June 05th, 2023

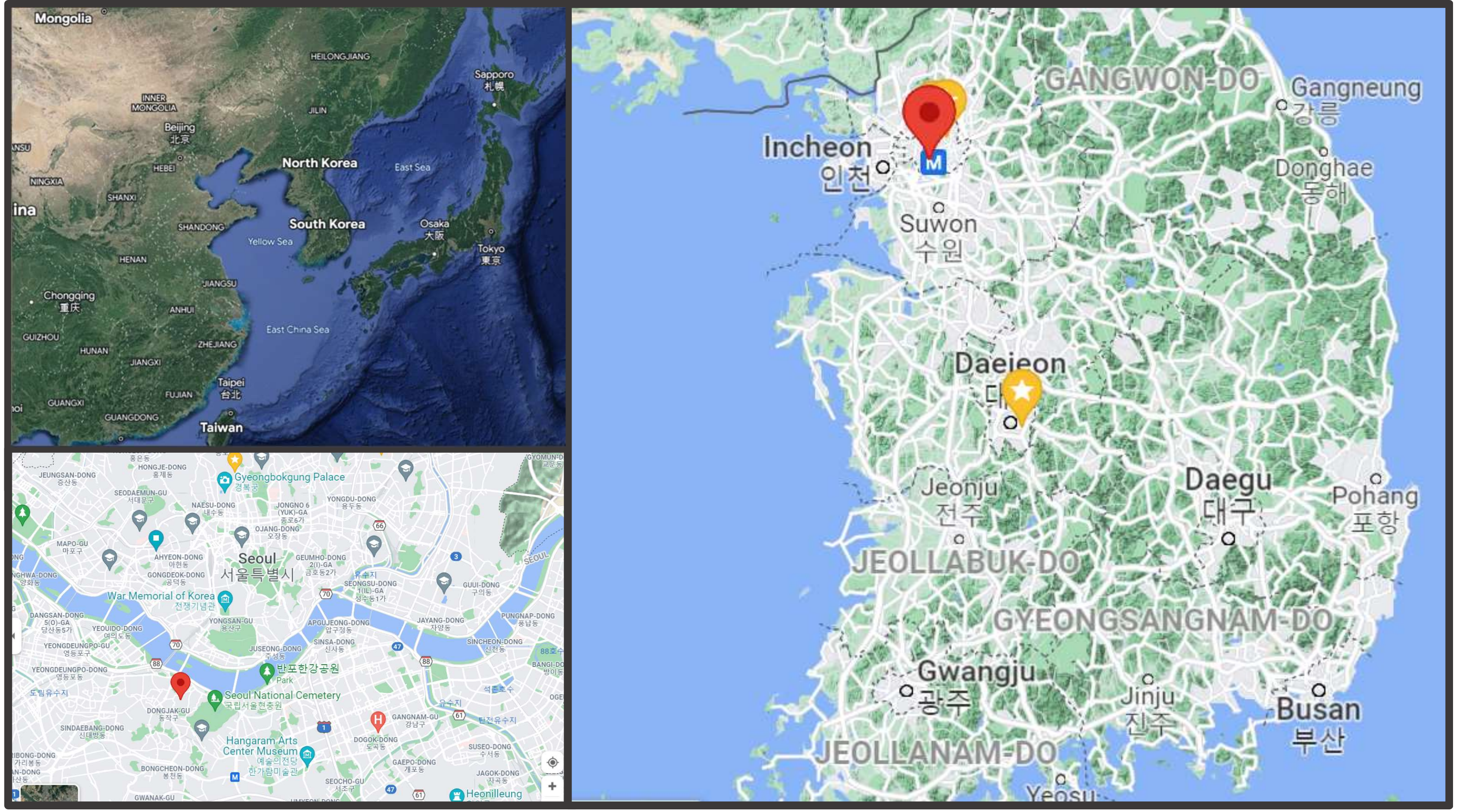
Topological metal arising from Strongly Disordered Floquet operators

Speaker: Kun Woo Kim (Chung-Ang University)

References

- https://arxiv.org/abs/2301.05428
- Physical Review X 13, 011003 (2023)
- Physical Review Research 3, 023183 (2021)
- Physical Review B 101, 165401 (2020)

Republic of Korea
Seoul,



Chung-Ang Univ.
April 2023

Outline of this talk

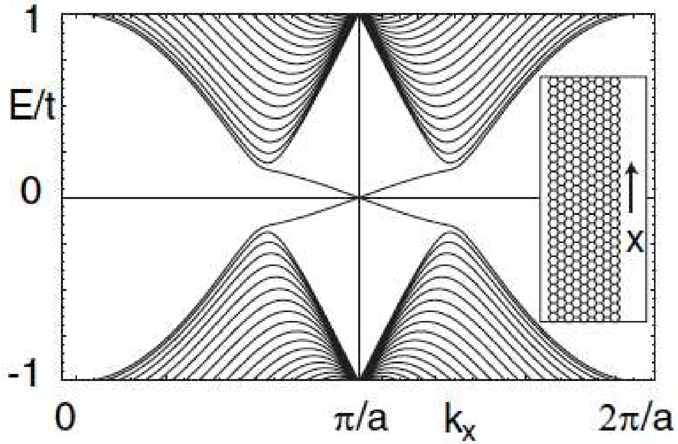
(Short) Introduction of background:
- Topology /Disorder / Floquet

(Main) Topological surface states in Isolation
- Analytical, and
- Numerical verifications
- Experiment proposal

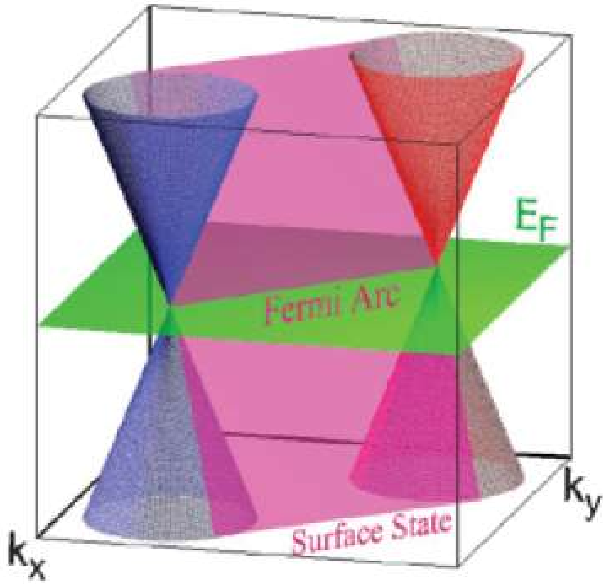
(Future) Directions

Condensed matter with topological protection

- insulator and conductor



Kane and Mele, PRL (2005)



Wan, Turner et al., PRB (2011)

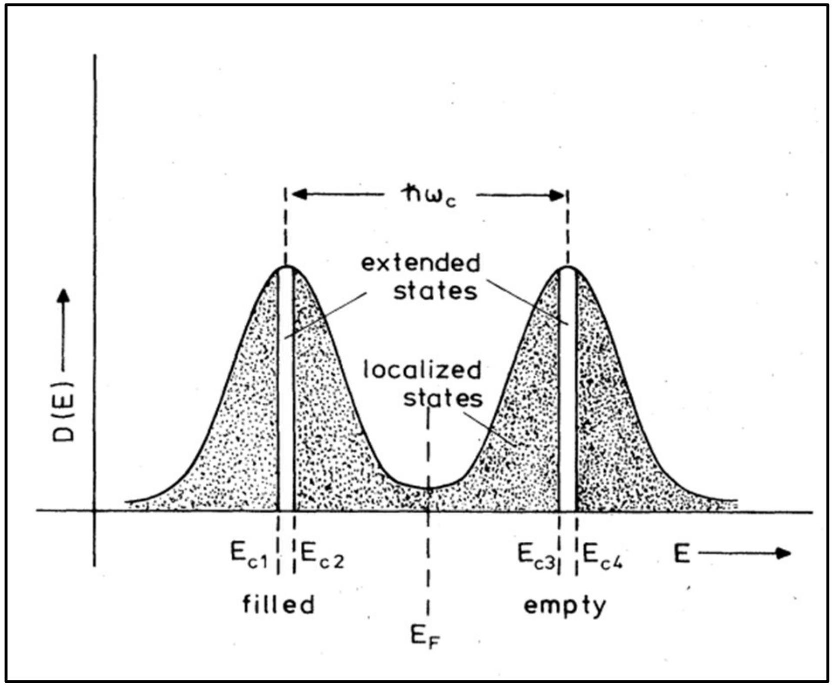
Validity:

(Insulator): When disorder-strength < Energy gap

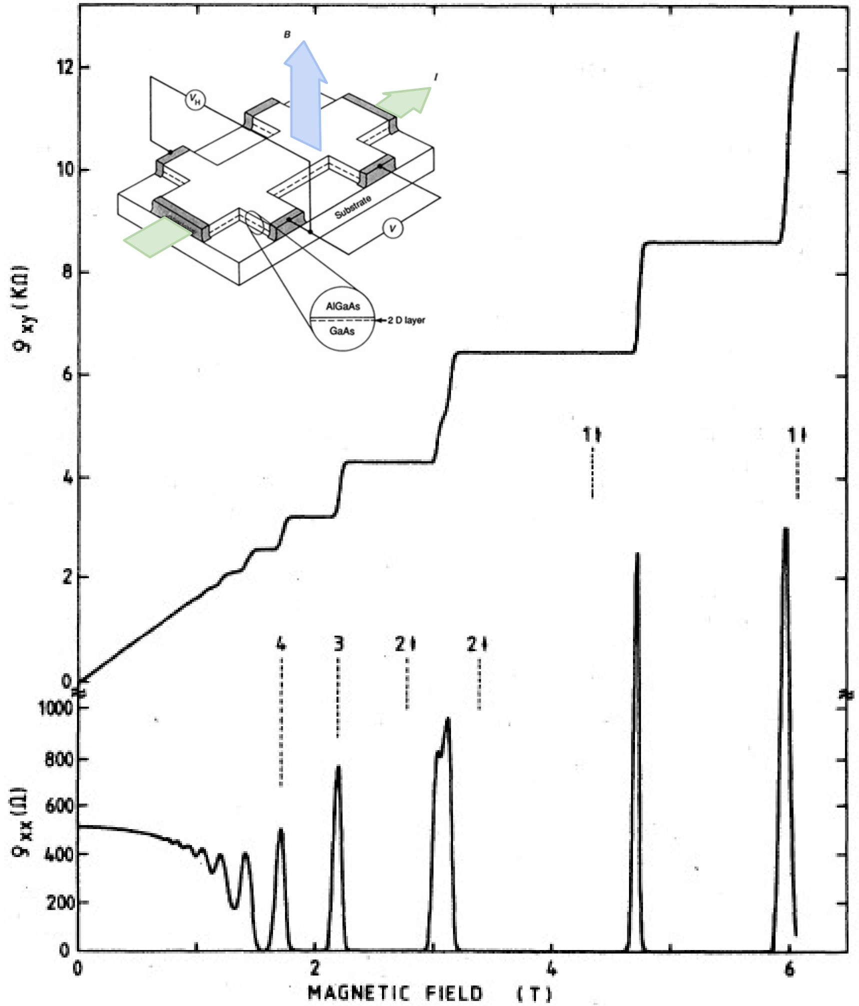
(Conductor): When disorder does not lead inter-valley scatterings

Stabilization of topological response in quantum Hall effect

- Only modes with topological protection can avoid the Anderson localization.



von Klitzing (RMP, 1986)
"The quantized Hall effect"



The meaning of stabilization in topological insulator/metal
- The convergence of a correlation function to the topological invariant.

Time periodic Hamiltonian (Floquet) and topology

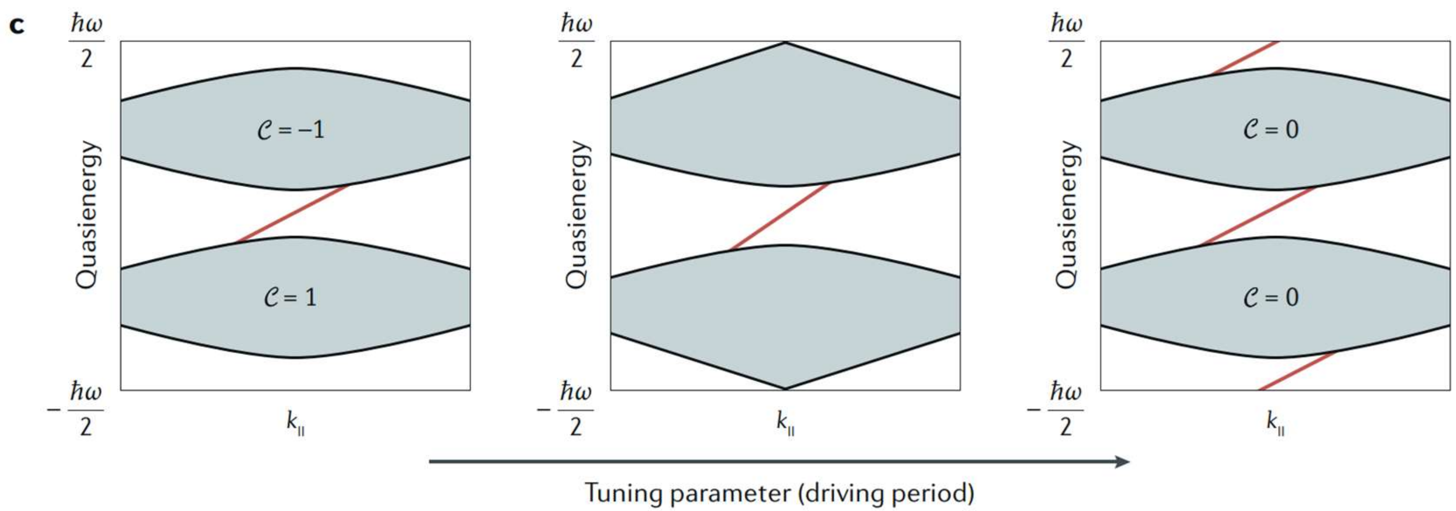
Hamiltonian in momentum space:

$$H = H(\vec{k})$$

Time periodic Hamiltonian in (momentumtime) space:

$$H = H\left(\vec{k}, t\right) = H\left(\vec{k}, t + T\right)$$

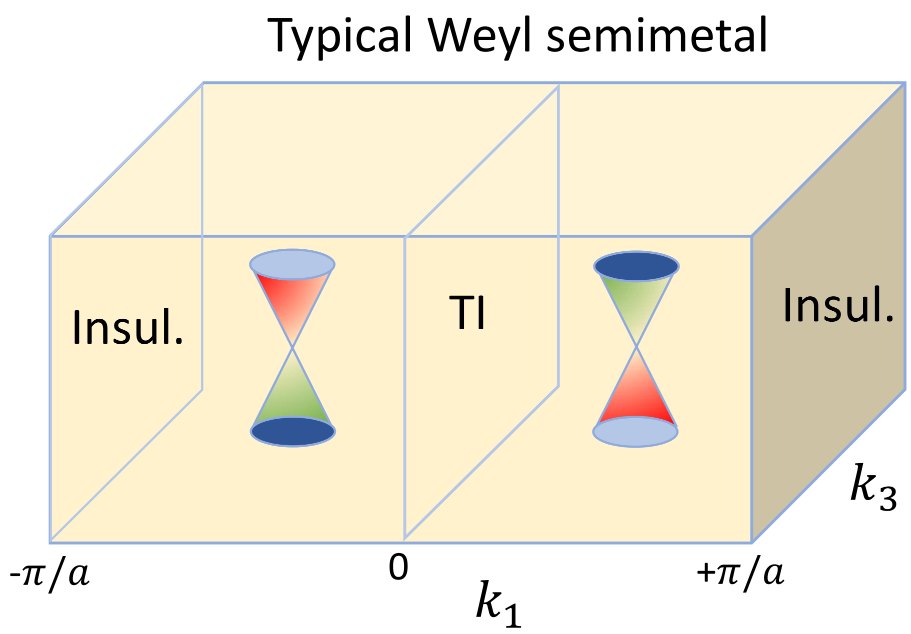
U_T contains the time-ordered information: $U_T = \mathcal{T} e^{-i\int_0^T H(t)dt}$

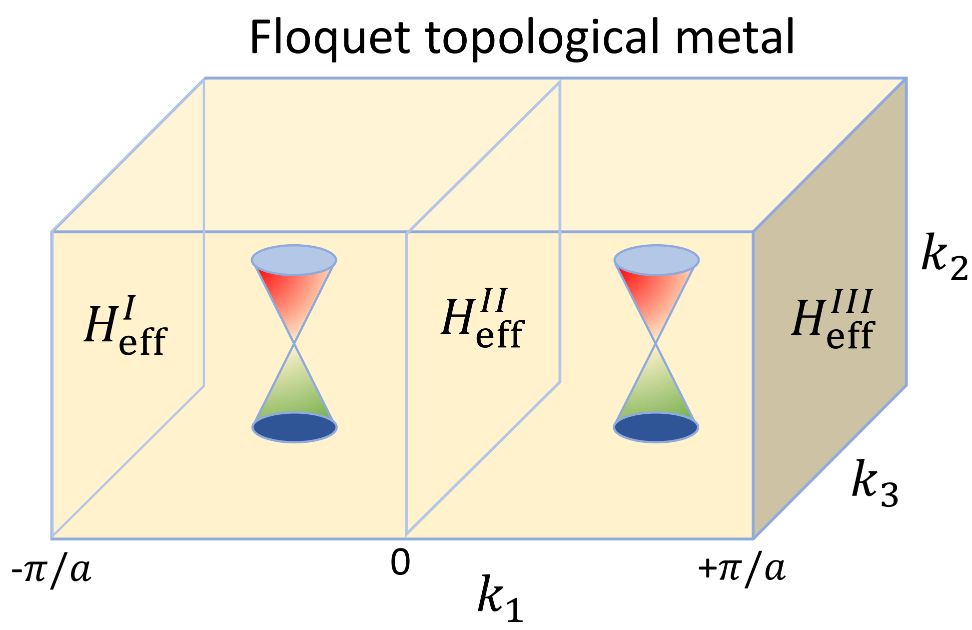


e.g. Anomalous Floquet Anderson Insulator

Rudner and Lindner, *Nature Reviews* **2** 229 (2020)

Topological Floquet metal in 3-dimension





A model in **3-dimension**

$$U_T(\vec{k}) = \exp[ik_1(\sigma_1 \cos k_2 \, |\sin k_3| + \sigma_2 \sin k_2 \sin k_3 + \sigma_3 \cos k_3)]$$

Topological conductor in isolation

- Build a **3-dim** model with nonzero $W = \frac{1}{24\ \ ^2} \iiint \epsilon_{ijk} \mathrm{Tr}\left(\hat{v}_i \hat{v}_j \hat{v}_k\right) d^3\vec{k}$

$$U_T = \exp[ik_1(\sigma_1 \cos k_2\, |\sin k_3| + \sigma_2 \sin k_2 \sin k_3 + \sigma_3 \cos k_3)]$$

velocity operator: $\hat{v}_j = iU_T^\dagger \partial_{k_j} U_T$

- The model contains long-range hoping: $(U_T)_{z_1 z_2} \sim |z_1 - z_2|^{-2}$

- Place them in the synthetic space by replacing $k_3 \to \phi_{3,n} = \phi_{3,0} + \omega_3 n$.

(2+1)-dimension

$$\mathcal{U}_{n,n-1} = \exp\left[ik_1\left(\sigma_1 \cos k_2\, \left|\sin \phi_{3,n}\right| + \sigma_2 \sin k_2 \sin \phi_{3,n} + \sigma_3 \cos \phi_{3,n}\right)\right]$$

(1+2)-dimension

$$\mathcal{U}_{n,n-1} = \exp\left[ik_1\left(\sigma_1 \cos \phi_{2,n}\, \left|\sin \phi_{3,n}\right| + \sigma_2 \sin \phi_{2,n} \sin \phi_{3,n} + \sigma_3 \cos \phi_{3,n}\right)\right]$$

When a model is sent to (1+d$_{synthetic}$)-dim, it comes with a quasi-periodic potential in synthetic space.

For (1+1)-dimension

$$\mathcal{U}_{n,n-1} = \hat{U}(\hat{x}_1, k_2 + \omega_2 n)$$

The time dependence can be taken out:

$$\mathcal{U}_{n,n-1} = e^{+\omega_2 n \partial_{k_2}} \hat{U}(\hat{x}_1, k_2) e^{-\omega_2 n \partial_{k_2}}$$

Wave function at time step 'n' is

$$\begin{aligned} |\psi_n\rangle &= \mathcal{U}_{n,n-1}\mathcal{U}_{n-1,n-2}\cdots\mathcal{U}_{1,0}|\psi_0\rangle \\ &= e^{+\omega_2 n \partial_{k_2}}\left[\mathcal{U}_{0,-1}e^{-\omega_2 \partial_{k_2}}\right]^n|\psi_0\rangle \end{aligned}$$

As a result, the quantum dynamics takes places in (1+1)-dim by the Floquet operator

$$\mathcal{U}_F = \mathcal{U}_{0,-1}e^{-\omega_2 \partial_{k_2}}$$

The verification of 3-dim topological metal

- The construction of the effective theory at low energy:

$$S_{\text{eff}}[Q] = \sigma \int \text{tr}[(\nabla Q)^2] + \frac{iW}{128\pi} \iint \text{tr}[Q(\wedge Q)^4]$$

- (i) Measuring the width of wave packet :

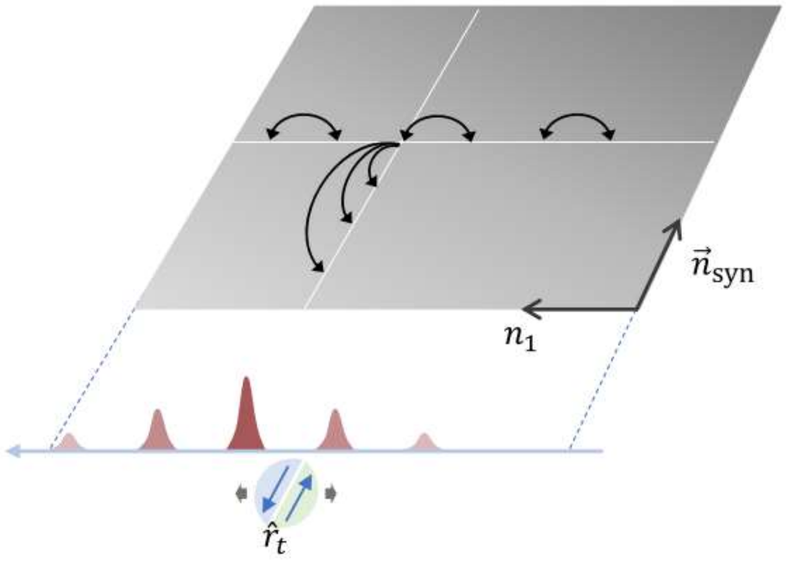
$$\langle \hat{X}^2 \rangle = \langle \psi_t | \hat{X}^2 | \psi_t \rangle$$

from $|\psi_n\rangle = \mathcal{U}_{n,n-1} \mathcal{U}_{n-1,n-2} \cdots \mathcal{U}_{1,0} |\psi_0\rangle$.

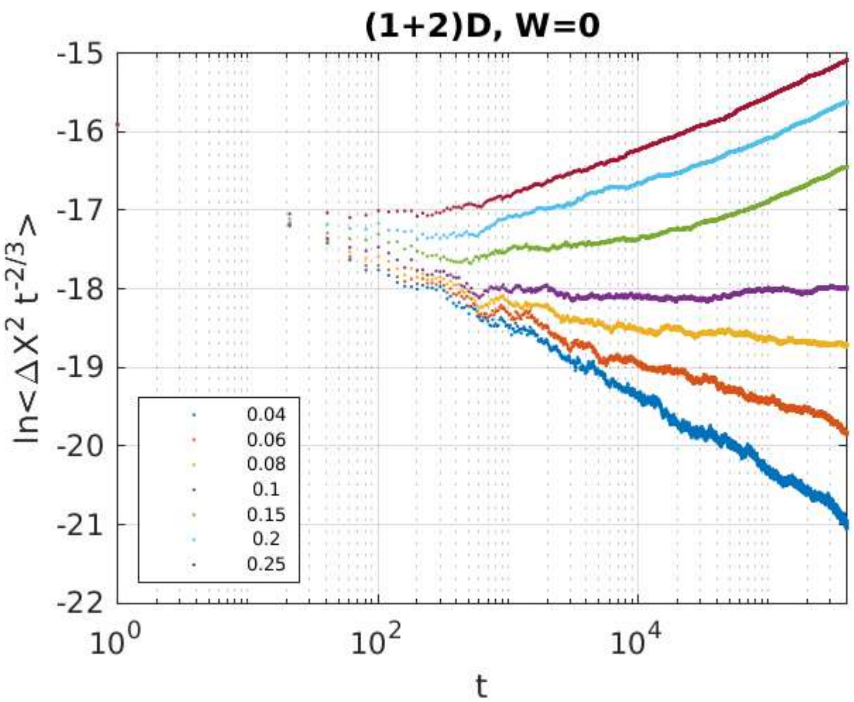
- (ii) Measuring a correlation function related to topological response -> Experimentally relevant proposal

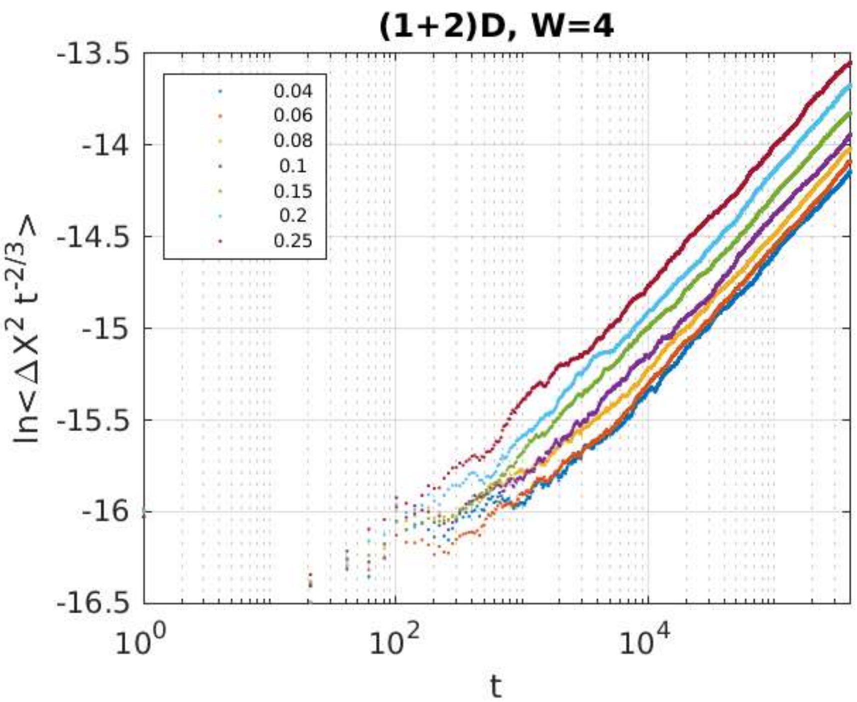
$$\text{tr}(x_i G^+ x_j G^-), \qquad \text{tr}(v_i G^+ v_j G^-), \qquad \text{tr}(x_i v_j), \qquad \text{tr}(x_i v_j G^+ x_k G^-),$$

Numerical check of Anderson Delocalization

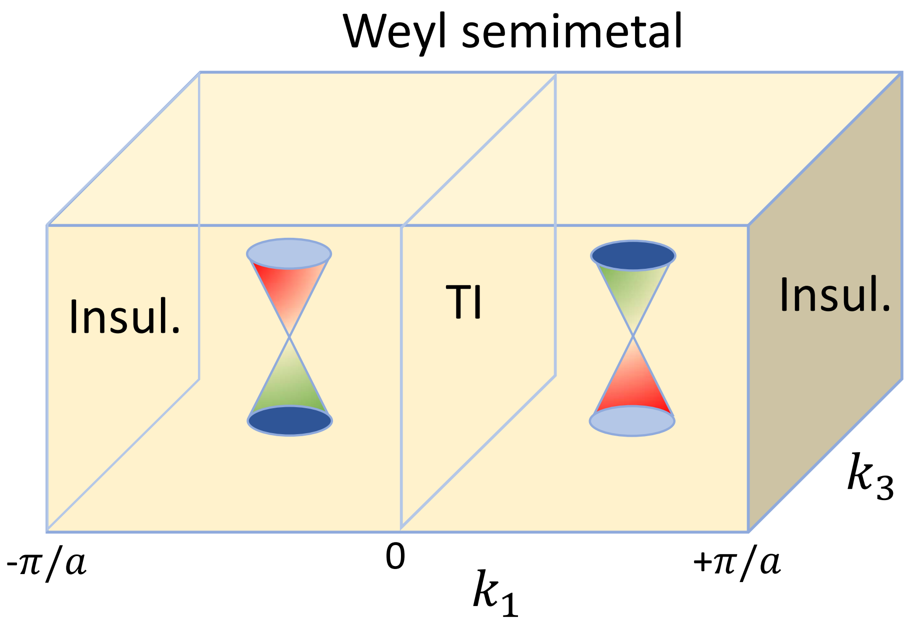


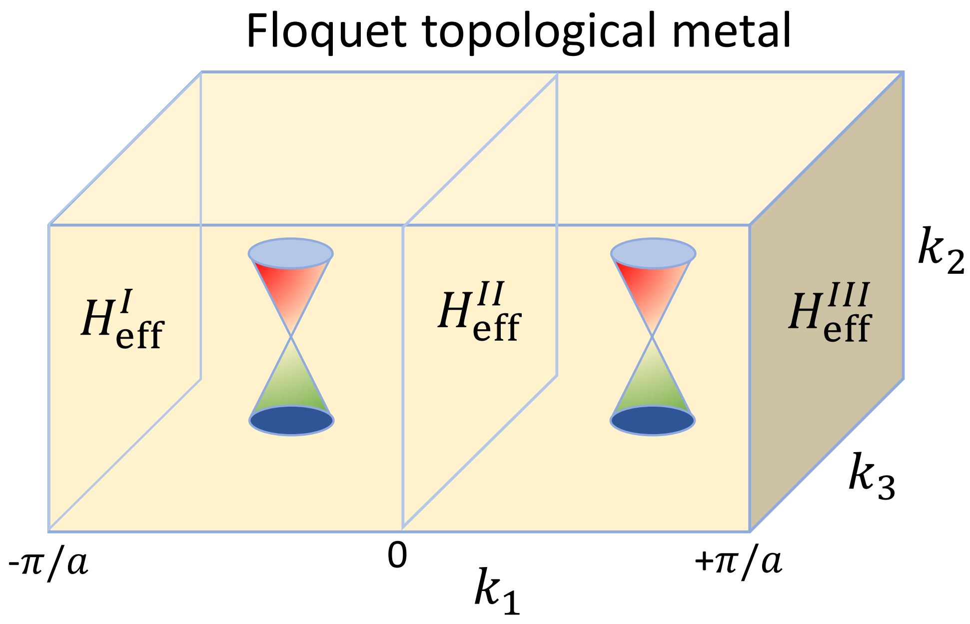
$$\langle \hat{X}^2 \rangle = \langle \psi_t | \hat{X}^2 | \psi_t \rangle$$





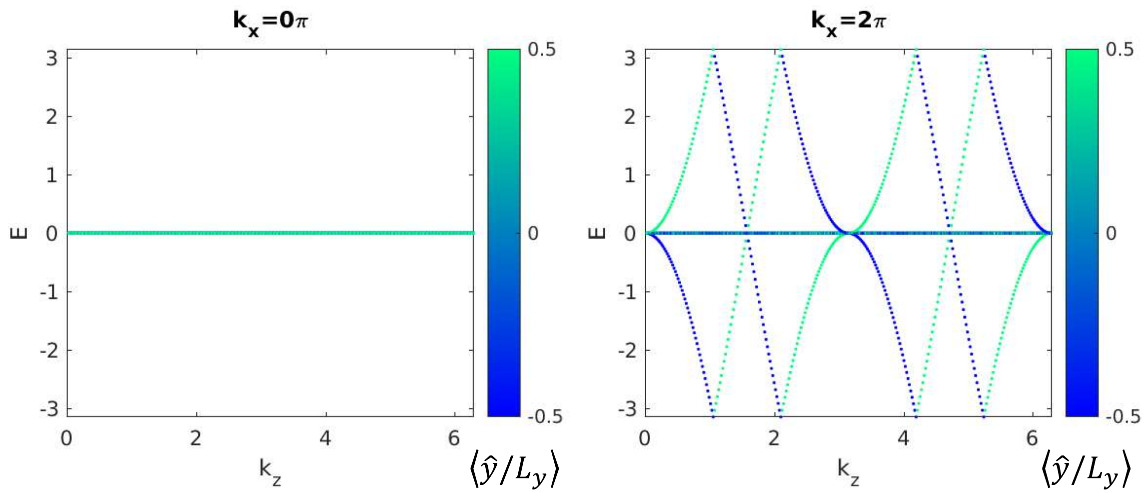
Topological Floquet metal in 3-dimension

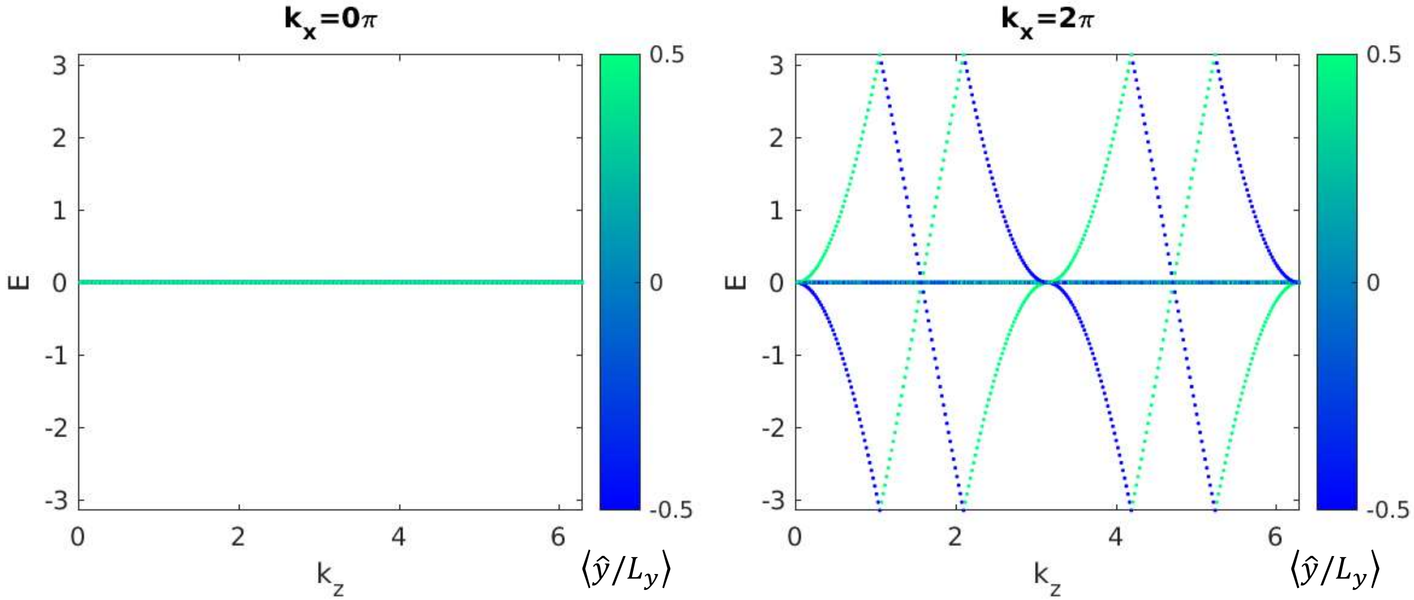




Recall the model in **(1+2)-dimension**

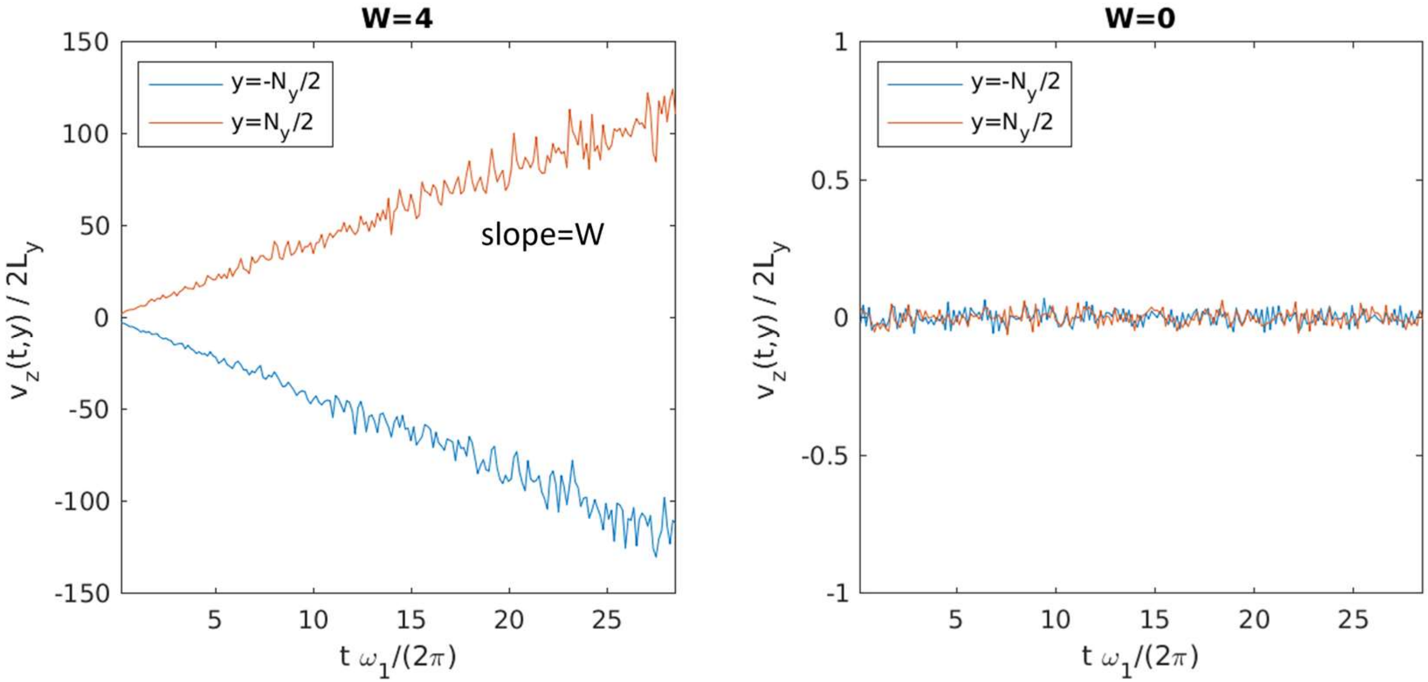
$$\mathcal{U}_{n,n-1} = \exp\left[ik_1(\sigma_1 \cos\phi_{2,n}\; |\sin\right.$$



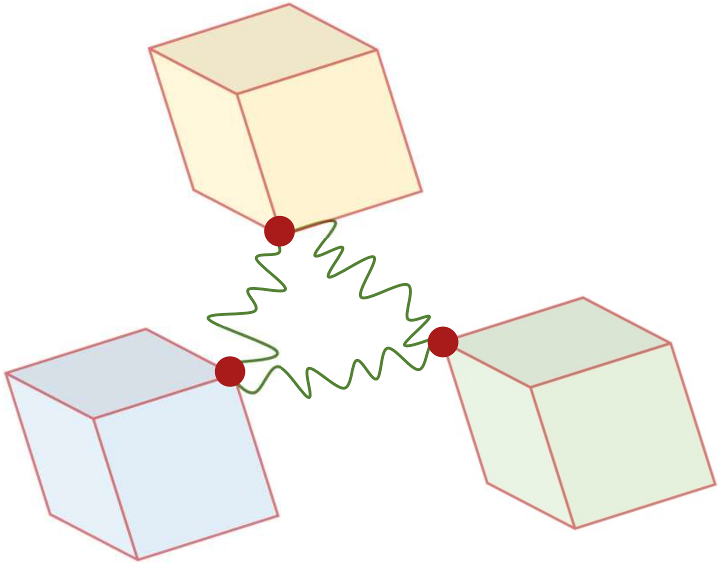


Measure the velocity operator: $v_3(t,y) = \langle \psi_t | \hat{v}_{3,t} | \psi_t \rangle_y$

Due to the flat bulk band in y and z direction, the system size can be minimal.



Future directions



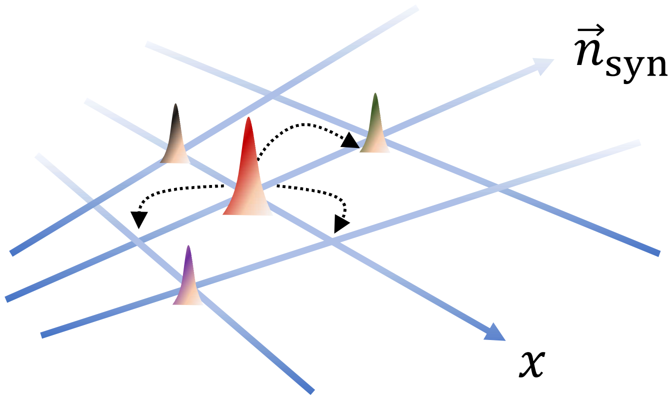
$$\hat{U}_{t+1,t} = \hat{U}_{t+1,t}\left(x^{(m)},\ \varphi_j^{(m)} + \omega_j^{(m)} t\right),$$

where $m \in \{1,2,3, \dots, N\}$ indicates particles.

I. Strongly interacting particles in synthetic space

- The stability test of single particle physics.
- Engineering the interaction via synthetic dimension.
- Computational and experimental advantages and challenges.

Future directions



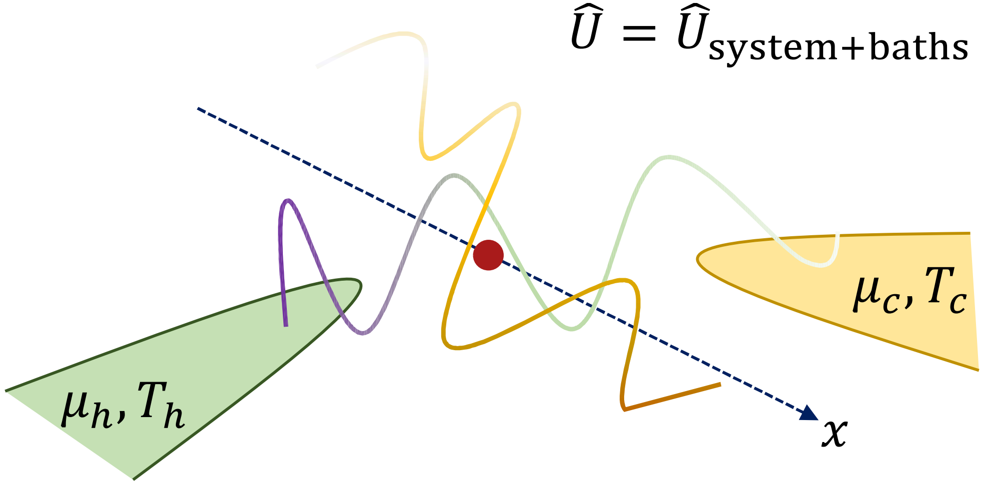
$$\psi_{t+T} = \hat{U}\ \psi_t$$

II. Quantum walk-based quantum algorithms

- The conventional quantum walk consists of a rotation operator and translation operator.
- Our study is a generalized version of quantum walk by incorporating topology, dimension, and disorder.
- Multiparticle quantum walk can perform the universal quantum computation.

Childs, Gosset, Webb (2013). Science 339, 791. “Universal computation by multiparticle quantum walk”

Future directions



III. Quantum thermal machine

- The upper bound of thermal efficiency: the Clausius relation.
- The upper bound of work-production rate: thermodynamic uncertainty relation.
- Quantum thermal machine to break the upper limit.

R. Lopez, J. Lim, **KWK.** Physical Review Research (2023), “Optimal superconducting thermal machine”

Summary of this talk

We shared the discussion of …

- Topological metal in synthetic dimensions
- Its proposal in experiments
- Possible extensions of the work

Thank you for your attention.