1}{\sqrt{2}}|eh\rangle + \frac{1}{\sqrt{2}}|vac\rangle$

Two type of particles causes the sign problem

How to verify purity of a dipole state?

$G^{(1)}$ for a pure state is idempotent:

indeed, for a Leviton, $G_L^{(1)}(t_1;t_2) = \Psi_L^*(t_1)\,\Psi_L(t_2)$, we have $G_L^{(1)} \bullet G_L^{(1)} = G_L^{(1)}$,

where $\bullet = \int d\tau (*,\tau)(\tau,*)$,

while for a hole (a removed particle, $\mathcal{G}_{on}^{(1)} < \mathcal{G}_{off}^{(1)} \Rightarrow G^{(1,ex)} < 0$), the extra sign is needed :

$$G_{aL}^{(1)}(t_1;t_2) = -\Psi_{aL}^*(t_1)\,\Psi_{aL}(t_2) \Rightarrow G_{aL}^{(1)} \bullet G_{aL}^{(1)} = -G_{aL}^{(1)},$$

Therefore, for an e-h state, $G^{(1,ex)}$ loses its ability to verify purity. For example:

$$G_{LaL}^{(1,ex)} = G_L^{(1)} + G_{aL}^{(1)} \Rightarrow G_{LaL}^{(1,ex)} \bullet G_{LaL}^{(1,ex)} = G_L^{(1)} - G_{aL}^{(1)} \neq G_{LaL}^{(1,ex)},$$

Auxiliary $G^{(1)}$ solves the sign problem

Let's go back from the quasi-particle (e-h) picture:

$$G^{(1,ex)}(t_1;t_2) = \sum_{j=1}^{n_e}\alpha_j \underbrace{\psi^*_{e,j}(t_1)\,\psi_{e,j}(t_2)}_{\text{poles in the upper semi-plane}} - \sum_{k=1}^{n_h}\beta_k \underbrace{\psi^*_{h,k}(t_1)\,\psi_{h,k}(t_2)}_{\text{poles in the lower semi-plane}} + \text{cross terms},$$

to the minimal real-particle picture (remembering that holes are nothing but removed electrons):

$$G^{(1,ex)} \to G^{(1,aux)}(t_1;t_2) = \underbrace{G^{(1,ex)}(t_1;t_2)}_{\mathcal{G}^{(1)}_{on}-\mathcal{G}^{(1)}_{off}} + \sum_{k=1}^{n_h}\psi^*_{h,k}(t_1)\,\psi_{h,k}(t_2).$$

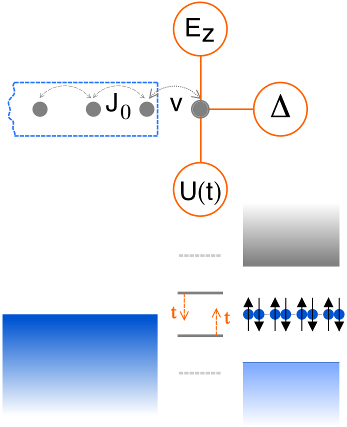
In equilibrium, $G^{(1,ex)}=0$, while $G^{(1,aux)}\neq 0$.

For the dipole state (matrix representation):

$$\hat{G}^{(1,ex)}_{dip} = \frac{1}{2}\begin{pmatrix}1 & i\\ -i & -1\end{pmatrix} \Rightarrow \hat{G}^{(1,ex)}_{dip}\hat{G}^{(1,ex)}_{dip} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \neq \hat{G}^{(1,ex)}_{dip},$$

$$\hat{G}^{(1,aux)}_{dip} = \hat{G}^{(1,ex)}_{dip} + \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix} \Rightarrow \hat{G}^{(1,aux)}_{dip}\hat{G}^{(1,aux)}_{dip} = \frac{1}{4}\begin{pmatrix}2 & 2i\\ -2i & 2\end{pmatrix} = \hat{G}^{(1,aux)}_{dip}$$

Andreev's ac pump : e-h pair from QD



$$\hat{\Psi} = \{\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger\}^T \leftarrow \textit{the Nambu spinor},$$
$$\textit{with } \psi = d \textit{ for a quantum dot and } \psi = c_n \textit{ for a lead}$$

$$\hat{\mathbf{H}}(t) = \hat{\mathbf{H}}_d(t) + \hat{\mathbf{H}}_L + \hat{\mathbf{H}}_T,$$

$$\hat{\mathbf{H}}_d(t) = \hat{d}^\dagger \{-\epsilon_d(t)\tau_z \otimes \sigma_0 - E_Z\tau_0 \otimes \sigma_z - \Delta\tau_x \otimes \sigma_0\} \hat{d},$$

$$\hat{\mathbf{H}}_L = -J_0 \sum_{n=1}^{\infty} \hat{c}_{n+1}^\dagger \hat{c}_n + H.c., \qquad \mathrm{WBA}: \; J_0 \to \infty,$$

$$\hat{\mathbf{H}}_T = J_T \hat{d}^\dagger \hat{c}_1 + H.c.,$$

$$\epsilon_d(t) = \epsilon_{d,0} + eU(t), \qquad eU(t) = ct \leftarrow \textit{a clean emission}$$

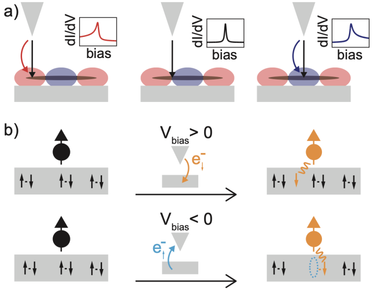


Figure 4. (a) Schematic HOMO wavefunction (red/blue) and spin-carrying molecular orbital (gray) on a surface for different tip positions, with the corresponding Kondo lineshapes in the insets. At the HOMO nodal plane, the lineshape is symmetric. (b) Tunneling into the superconducting substrate at positive (top) and negative bias (bottom) populating the YSR excitation (right) from an unscreened-spin ground state (left). Due to spin polarization of the YSR state and singlet Cooper pairing, the spin of electrons tunneling (into) from the substrate is (anti)parallel to the impurity spin.

L. Faribacci, et al., PRL **125**, 256805 (2020)

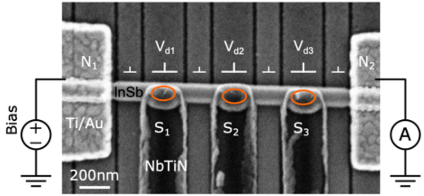
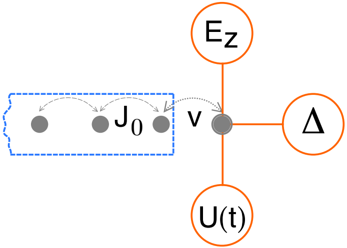


FIG. 1. Scanning electron micrograph (SEM) of the triple-dot device. Orange circles indicate positions of quantum dots within the InSb nanowire, underneath three superconducting leads (S_1, S_2, S_3, NbTiN), electrostatic gates that separate and tune dots are marked with inverted T's. The circuit

H. Wu, et al., arXiv:2105.08636

Adiabatic Nambu scattering matrix



$$\hat{S}(t) = \hat{I} - 2\pi i \hat{W}^{\dagger} \hat{G}^{r}(t, \mu) \hat{W},$$

$$\left[\hat{G}^{r}(t, E)\right]^{-1} = (E - \mu + i0)\hat{I} - \hat{H}_{d}(t) - \hat{\Sigma},$$

$$\hat{\Sigma} = -iv\hat{I}, \qquad \hat{W}^{\dagger} = \sqrt{\frac{i}{2\pi}\left(\hat{\Sigma} - \Sigma^{\dagger}\right)}, \quad v = \frac{|J_T|^2}{J_0},$$

$$\hat{S} = \begin{pmatrix} r_{\uparrow\uparrow} & 0 & r_{\uparrow\downarrow,a} & 0 \\ 0 & r_{\downarrow\downarrow} & 0 & r_{\downarrow\uparrow,a} \\ -r^{*}_{\downarrow\uparrow,a} & 0 & r^{*}_{\downarrow\downarrow} & 0 \\ 0 & -r^{*}_{\uparrow\downarrow,a} & 0 & r^{*}_{\uparrow\uparrow} \end{pmatrix}, \quad \hat{S}^{\dagger}\hat{S} = \hat{I},$$

$$r_{\uparrow\uparrow} = \frac{\epsilon_d^2 - \mathcal{E}^2 - 2v^2 - i2v(\epsilon_d + E_Z)}{\epsilon_d^2 - \mathcal{E}^2}, \quad r_{\uparrow\downarrow,a} = -\frac{i2v\Delta}{\epsilon_d^2 - \mathcal{E}^2}, \quad r_{\downarrow\downarrow} = r_{\uparrow\uparrow}(-E_Z), \; r_{\downarrow\uparrow,a} = r_{\uparrow\downarrow,a}(-E_Z),$$

$$\mathcal{E}^2 = E_Z^2 - \Delta^2 - v^2 - i2vE_Z,$$

Nambu $G^{(1)}$ and the emitted state

$\hat{G}^{(1,ex)}(t_1;t_2) = \frac{1}{2\pi i}\frac{\hat{S}^*(t_1)\hat{S}^T(t_2)-\hat{I}}{t_1-t_2}$, $\quad \epsilon_d(t) = ct$,

$$\hat{G}^{(1,ex)}(t_1;t_2) = \begin{pmatrix} G^{(1)}_{\uparrow\uparrow} & 0 & G^{(1)}_{\uparrow\downarrow,a} & 0 \\ 0 & G^{(1)}_{\downarrow\downarrow} & 0 & G^{(1)}_{\downarrow\uparrow,a} \\ G^{(1)*}_{\downarrow\uparrow,a} & 0 & -G^{(1)*}_{\downarrow\downarrow} & 0 \\ 0 & G^{(1)*}_{\uparrow\downarrow,a} & 0 & -G^{(1)*}_{\uparrow\uparrow} \end{pmatrix},$$

$$G^{(1)}_{\uparrow\uparrow}(t_1;t_2) = \frac{v}{c\pi}\frac{\frac{\Delta^2}{c^2} - \left[t_1 + \frac{E_Z}{c} + i\frac{v}{c}\right]\left[t_2 + \frac{E_Z}{c} - i\frac{v}{c}\right]}{\left(t_1^2 - \mathcal{E}^{*2}/c^2\right)\left(t_2^2 - \mathcal{E}^2/c^2\right)}, \quad G^{(1)}_{\downarrow\downarrow}(E_Z) = G^{(1)}_{\uparrow\uparrow}(-E_Z),$$

$$G^{(1)}_{\uparrow\downarrow,a}(t_1;t_2) = -\Delta\frac{v}{c\pi}\frac{t_1 + t_2 + i\frac{2v}{c}}{\left(t_1^2 - \mathcal{E}^{*2}/c^2\right)\left(t_2^2 - \mathcal{E}^2/c^2\right)}, \quad G^{(1)}_{\downarrow\uparrow,a}(E_Z) = G^{(1)}_{\uparrow\downarrow,a}(-E_Z),$$

The two-particle state is excited in each spin sector:

$$G^{(3)}_{\sigma\sigma} \equiv 0 \Rightarrow G^{(1)}_{\sigma\sigma}(t_1;t_2) = \sum_{j=1}^{2}\alpha_{j,\sigma}\Psi^*_{j,\sigma}(t_1)\Psi_{j,\sigma}(t_2)$$

Auxiliary Nambu $\hat{G}^{(1)}$ reveals a pure 8-particle state (describing 2e and 2h)

Basis wave functions: $\psi_e(t) = \frac{\sqrt{r\sin(\phi)/\pi}}{t + re^{-i\varphi}}$, $\psi_h(t) = \frac{\sqrt{r\sin(\phi)/\pi}}{t - re^{-i\varphi}}$, where $\mathcal{E}/c = re^{-i\varphi}$

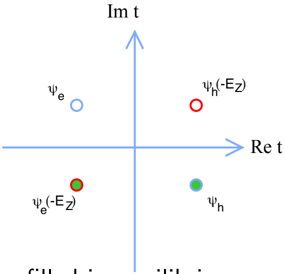
Quasi-particle basis (not Nambu!) : $\hat{\Psi} = (\psi_e, \psi_h)^T \Rightarrow$

$$G^{(1)}_{\uparrow\uparrow}(t_1;t_2) = \hat{\Psi}^\dagger(t_1)\,\hat{G}^{(1)}_{\uparrow\uparrow}\hat{\Psi}(t_2)\,,\quad G^{(1)}_{\downarrow\downarrow}(t_1;t_2) = \hat{\Psi}^\dagger(t_1,-E_Z)\,\hat{G}^{(1)}_{\downarrow\downarrow}\hat{\Psi}(t_2,-E_Z)\,,\quad etc.$$

Nambu spinor : $\hat{\Psi}_N = \{d_\uparrow, d_\downarrow, d_\downarrow^\dagger, -d_\uparrow^\dagger\}^T$

Quasi-particle Nambu basis (in this model $^\dagger \equiv E_Z \to -E_Z$) :

$$\hat{\Psi}_{qN} = \{\psi_e, \psi_h, \psi_e(-E_Z), \psi_h(-E_Z), \psi_e, \psi_h, \psi_e(-E_Z), \psi_h(-E_Z)\}^T$$



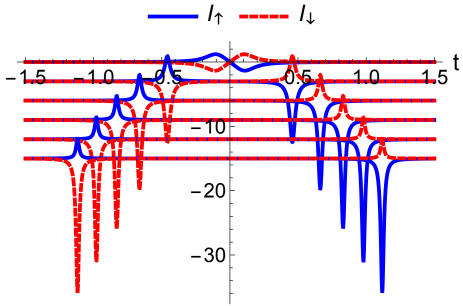
Following the poles : (the lower semi-plane: $E < \mu$, the upper semi-plane: $E > \mu$)

$$\hat{G}^{(1,aux)} = \hat{G}^{(1,ex)} + diag(0,1,1,0,0,1,1,0)\,,\quad 0 - \text{empty in equilibrium}, 1 - \text{filled in equilibrium},$$

The emitted Nambu state is a pure quantum state (effectively 8-particle state):

$\hat{G}^{(1,aux)}\hat{G}^{(1,aux)} = \hat{G}^{(1,aux)}$ but any sub-matrix is not idempotent, e.g., $\hat{G}^{(1,aux)}_{\uparrow\uparrow}\hat{G}^{(1,aux)}_{\uparrow\uparrow} \neq \hat{G}^{(1,aux)}_{\uparrow\uparrow}$

A time-dependent current, $I_\sigma = eG^{(1)}_{\sigma\sigma}(t;t)$, reveals a fractional charge



$E_Z/\Delta = 1.0;\ 1.1;\ 1.2;\ 1.3;\ 1.4;\ 1.5$

"fractional" - due to competing processes:

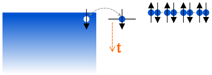
(i)N, (ii)CPS, (iii)no emission

(nothing happens or

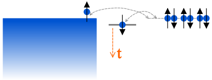
CP absorption)

left pulses due to ↓-level:

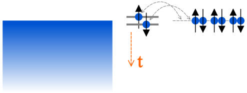
N :



CPS :

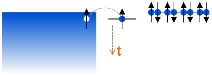


CP absorption :

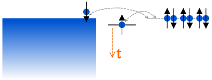


right pulses due to ↑-level:

N :



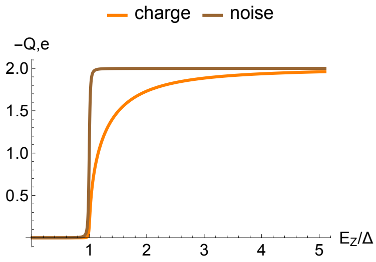
CPS :



Position of the sub-gap eigenstates of H_d and the emission regimes

$$\frac{Q}{e} = \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{\infty} dt G^{(1)}_{\sigma\sigma}(t;t) = n_e - n_h,$$

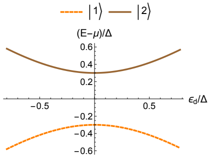
$$\frac{-\mathcal{P}^{sh}}{e^2 T(1-T)} = \frac{1}{2}\iint_{-\infty}^{\infty} dt_1 dt_2 \mathrm{Tr}\left[\hat{G}^{(1)\dagger}(t_1;t_2)\,\hat{G}^{(1)}(t_1;t_2)\right] = n_e + n_h$$



$v/\Delta = 0.02, \quad U(t) = ct, \quad c > 0$

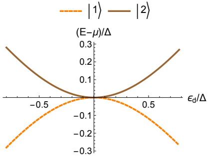
sub-gap states (each provides the same information):

$$|1\rangle = \{u_\uparrow |e\uparrow\rangle, 0, v_\downarrow |h\downarrow\rangle, 0\} \to e - h,$$
$$|2\rangle = \{0, u_\downarrow |e\downarrow\rangle, 0, v_\uparrow |h\uparrow\rangle\} \to e - h$$



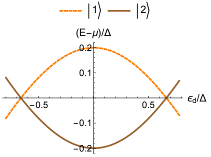
$E_Z/\Delta = 0.7$, no crossings

no emission



$E_Z/\Delta = 1.0$, touch

two dipoles

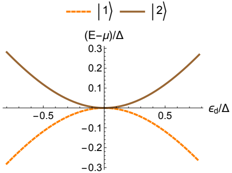


$E_Z/\Delta = 1.2$ two crossings

four fr. particles

QD population: $0 \to 1 \to 2$

Touch Two dipoles are emitted at $E_Z = \sqrt{v^2 + \Delta^2}$ with probability $\frac{1}{2}$



$$G^{(1)}_{\uparrow\uparrow}(t_1;t_2) = G^{(1)}_{\uparrow\downarrow}(t_1;t_2) = -\frac{1}{2} G^{(1)*}_{dip}(t_1;t_2), \text{ with } \Gamma_\tau = \sqrt{2v\Delta}/c \ at\ v \to 0,$$

$$G^{(1)}_{\downarrow\downarrow}(t_1;t_2) = -G^{(1)}_{\downarrow\uparrow}(t_1;t_2) = \frac{1}{2} G^{(1)}_{dip}(t_1;t_2), \quad \text{and} \quad \hat{G}^{(1)}_{dip} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix},$$

$$\langle n_{dip} \rangle = 1 \quad \Rightarrow \quad \langle n_\sigma \rangle = \frac{1}{2} \quad \Rightarrow \quad \langle n_{e,\sigma} \rangle = \langle n_{h,\sigma} \rangle = \frac{\langle n_\sigma \rangle}{2} = \frac{1}{4},$$

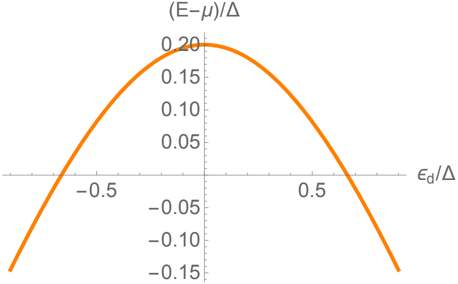
$$\frac{-\mathcal{P}^{sh}}{e^2 T(1-T)} = N_{tot} = \langle n_\uparrow \rangle + \langle n_\downarrow \rangle = 1,$$

$$\left.\begin{aligned} I^{(q)}_{tot} &\sim \langle n_e \rangle - \langle n_h \rangle = 0, \\ I^{(\sigma)}(t) &\sim G^{(1)}_{dip}(t;t), \end{aligned}\right\} \text{a charge-neutral spin-dipole quartet}$$

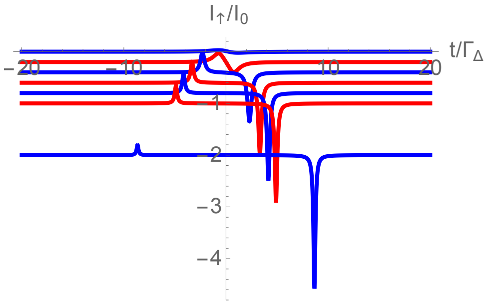
Crossings Two particles are emitted for each σ

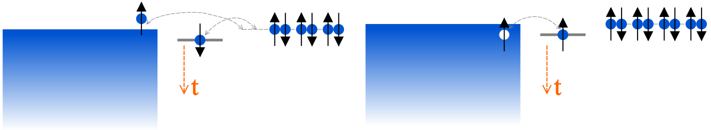
$$I_{\uparrow}(t) = eG^{(1)}_{\uparrow\uparrow}(t;t),$$

$$|1\rangle = \{u_{\uparrow}|e\uparrow\rangle, 0, v_{\downarrow}|h\downarrow\rangle, 0\}$$



el. emission / el. absorption



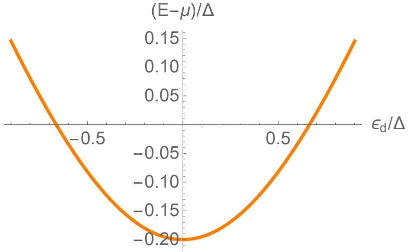


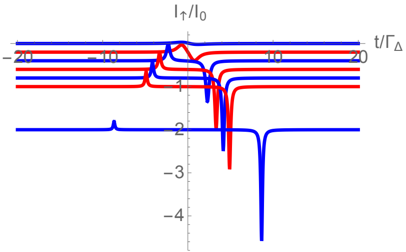
Another level the same info (Nambu doubling at work)

$$I_{\uparrow}(t) = eG^{(1)}_{\uparrow\uparrow}(t;t),$$

$$|2\rangle = \{0, u_{\downarrow}|e\downarrow\rangle, 0, v_{\uparrow}|h\uparrow\rangle\}$$

hole absorption / hole emission

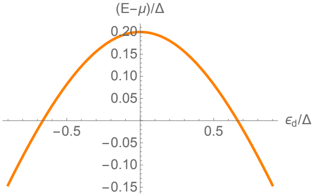




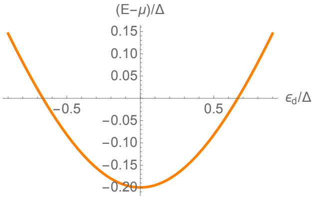
Another parts of eigenlevels give us a spin-down current

Population of the sub-gap eigenstates of H_d at the crossing points

$\{u_\uparrow |e\uparrow\rangle, 0, v_\downarrow |h\downarrow\rangle, 0\}$



$\{0, u_\downarrow |e\downarrow\rangle, 0, v_\uparrow |h\uparrow\rangle\}$



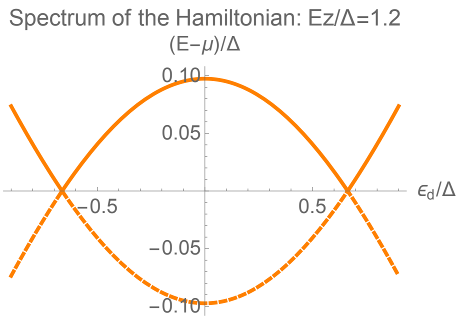
$$u_\uparrow = \frac{\epsilon_d + \sqrt{\Delta^2 + \epsilon_d^2}}{\sqrt{2\Delta^2 + 2\epsilon_d\left(\epsilon_d + \sqrt{\Delta^2+\epsilon_d^2}\right)}},$$

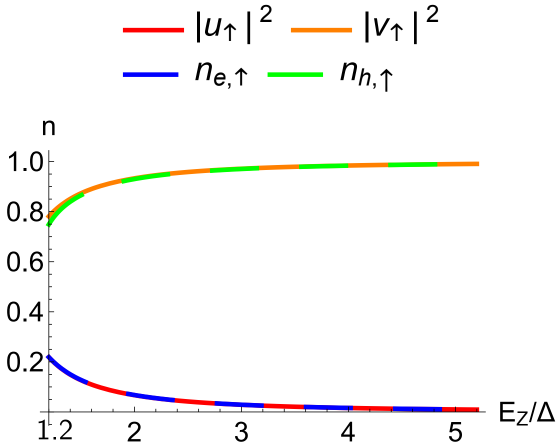
$$v_\uparrow = \frac{\Delta}{\sqrt{2\Delta^2 + 2\epsilon_d\left(\epsilon_d - \sqrt{\Delta^2+\epsilon_d^2}\right)}},$$

$$|u_\sigma|^2 + |v_{\bar\sigma}|^2 = 1 :$$

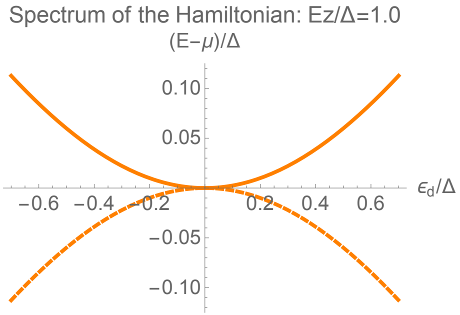
$$\langle n\rangle_{e,\uparrow} \Leftrightarrow \left|u_\uparrow\left(\epsilon_d^{crossing,L}\right)\right|^2 \left(\equiv \left|v_\uparrow\left(\epsilon_d^{crossing,L}\right)\right|^2\right), \qquad \langle n\rangle_{h,\uparrow} \Leftrightarrow \left|v_\uparrow\left(\epsilon_d^{crossing,R}\right)\right|^2 \left(\equiv \left|u_\uparrow\left(\epsilon_d^{crossing,R}\right)\right|^2\right)$$

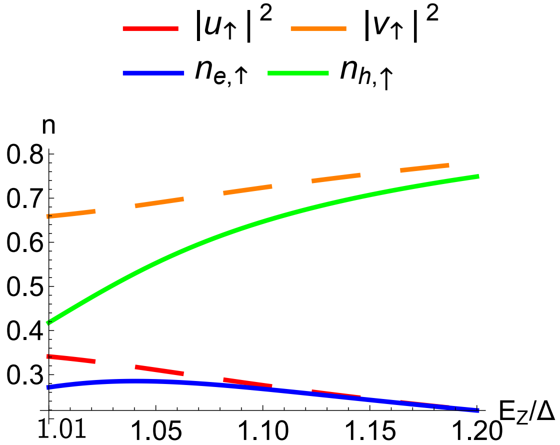
Population vs emission: Full emission, $E_Z > \Delta$, $\frac{v}{\Delta} = 0.1$





Population vs emission: Partial emission, $E_Z \sim \Delta$, $\frac{v}{\Delta} = 0.1$





The mean number of particles emitted is less than the population
because the widened level crosses the Fermi level partially and contributes to emission partially

Summary

- A driven Andreev's level allows us to create :
  - Excitations with variable charge
  - A charge-neutral spin-dipole quartet

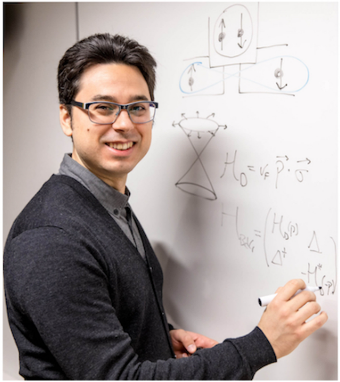
- A charge-dipole excitation can be created using a voltage pulse

- Methodology : "ex" versus "aux"
  - $G^{(1),ex}$ provides Ψ for created quasi-particles (injected, removed and/or excited electrons),
  - $G^{(1),aux}\left(= G^{(1),ex} + G^{(1),removed} + G^{(1),excited}\right)$ verifies the number of relevant DoF via

$$G^{(1),aux}G^{(1),aux} = G^{(1),aux}$$

Thank to the co-authors!

Benjamin Roussel



Pablo Burset

Christian Flindt

Thank you!

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